

Topological Origin of  
Dynamical Breaking of  
Chiral Symmetry in  
QCD and in Gravity.

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arXiv: 1705.06317

# Previous work:

Dvali, hep-th/0507215  
0510053 PRD 74

Dvali, Jackiw, Pi, PRL 96, 2006  
hep-th/0511175

Dvali, Folkerh, Franca, hep-th/13127273  
PRD 89, 2014

Dvali, Funcke, hep-ph/1602.03191  
1608.08969  
PRD 93, 2016

Topological susceptibility  
of vacuum (TSV) in  
QCD (pure glue):

$$\int \tilde{F} \tilde{F} \equiv \int *dC \equiv E$$

$$C \equiv A dA + \frac{2}{3} A A A$$

under QCD gauge transformation:

$$C \rightarrow C + d\Omega$$

↑ 2-form

$$\langle EE \rangle_{p \rightarrow 0} \equiv$$

$$\equiv \lim_{p \rightarrow 0} \int d^4x e^{ipx} \langle T[\tilde{F}\tilde{F}(x), \tilde{F}\tilde{F}(0)] \rangle \neq 0$$

In gravity:

$$E_g \equiv R\tilde{R} = dC_g$$

$$C_g \equiv \Gamma d\Gamma + \frac{2}{3} \Gamma \Gamma \Gamma$$

TSV in fermion-free gravity

$$\langle E_g, E_g \rangle_{p \rightarrow 0} \equiv \langle R\tilde{R}, R\tilde{R} \rangle_{p \rightarrow 0} = ?$$

# Role of TSV in QCD

(Witten; Veneziano;

Lüscher;  
Anirbia, Takahashi, Townsend,  
others...)

①\* What is the connection  
with chiral symmetry breaking?

②\* Can be generalized to  
gravity?

We would like to show  
that if in massless-fermion-free  
version of theory (QCD or  
Gravity)  $TSV \neq 0$ , i.e.,  
in QCD

$$\langle FF, FF \rangle_{p \rightarrow 0} = \text{const} \neq 0,$$

or in Gravity

$$\langle RR, RR \rangle_{p \rightarrow 0} = \text{const} \neq 0,$$

then when massless fermions  
are added, the following  
takes place:

① The chiral flavor symmetry is broken dynamically down to anomaly free subgroup;

② Pseudo-Goldstone of broken  $U(1)_A$  is becoming massive;

③ Both, in QCD and in gravity massless fermions (elementary or composite) are eliminated from the spectrum.

Implications:

① Role of confinement in chiral symmetry breaking - Is topology central?

② Fundamental resolution of incompatibility of global symmetries with black holes:

Dynamical breaking of chiral symmetry via contribution of micro-BH-s in TSV!



$$\langle R\tilde{r}, R\tilde{r} \rangle =$$

$$= \sum_{BH} \langle R\tilde{r} | BH \rangle \langle BH | R\tilde{r} \rangle \neq 0$$



Micro-BH-s

~~TSV~~  $\neq 0$

Macro-BH-s

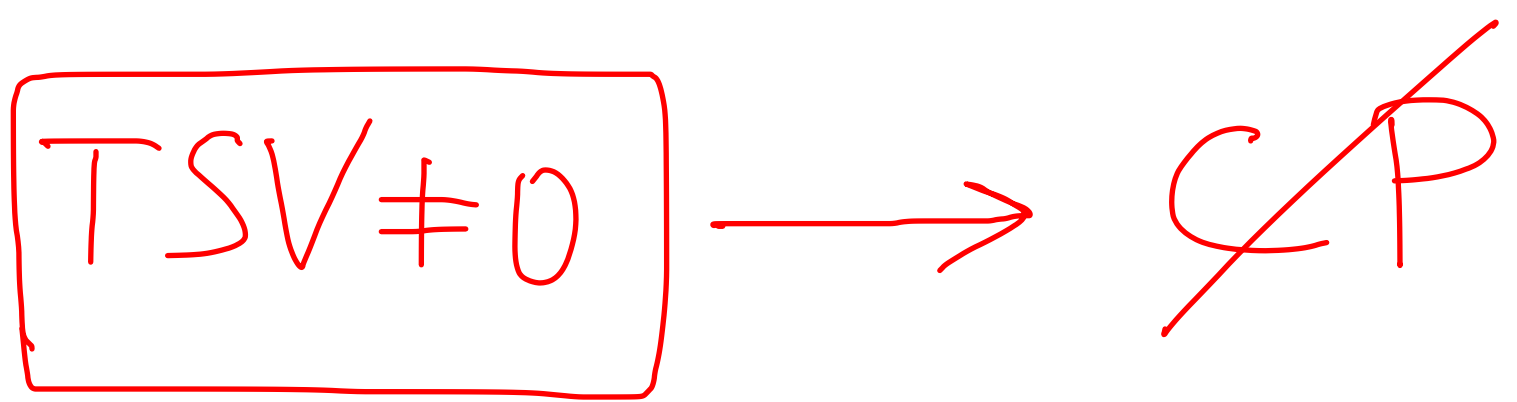
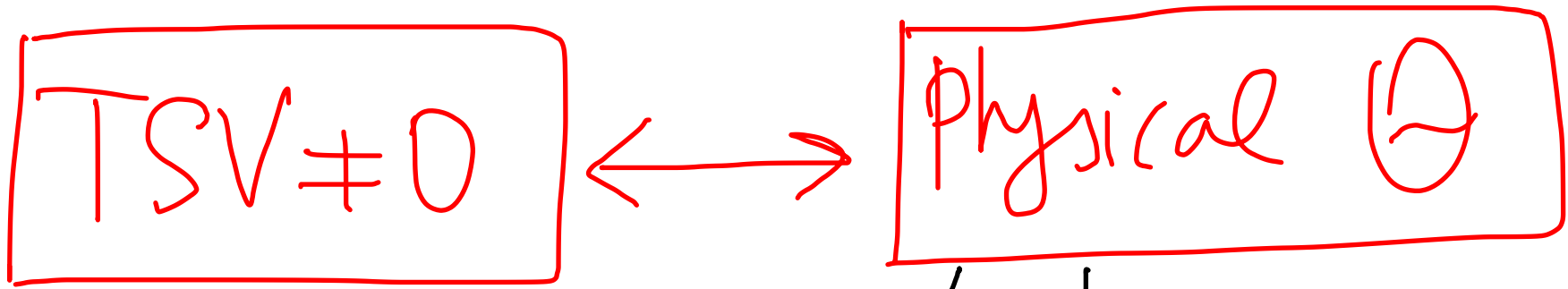
Dynamical  
breaking of  
chiral symmetry

③ Origin of neutrino masses from neutrino condensate.

$$\langle \bar{\nu}\nu \rangle \neq 0 \rightarrow M_\nu \neq 0.$$

Induced by gravity,  
(Leha Funcke & G.D.)

④ Protection of axion from gravity by neutrinos  
(Sarah Folkerts, Andre Franca,  
& G.D.)



Alice and Bob  
 must agree on breaking of  
 CP - symmetry

$$\langle E, E \rangle_{p \rightarrow 0} \equiv \langle F\tilde{F}, F\tilde{F} \rangle_{p \rightarrow 0} = \cos\theta \neq 0$$

For Alice this means  
that there exist  $\theta$ -values

With measurable  
CP-violation.

Bob's view:

$$\langle E, E \rangle_{p \rightarrow 0} = \text{const} \neq 0$$

$$E \equiv dC$$

$$\langle C, C \rangle_{p \rightarrow 0} = \frac{1}{p^2}$$

$$C = \underbrace{\text{massless 3-form}}_C + \sum \text{massive glueballs, ...}$$

$$\mathcal{L}_{\text{Bob}} = \frac{1}{2} E^2 + \dots$$

$$\partial_m E = 0 \rightarrow E = \text{const} \leftarrow \begin{matrix} \text{integration} \\ \text{constant} \end{matrix}$$

Thus,

$$(\theta\text{-vacua})_{\text{Alice}} = (E\text{-vacua})_{\text{Bob}}$$

$$\Delta\theta_{\text{Alice}} = \Delta E_{\text{Bob}}$$

CP-odd "electric"  
field  $\mathbf{E}$ .

Now let us introduce  
 $N_f$  massless quark flavors  
(all left-handed)  $\psi_j, \bar{\psi}_j$   
 $j = 1, \dots, N_f$ .

$$\langle F\bar{F}, F\bar{F} \rangle_{p \rightarrow 0} = 0.$$

Alice and Bob agree  
that CP is conserved,  
But interpret differently.

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Alice:  $U(1)_A$ -transformation

$$\psi \rightarrow e^{i\alpha} \psi$$



$$S\mathcal{L}_{Alice} = \alpha F \tilde{F}$$



$$\Theta \rightarrow \Theta + \alpha$$



$\Theta$ -unphysical!



Bob: Higgs effect for 3-form!

$$\langle E, E \rangle_{p \rightarrow 0} = 0 \rightarrow \langle C, C \rangle_{p \rightarrow 0} = \frac{1}{p^2 + m^2}$$

$C$  became massive. Needs a pseudoscalar!

$$\mathcal{L}_{\text{Bob}} = \frac{1}{2} E^2 - \frac{\eta'}{f_\eta} + \frac{1}{2} (\partial_\mu \eta')^2$$

$$\partial_\mu \left( E - \frac{\eta'}{f_\eta} \right) = 0 \rightarrow E = \frac{\eta'}{f_\eta} - \theta$$

$$\square \eta' - \frac{1}{f_\eta} E = 0$$

$$\square \eta' + \frac{1}{f_\eta} \left( \frac{\eta'}{f_\eta} - \theta \right) = 0$$

Thus,

$$\square \eta' + \frac{1}{f_h} \left( \frac{\eta'}{f_h} - \theta \right) = 0$$

$$\boxed{E = \frac{\eta'}{f_h} - \theta}$$

$$M_{\eta'} = \frac{1}{f_h^2}$$

under  $U(1)_A: \psi \rightarrow e^{i\alpha} \psi$

$$\theta \rightarrow \theta + \alpha,$$

$$\boxed{\frac{\eta'}{f_h} \rightarrow \frac{\eta'}{f_h} + \alpha}$$

$$\frac{\eta'}{f_h} \equiv \frac{1}{2N_f} \arg(\det(\psi\psi))$$

Thus,  $\langle \det(\psi\psi) \rangle \neq 0$

and  $\psi'$  is a (pseudo)-  
Goldstone of  $U(1)_A$ .

But, it is also a  
Stückelberg field  
for 3-form!

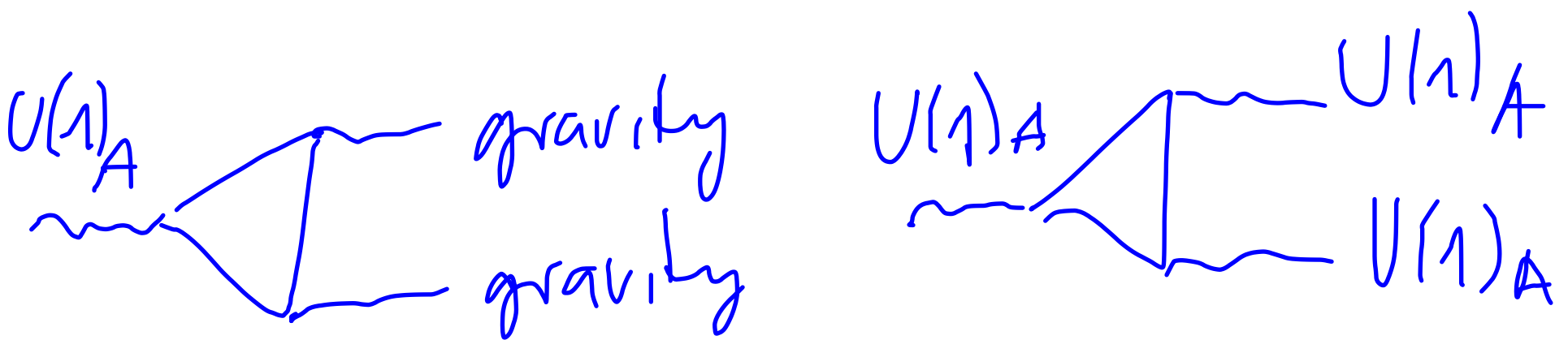
Next step: Gauge  $U(1)_A$

and use gravity as spectator.

$$\psi \rightarrow e^{i\alpha} \psi, \quad X_\mu \rightarrow X_\mu + \frac{1}{g_x} \partial_\mu \alpha(x)$$

$$\frac{\eta'}{f_\eta} \rightarrow \frac{\eta'}{f_\eta} + \alpha$$

$$GS\text{-axion} \quad \frac{a}{f_a} \rightarrow \frac{a}{f_a} - \alpha$$



$$\frac{a}{f_a} (R \tilde{R} + F_x \tilde{F}_x + F \tilde{F})$$

Anomaly matching:

Alice:  $U(1)_A \quad \psi \rightarrow e^{i\alpha} \psi$

$$\frac{a}{f_a} \rightarrow \frac{a}{f_a} - \alpha$$

$$\mathcal{L}_{\text{Alice}} = \frac{a}{f_a} (R\tilde{R} + F_x \tilde{F}_x + FF)$$

Bob:  $U(1)_A \quad \eta' \rightarrow \frac{\eta'}{f_h} + \alpha$

$$\frac{a}{f_a} \rightarrow \frac{a}{f_a} - \alpha$$

$$\mathcal{L}_{\text{Bob}} = \left( \frac{b'}{f_h} + \frac{a}{f_a} \right) (R\tilde{R} + F_x \tilde{F}_x + FF)$$

No room for massless fermions!

Because  $h'$  is Stückelberg,  
it takes care of anomaly  
cancellation.

Thus, in theory of Bob  
there is no room for  
massless fermions due  
to 't Hooft's anomaly  
matching!

Absence of massless fermions

$U(N_f)_L \otimes U(N_f)_R$  flavor symmetry  
is dynamically broken  
down to an anomaly-free  
subgroup.

Follows from 't Hooft  
anomaly matching

The same conclusion applies  
to gravity: Flavor  
group  $U(N)$  must be  
broken to an anomaly-free  
subgroup.

E.g.

$$U(N) \rightarrow SO(N)$$



Universal fermion condensate  
in gravity?