# New applications of gauge/gravity duality

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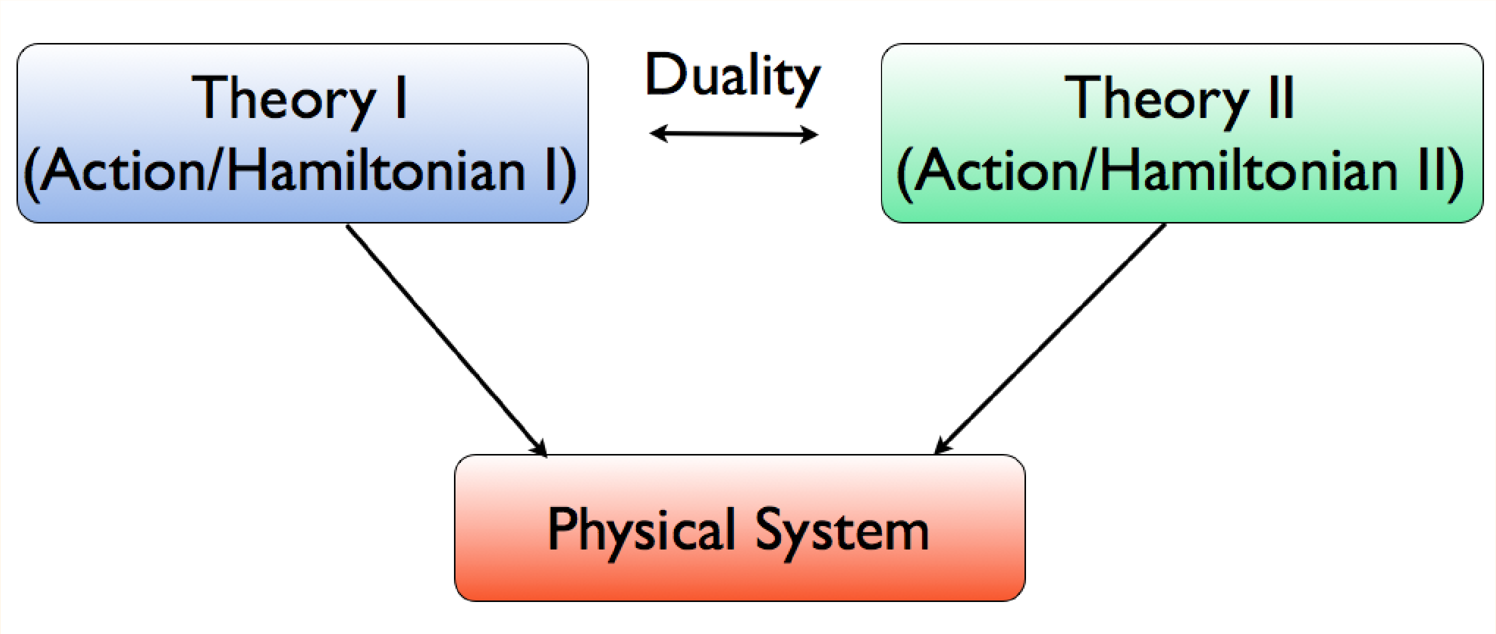
# Gauge/Gravity Duality: Foundations

## Duality:



Theory I
(Action/Hamiltonian I)

Duality

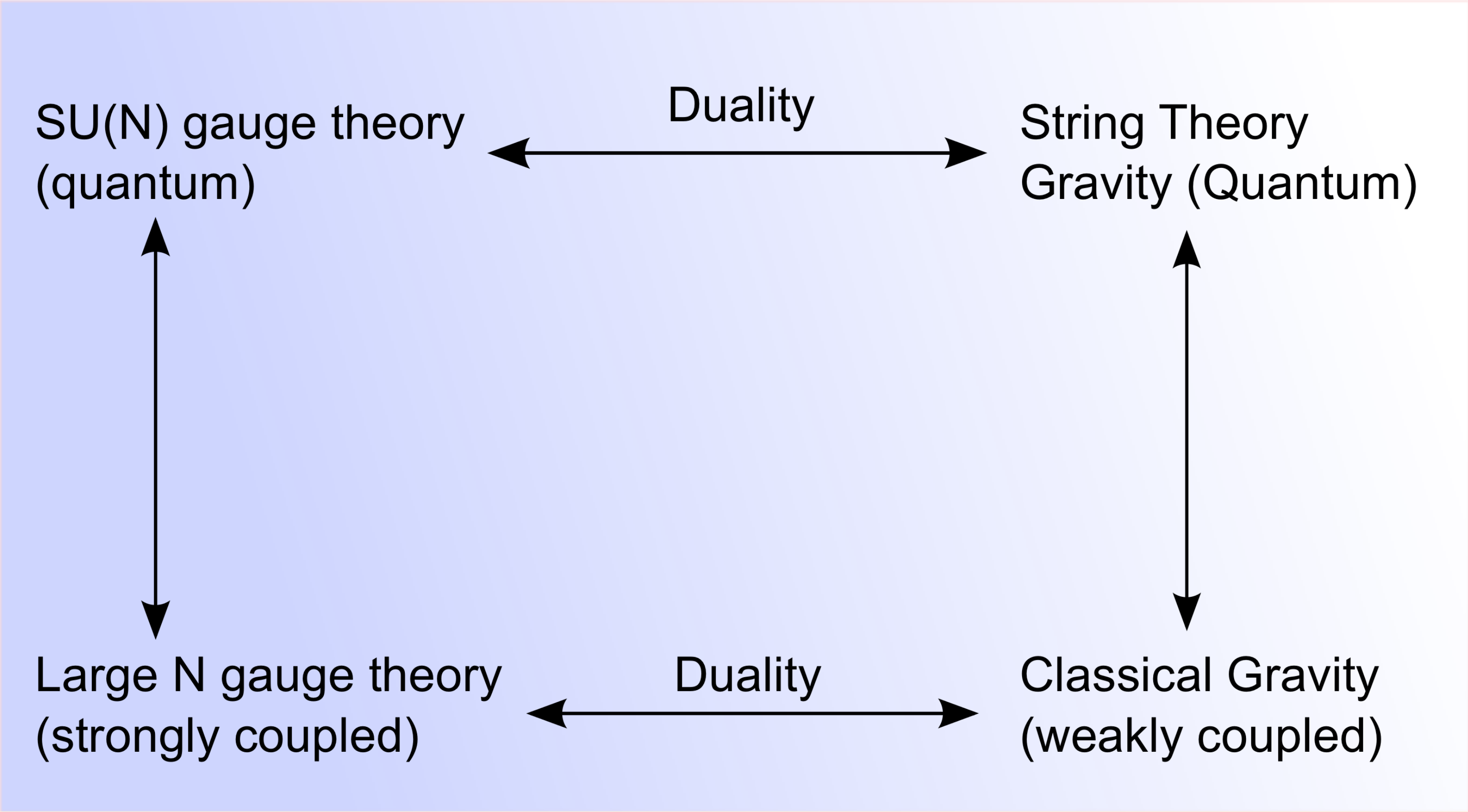
Theory II
(Action/Hamiltonian II)

Physical System

A physical theory has two equivalent formulations

Same dynamics, one-to-one map between states

# Gauge/Gravity Duality: String Theory Origin



SU(N) gauge theory
(quantum)

Duality

String Theory
Gravity (Quantum)

Large N gauge theory
(strongly coupled)

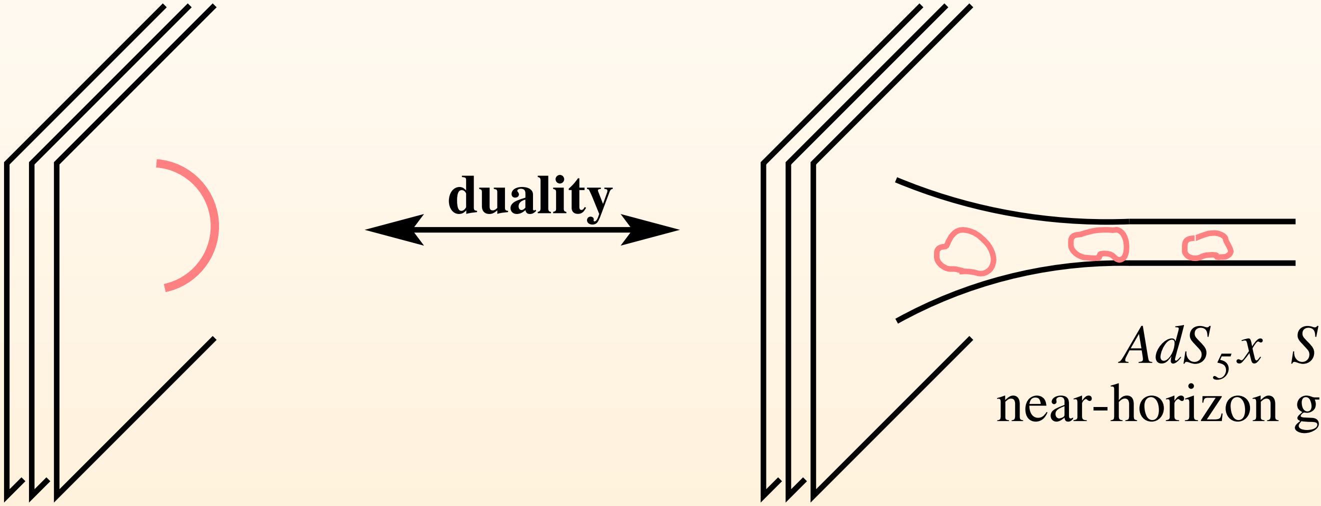
Duality

Classical Gravity
(weakly coupled)

# String theory origin of the AdS/CFT correspondence

Maldacena 1997

D3 branes in 10d



**duality**

$AdS_5 x\ S^5$
near-horizon geometry

$\Downarrow$ Low energy limit

$\mathcal{N} = 4\ SU(N)$ Super Yang-Mills theory ($N \to \infty$)

IIB Supergravity on $AdS_5 \times S^5$

# Gauge/Gravity Duality

Limits $N \to \infty$, 't Hooft coupling $\lambda \equiv g^2_{\mathrm{YM}} N \to \infty$

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Gauge/gravity duality valid more generally than string theory examples?

# Gauge/gravity duality: Examples

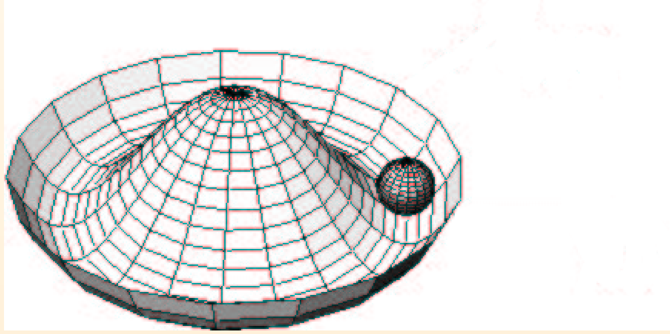
1. Mesons in large $N$ QCD
2. Low-x physics
3. Kondo model: Simple condensed matter model similar to QCD
4. Gauge/Gravity Duality and Quantum Information

# Example 1: Light mesons

Babington, J.E., Evans, Guralnik, Kirsch
Phys.Rev. D69 (2004) 066007

Gauge/gravity duality realization of

spontaneous chiral symmetry breaking and light mesons



Mesons: quark-antiquark bound states

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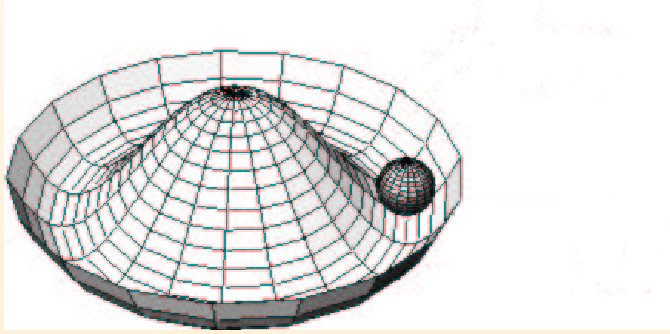


Mesons: quark-antiquark bound states

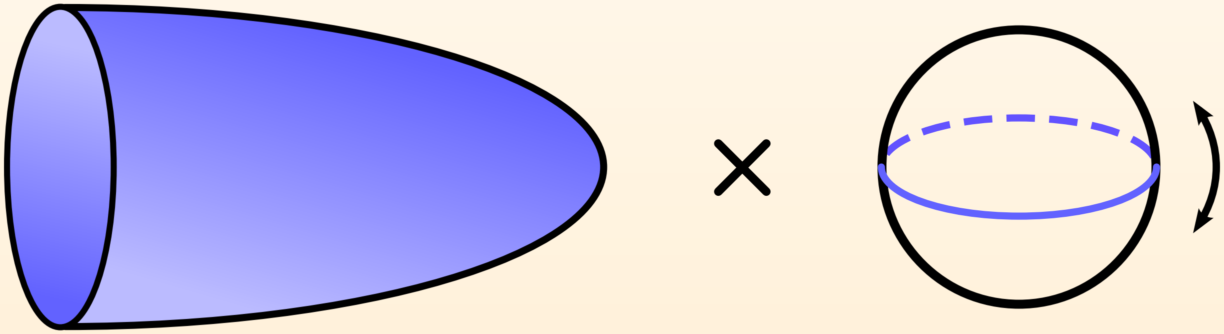
Meson masses

from solving classical equations of motion in higher-dimensional gravity

# Light mesons

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Meson masses obtained from fluctuations of probe D7-brane

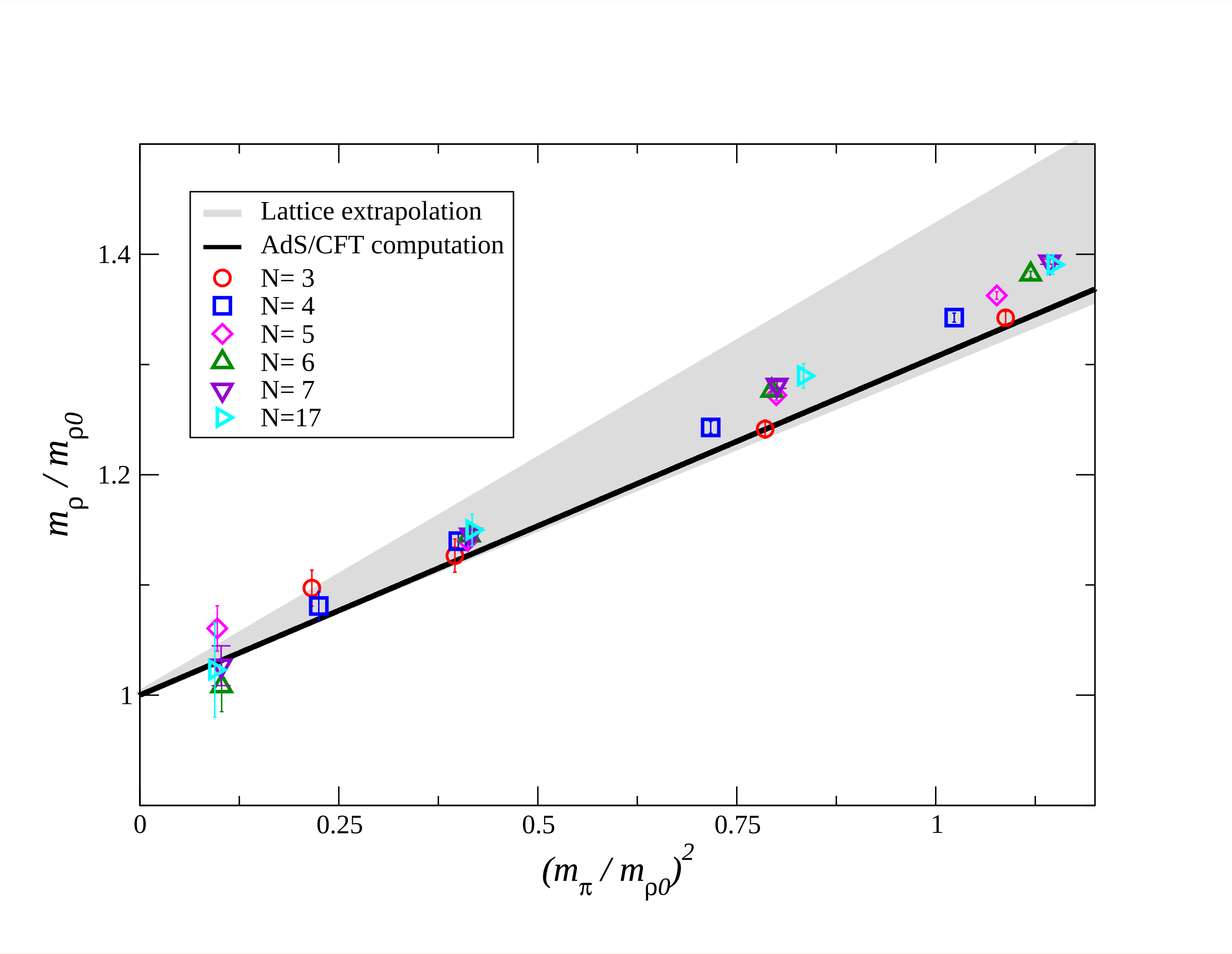


$\pi$ pseudoscalar meson mass: From fluctuations of probe D7-brane embedding

$\rho$ vector meson mass: From fluctuations of gauge field on D7-brane

# Comparison to lattice gauge theory

Mass of $\rho$ meson as function of $\pi$ meson mass$^2$ (for $N \to \infty$)



# Comparison to lattice gauge theory

Gauge/Gravity Duality: J.E., Evans, Kirsch, Threlfall ’07, review EPJA

Lattice gauge theory: Lucini, Del Debbio, Bali, Panero et al ’13

Result Gauge/Gravity Duality:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left( \frac{m_\pi}{m_\rho(0)} \right)^2$$

Result Lattice Gauge Theory (Bali, Bursa ’08): Slope $0.341 \pm 0.023$

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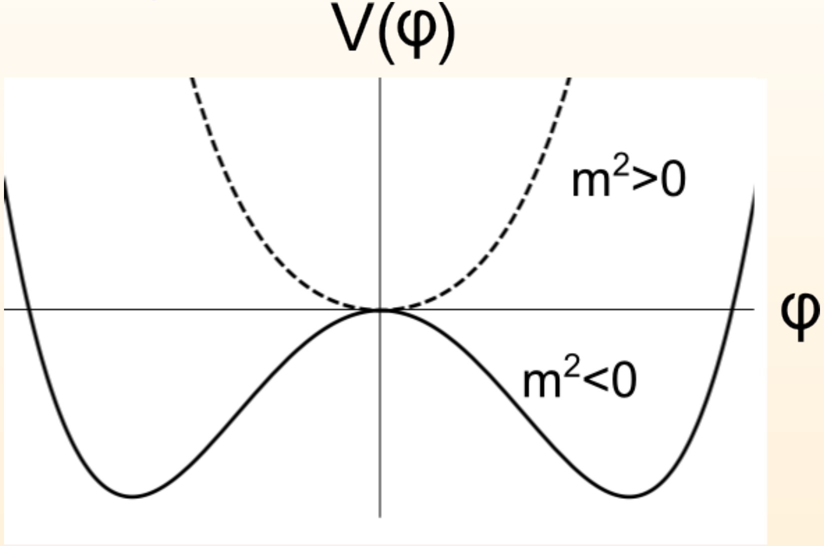
Why is the agreement so good?

# Comparison to lattice gauge theory

J.E., Evans, Scott PRD 91 (2015) 085004

Spontaneous symmetry breaking
in flat space:



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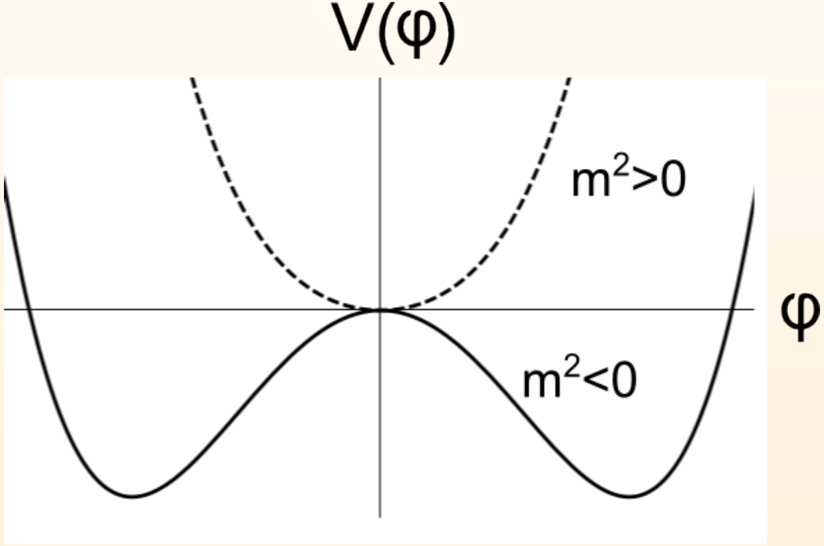

In Anti-de Sitter space,
instability occurs if quark mass is below the
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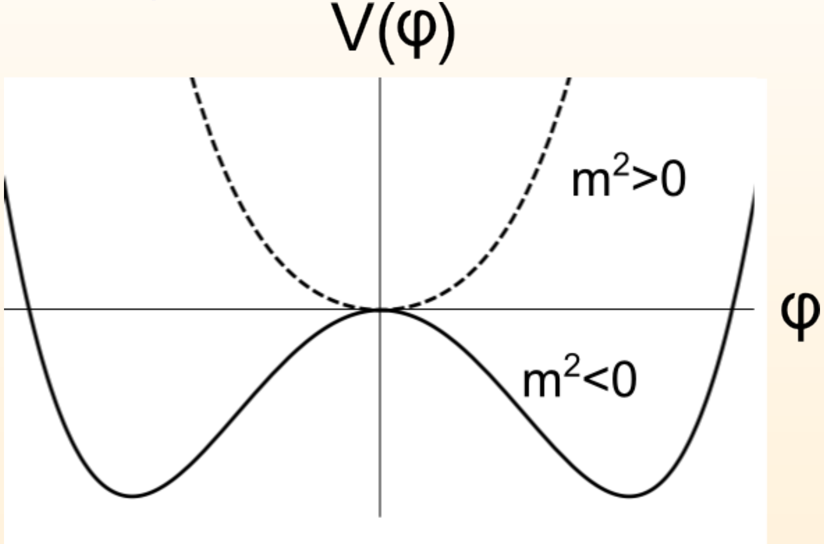

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Classical dimension of quark bilinear $\bar{\psi}\psi$: $\Delta = 3$

AdS/CFT: Mass of dual scalar $m^2L^2 = \Delta(\Delta - d)$,    here: $m^2L^2 = -3 \geq -4$

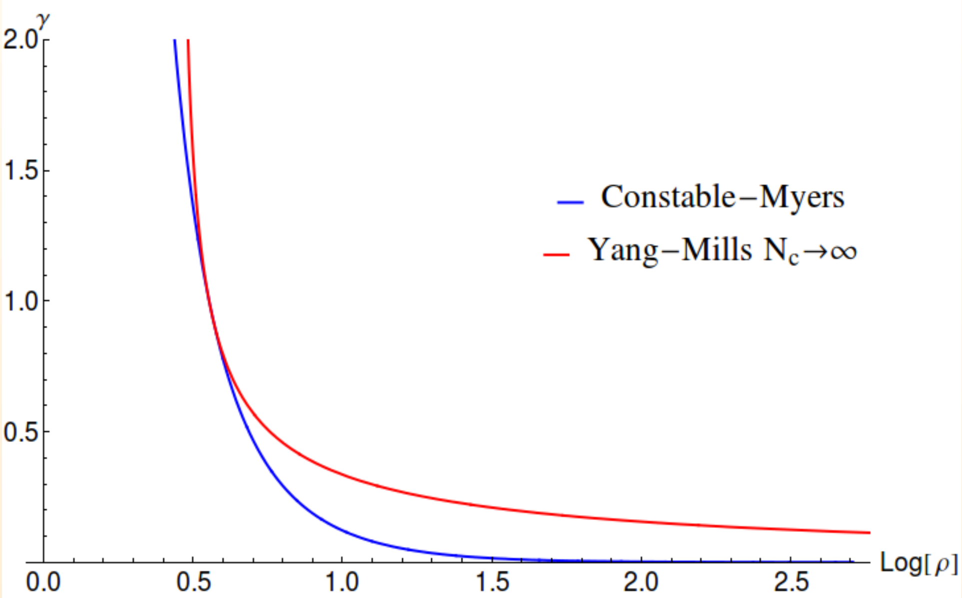
Along the RG flow, $\Delta \to \Delta' = \Delta - \gamma$

When $\Delta' < 2$, $m^2L^2 < -4$ below BF bound $\Rightarrow$ Instability

# Anomalous dimension

Anomalous dimension leads to violation of BF bound

Anomalous dimension in gauge/gravity model originating from string theory:



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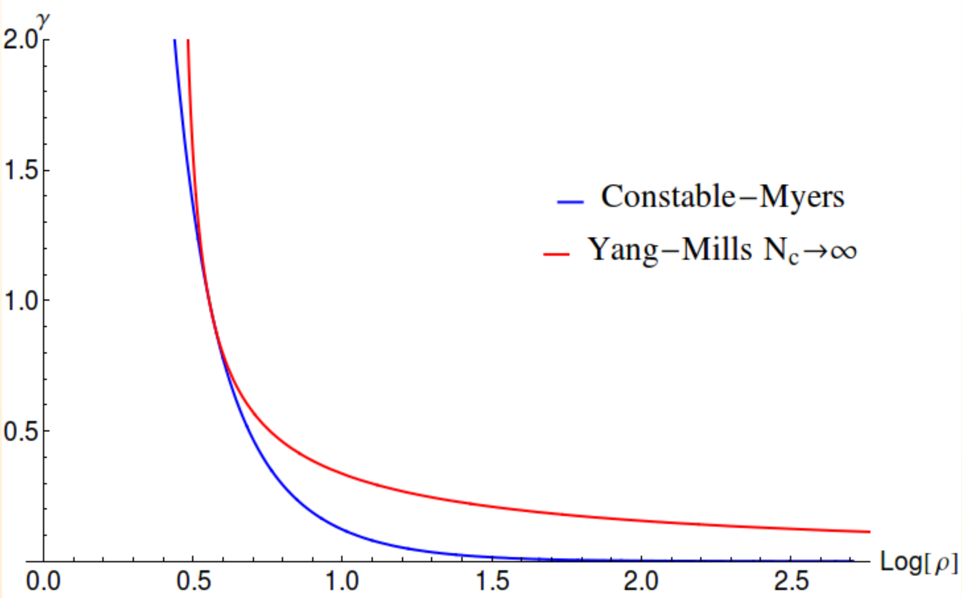
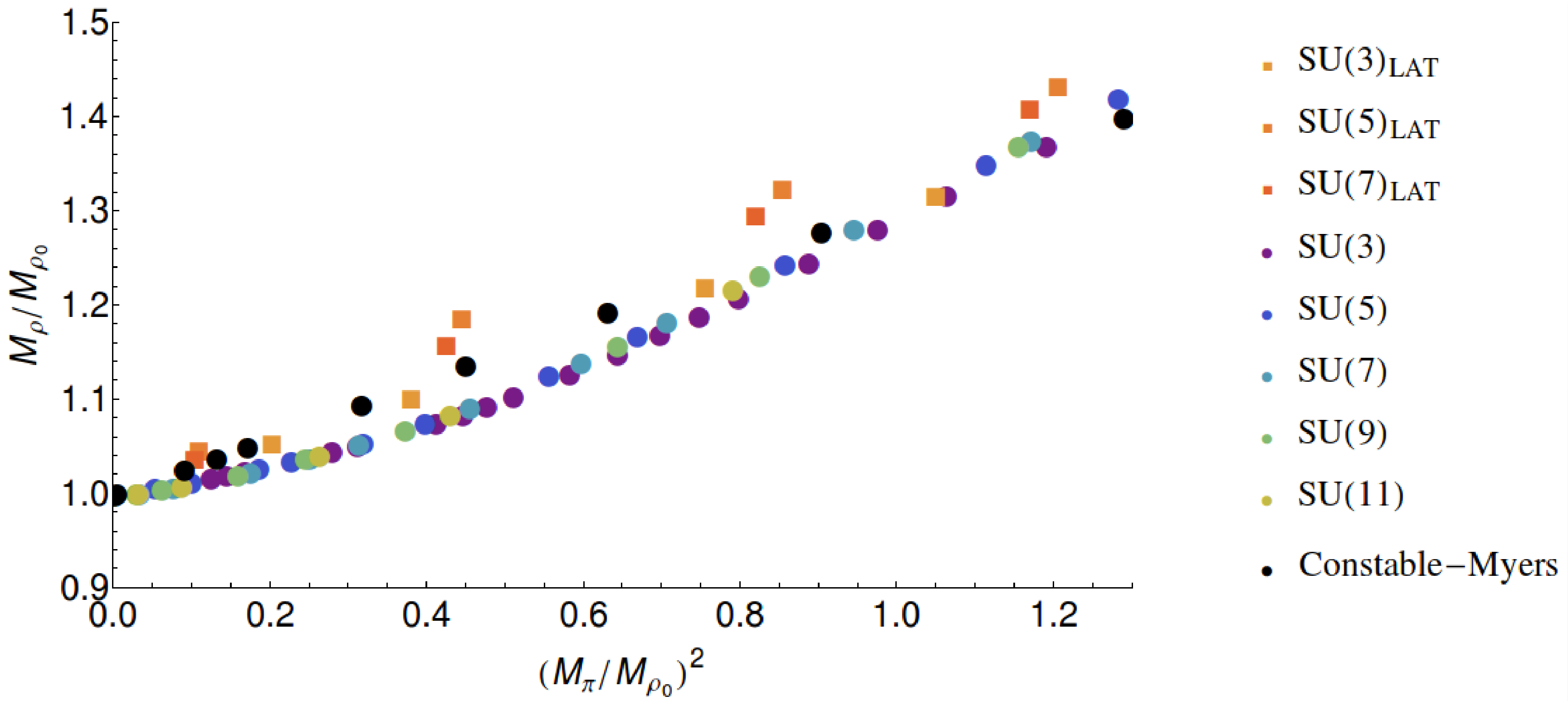


Phenomenological model: Make contact with QCD by chosing

$$\gamma = \frac{3(N^2-1)}{2N\pi}\alpha_s$$

# $\rho$ meson vs. $\pi$ meson mass$^2$

J.E., Evans, Scott PRD 91 (2015) 085004



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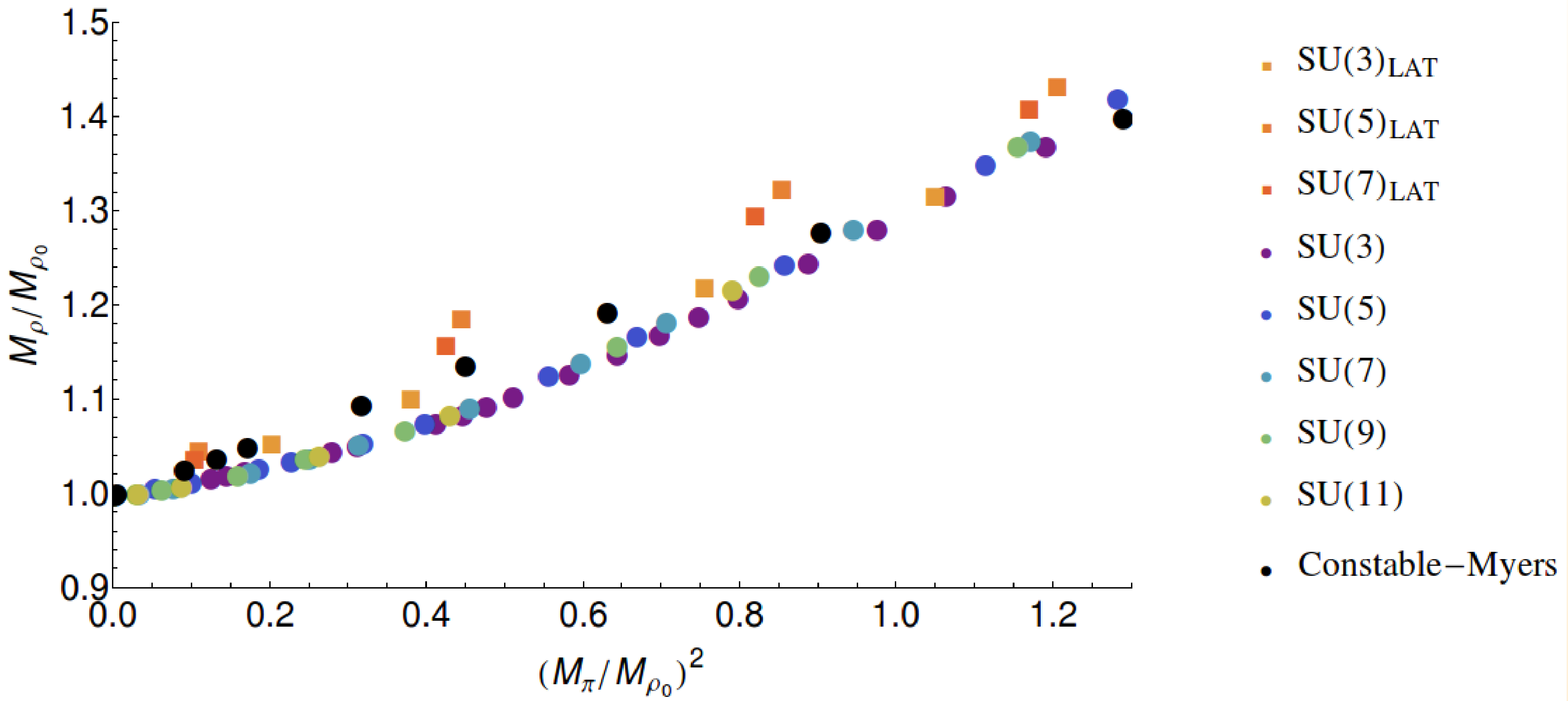

Gravity side: Anomalous dimension of operator $\bar{\psi}\psi$ leads to instability

$\Rightarrow$ Spontaneous chiral symmetry breaking as in QCD

Example for Universality

# Example 2: Deep inelastic scattering and Froissart bound

Froissart 1961:

At high energies, the total cross-section for two-particle scattering (protons) has an upper bound

$$\sigma \propto \ln^2 \frac{s}{s_0}$$

$s$ centre-of-mass energy, $s_0$ energy scale

General argument based on unitarity of S matrix and
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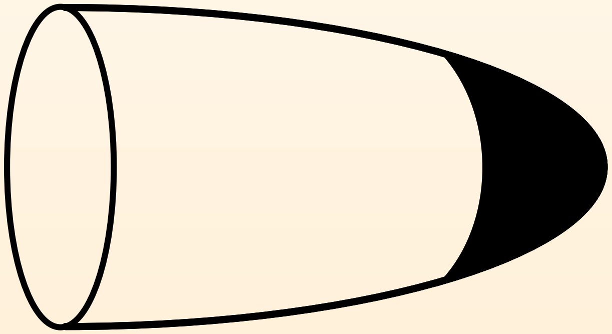
QCD considerations link the Froissart bound at high energies to the dynamics of ultra-soft gluons (strongly coupled)

# Example: Froissart bound in gauge/gravity duality

Giddings Phys.Rev. D67 (2003) 126001; Kang, Nastase Phys.Rev. D72 (2005) 106003

AdS metric with IR cutoff ('hard wall'), point mass $m$ is placed on this IR wall

This creates perturbations of the AdS space which may lead to the formation of a black hole in AdS space



Geometrical cross section of this black hole $\Leftrightarrow$
maximum possible scattering cross section in the field theory

$$\sigma \le \sigma_{\mathrm{BH}} = \pi r_h^2 \propto \ln^2 \frac{E}{E_0}$$

# Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

Subleading corrections $\propto -\ln(s/s_0)$ and $\propto \ln s/s_0 \ln\ln s/s_0$,
from higher curvature corrections

improve fits to cosmic ray and LHC data

# Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

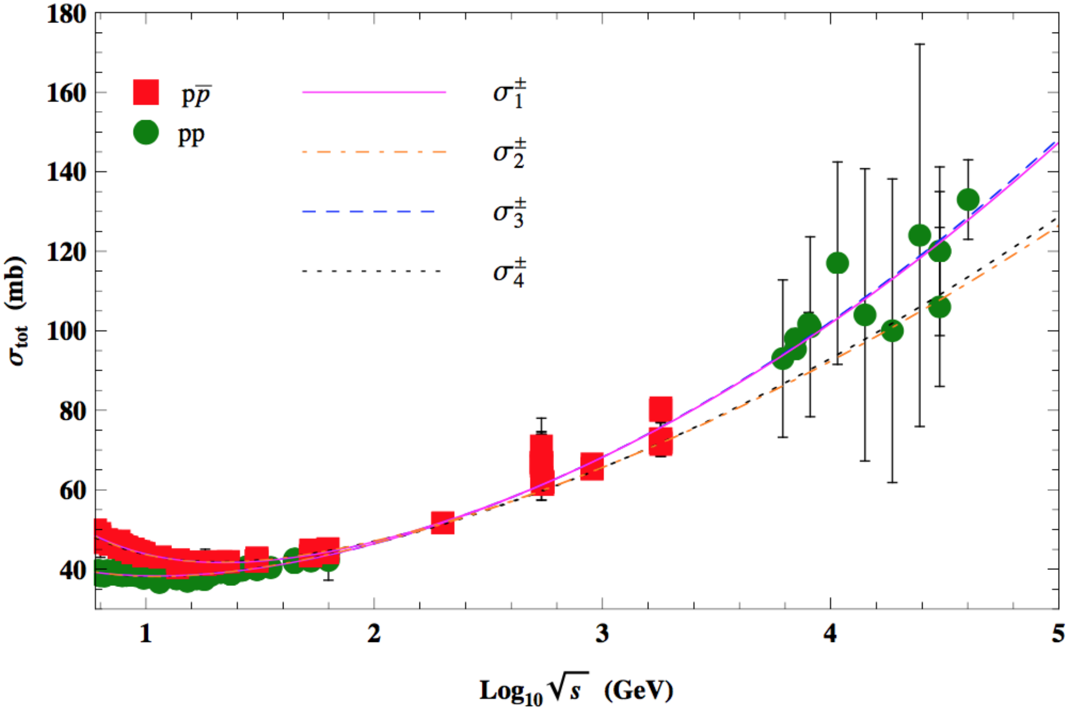
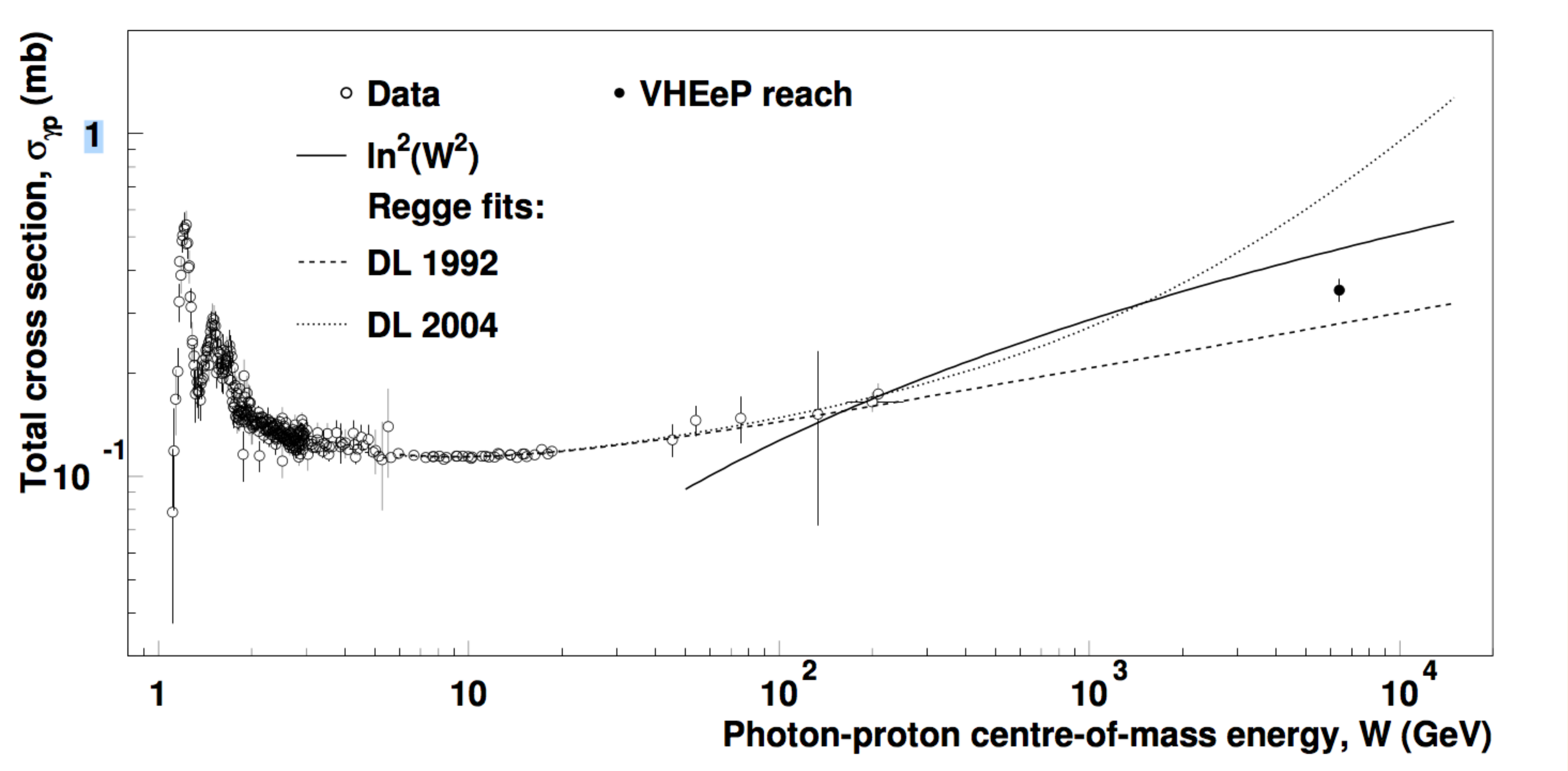


FIG. 1: (Colour online.) Fit results to experimental values of $\sigma_{tot}^{pp}$ and $\sigma_{tot}^{p\bar{p}}$. The magenta solid, orange dot-dashed, blue dashed and black dotted curves are the (57)-(60) fits to the $pp$ (green circles) and $p\bar{p}$ (red squares) data points, respectively. The data are from CDF, E710, E811, UA1, UA4, UA5 experiments [35–42]. The $pp$ data points also include $\sigma_{tot}^{pp}$ results from the LHC (at $\sqrt{s} = 7, 8$ TeV) [43–45] and cosmic-ray data [46].

# Total $\gamma p$ cross section

Case for plasma wakefield acceleration    Caldwell, Wing Eur.Phys.J. C76 (2016) 463



# Example 3: Gauge/Gravity Dual of a Kondo model

A simple model in which perturbation theory breaks down at low energies just as in QCD:

Kondo model of condensed matter physics

Magnetic impurity interacting with electron gas

- Negative beta function
- Asymptotic freedom and confinement
- Dynamical scale generation

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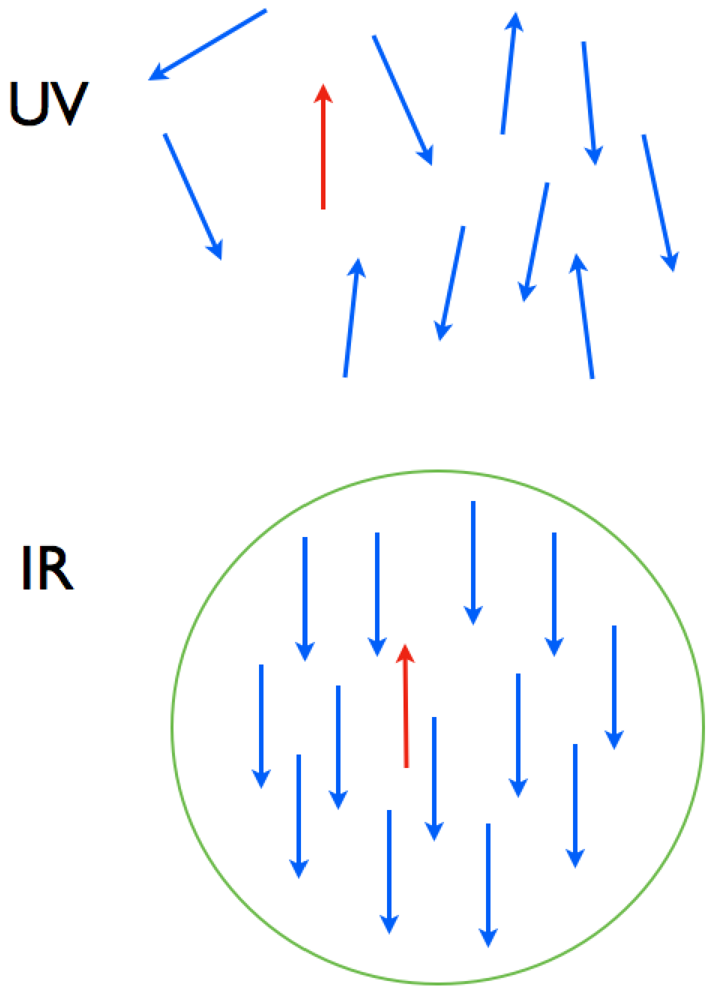

UV

Free electrons
+
impurity spin

IR

Impurity screened

# Example 3: Kondo model

Gravity dual:

Based on probe brane construction with D7 and D5 probe branes

Essential features captured by

Three-dimensional Chern-Simons theory in AdS$_3$ black hole background coupled to matter fields on AdS$_2$ subspace

Electron gas strongly coupled

J.E., Flory, Hoyos, Newrzella, O'Bannon, Wu JHEP 2013, 2014, 2015, 2017
Fortschr. Phys. 2016

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Recent new duality example:

Sachdev 2015, Maldacena, Stanford 2016

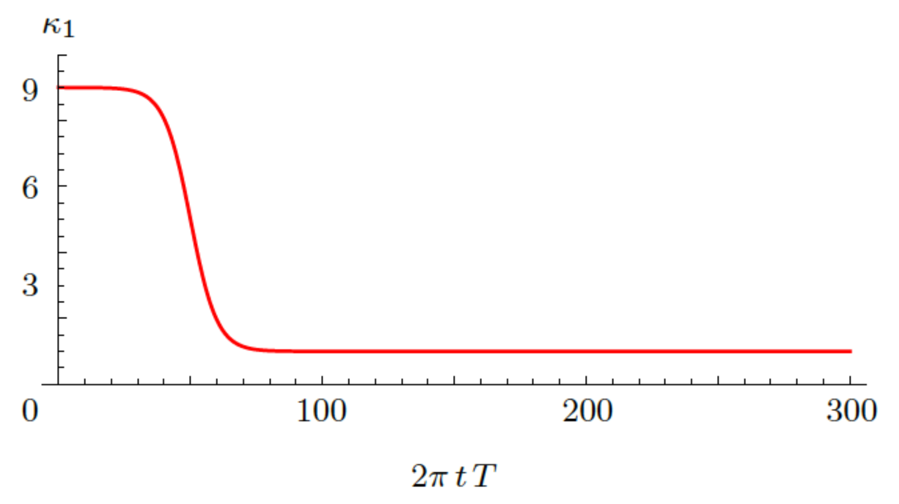
Quantum mechanics for strange metal
with infinite range interactions $\Leftrightarrow$ Charged AdS$_2$ black hole
(Sachdev-Ye Kitaev model)

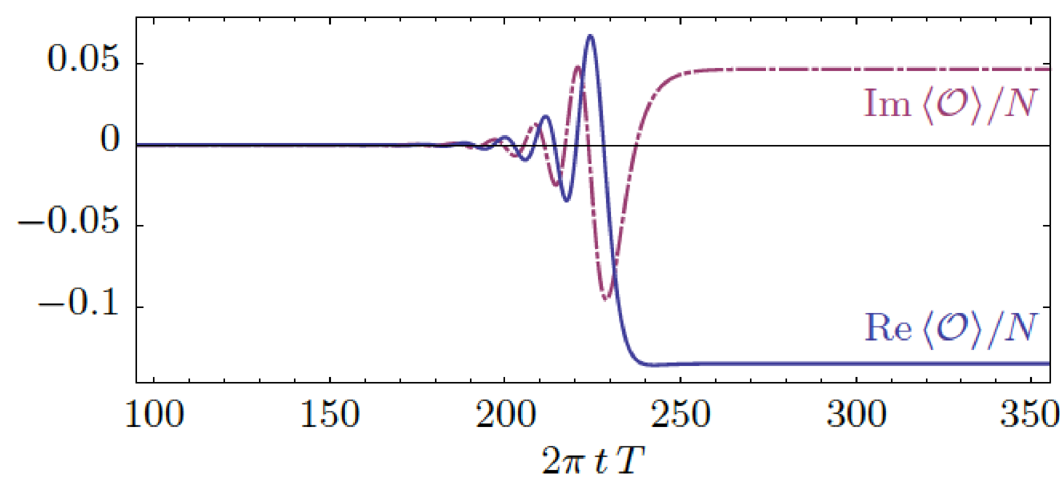
# Kondo effect within gauge/gravity duality

- Magnetic impurity coupled to strongly correlated electron system
- Model J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 2013
- Screening, resistivity
- Quantum quenches J.E., Flory, Newrzella, Strydom, Wu JHEP 2017
- Entanglement entropy J.E., Flory, Newrzella 1410.7811, JHEP 2014
  J.E., Flory, Hoyos, Newrzella, Wu 1511.03666, Fortschr. Phys. 2016
- Two-point correlators J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu JHEP 2017

# Quantum quench in Kondo model within gauge/gravity duality

J.E., Flory, Newrzella, Strydom, Wu JHEP 2017



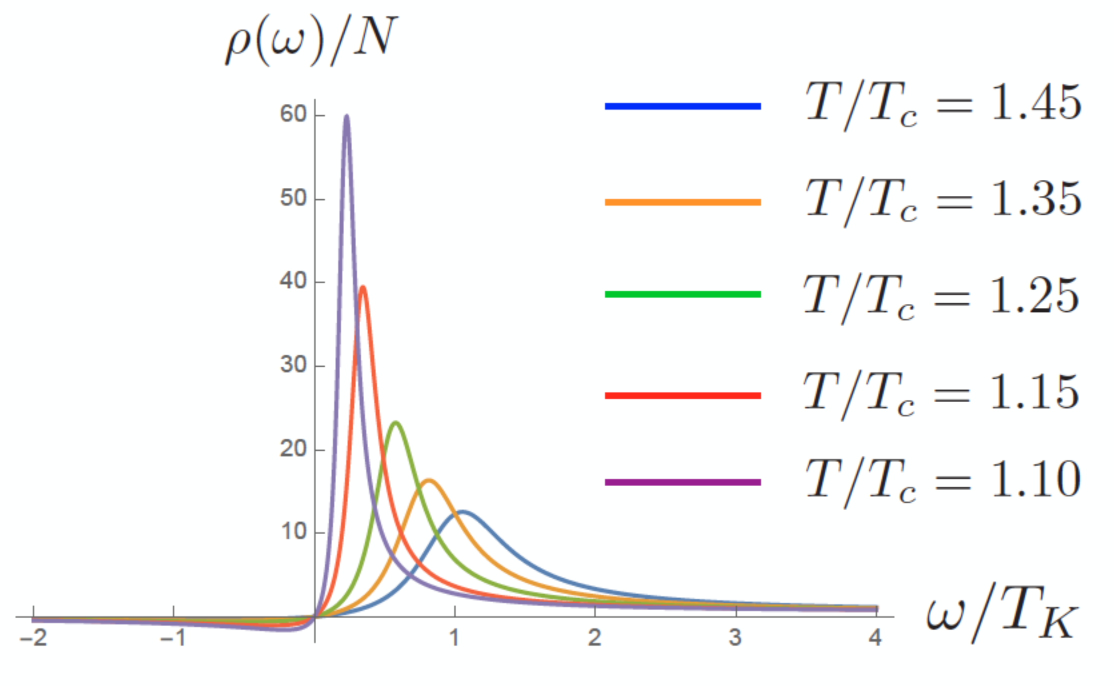


Formation of screening cloud:

Exponential fall-off of number of degrees of freedom at impurity

Timescales set by eigenmodes of gravitational system (QNM’s)

# Spectral function $-\mathrm{Im}\langle\mathcal{O}^\dagger\mathcal{O}\rangle$



$$\rho_{\text{peak}} \propto \frac{1}{T - T_c}$$

Fano resonance

A continuum scatters off a discrete set of resonant states

Observed in side-coupled quantum dots

Göres et al PRB 62 2188

# Example 4: Gauge/Gravity Duality and Quantum Information

Use of geometry widespread in information theory

Make use of this to gain further understanding of gauge/gravity duality

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Fisher metric in information theory: Metric on space of probability distributions

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Make use of this to gain further understanding of gauge/gravity duality

Fisher metric in information theory: Metric on space of probability distributions

Probability distribution $p(x,\vec{\theta})$, $x$ a stochastic variable, $\vec{\theta}$ a set of $n$ external parameters

Spectrum $\gamma(x,\vec{\theta}) \equiv -\ln p(x,\theta)$

Fisher metric

$$g_{\mu\nu}(\vec{\theta}) \;=\; \int dx\, p(x,\vec{\theta}) \frac{\partial\gamma(x,\theta)}{\partial\theta^{\mu}} \frac{\partial\gamma(x,\theta)}{\partial\theta^{\nu}} \;=\; \langle \partial_\mu \gamma \partial_\nu \gamma \rangle$$

For Gaussian distribution (saddle point approximation)

$$p(x_1,\ldots,x_n) = \frac{1}{(\sqrt{2\pi}\sigma)^n} \exp\left( - \sum_{i=1}^{n} \frac{(x_i - \bar{x}_i)^2}{2\sigma^2} \right)$$

Fisher metric gives Anti-de Sitter space:

$$ds^2 = \frac{1}{\sigma^2}\left( d\bar{x}_i d\bar{x}^i + 2n d\sigma^2 \right)$$

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Example: Dual Fisher information for mixed states

Banerjee, J.E., Sarkar 1701.02329

Proposal: FIsher information $\partial_m \partial_m \mathcal{F}$

$\mathcal{F}$ given by regularized volume under Ryu-Takayanagi surface

Agrees with field-theory result Ugajin, Sarosi 2016

# Conclusion and outlook

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Future of Gauge/Gravity Duality

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## Future of Gauge/Gravity Duality

- Understanding the foundations of gauge/gravity duality

  String theory ⇔ Black holes and quantum information

- Beyond the large $N$ limit: Quantum gravity

- Obtain further knowledge about strongly coupled systems and about quantum field theory beyond perturbation theory

- Further relations between particle physics, gravity and other branches of physics