

Singularities

in the Universe
in BH

Energy dominance
conditions

$$\varepsilon > 0$$

$$\varepsilon + p > 0$$

$$\varepsilon + 3p > 0 \quad ?$$

$$p = -\varepsilon \text{ for de Sitter}$$

80-90 th

- Starobinsky

$$V(\varphi), R^2 \rightarrow p \approx -\varepsilon$$

$$\varepsilon + 3p < 0$$

nonsingular Universe?

- Markou (82)

limiting density

$$3\left(\frac{\dot{a}}{a}\right)^2 = \varepsilon \left(1 - \frac{\varepsilon}{\varepsilon_{\text{crit}}}\right)$$

$$\varepsilon \propto \frac{1}{a^3}$$

???

MB (92)

$$S = \int \left(-\frac{1}{2} R + \lambda (4R_{\mu\nu} R^{\mu\nu} - R^2) + V(\lambda) + \dots \right) \dots \Rightarrow$$

nonsingular isotropic
Universe

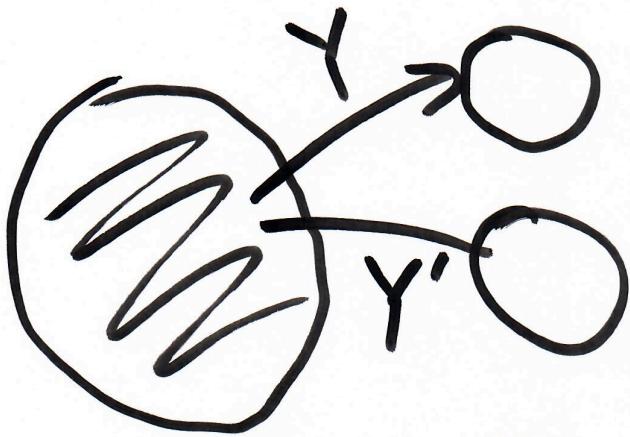
nonsingular 2d BH

but

What about Kasner,
4d. B.H. ?

90th - Strings, LQG...

NCG



$$Y = Y^A \Gamma_A$$

$$Y' = Y'^B \Gamma_B$$

$$Z = \frac{1}{2} (Y+1) (Y'+1)$$

$$\langle Z, (\partial Z)^* \rangle = \gamma$$



$$\sqrt{g} = \varepsilon_{\dots} \varepsilon^{\dots} Y \partial Y J Y C Y \partial Y$$



- Cosmol. const as integration const, 4 volume is quantized

- 3 volume is quantised

$$Y^5 = \varepsilon^{\prime \prime} \varphi$$

$$x^\mu = \varepsilon t_0^\mu, \quad \partial_\mu \varphi \partial^\mu \varphi = 1 ?$$

$$8\pi G = 1$$

$$S = \int \left(\frac{1}{2} R + \lambda ((\partial \phi)^2 - 1) + f(\square \phi) \right) \sqrt{-g} d^4x$$

$$g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = 1.$$

- Synchronous coordinates

$$ds^2 = dt^2 - \gamma_{ik} dx^i dx^k$$

- $\square \phi = \frac{\dot{x}}{2\gamma} !$

x

$f \cdot ?$

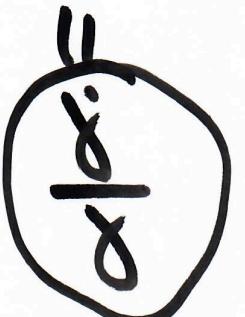
$$f(\Box \varphi) = 1 - \sqrt{1 - \frac{(\Box \varphi)^2}{\varepsilon_m}} + \dots$$

" "
 X

$$\begin{aligned}
 f(x) &= x_m^2 \left(1 + \frac{1}{3} \left(\frac{x}{x_m} \right)^2 - \right. \\
 &\quad - \sqrt{\frac{2}{3}} \frac{x}{x_m} \arcsin \left(\sqrt{\frac{2}{3}} \frac{x}{x_m} \right) - \\
 &\quad \left. - \sqrt{1 - \frac{2}{3} \frac{x^2}{x_m^2}} \right)
 \end{aligned}$$

$$\partial e^i_k = \gamma^m \dot{e}_{mk}$$

$i-k$ Einstein eqs.

$$\partial e^i_k = \frac{1}{3} \partial e \delta^i_k + \frac{\lambda^i_k}{\sqrt{\gamma}} \text{ const no sp. curv.}$$


0-0 eq. \Rightarrow

$$\frac{1}{12} \left(\frac{\dot{e}}{\gamma} \right)^2 = \varepsilon \left(1 - \frac{\varepsilon}{\varepsilon_m} \right) \rightarrow 2x_m^2$$

where

$$\varepsilon = \frac{\lambda^i_k \lambda^k_i}{\sqrt{\gamma}} + \frac{C}{\sqrt{\gamma}} + T_0^0 \quad \text{= kinetic}$$

Friedmann Universe

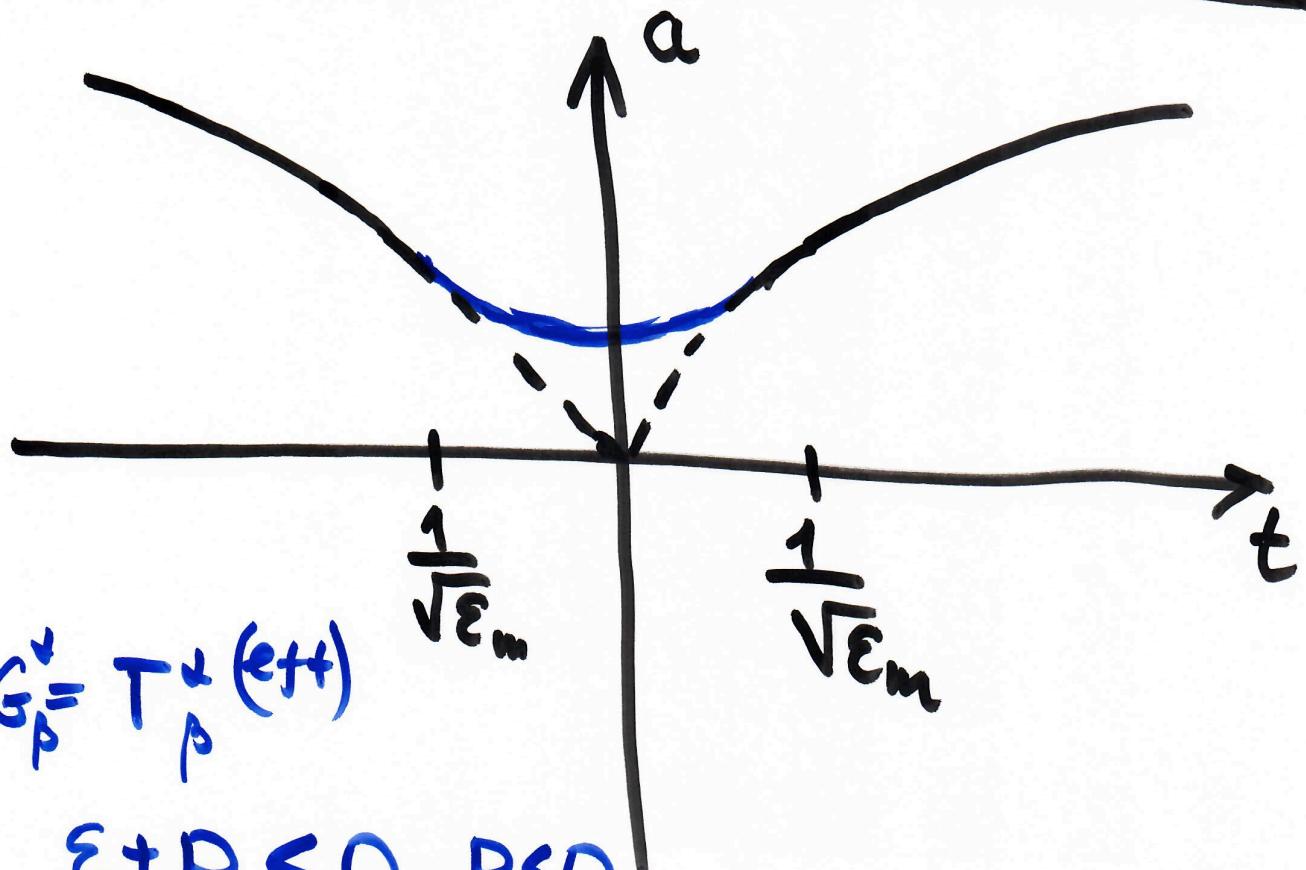
$$ds^2 = dt^2 - a^2(t) \delta_{ik} dx^i dx^k$$

$$\gamma = a^6, \gamma^i_k = 0$$

add matter with

$$\rho = +w \epsilon$$

$$a = \left(1 + \frac{3}{4} (1+w)^2 \epsilon_m t^2 \right)^{\frac{1}{3(1+w)}}$$



Kasner Universe

$$ds^2 = dt^2 - t^{2P_1} dx_1^2 - t^{2P_2} dy^2 - t^{2P_3} dz^2$$

$$P_1 + P_2 + P_3 = 1, \quad P_1^2 + P_2^2 + P_3^2 = 1$$

$$\bullet R_{\mu\nu\sigma\tau} R^{\mu\nu\sigma\tau} = -\frac{16}{t^4} P_1 P_2 P_3$$

$$\bullet \underline{\underline{\gamma \propto t^2}}$$

$$\varepsilon = \frac{\lambda_k^i \lambda_i^k}{\gamma} = \frac{\bar{\lambda}^2}{\gamma}$$

$$\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \frac{3\bar{\lambda}^2}{2\gamma} \left(1 - \frac{\bar{\lambda}^2}{8\varepsilon_m \gamma}\right)$$



$$\gamma = \frac{\bar{\lambda}^2}{8\varepsilon_m} \left(1 + 3\varepsilon_m t^2\right)$$

At $t=0$ $\gamma \neq 0$

$$\chi_{ik} = \chi_{(i)} s_{ik}$$

$$\chi_{(i)} = \left[\left(\frac{\pi^2}{8\epsilon_m} \right) (1 + 3\epsilon_m t^2) \right]^{1/3} \times$$

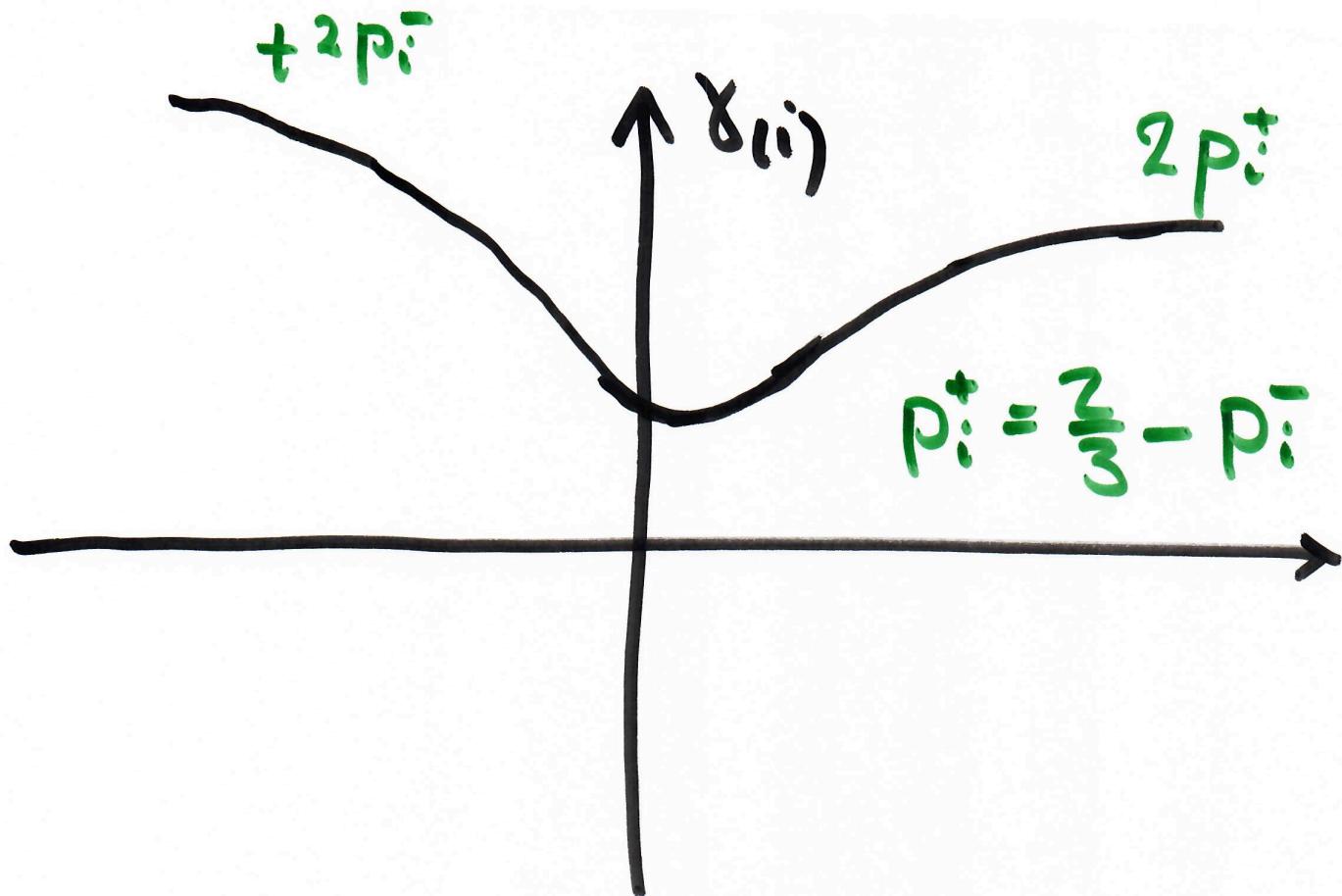
$$\times \exp \left(2 \sqrt{\frac{2}{3}} \frac{\lambda_{i1}}{\lambda} \sinh^{-1}(\sqrt{3}\epsilon_m t) \right)$$

Exact Solution

$$|t| \gg \frac{1}{\sqrt{\epsilon_m}}$$

$$\chi_{(i)} \propto (\epsilon_m t^2)^{p_i^{\pm}}$$

$$\sum_i p_i^{\pm} = 1 \quad \sum (p_i^{\pm})^2 = 1.$$



Kasner with $\bar{p}_i = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$
 turns to Kasner with
 $\bar{p}_i^* = (1, 0, 0) \equiv$ Minkowski

BH

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt_s^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 d\Omega^2$$

$r < r_g$ τ is time coordinate
 $t_s \rightarrow R$ space coord.

Inside BH

$$ds^2 = dt^2 - a^2(t) dR^2 - b^2(t) d\Omega^2$$

$$\parallel t = r_g (\arcsin \tau - \tau \sqrt{1-\tau^2})$$

$$\parallel a^2(t) = \frac{1-\tau^2(t)}{\tau^2(t)}, \quad b^2 = r_g^2 \tau^4(t)$$

$$\varphi(t) = t + \text{const} \quad ?$$

$$\square \varphi = \frac{\dot{x}}{2\gamma} \rightarrow \infty \quad (\text{firewall?})$$

Go to Lemaître coordinate system

$$T = R + \int \frac{\sqrt{1+a^2}}{a} dt^2, \quad \bar{R} = R + \int \frac{dt}{a\sqrt{1+a^2}}$$

$$ds^2 = dT^2 - (1+a^2) d\bar{R}^2 - b^2 d\Omega^2$$

\downarrow
 $a(T-R)$

$$\varphi = T \quad ! \quad (\text{no firewall!})$$

At late times $t \underset{\equiv}{\approx} T$

Near horizon region

• $\frac{r_g - R}{r_g} \ll 1$

$$ds^2 \approx dt^2 - \frac{1}{4} \left(\frac{\bar{t}}{r_g} \right)^2 dR^2 - r_g^2 d\Omega^2$$

Similar to Kastner with

$$P_i = (1, 0, 0) \equiv \text{Minkowski}$$

Near singul. region

• $R \ll r_g$

$$ds^2 \approx dt^2 - \left(\frac{t}{t_0} \right)^{-2/3} dR^2 - \left(\frac{t}{t_0} \right)^{4/3} r_g^2 d\Omega^2$$

$$\underline{t_0 = 2r_g/3}$$

Similar to Kasner with

$$P = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right)$$

Spatial curvature term from dR^3
is relevant only at $r \sim r_g/2$.

In our theory singularity
which would happen at
 $t=0$ is avoided and

"Kasner" with $p^- = (-\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$
goes to $p^+ = (1, 0, 0)$

Metric at $t > 1/\sqrt{\epsilon_m}$

$$ds^2 = dt^2 - Q_0^2 \left(\frac{t}{t_0}\right)^2 dR^2 - \frac{r_g^2}{Q_0^2} d\Omega^2$$

$$Q_0 = \left(\frac{16}{3} \epsilon_m R_g^2\right)^{2/3}$$

$$R_{g1} = \frac{r_g}{Q_0^{1/2}} \propto r_g^{1/3}$$

Then

$P = (1, 0, 0)$ in BH of size $r_g^{1/3}$

$\rightarrow P = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \rightarrow$ next bounce

$\rightarrow P = (1, 0, 0)$ but $R_{g_2} = \frac{R_{g_1}}{Q_1} \propto r_g^{\frac{1}{3}}$

et. cetera.

Finally we come to BH.

of radius $R_{gn} \propto r_g^{\frac{1}{3n}} \dots$

and end up at limiting
curvature