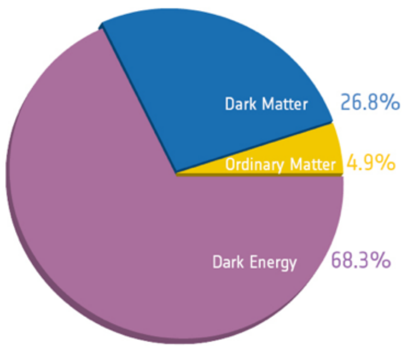
The mildly non-linear regime of structure formation

June 2017

Outstanding questions in physical cosmology

Composition



Example: Neutrino masses

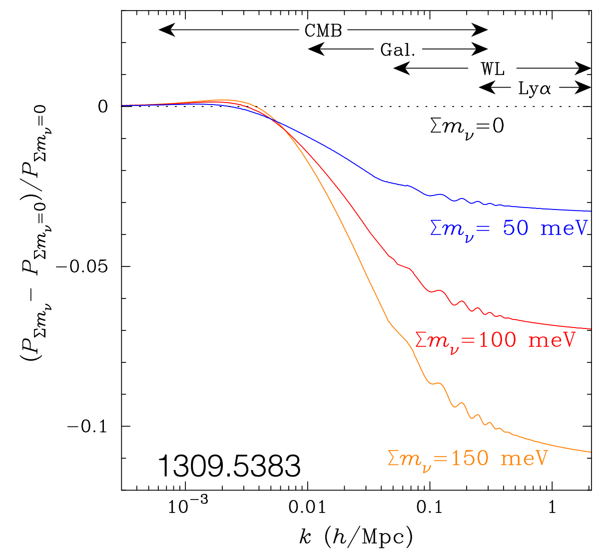


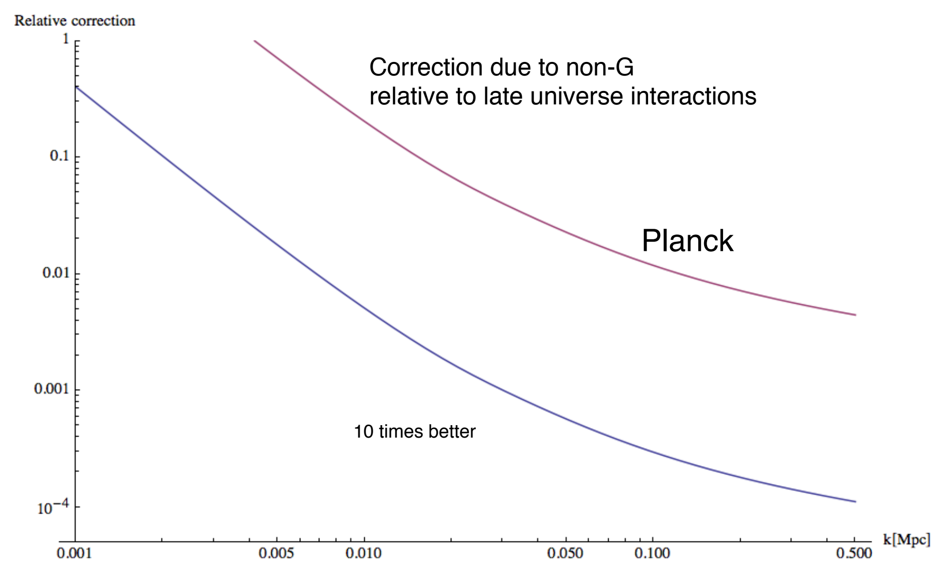
Figure 1. Fractional change in the matter density power spectrum as a function of comoving wavenumber k for different values of $\sum m_\nu$. Neutrino mass suppresses the power spectrum due to free streaming below the matter-radiation equality scale. The shape of the suppression is highly characteristic and precision observations over a range of scales can measure the sum of neutrino masses (here assumed all to be in a single mass eigenstate). Also shown are the approximate ranges of experimental sensitivity in the power spectrum for representative probes: the cosmic microwave background (CMB), galaxy surveys (Gal.), weak lensing of galaxies (WL), and the Lyman-alpha forest (Lyα). The CMB lensing power spectrum involves (an integral over) this same power spectrum, and so is also sensitive to neutrino mass.

The history of the Universe before the hot big bang

Example: Primordial non-Gaussianity

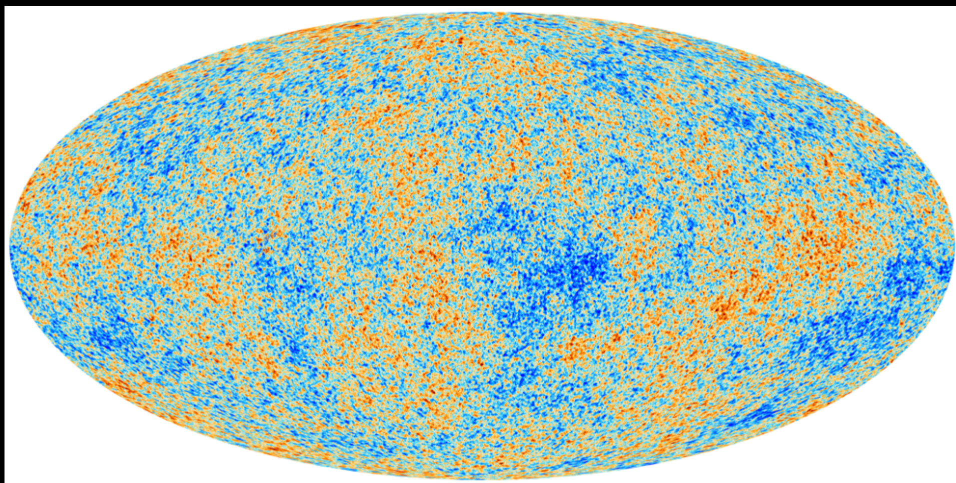
Planck constraints

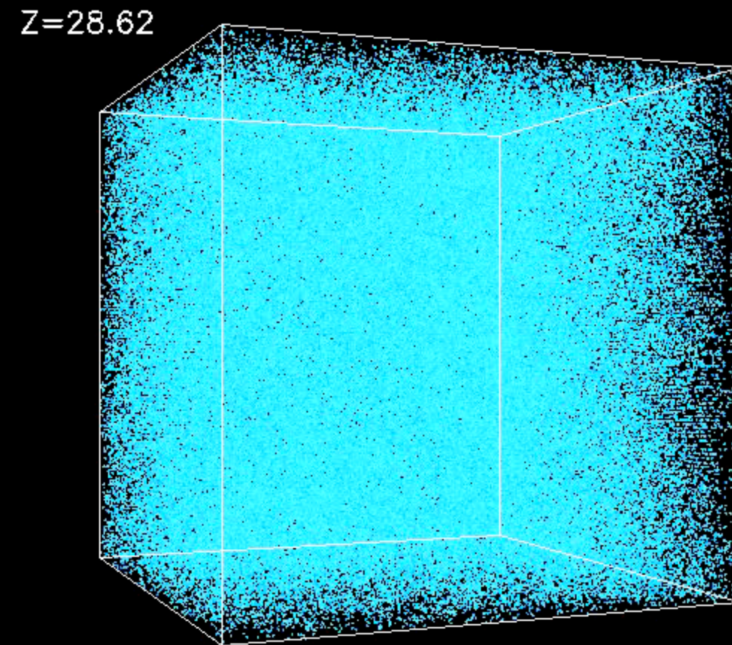
$$f_{\rm NL}^{\rm loc.} = 0.8 \pm 5.0\,, \quad f_{\rm NL}^{\rm eq.} = -4 \pm 43\,, \quad (68\%\ {\rm CL})\,.$$

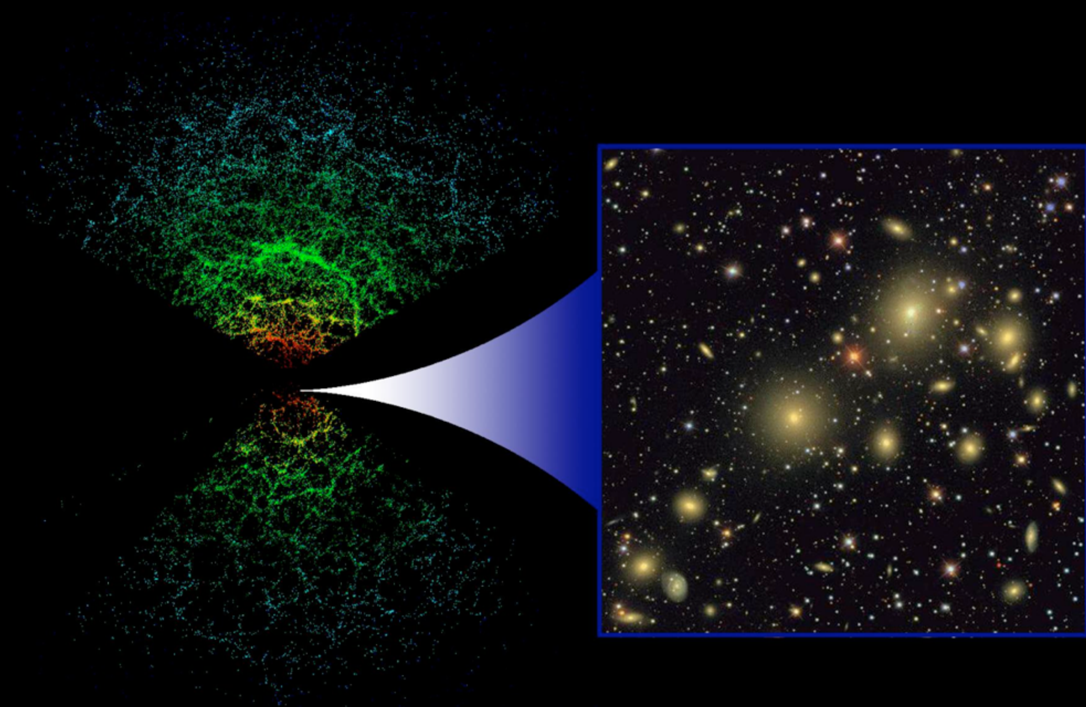


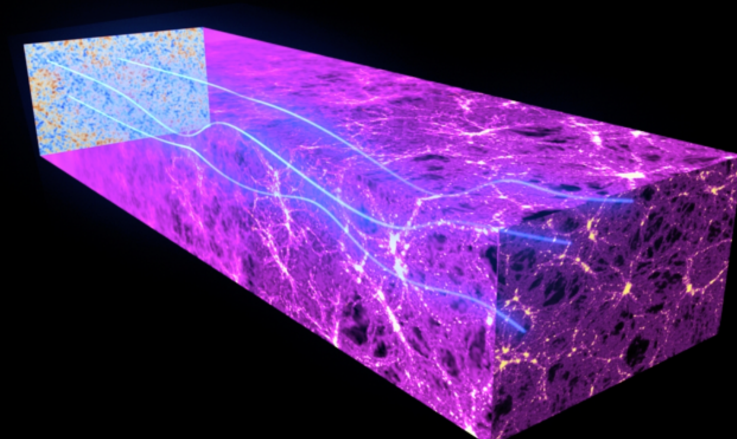
We are looking for extremely small effects but need to have enough modes. We are forced towards the non-linear regime.

The growth of structure

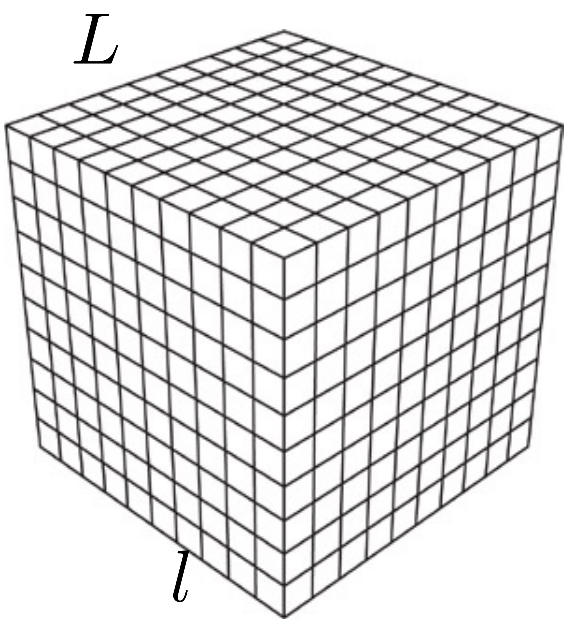


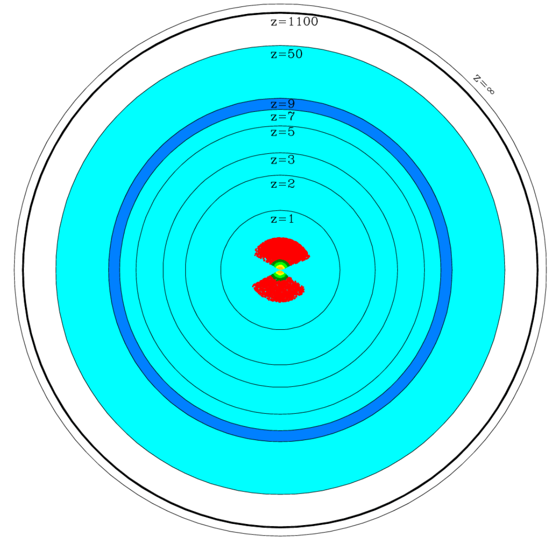






Errors on parameters scale as $1/N_{modes}^{-1/2}$





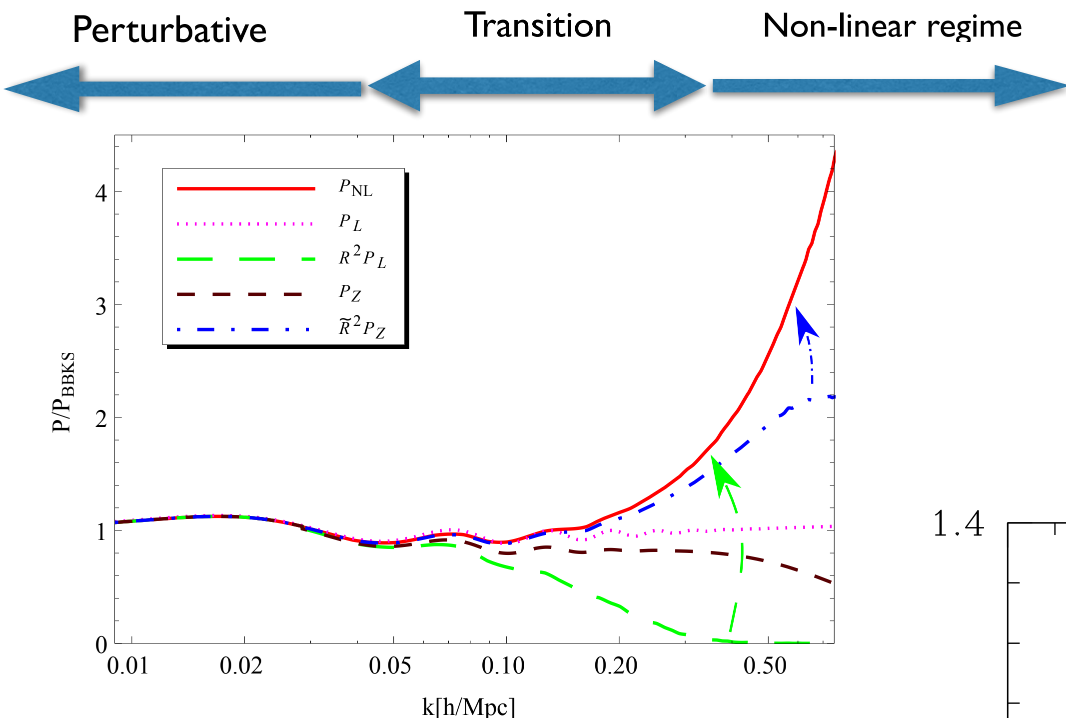
$$k_{\rm max} = \frac{\pi}{l}$$

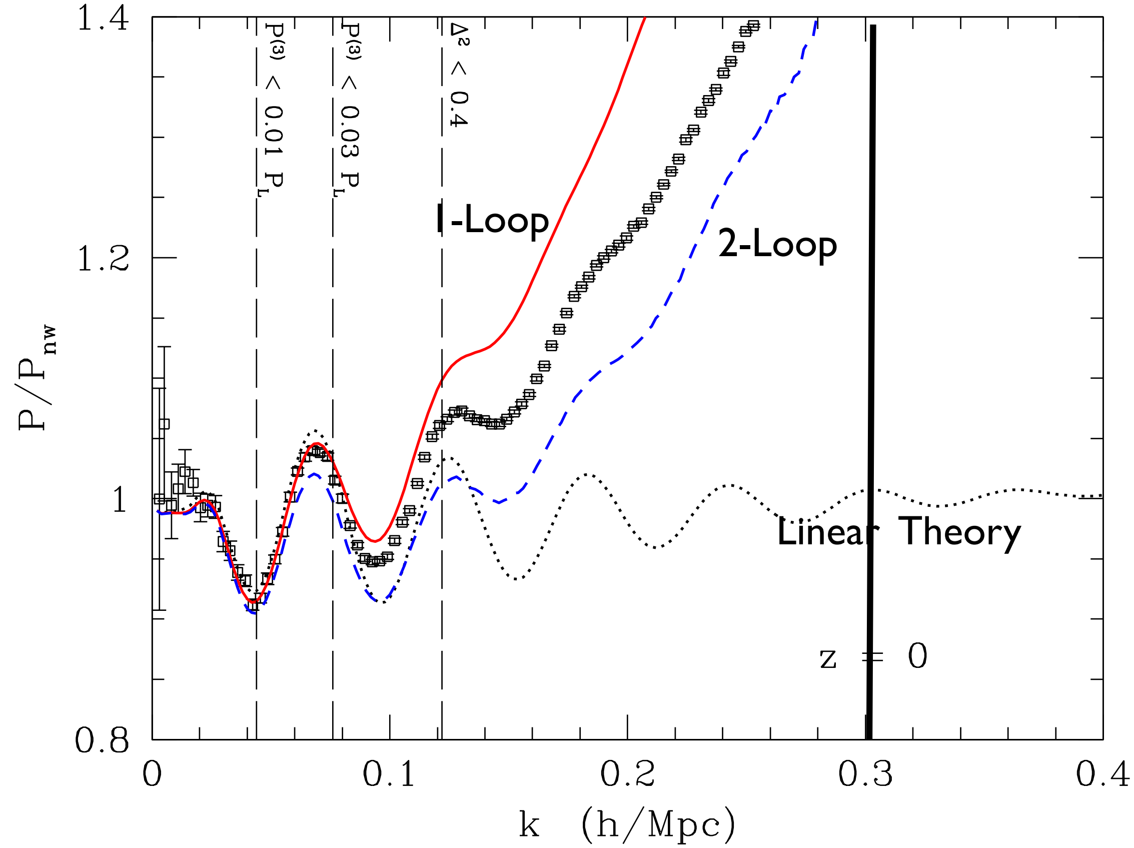
$$N_{\rm modes} = (\frac{L}{l})^3 = \frac{V k^3_{\rm max}}{\pi^3}$$

Number of modes in Planck approximately 10^6

redshift range	Volume Gpc^3	kmax h Mpc^{-1} (Planck x 10)
0-1	50	0.4
1-2	140	0.3
2-3	160	0.3

Non-linear corrections in LSS: Power spectrum





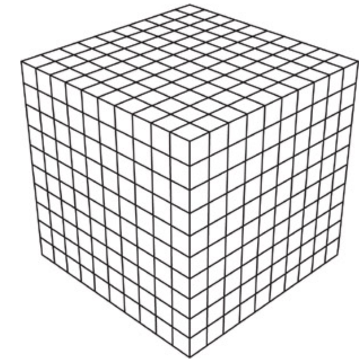
Recent results using the EFT of LSS

Reconstructing the initial conditions, undoing the effects of the non-linearities

Baldauf, Mercolli, Mirbabayi, Schaan, Schmitfull, Senatore, Simonovic

EFT of LSS

Describe the dynamics on large scales, after integrating out the short scale modes.



$$\partial_\tau \delta + \partial_i[(1+\delta)v^i] = \partial_i u^i \ ,$$

$$\partial_\tau v^i + \mathcal{H} v^i + \partial^i \phi + v^j \partial_j v^i = -\frac{1}{a\rho}\partial_j \tau^{ij}$$

$$\triangle \phi = \frac{3}{2}\mathcal{H}^2 \Omega_{\rm m} \delta \ .$$

$$\delta = \delta_{(1)} + \delta_{(2)} + \delta_{(3)} + \delta_{(4)} + \delta_{(5)} + \cdots$$

$$\tau_\theta \equiv -\partial_i \left[\frac{1}{a\rho}\partial_j \tau^j\right] = \tau_\theta^{\rm det} + \tau_\theta^{\rm stoch}$$

$$\tau_\theta^{\rm det} = \tau_\theta^{\rm det}[\partial_i \partial_j \bar{\phi}].$$

$$\tau_\theta^{\rm det}\big|_{\rm LO} = -d^2 \triangle \delta_{(1)} = -d^2 \triangle\triangle \bar{\phi}_{(1)}$$

$$\tau_\theta^{\rm det}\big|_{\rm NLO} = -d^2\triangle[\delta_{(1)} + \delta_{(2)}] - e_1 \triangle \delta_{(1)}^2 - e_2 \triangle (s_{ij(1)} s_{(1)}^{ij}) - e_3 \partial_i s_{(1)}^{ij} \partial_j \delta_{(1)},$$

$$s_{ij} = \left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}^{({\rm K})} \triangle\right)\bar{\phi}.$$

EFT of LSS

- Study regime of small corrections
- Characterize terms
- Calculable vs non-calculable (counter terms)
- How many terms to achieve a desired accuracy?
- What is the relation between results for different statistics

EFT terms

- Write all terms consistent with symmetries: Mass & momentum conservation, equivalence principle
- Non-locality in time

Examples:

$\delta_0(\boldsymbol{k})$ Initial conditions

$$\delta^{(2)}(\boldsymbol{k}) = \int_p \left[\frac{3}{14}\left(1 - \frac{(\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2}{\boldsymbol{p}_1^2\boldsymbol{p}_2^2}\right) + \frac{1}{2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_1^2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_2^2}\right]\delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

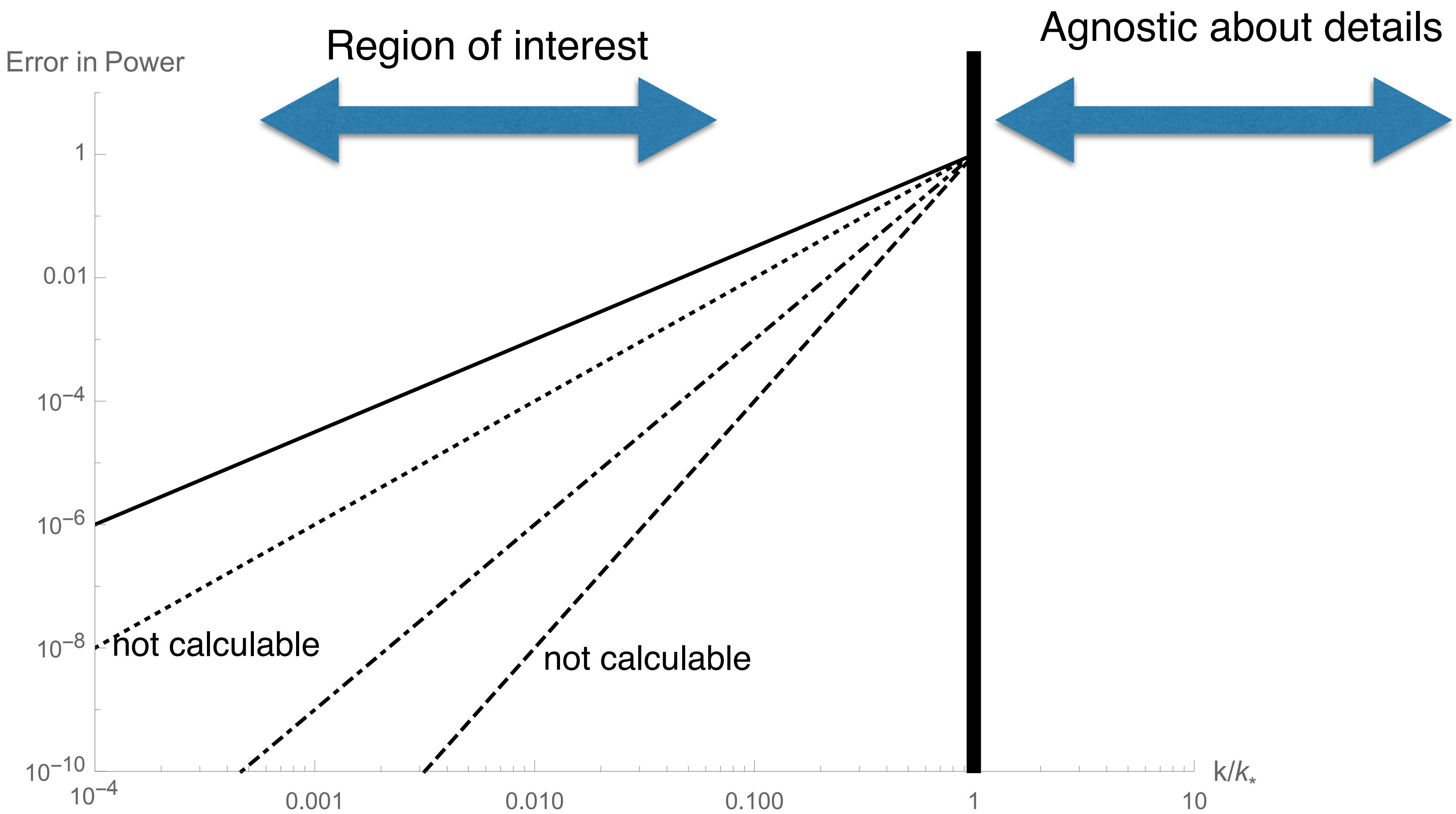
First correction

$$\delta^{ct(1)}(\boldsymbol{k}) = l_1^2\boldsymbol{k}^2\delta_0(\boldsymbol{k}) + l_1^2\int_p \frac{\boldsymbol{k}\cdot\boldsymbol{p}_1}{\boldsymbol{p}_1^2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_2^2}\boldsymbol{p}_1^2\delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

first “un-calculable” piece (starts linear)

$$\delta^{ct(2)}(\boldsymbol{k}) = \int_p \left[l_{21}^2\boldsymbol{k}^2 + l_{22}^2\boldsymbol{k}^2(1 - \frac{(\boldsymbol{p}_1\cdot\boldsymbol{p}_2)^2}{\boldsymbol{p}_1^2\boldsymbol{p}_2^2}) + l_{23}^2\frac{\boldsymbol{k}\cdot\boldsymbol{p}_1\ \boldsymbol{k}\cdot\boldsymbol{p}_2\ \boldsymbol{p}_1\cdot\boldsymbol{p}_2}{\boldsymbol{p}_1^2\boldsymbol{p}_2^2}\right]\delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

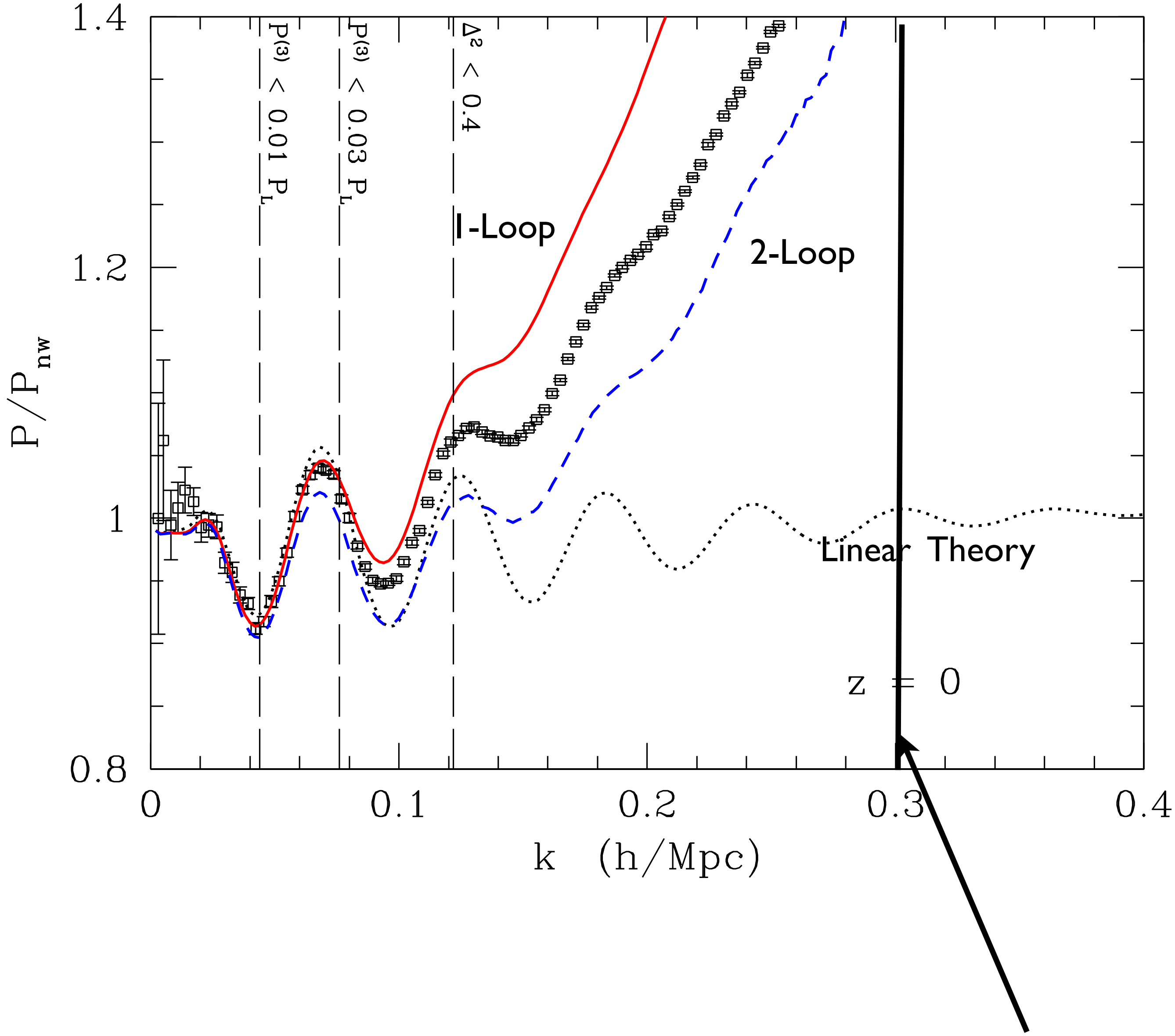
“un-calculable” pieces that starts quadratic



There are contributions whose size cannot be computed within the large scale theory, they depend on the details of the small scale dynamics. However there k dependence is known.

$$\delta^{ct(1)}(\boldsymbol{k}) = l_1^2 \boldsymbol{k}^2 \delta_0(\boldsymbol{k}) + l_1^2 \int_p \frac{\boldsymbol{k}\cdot\boldsymbol{p}_1}{\boldsymbol{p}_1^2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_2^2}\boldsymbol{p}_1^2 \delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

Standard Perturbation Theory

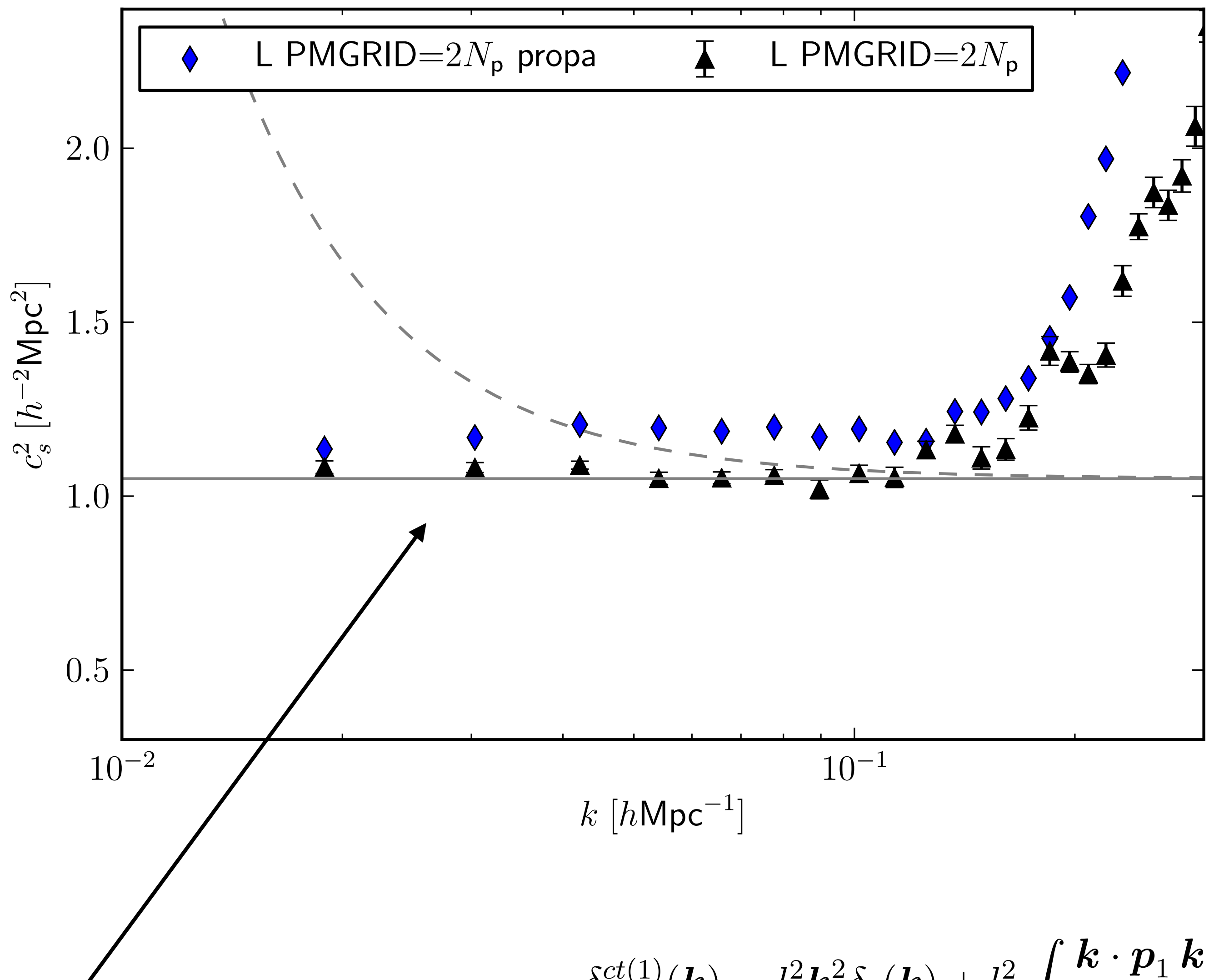


At this scale the 2-loop EFT is good to 1 %

Carlson et al 0905.0479

Amplitude of the first non-calculabe term:

l_1^2



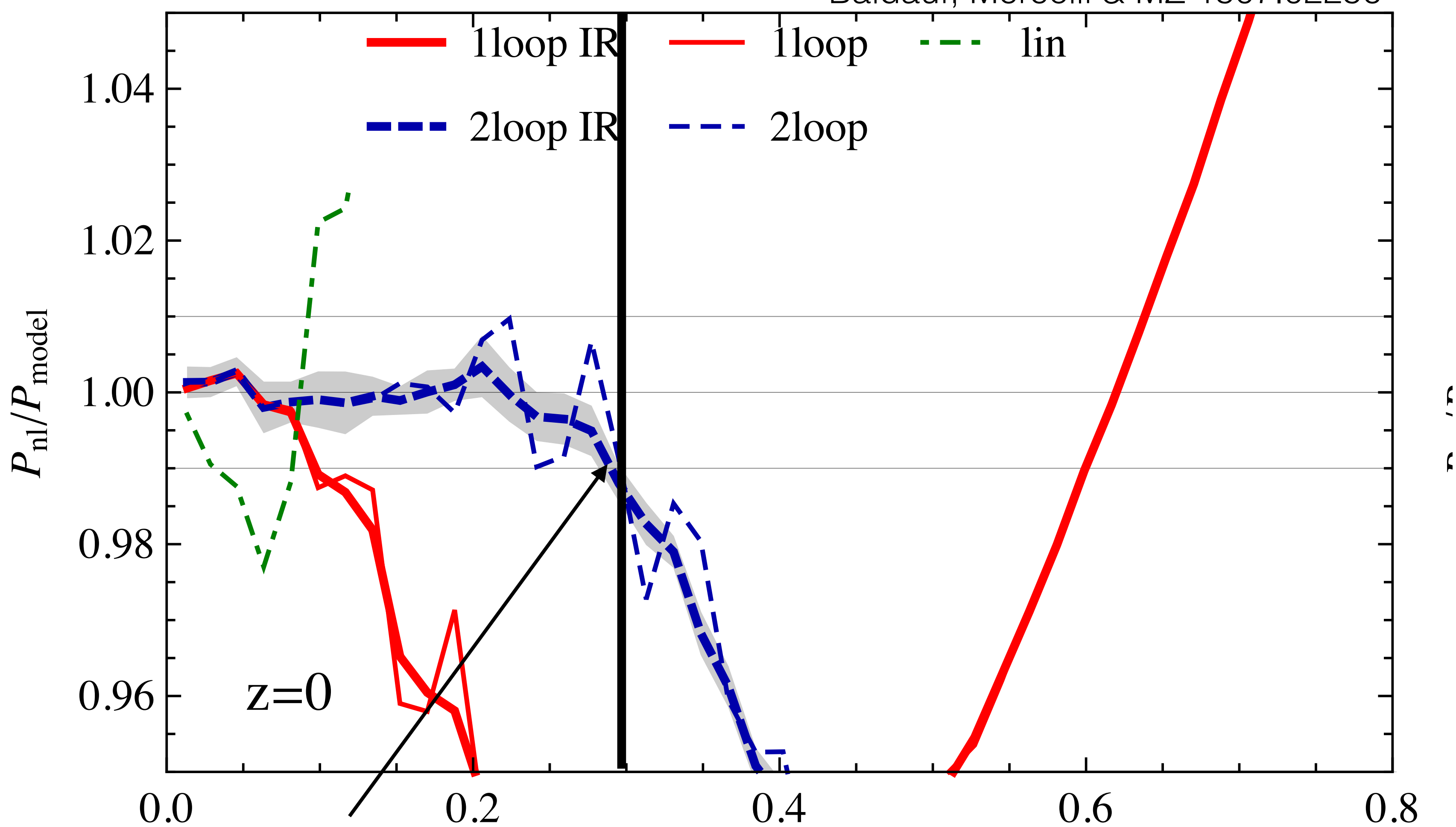
$$\delta^{ct(1)}(\boldsymbol{k}) = l_1^2 \boldsymbol{k}^2 \delta_0(\boldsymbol{k}) + l_1^2 \int_p \frac{\boldsymbol{k}\cdot\boldsymbol{p}_1}{\boldsymbol{p}_1^2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_2^2}\boldsymbol{p}_1^2 \delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

Measured on large scales

Baldauf, Mercolli & MZ 1507.02256

Comparison with sims once non-calculable term measured on large scales

Baldauf, Mercolli & MZ 1507.02256



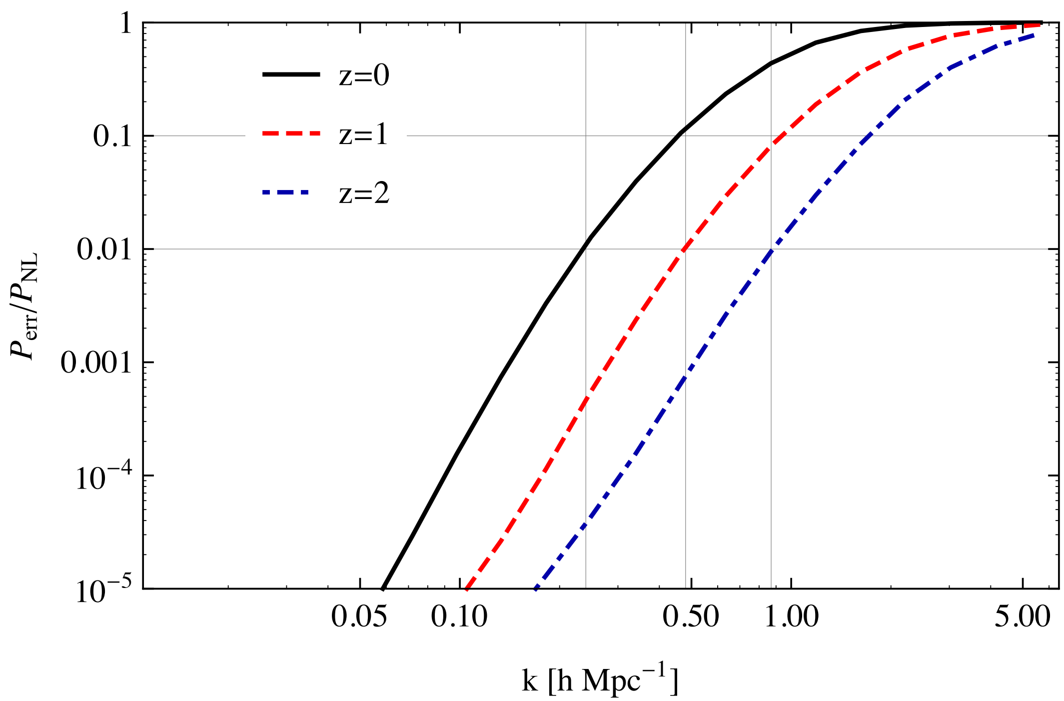
Improvements in the mildly non-linear regime

$$\delta^{ct(1)}(\boldsymbol{k}) = l_1^2 \boldsymbol{k}^2 \delta_0(\boldsymbol{k}) + l_1^2 \int_p \frac{\boldsymbol{k}\cdot\boldsymbol{p}_1}{\boldsymbol{p}_1^2}\frac{\boldsymbol{k}\cdot\boldsymbol{p}_2}{\boldsymbol{p}_2^2}\boldsymbol{p}_1^2\delta_0(\boldsymbol{p}_1)\delta_0(\boldsymbol{p}_2)$$

Amplitude determined at k=0.02,
shape known theoretically
improvement at k=0.3

Comparison realization by realization

Baldauf, Schaan & MZ 1505.07098, 1507.02255



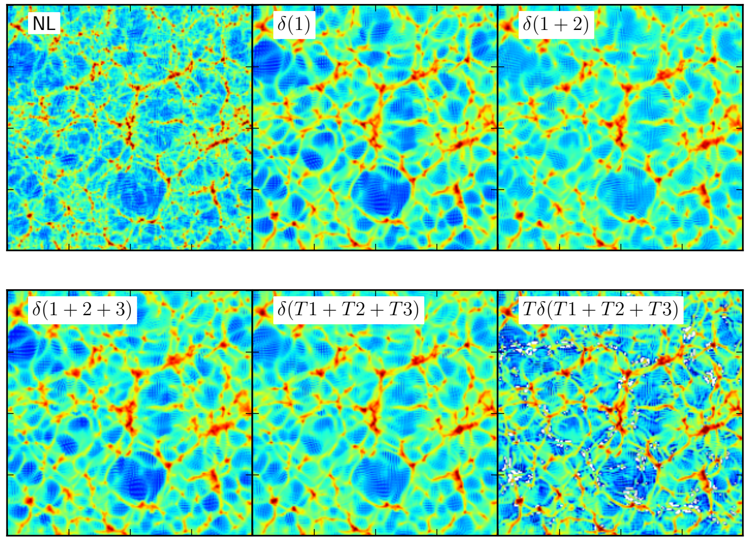
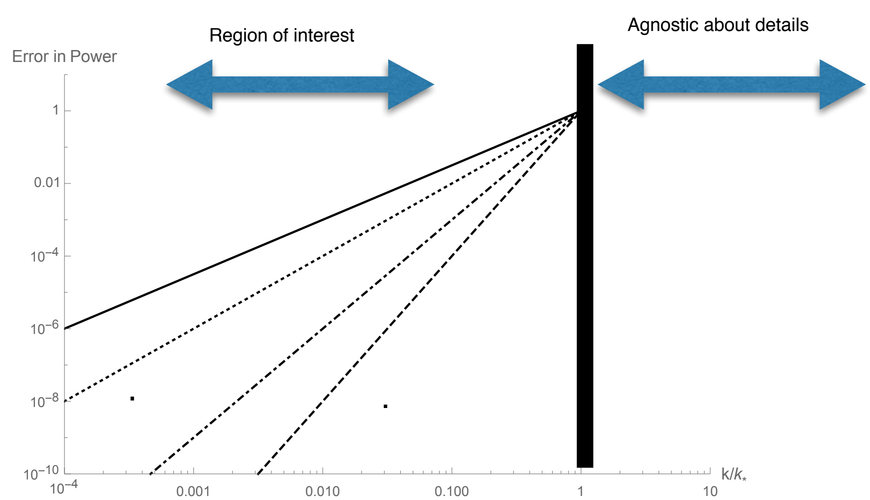


Figure 7. Non linear transformation of the density field in a patch of 300 h^{-1}Mpc length and 15 h^{-1}Mpc depth.



Theoretical errors & neutrino masses

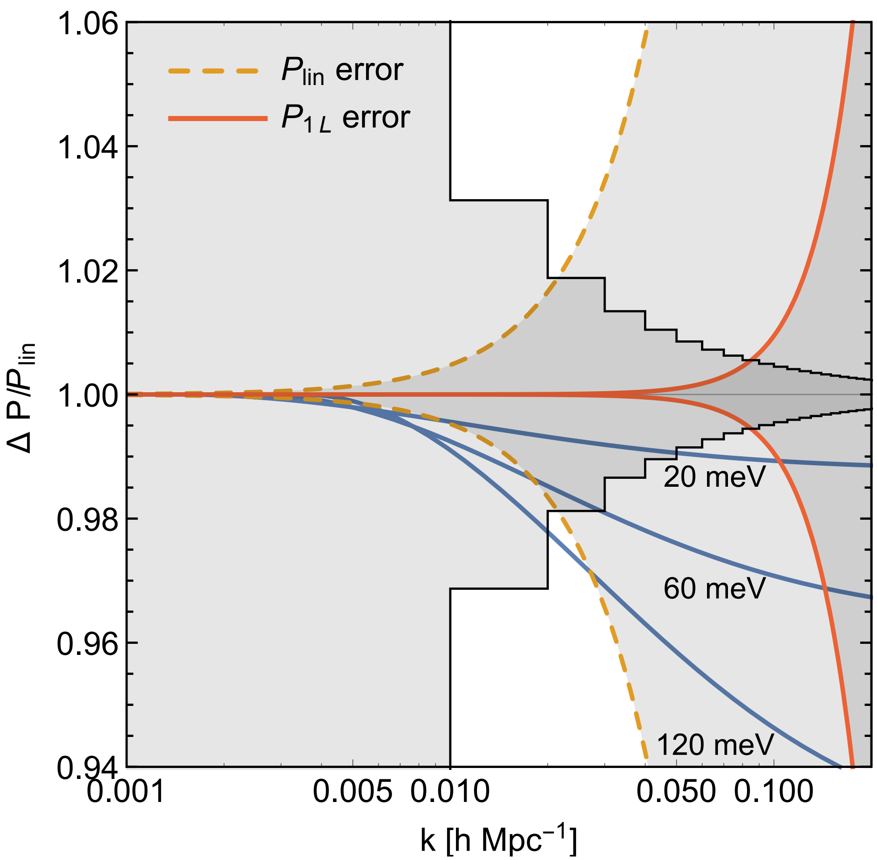
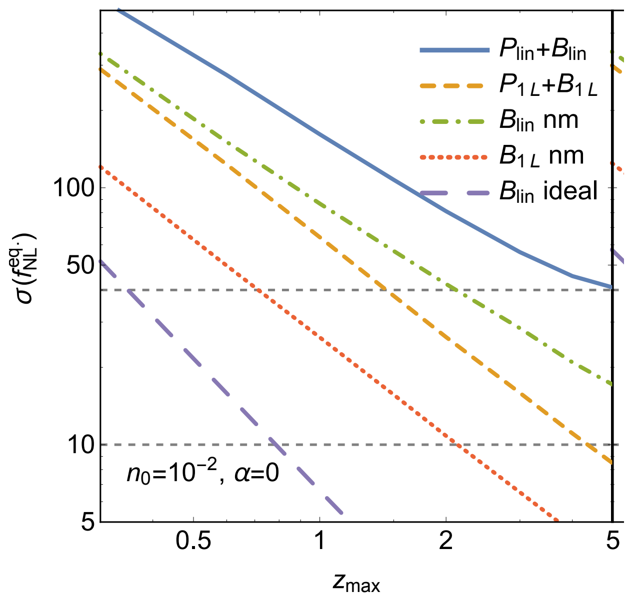
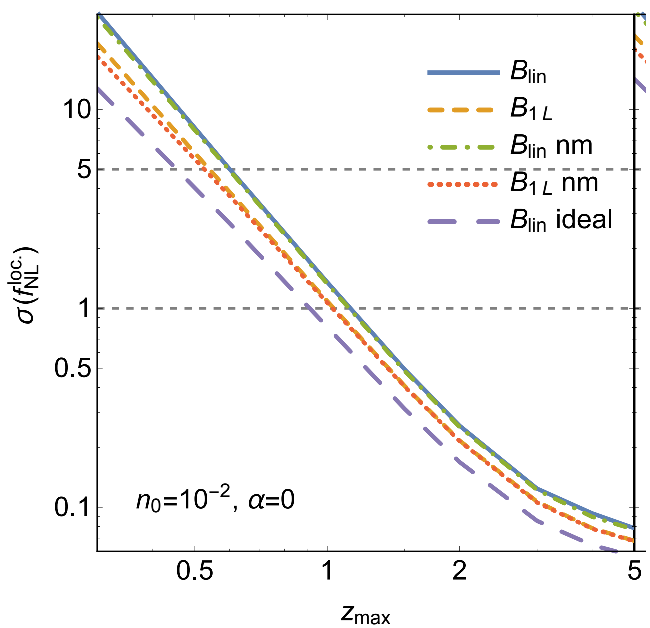


FIG. 2: Theoretical errors for the linear theory and one-loop power spectrum (see Eq. (42)) as a function of k. The cosmic variance is plotted for the redshift bin $1 < z < 2$. Three solid lines are relative suppression of the power spectrum for three different M_ν.

Theoretical errors & non-Gaussianity



Improvement looks very difficult



Improvement seems likely

General lessons from EFT

- The small scale dynamics that is not captured by perturbation theory introduces a small number of free parameters that need to be fitted from simulation or data
- We understand the structure of these new terms, their dependence with scale is fixed.
- Calculations come with theoretical error bars.
- We are not strangers to these type of things, bias, higher dimension operators in particle physics.

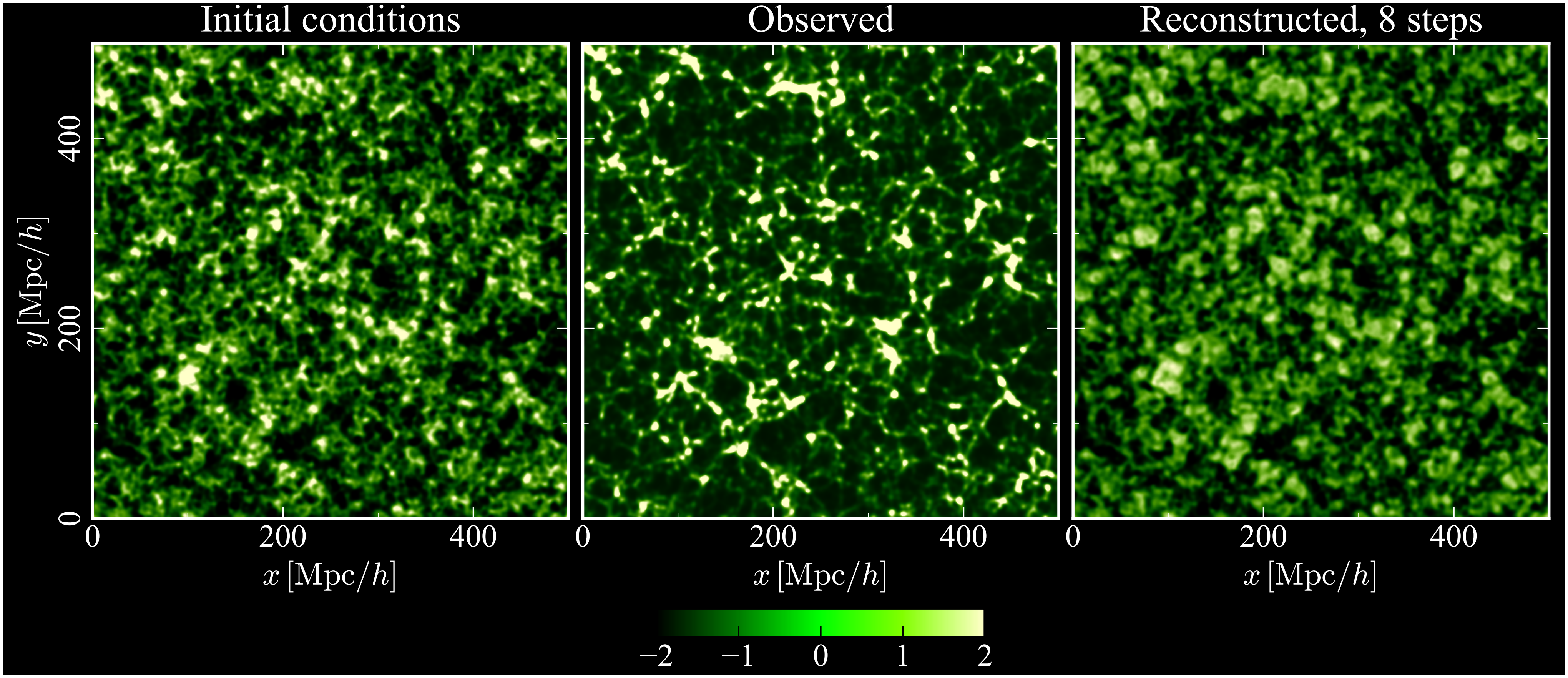
Interesting conceptual differences to standard QFT set up

- Non-locality in time
- Prevalence of composite operators

Additional things to consider

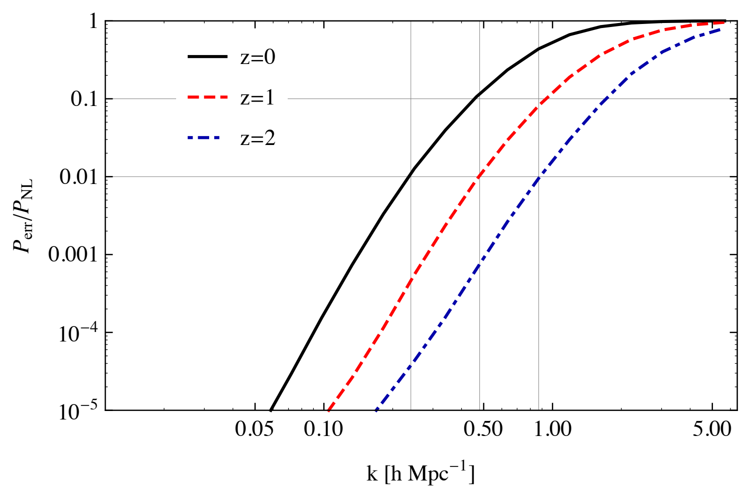
- Biased tracers, redshift space distortions, bispectrum
- Better comparison with simulations to cross the percent level accuracy
- Where is the information on parameters of interest?

Backward modeling/reconstruction

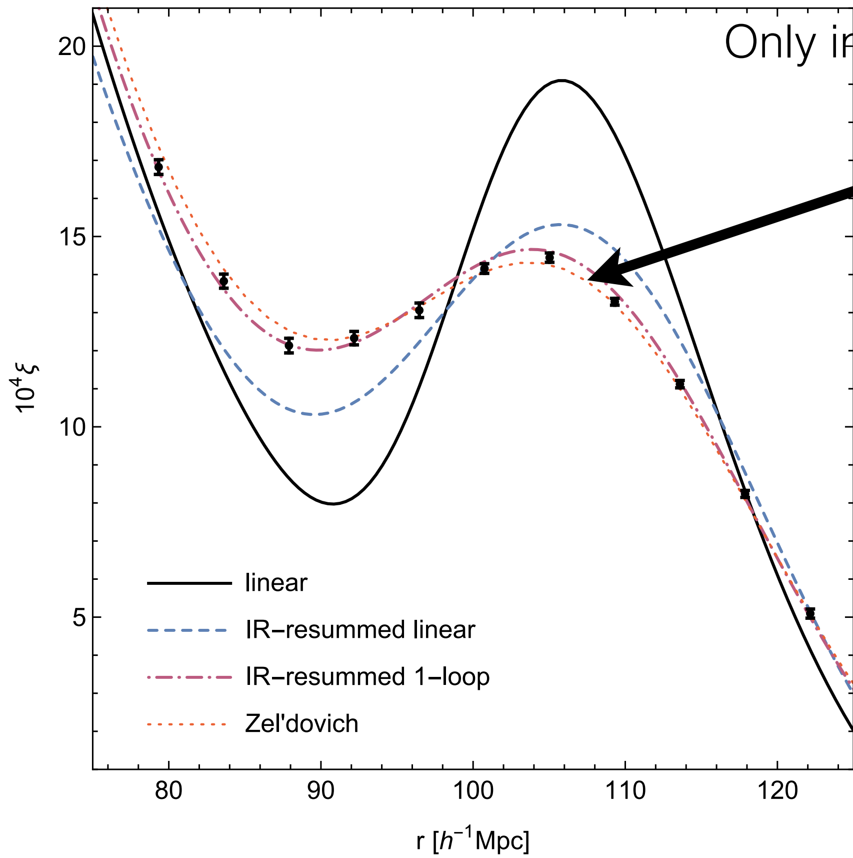


$$\delta_{NL} = \delta_{PT}[\delta_{lin}] + \text{error}$$

Filter the non-linear density
and solve for the linear density



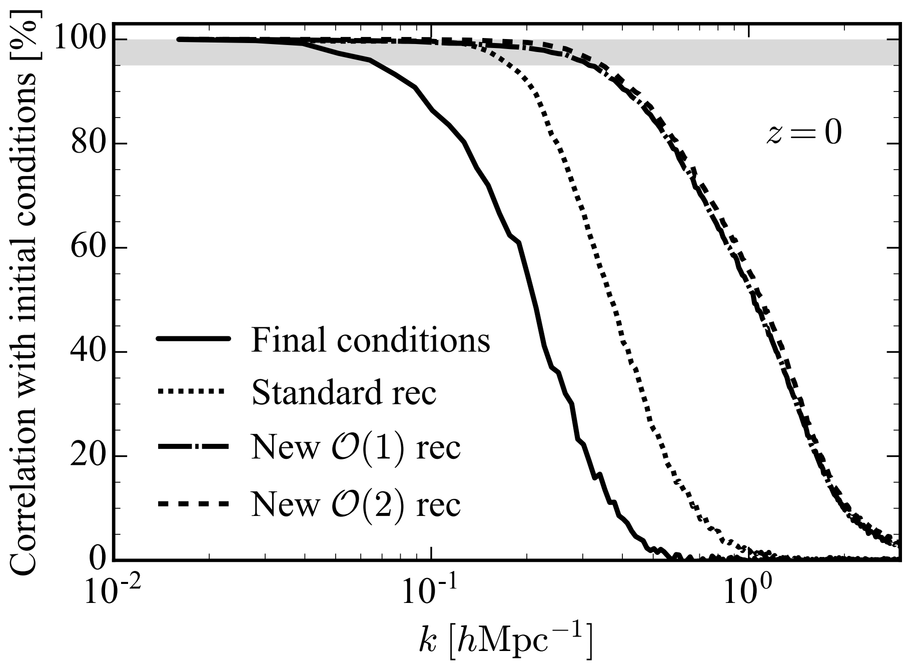
The smoothing of the BAO peak



Only include displacement non-linearities

FIG. 5. *Various theoretical approximations to the acoustic peak in the correlation function as well as simulation measurements. Solid: linear, dashed: IR-resummed linear, dot-dashed: IR-resummed 1-loop, and dotted: Zel'dovich.*

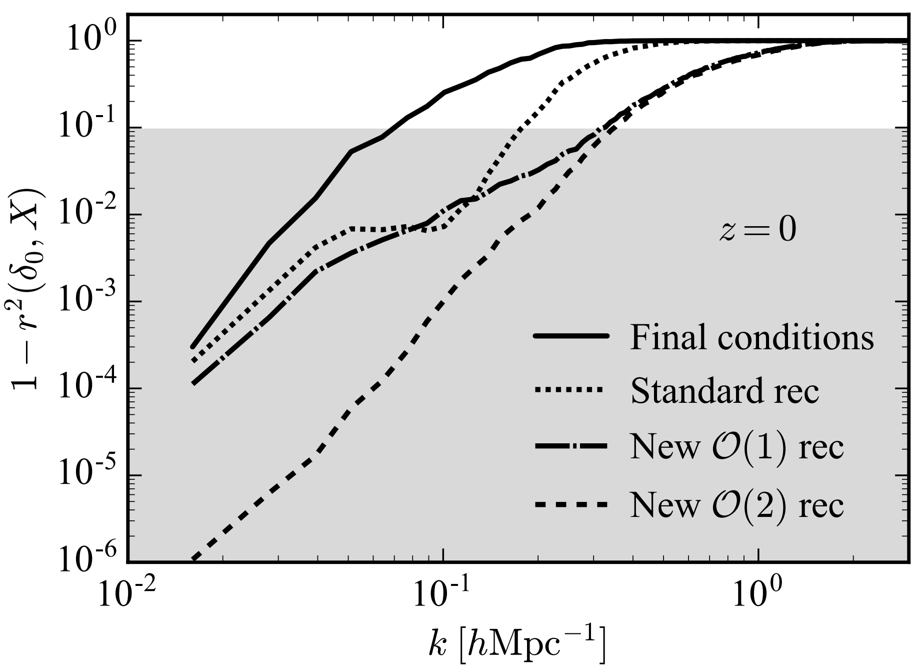
Width 20 Mpc
Displacements due to LSS 10 Mpc

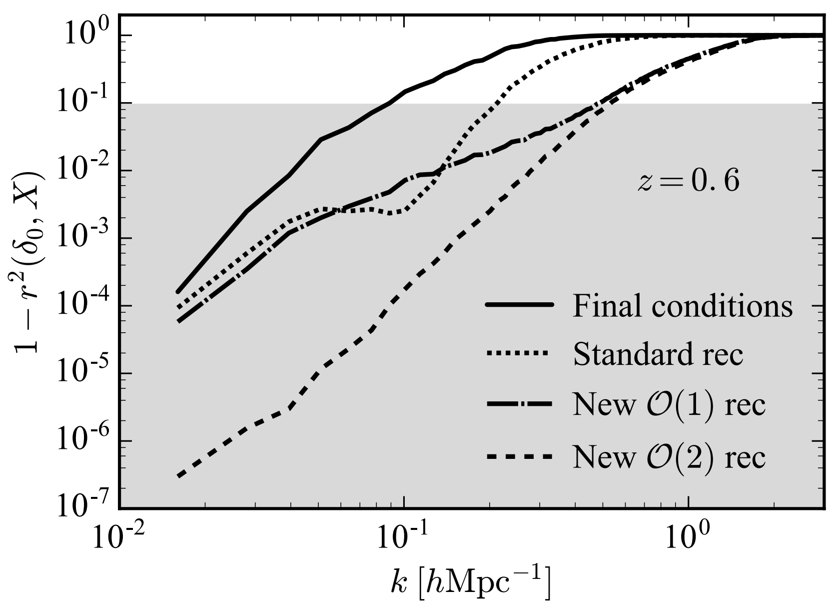


Cross correlation between initial and reconstructed field

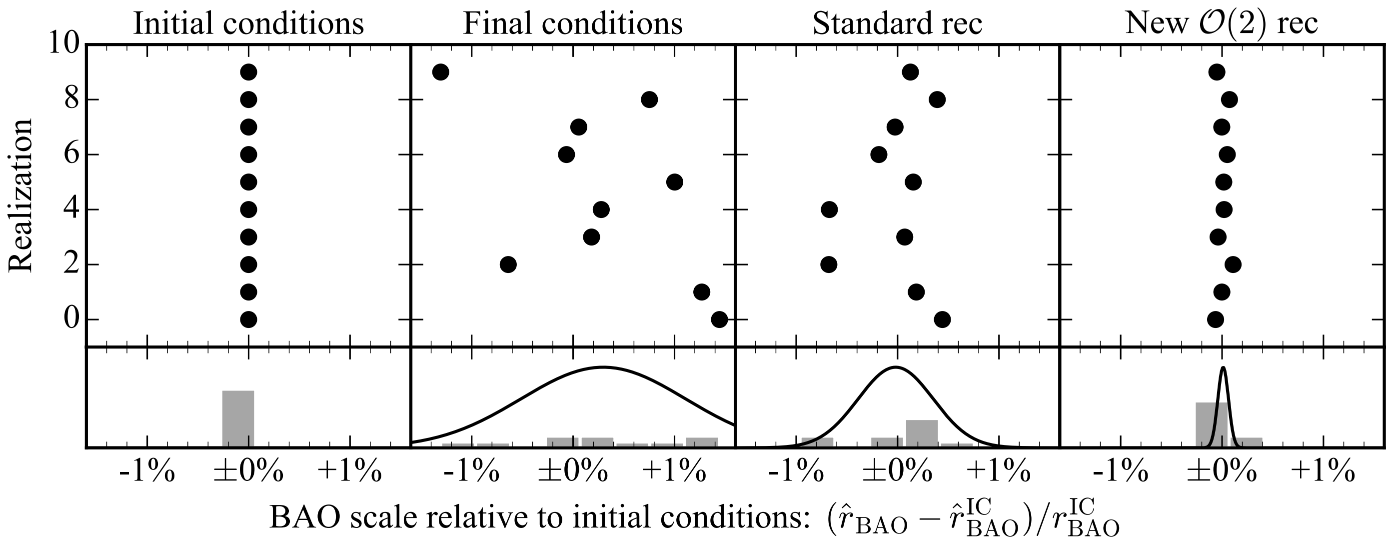
$$\delta_{NL} = \delta_{PT}[\delta_{lin}] + \text{error}$$

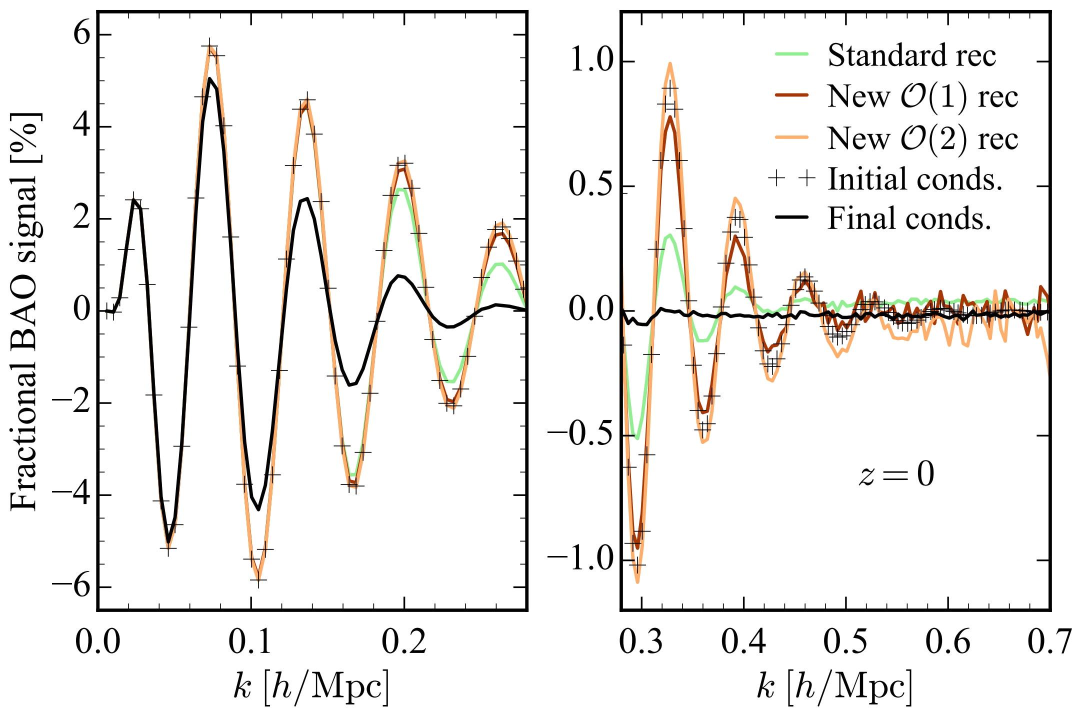
Filter the non-linear density and solve for the linear density

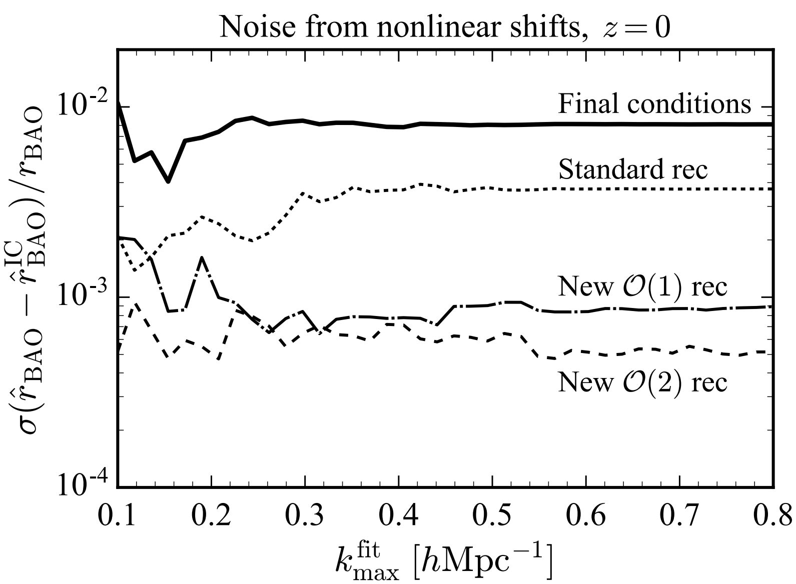




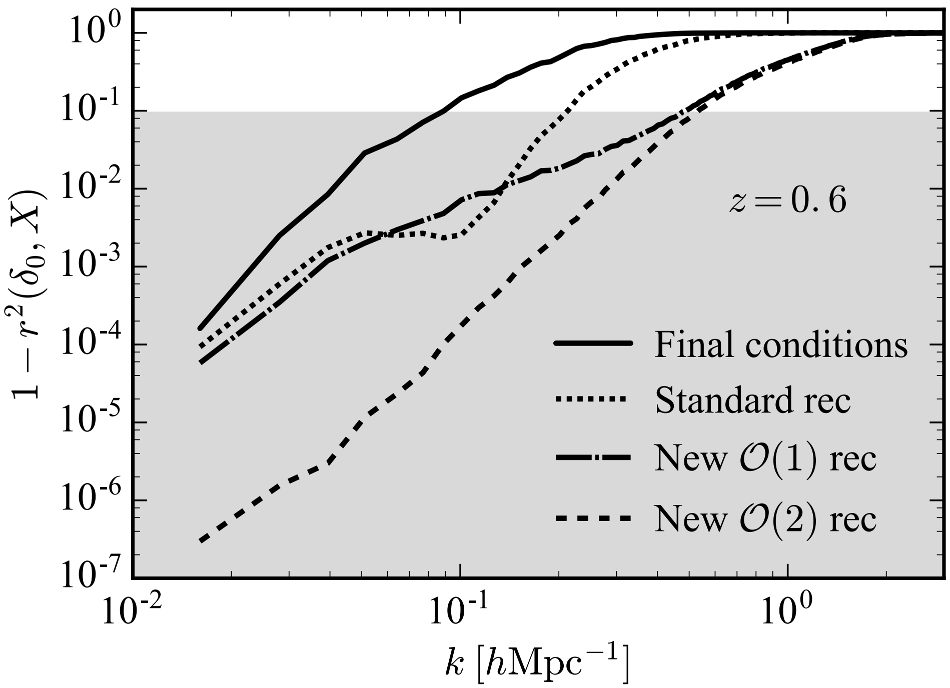
BAO reconstruction

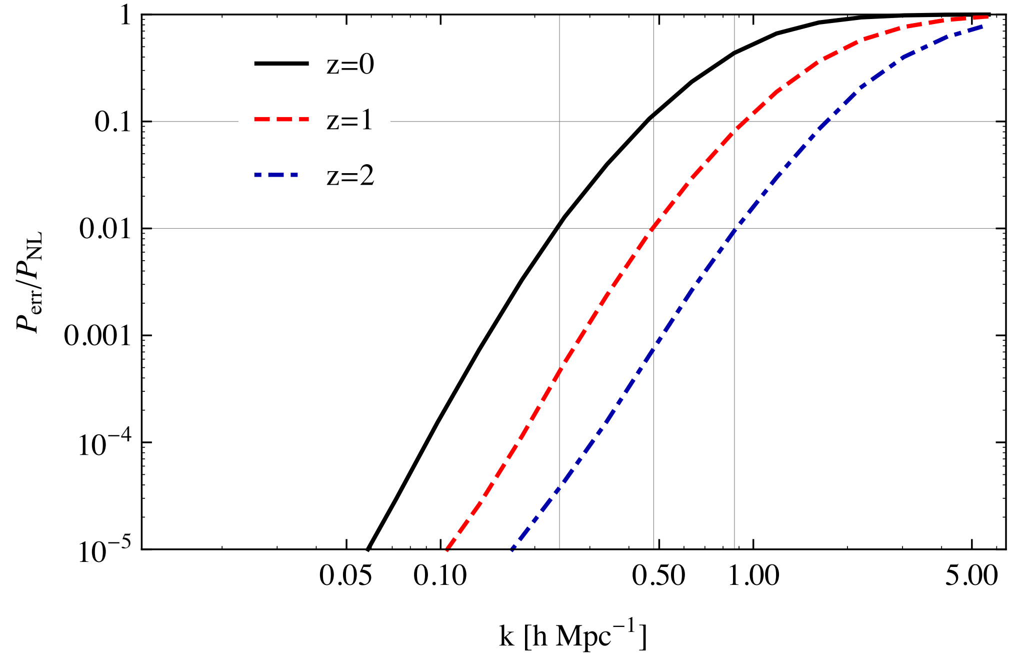






Summary





Many of the questions in cosmology require looking for very small effect.
Improving constraints using LSS is both experimentally and theoretically challenging.
We have made interesting progress in our understanding of the mildly non-linear regime but additional work is required.

The End