Black holes don't explode!

Stable Black Holes in Higher-Curvature Gravity

Pablo Bueno & Pablo A. Cano

Instituut voor Theoretische Fysica, KU Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium

Instituto de Física Teórica UAM/CSIC, C/ Nicolás Cabrera, 13-15, C.U. Cantoblanco, 28049 Madrid, Spain

pablo@itf.fys.kuleuven.be

pablo.cano@uam.es

Abstract

We show that four-dimensional black holes become stable below certain mass when the Einstein-Hilbert action is supplemented with higher-curvature terms. We prove this to be the case for an infinite family of ghost-free theories involving terms of arbitrarily high order in curvature. The new black holes, which are non-hairy generalizations of Schwarzschild's solution, present a universal thermodynamic behavior for general values of the higher-order couplings. In particular, the temperature of black holes is bounded from above and they have infinite lifetimes. When the semiclassical approximation breaks down, the resulting object still has a large entropy, in stark contrast with the Schwarzschild case. Based on [1].

Introduction

As proven by Hawking [2], a black hole with surface gravity κ emits thermal radiation with a temperature $T_{\rm H} = \kappa/(2\pi)$. In the prototypical case of a Schwarzschild black hole of initial mass M_0, the temperature increases as the black hole radiates, as a consequence of its negative specific heat. After a finite time of order $\sim M_0^3/M_{\rm P}^4$, where $M_{\rm P}$ is the Planck mass, the black hole evaporates down to an order-$M_{\rm P}$ object of order-one entropy. This suggests a violent ending for the evaporation process, and gives rise to the information paradox. However, in the final stages the curvature becomes very high, and the presence of higher-order curvature corrections in the gravitational field equations could drastically change the picture.

Higher-order corrections

We consider the Einstein-Hilbert action extended with higher-curvature terms:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\left[R + \sum_{n=3}^{\infty}\frac{\lambda_n}{M_c^{2(n-1)}}\mathcal{R}_{(n)}\right], \tag{1}$$

where $G = 1/M_{\rm P}^2$ is the Newton constant, M_c is some new energy scale, λ_n are dimensionless couplings and n is the order in curvature of each invariant $\mathcal{R}_{(n)}$. These invariants are defined by the property that the theory above allows for solutions of the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{(2)}^2, \tag{2}$$

i.e., with $g_{tt}g_{rr} = -1$. Apart from simplifying the problem of finding solutions, this property implies that the spectrum of the theories is Einstein-like on flat/de Sitter/anti de Sitter backgrounds [3]. The first densities read

$$\begin{aligned}
\mathcal{R}_{(3)} &= -\frac{1}{6}(12 R_{a\ c}^{\ b\ d}R_{b\ d}^{\ e\ f}R_{e\ f}^{\ a\ c} + R_{ab}^{cd}R_{cd}^{ef}R_{ef}^{ab} - 12R_{abcd}R^{ac}R^{bd} + 8R_a^bR_b^cR_c^a),\\
\mathcal{R}_{(4)} &= +\frac{2}{3}(10R^{abcd}R_{a\ c}^{\ e\ f}R_{e\ b}^{\ g\ h}R_{fgdh} + 4R^{ab}R^{cdef}R_{c\ ea}^{\ h}R_{dhfb} - 14R^{abcd}R_{ab}^{\ \ ef}R_{c\ e}^{\ g\ h}R_{dgfh}\\
&\quad - 5R^{abcd}R_{a\ c}^{\ e\ f}R_{e\ f}^{\ g\ h}R_{bgdh}),\\
\mathcal{R}_{(5)} &= \ldots
\end{aligned}$$

We start at $n = 3$ because there is no quadratic theory with the properties we are searching.

Black holes

The theories (1) allow for solutions of the form (2) where $f(r)$ satisfies the following second-order differential equation

$$(1-f)r - 2GM = \sum_{n=3}^{\infty}\frac{\lambda_n}{M_c^{2(n-1)}}\left(\frac{f'}{r}\right)^{n-3}\left[\frac{f'^3}{n} + \frac{(n-3)f+2}{(n-1)r}f'^2 - \frac{2}{r^2}f(f-1)f' - \frac{1}{r}ff''\left(f'r - 2(f-1)\right)\right]. \tag{3}$$

This equation has a unique solution representing an asymptotically flat black hole. We plot $f(r)$ in figure 1 for several values of the scale M_c.

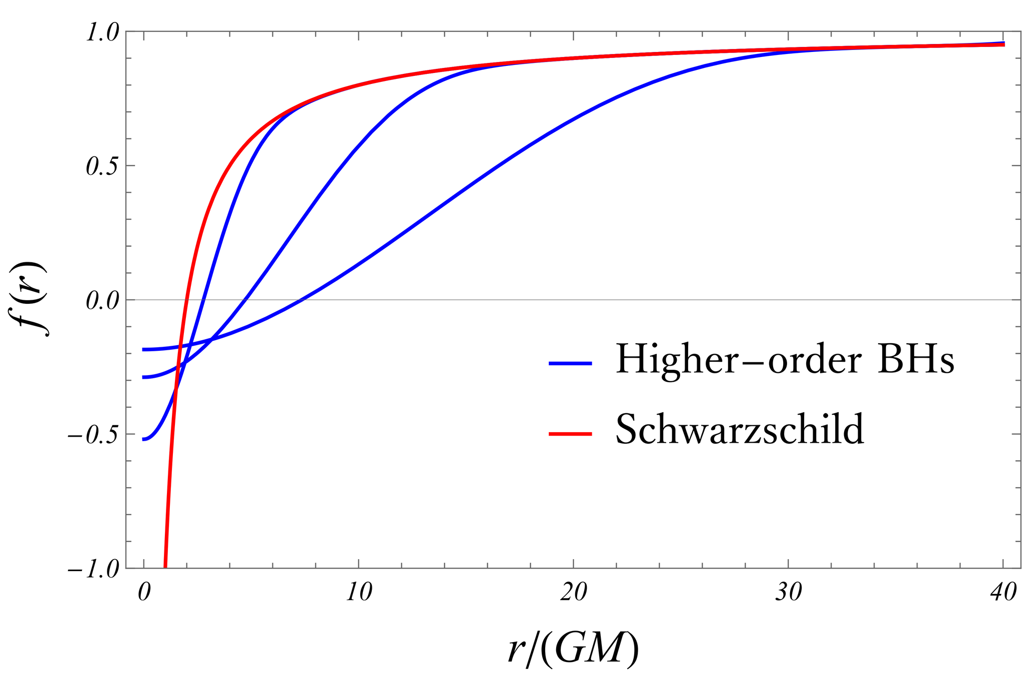

Figure 1: Metric function $f(r)$ for Schwarzschild's solution (red) and for the new higher-order black holes (blue), with $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 1$, $\lambda_{n>6} = 0$ and $GMM_c = 0.5\,, 0.2\,, 0.1$ respectively, (from left to right).

Thermodynamics

The thermodynamics of these black holes can be studied analytically. Near the horizon $r = r_h$ we perform the expansion $f(r) = 4\pi T(r - r_h) + \mathcal{O}((r - r_h)^2)$, where T is the black hole temperature. Solving (3) for the first two orders in $(r - r_h)$ gives rise to the following relations:

$$2GM = r_h - \sum_{n=3}^{\infty}\frac{\lambda_n(4\pi T)^{n-1}}{M_c^{2n-2}r_h^{n-2}}\frac{(2n + (n-1)4\pi T r_h)}{n(n-1)}, \tag{4}$$

$$1 = 4\pi T r_h + \sum_{n=3}^{\infty}\frac{\lambda_n(4\pi T)^{n-1}}{M_c^{2n-2}r_h^{n-1}}\frac{(2n + (n-3)4\pi T r_h)}{n(n-1)}. \tag{5}$$

These equations fix r_h and T in terms of the black hole mass M. Making use of Wald's formula [4], it is possible to compute the entropy of the black holes, which yields

$$S = \frac{\pi r_h^2}{G}\left[1 - 2\sum_{n=3}^{\infty}\frac{\lambda_n(4\pi T)^{n-1}}{M_c^{2n-2}r_h^{n-1}}\left(\frac{2}{(n-2)4\pi T r_h} + \frac{1}{n-1}\right)\right] + \frac{4\pi}{GM_c^2}\sum_{n=3}^{\infty}\frac{\lambda_n\chi^{n-2}}{(n-2)}, \tag{6}$$

where χ is defined as $\sum_{n=3}^{\infty}\frac{2\lambda_n\chi^{n-1}}{(n-1)} \equiv 1$. These relations exactly satisfy the 1st law of thermodynamics

$$dM = TdS. \tag{7}$$

In figure 2 we show the temperature $T(M)$ and the specific heat $C(T) = \frac{dM}{dT}$. The temperature becomes maximum for $M \sim M_{\rm P}^2/M_c$ and below this mass the specific heat becomes positive $C > 0$, so these small black holes are stable. The behavior is very similar for any choice of the couplings $\lambda_n \neq 0$.

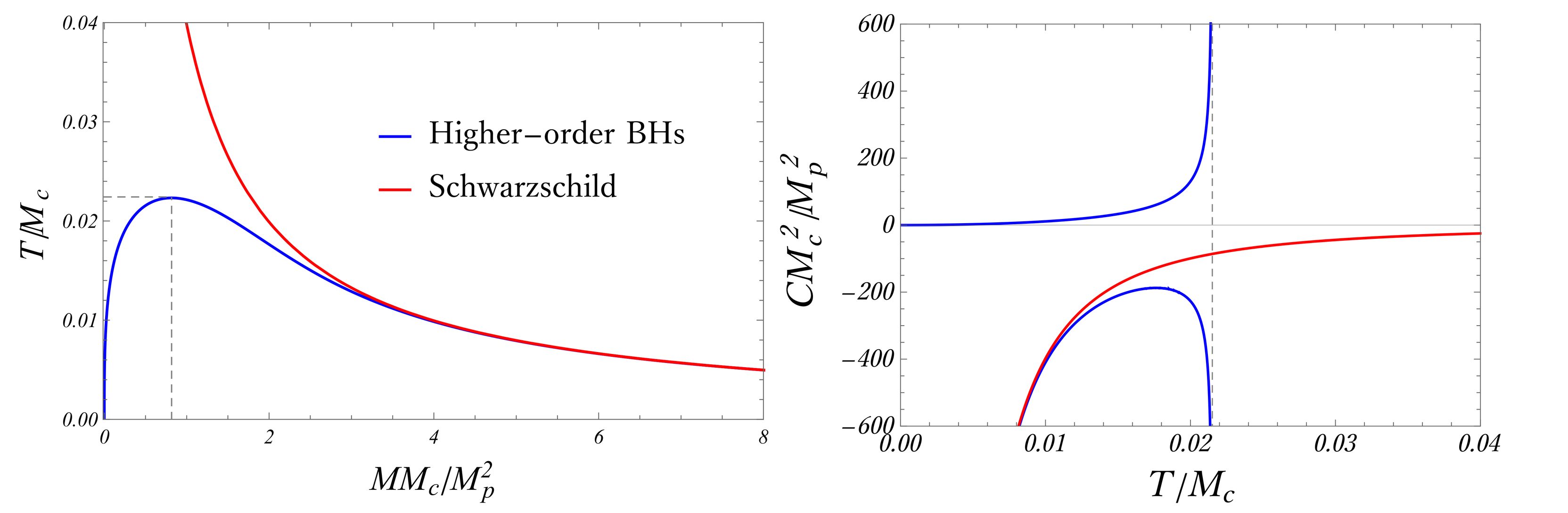

Figure 2: Left: Black hole temperature as a function of the mass for Schwarzschild's solution (red) and for the higher-order black holes with $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 1$, $\lambda_{n>6} = 0$. Right: Specific heat. The lower branch, with $C < 0$, corresponds to BHs with large mass while the upper one represents small BHs, which have $C > 0$. The transition happens for $M \sim M_{\rm P}^2/M_c$. Below this mass, higher-order black holes become stable. The shape of these curves is qualitatively the same for any other choice of couplings (except $\lambda_n = 0$ for all n).

For small masses, $M \ll M_{\rm P}^2/M_c$, the expressions for $r_h(M)$, $S(M)$ and $T(M)$ are approximately given by

$$r_h = \left[\frac{M}{\zeta\chi^3M_c^2M_{\rm P}^2}\right]^{1/3}, \quad S = 6\pi\left[\frac{\zeta^{1/2}MM_{\rm P}}{M_c^2}\right]^{2/3}, \quad T = \frac{1}{4\pi}\left[\frac{MM_c^4}{\zeta M_{\rm P}^2}\right]^{1/3}, \tag{8}$$

where $\zeta \equiv \sum_{n=3}^{\infty}\frac{\lambda_n\chi^{n-3}}{n}$. In this regime, the solutions satisfy the Smarr relation

$$M = \frac{2}{3}TS. \tag{9}$$

This relation, which describes the thermodynamics of these stable black holes, holds for all theories as long as any $\lambda_n \neq 0$, so the only exception is Einstein gravity.

Black hole evaporation

Let us explore the evaporation process of these black holes in the small mass regime. The rate of mass-loss of a black hole in the vacuum can be computed using the Stefan-Boltzmann law, $\frac{dM(t)}{dt} = -4\pi r_h^2\sigma\cdot T^4$ where $\sigma = \pi^2/60$. Using (8), we can easily integrate this expression for $M \ll M_{\rm P}^2/M_c$. The result is

$$M(t) = \frac{M_0}{1 + t/t_{1/2}}, \quad \text{where} \quad t_{1/2} = \frac{3840\pi\chi^2\zeta^2M_{\rm P}^4}{M_0M_c^4}. \tag{10}$$

Hence, the mass never vanishes and the black holes have an infinite lifetime!

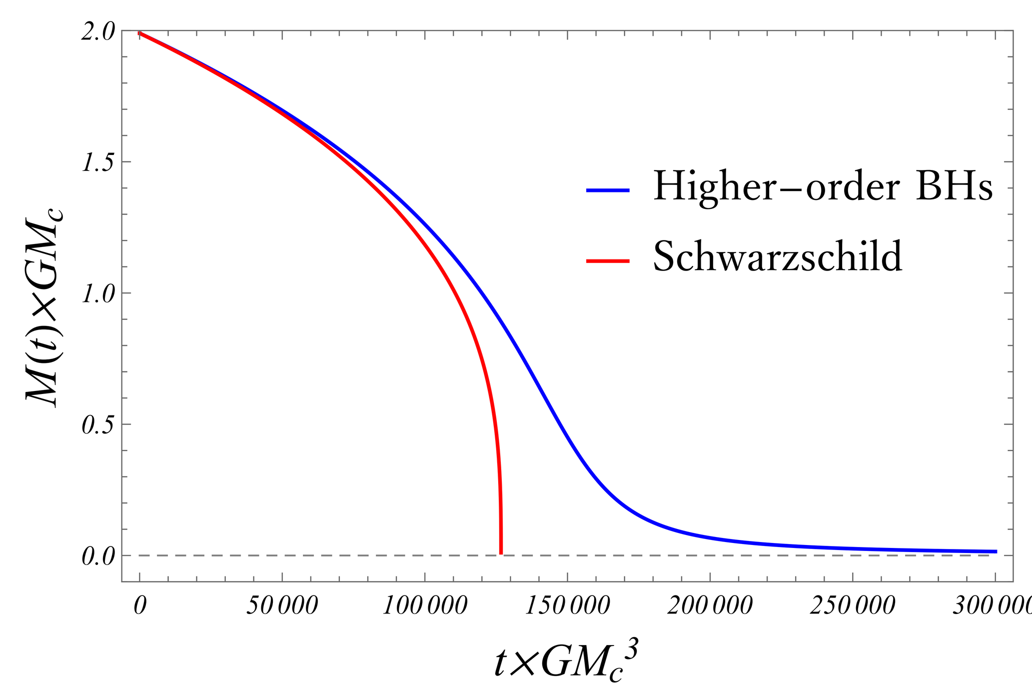

Figure 3: Time evolution of the mass due to Hawking evaporation. Schwarzschild black holes "explode" after a finite time, while higher-order black holes never evaporate completely.

After a time $\Delta t \sim M_{\rm P}^{7/2}/M_c^{9/2}$ the mass becomes $M \sim \sqrt{M_{\rm P}M_c}$ and the semiclassical description breaks down. The entropy at that moment is $S \sim M_{\rm P}/M_c$. If we choose $M_c << M_{\rm P}$, the time to reach the semiclassical breakdown becomes huge (even for a microscopic black hole) and the final entropy is large, $S >> 1$, in contrast with the Schwarzschild's case. This shows that the last stages of the evaporation process are seriously affected by higher-order corrections.

Conclusions

- We have constructed a new family of higher-order gravities in four dimensions which allow to study generalizations of Schwarzschild solution
- The new black holes have a universal thermodynamic behavior and they become stable below certain mass. For small masses the thermodynamics is universally characterized by the relation $M = 2/3TS$.
- These black holes have infinite lifetimes. They do not explode and the temperature vanishes when the mass goes to zero. This could have consequences for the information problem.

References

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