Oscillating Chiral Primordial Tensor Spectrum from Axionic Inflation

Ippei Obata (Kyoto Univ.)

Collaborator: **Jiro Soda** (Kobe Univ.)

@Pascos Workshop (Madrid)

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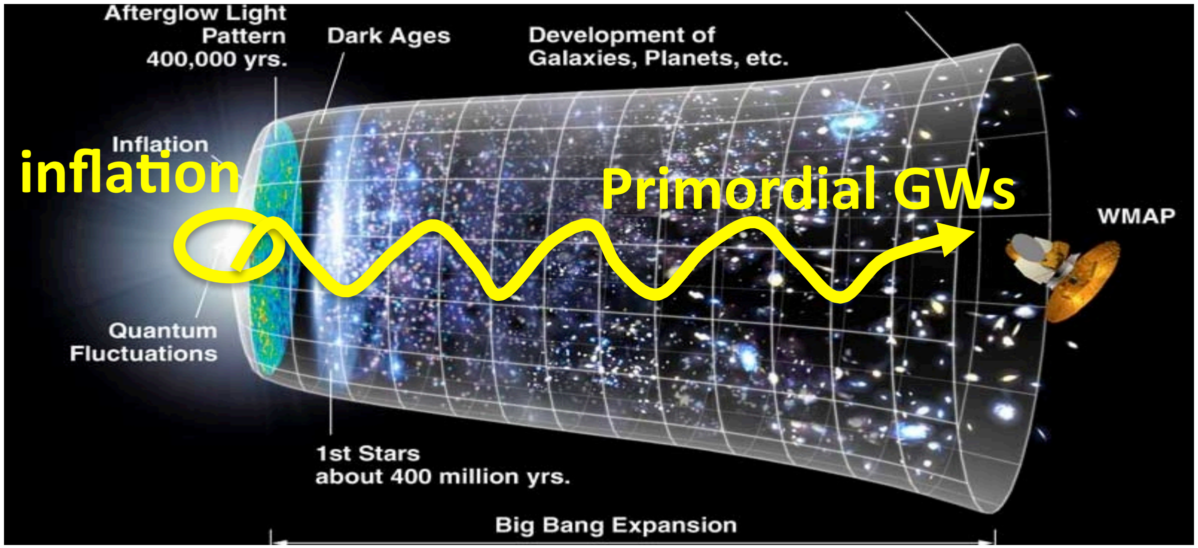
Probing early Universe by GWs!



✧ **energy scale of inflation**

$$E_{\rm inf} \simeq \underline{10^{16}}{\rm GeV}\left(\frac{r}{0.01}\right)^{1/4} \gg \text{Electroweak scale } (\sim 100\text{GeV})$$

Research interest

✓ **Seeking a novel feature of primordial GWs predicted by high energy physics.**

string

Large PGWs from Axionic inflation?

(string) axion is…

- (pseudo) NG boson ← posessing a **shift symmetry**!

$$\varphi \to \varphi + \mathrm{const.}$$ (protects against UV corrections)

- Shift symmetry is broken by non-perturbative effects: $\mu^4 e^{-S_E} e^{i\varphi/f}$

$$V(\varphi) \supset \Lambda^4 \cos(\varphi/f)$$ → **Natural inflation!**

Freese et al 1990, …

- Axion couples to gauge fields through Chern-Simons terms:

$$\frac{\varphi}{f} F\tilde{F}$$ → **Parity-violated phenomenology!** (CMB、GWs、Magnetic fields、…) .

Axionic inflation with SU(2) gauge field

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$$

Chromo-natural inflation

P.Adshead & M.Wyman 2012

$$S = S_{\rm EH} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS}$$
$$= \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\varphi}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

$$V(\varphi) = \Lambda^4\left[1 - \cos\left(\frac{\varphi}{f}\right)\right]$$

$$\tilde{F}^{a\mu\nu} \equiv \frac{1}{2}\sqrt{-g}\epsilon^{\mu\nu\lambda\sigma}F^a_{\lambda\sigma}$$

Background configurations

space-time (flat FLRW) : $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$

inflaton + SU(2) gauge field : $\varphi = \varphi(t)$

$$A^a_i = a(t)Q(t)\delta^a_i$$

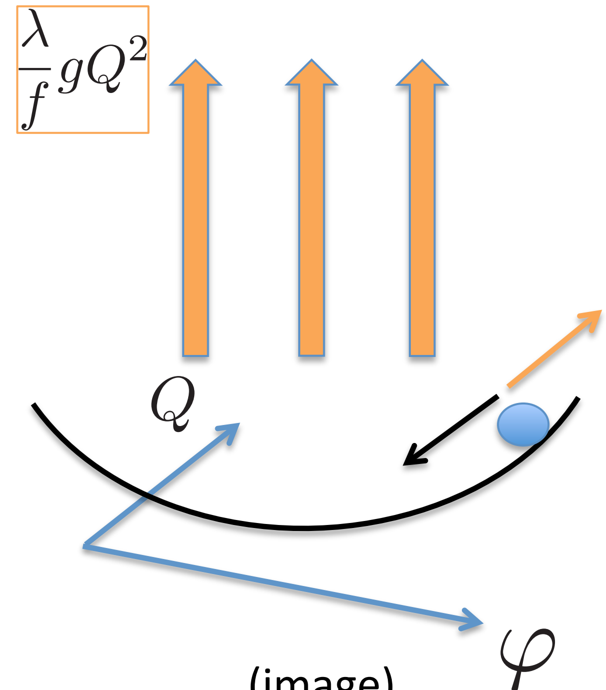
Solving background equations for not only ϕ(t) but Q(t) !

Background dynamics

Equations of motions for ϕ(t) and Q(t)

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{\varphi} = -3\frac{\lambda}{f}gQ^2(\dot{Q} + HQ)$$

$$\ddot{Q} + 3H\dot{Q} + (2H^2 + \dot{H})Q + 2g^2Q^3 = \frac{\lambda}{f}gQ^2\dot{\varphi}$$



(image)

Assuming $\lambda \gg 1$

``Magnetic drift force" dominates over Hubble friction!

(gauge field assists slow-roll motion of inflaton !)

Attractor solutions

$$Q(t) \simeq Q_{\rm min} \equiv -\left(\frac{V_{\tilde{\varphi}}}{3\lambda g H}\right)^{1/3} \qquad \frac{\lambda\dot{\varphi}}{2fH} \simeq m_Q + \frac{1}{m_Q}$$

$$m_Q \equiv \frac{gQ}{H} \qquad \tilde{\varphi} \equiv \frac{\varphi}{f}$$

Analysis of tensor modes

$$S = S_{\rm EH} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS}$$
$$= \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\varphi}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

Considering perturbations…

$$\varphi = \bar{\varphi}(t) + \delta\varphi$$
$$A^a_i = a(t)Q(t)\delta^a_i + \delta A^a_i$$

$$\delta A^a_i \supset t^a_i \leftrightarrow \delta g_{\mu\nu}$$

Couples to metric tensor mode at linear level!

Parity-violating interaction

Fourier mode

$$\lambda\frac{\varphi}{f}{\rm Tr}F\tilde{F} \supset \pm\lambda k\frac{\bar{\varphi}'}{f}\delta A^{\pm}_k\delta A^{\pm}_k$$

$$|\delta A^+_k| \neq |\delta A^-_k|$$

Analysis of tensor modes

Quantization of “tensor type” of fluctuations of SU(2) gauge field

$$t_{ij}(\boldsymbol{x},\tau) = \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} e^A_{ij}(\hat{\boldsymbol{k}}) t^A_{\boldsymbol{k}}(\tau) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$= \sum_{A=\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \left[e^A_{ij}(\hat{\boldsymbol{k}}) t^A_k(\tau) b^A_{\boldsymbol{k}} + e^{A*}_{ij}(-\hat{\boldsymbol{k}}) t^{A*}_k(\tau) b^{A\dagger}_{-\boldsymbol{k}} \right] e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$[b^A_{\boldsymbol{k}}, b^{B\dagger}_{-\boldsymbol{k}'}] = (2\pi)^3 \delta_{AB} \delta^3(\boldsymbol{k}+\boldsymbol{k}')$$

EOM for free gauge particle $(x \equiv -k\tau$: dimensionless time variable$)$

$$\frac{d^2 t^\pm_k}{dx^2} + \left(1 + \frac{A}{x^2} \mp \frac{2B}{x}\right) t^\pm_k \simeq 0$$

$$A = 2(m_Q^2 + 1) > 0$$
$$B = 2m_Q + m_Q^{-1} > 0$$

$$t^+_k \; : \quad m^2 < 0 \quad \text{for} \quad \frac{1}{2}(B - \sqrt{B^2 - A}) < x < \frac{1}{2}(B + \sqrt{B^2 - A})$$

One helicity mode is enhanced due to a tachyonic instability!

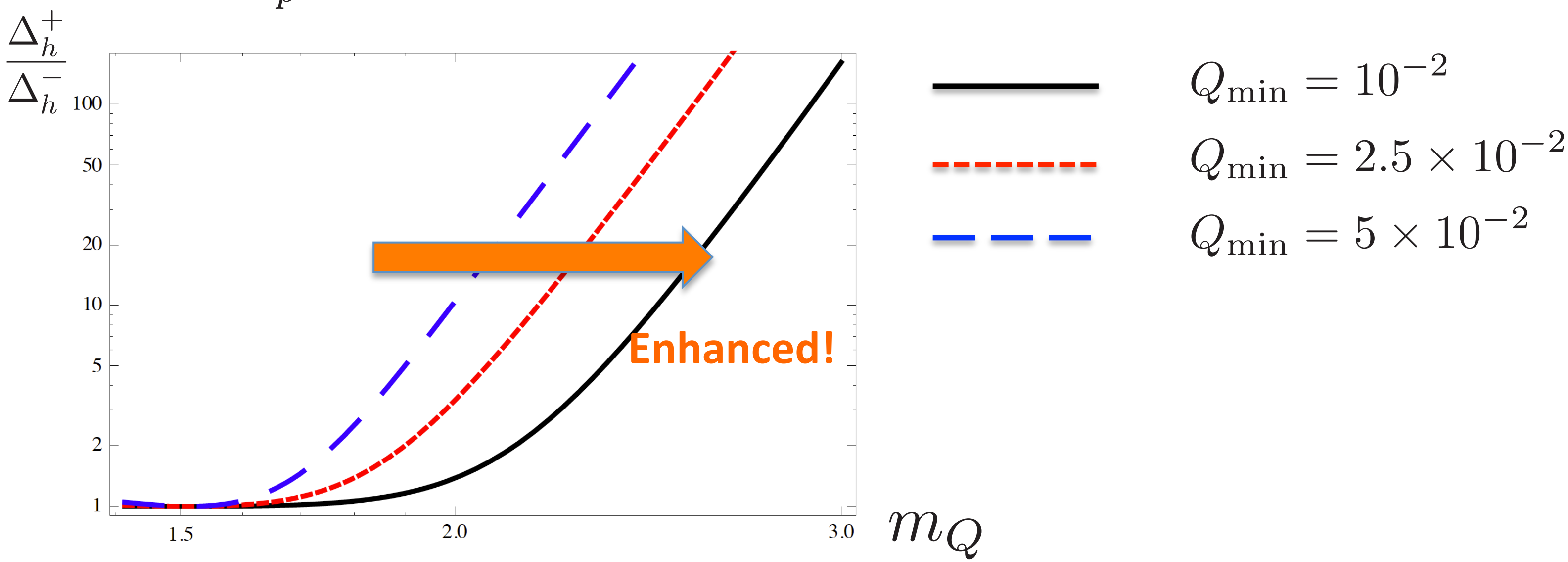
⟶ producing parity-violated gravitational tensor modes!

Generation of chiral GWs

Power spectrum of GWs in this model

$$\Delta_h^- \simeq \frac{H^2}{\pi^2 M_p^2},$$

$$\Delta_h^+ \simeq \frac{H^2}{\pi^2 M_p^2}\left[1 + |Q|^2 f(m_Q) e^{\pi B(m_Q)}\right]$$



P.Adshead, E.Martinec & M.Wyman 2013

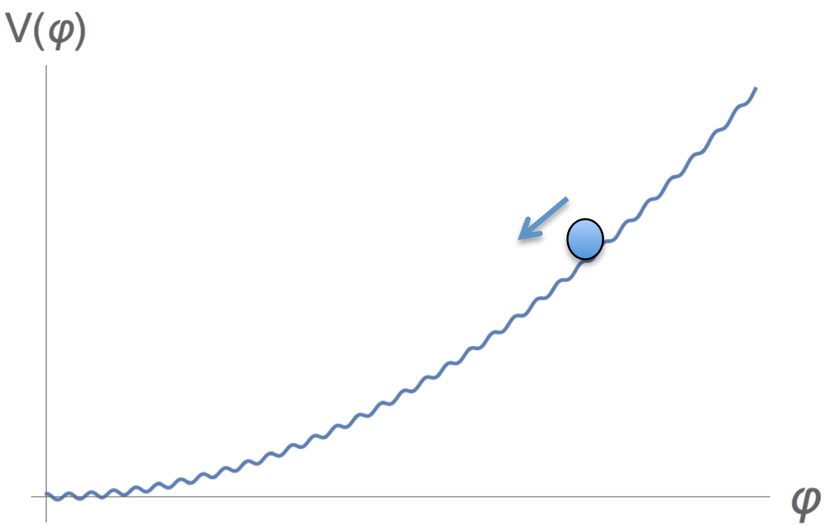
Today's work

Exploring Axionic inflation with modulation

L.McAliister, E.Silverstein & A.Westphal 2008,…
A. de la Fuente *et al* 2015, …

$$V(\varphi) = V_0(\varphi) + V_{\rm mod}(\varphi)$$

$$\Lambda^4_{\rm mod} \cos(\varphi/f)$$



✧ This modulation affects both scalar and tensor modes:

$$\mathcal{R} \to \mathcal{R}_{(0)} + \delta\mathcal{R}_{(\rm mod)}$$
$$h_{ij} \to h_{ij(0)} + \delta h_{ij(\rm mod)}$$

Scalar spectrum … has been explored by CMB observations (no evidence so far)

R.Easther & R.Flauger 2013

Tensor spectrum … will be explored by not only CMB observations but GW experiments!

Single field inflation with modulation

Potential : $V(\varphi) = \mu^{4-n}\varphi^n + \Lambda_{\rm mod}^4\left[1 - \cos\left(\frac{\varphi}{f_{\rm mod}} + \delta\right)\right]$

Amplitude of modulation : $b \equiv \Lambda_{\rm mod}^4 / V_{0\varphi} f_{\rm mod}$ $\left(V_\varphi = V_{0\varphi}\left[1 + b\sin\left(\frac{\varphi}{f_{\rm mod}} + \delta\right)\right]\right)$

Upper bound on b : $b \lesssim 1$ (inflaton does not get trapped)

Tensor spectrum with modulation

$$\langle h^\pm_{\boldsymbol{k}} h^\pm_{\boldsymbol{k}'}\rangle = (2\pi)^3\delta(\boldsymbol{k}+\boldsymbol{k}')\frac{2\pi^2}{k^3}\Delta_h^\pm(k), \qquad \Delta_h(k) = \Delta_h^+(k) + \Delta_h^-(k) = \left.\frac{2H^2}{\pi^2 M_p^2}\right|_{k=aH}$$

$$\Delta_h(k) \simeq \Delta_{h0}(k)\left[1 + \boxed{\sqrt{2\epsilon_{V0}}\frac{f}{M_p}} b\left(1 - \cos\left(\frac{\varphi_k}{f} + \delta\right)\right)\right]_{k=aH}$$

$\ll 1$ hard to detect...

Inflaton + gauge field with modulation

IO and J. Soda 2016

Ex: Chromo-natural inflation

$$S = S_{\rm EH} + S_{\rm axion} + S_{\rm gauge} + S_{\rm CS}$$
$$= \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi) - \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\lambda\frac{\varphi}{f}\tilde{F}^{a\mu\nu}F^a_{\mu\nu}\right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu \qquad A^a_i = a(t)Q(t)\delta^a_i \qquad \lambda \gg 1$$

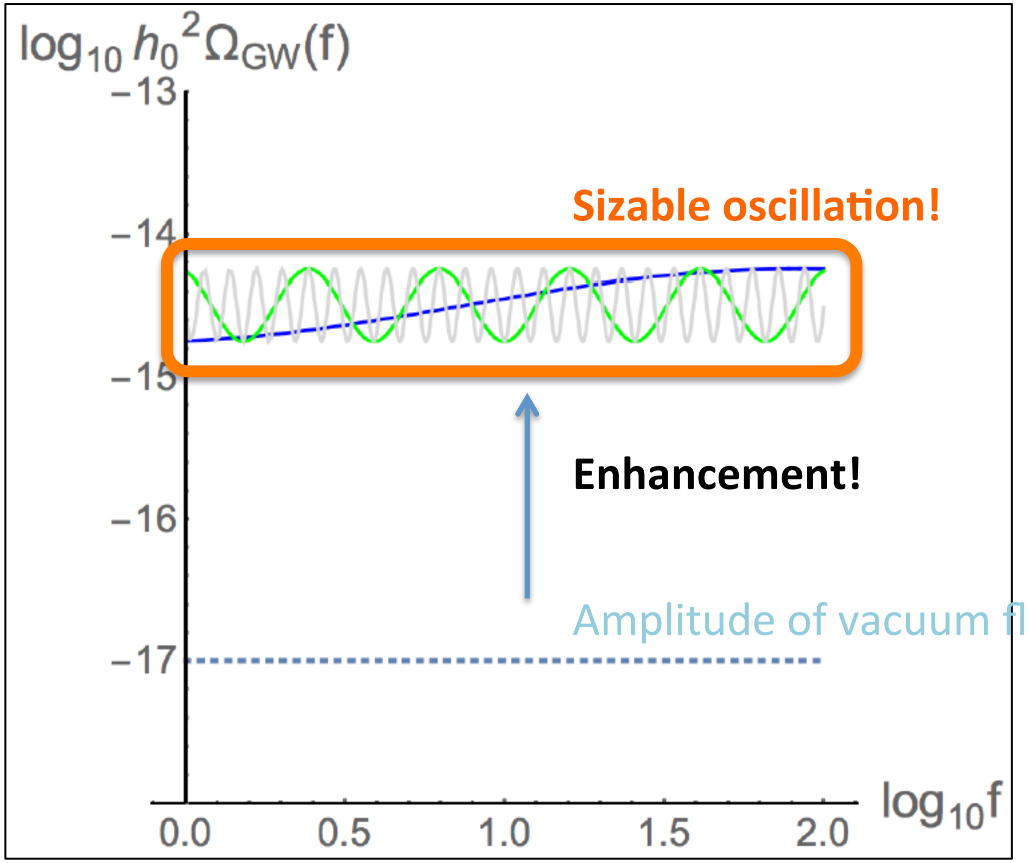
✧ Vev of gauge field has an attractor solution (supports slow-roll motion of inflaton!)

$$Q(t) \simeq -\left(\frac{fV_\varphi}{3\lambda gH}\right)^{1/3}, \quad \frac{\lambda\dot{\varphi}}{2fH} \simeq \frac{gQ}{H} + \frac{H}{gQ}$$

With modulation: $$Q \equiv Q_0\left(1 + b\sin\left(\frac{\varphi}{f_{\rm mod}} + \delta\right)\right)^{1/3}$$

Inflaton + gauge field with modulation

Chiral tensor spectrum with modulation $(H = 10^{13}\mathrm{GeV}\ ,\ b = 0.1)$



$$\Delta_h^+ \propto e^{\pi B(m_Q)}$$

$$m_Q \simeq m_{Q_0}\left[1 + \frac{b}{3}\sin\left(\frac{\varphi_k}{f_{\rm mod}}\right)\right]$$

$$\varphi_k \simeq \varphi_* + \frac{2f}{\lambda}\left(m_{Q_0} + \frac{1}{m_{Q_0}}\right)\ln\left(\frac{k}{k_*}\right)$$

IO and J.Soda 2016

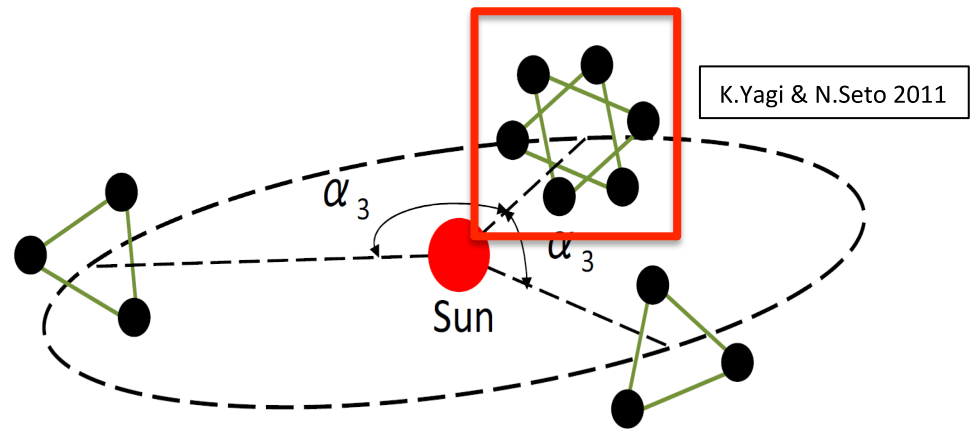
$(H, Q_0, m_{Q_0}, b) = (10^{-5}M_p, 10^{-2}M_p, 3, 0.1)$

Detectable size of modulation by BBO and DECIGO (star-like detector)

Deta stream of I-th detector:

$$V_I(f) = s_I(f) + n_I(f) \quad (I = 1, 2, 1', 2')$$

$$s_I(f) = \sum_{A=\pm} \int d\Omega h^A(f, \Omega) F_I^A(\Omega)$$



Correlation of data streams:

$$\mu_{IJ}(f) \equiv V_I(f)^* V_J(f)\, \delta f \ (I \neq J) \quad \langle \mu_{IJ}(f) \rangle = \langle s_I(f)^* s_J(f) \rangle \delta f = T_{obs} \frac{3H_0^2}{20\pi^2} f^{-3} \gamma_{IJ} \Omega_{GW}(f)\, \delta f$$

$$\sigma_{IJ}^2(f) \simeq T_{obs} S_I(f) S_J(f)\, \delta f / 4 \quad \langle n_I(f)^* n_I(f') \rangle = \frac{1}{2} S_I(f) \delta(f - f')$$

$$\gamma_{IJ} = \frac{5}{2} \sum_{A=\pm} \int \frac{d\Omega}{4\pi} F_I^A(\Omega) F_J^A(\Omega)$$

Signal-to-noise ratio:

$$(\Delta x = 0)$$

$$(SNR)^2 = \sum_{I \neq J} \sum_f \frac{\langle \mu_{IJ} \rangle^2}{\sigma_{IJ}^2} \simeq \left(\frac{3H_0^2}{10\pi^2} \right)^2 T_{\rm obs} \left[\sum_{I \neq J} \int_{f_{min}}^{f_{max}} df \frac{\gamma_{IJ} \Omega_{GW}^2}{f^6 S_I S_J} \right]$$

Detectable size of modulation by BBO and DECIGO (star-like detector)

Estimation error (evaluated by Fisher information quantity)

$$(\Delta b)^{-2} = \Gamma = \left(\frac{3H_0^2}{10\pi^2}\right)^2 T_{\rm obs} \left[\sum_{I \neq J} \int_{f_{min}}^{f_{max}} df \frac{(\partial_b \Omega_{GW}|_{b=0})^2}{f^6 S_I S_J}\right]$$

$Q_0 = f_{\rm mod} = 10^{-2} M_p$

Detectable condition

$$b \gtrsim \Delta b$$

(Single field inflation case)

$$b \gtrsim 55 \left(\frac{T_{obs}}{10\mathrm{yr}}\right)^{-1/2} \left(\frac{H}{10^{-5} M_p}\right)^{-2} \quad \text{(DECIGO)},$$

$$b \gtrsim 21 \left(\frac{T_{obs}}{10\mathrm{yr}}\right)^{-1/2} \left(\frac{H}{10^{-5} M_p}\right)^{-2} \quad \text{(BBO)}$$

(inflaton + gauge field case)

$$b \gtrsim 10^{-3.5} \left(\frac{T_{obs}}{10\mathrm{yr}}\right)^{-1/2} \left(\frac{H}{10^{-5} M_p}\right)^{-2} \quad \text{(DECIGO)},$$

$$b \gtrsim 10^{-4} \left(\frac{T_{obs}}{10\mathrm{yr}}\right)^{-1/2} \left(\frac{H}{10^{-5} M_p}\right)^{-2} \quad \text{(BBO)}$$

Upper bound on b

$$b \lesssim 1$$

Summary and Outlook

- We explored an axionic inflation with modulation and examined if primordial tensor power spectrum exhibits an oscillatory feature.

- We found that in the case of the axionic inflation coupled to gauge fields the sizable oscillation can occur due to an enhancement of chiral gravitational waves sourced by gauge fields, which is testable by future gravitational wave experiments.

- We need to numerically explore that oscillating chiral GWs actually appear in a frequency range higher than nHz and CMB constraints can be also satisfied.

Thank you so much!