Cosmology Basics

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SLAC Summer Institute
August 2017

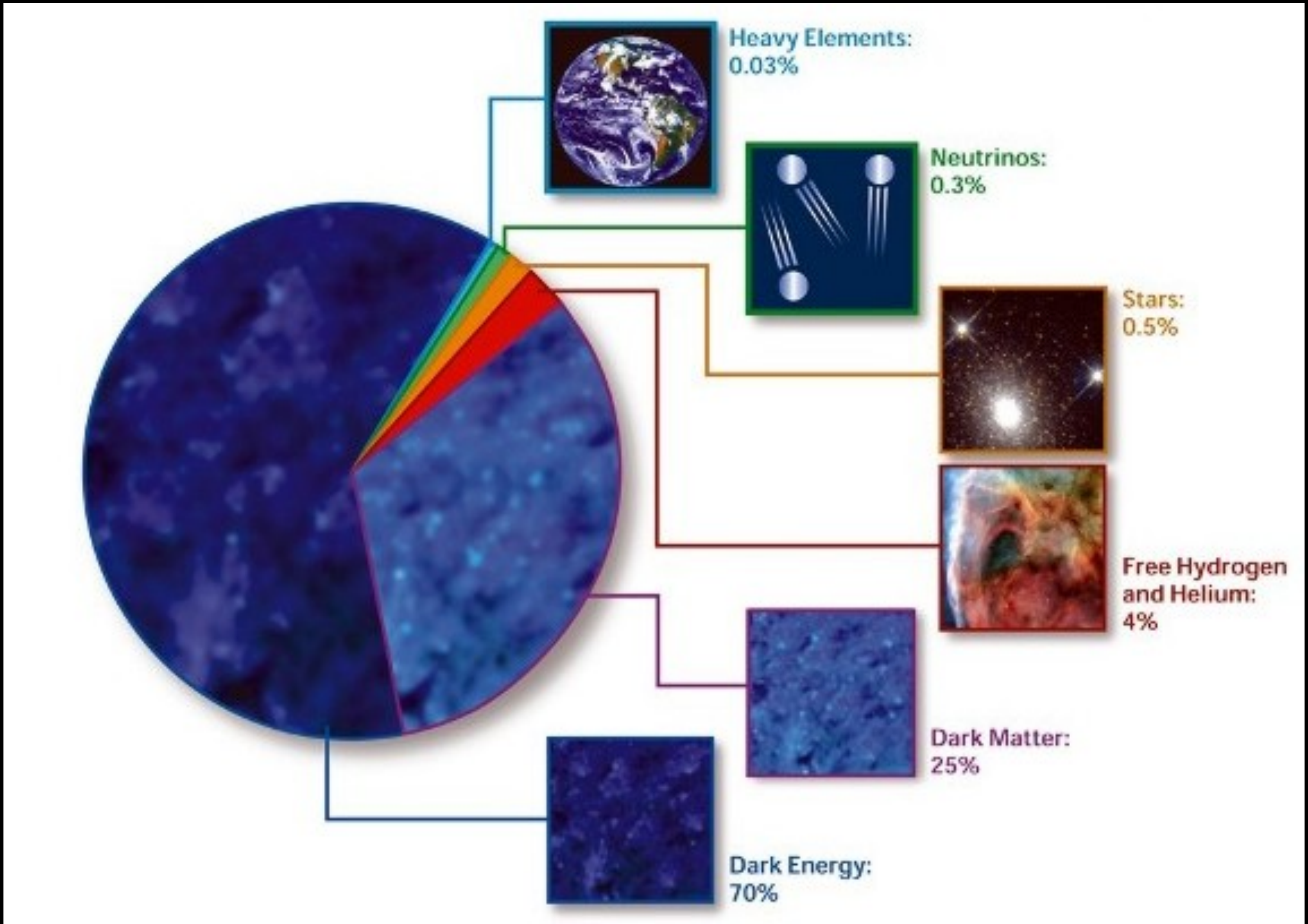
Outline

- The ΛCDM Cosmology: Overview
- Expansion Kinematics and H_0
- Expansion Dynamics
  - dark matter, dark energy
- The Hot Big Bang
  - BBN, CMB, relic dark matter particles
- Primordial Inflation
- Structure Formation in ΛCDM
- Probing Cosmology with Large-scale Structure

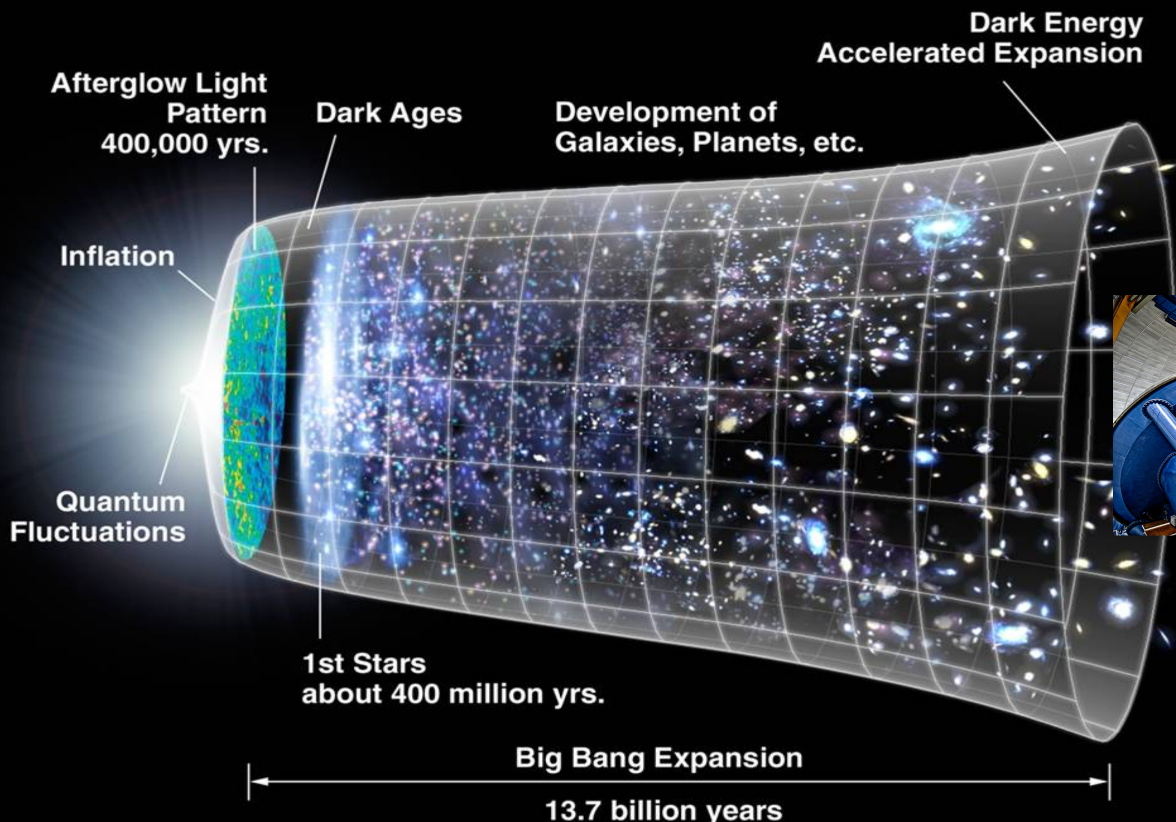
Cosmology 2017: ΛCDM

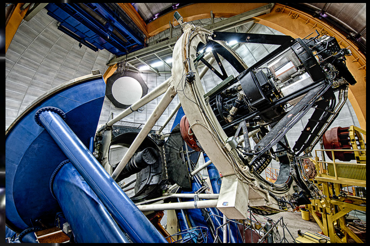
- A well-tested (6-parameter) cosmological model:
  - Universe is expanding from hot, dense early phase (Big Bang) 13.8 Gyr ago.
  - Early epoch of accelerated expansion (inflation) generated large-scale, nearly scale-invariant, nearly Gaussian density perturbations from quantum fluctuations and produced nearly flat & smooth observed spatial geometry
  - From these, structure formed from gravitational instability of cold dark matter (CDM, 25%) in currently Λ-dominated (70%) universe, which is again accelerating.
- Consistent with all data from the CMB, large-scale structure, galaxies, lensing, supernovae, clusters, light element abundances (BBN), expansion,…

Contents of the Universe



Brief History of the Universe





Dark Energy Survey

Evidence for <u>two</u> epochs of accelerated expansion
What are their physical origins?

NASA/WMAP Science Team

We have been very lucky so far

- Over the last 25 years, determination of a number of cosmological parameters has gone from ~100+% to ~1% precision.
- At each new stage of experimental precision, a simple (few-parameter) cosmological paradigm has been confirmed: it didn’t have to turn out that way.

Cosmological Physics

- Despite remarkable success of ΛCDM, we don't understand the *physics* of dark matter, dark energy, or inflation.
- What is the Dark Matter?
- Did inflation occur? Who is the Inflaton?
- What is the origin of Cosmic Acceleration today?
  - Dark Energy or Modified Gravity?
  - Nature of Dark Energy: Λ or dynamical component?
- Many of the SSI lecturers will focus on these questions.

Cosmic Surveys & Opportunities

- We don't understand the *physics* of dark matter, dark energy (late acceleration), or inflation (early acceleration).
- Is ΛCDM the correct model?
- Stress-test ΛCDM with improved precision & accuracy: new experiments and surveys, multiple probes that can be intercompared, novel tests.
- Need more, better, and different kinds of data to reduce statistical errors and control systematics.
- Route to potential new fundamental physics

The Big Bang Theory

- The Universe is expanding isotropically from a hot, dense beginning—the Big Bang---13.8 Gyr ago.
- This model provides a well-tested framework that explains key cosmological observations:
  - Thermal spectrum of Cosmic Microwave Background
  - Cosmic abundances of the light elements
    - Hydrogen, Helium, Deuterium, Lithium, formed in nuclear reactions in first 3 minutes: BBN
  - Formation and evolution of galaxies and large-scale structure from primordial perturbations

Logarithmic view of Cosmic History

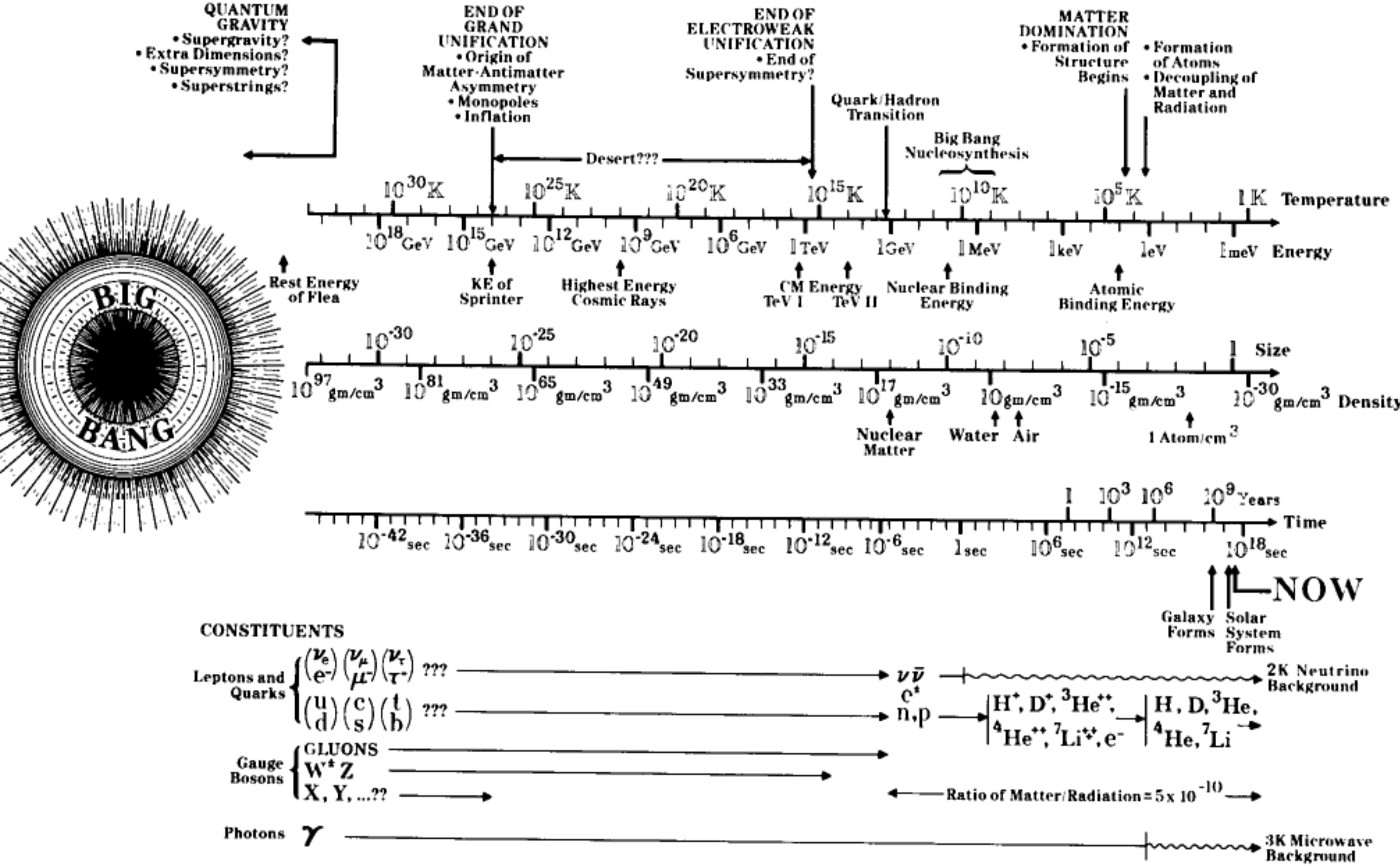
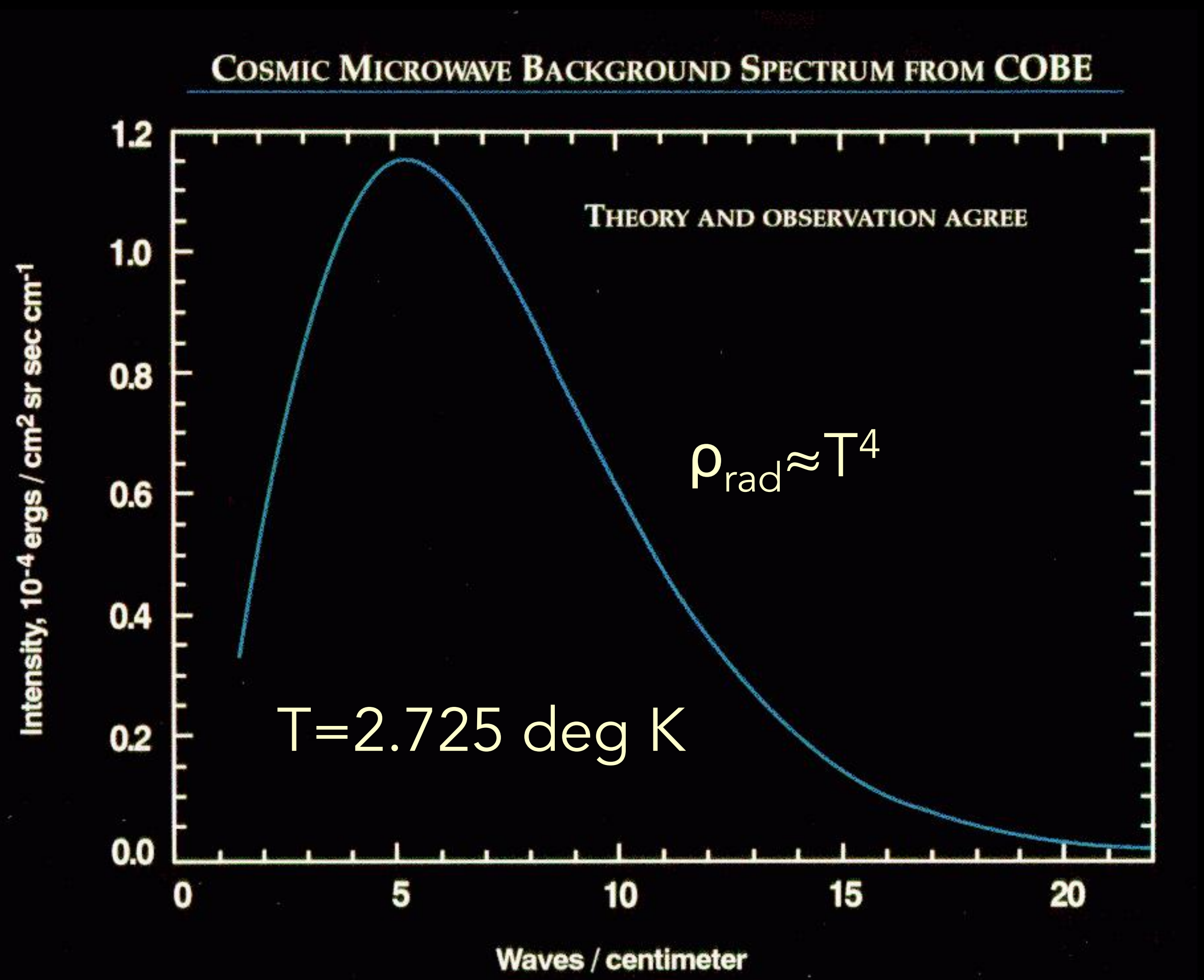


Fig. 1.5.

Cosmic Microwave Background

- Universe is filled with thermal electromagnetic radiation: the Cosmic Microwave Background (CMB), remnant from the hot early Universe.
- Precisely blackbody spectrum.

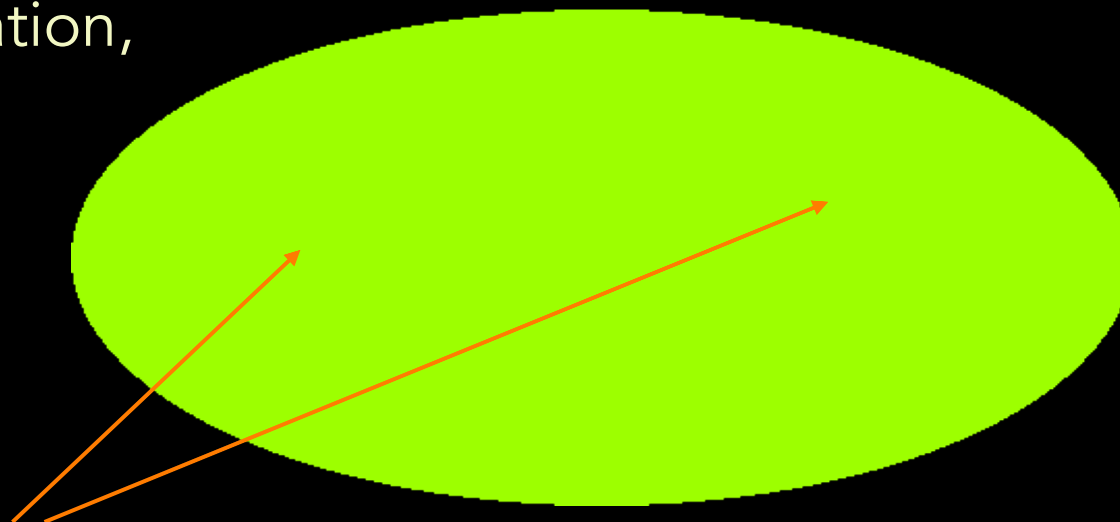


ν – frequency

Cosmic Microwave Background Radiation

The Universe is filled with a bath of thermal radiation, discovered by Penzias & Wilson (1965)

Map of the CMB temperature



On large scales, the CMB temperature is nearly isotropic around us (the same in all directions): snapshot of the young Universe, *t* ~ 380,000 years

T = 2.725 deg K above absolute zero

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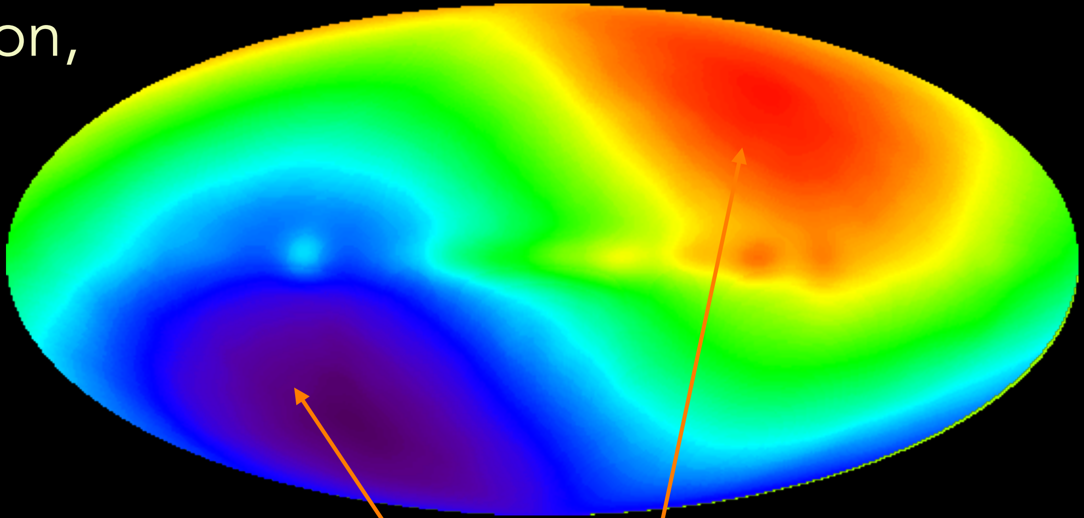

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Temperature fluctuations $\delta T/T \sim 10^{-3}$ due to dipole motion

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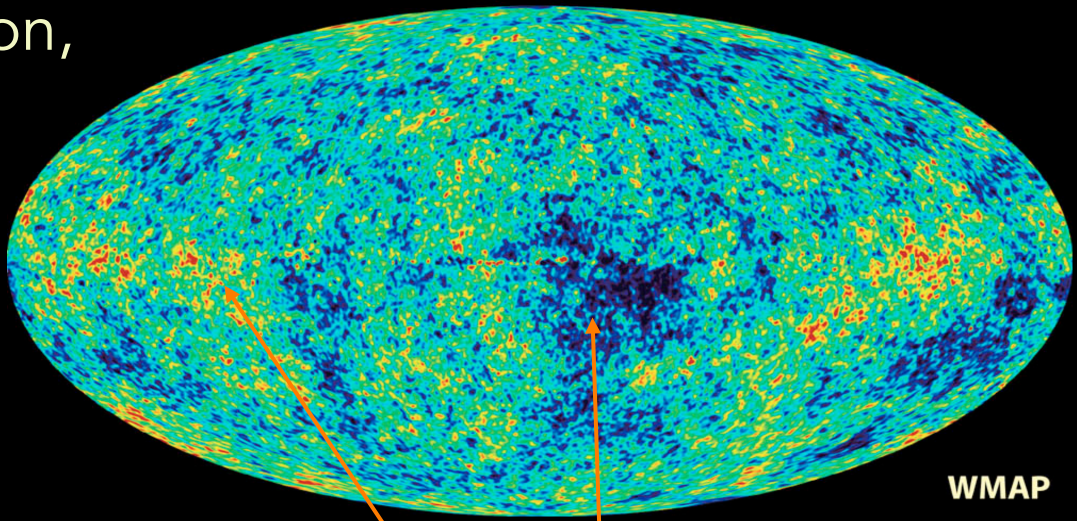
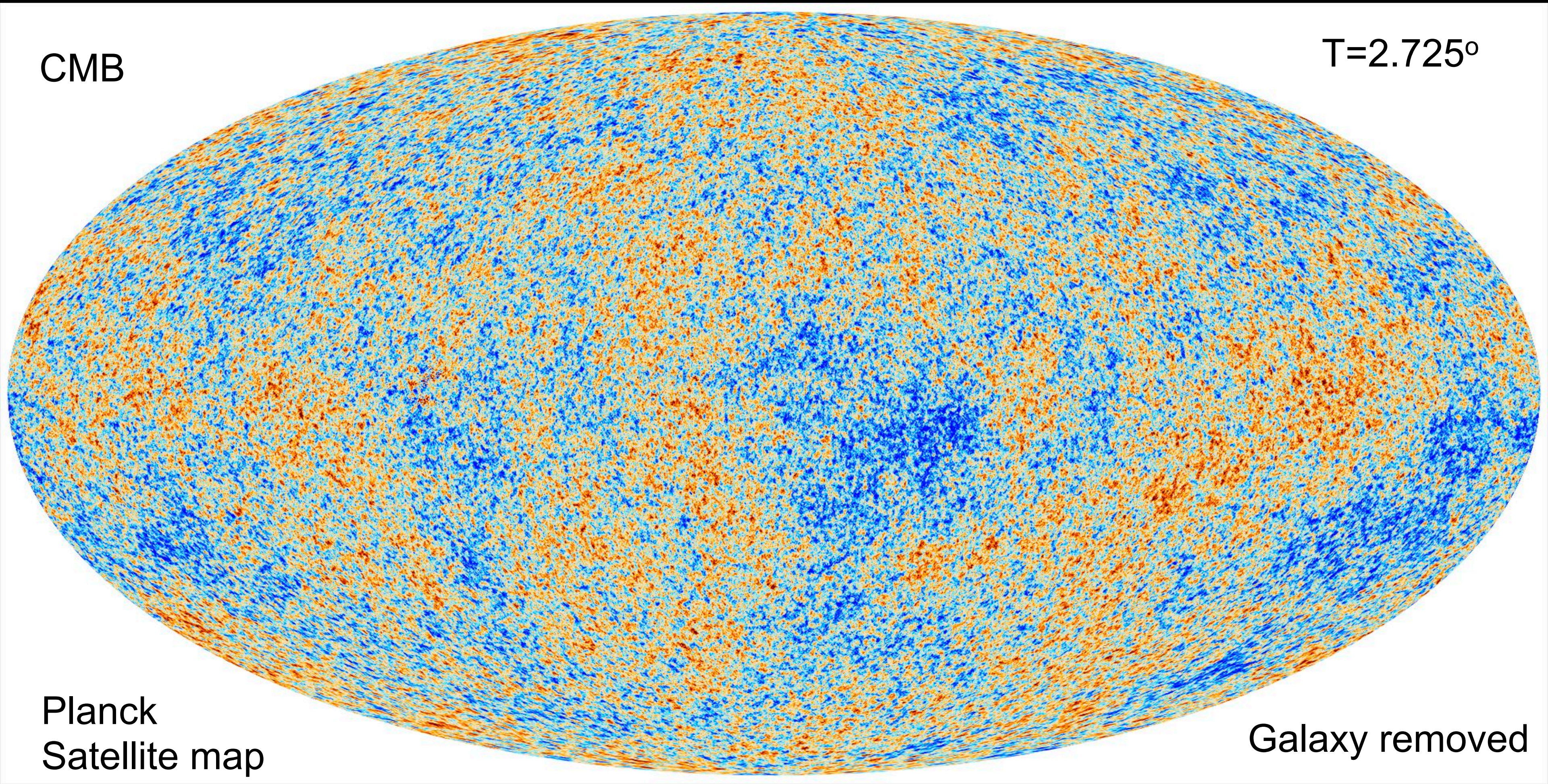


On large scales, the CMB temperature is nearly isotropic around us (the same in all directions): snapshot of the young Universe, t ~ 380,000 years

T = 2.725 deg K above absolute zero

Temperature fluctuations $\delta T/T \sim \delta\rho_{rad}/\rho_{rad} \sim 10^{-5}$ (dipole subtracted)

Cosmic Microwave Background Radiation

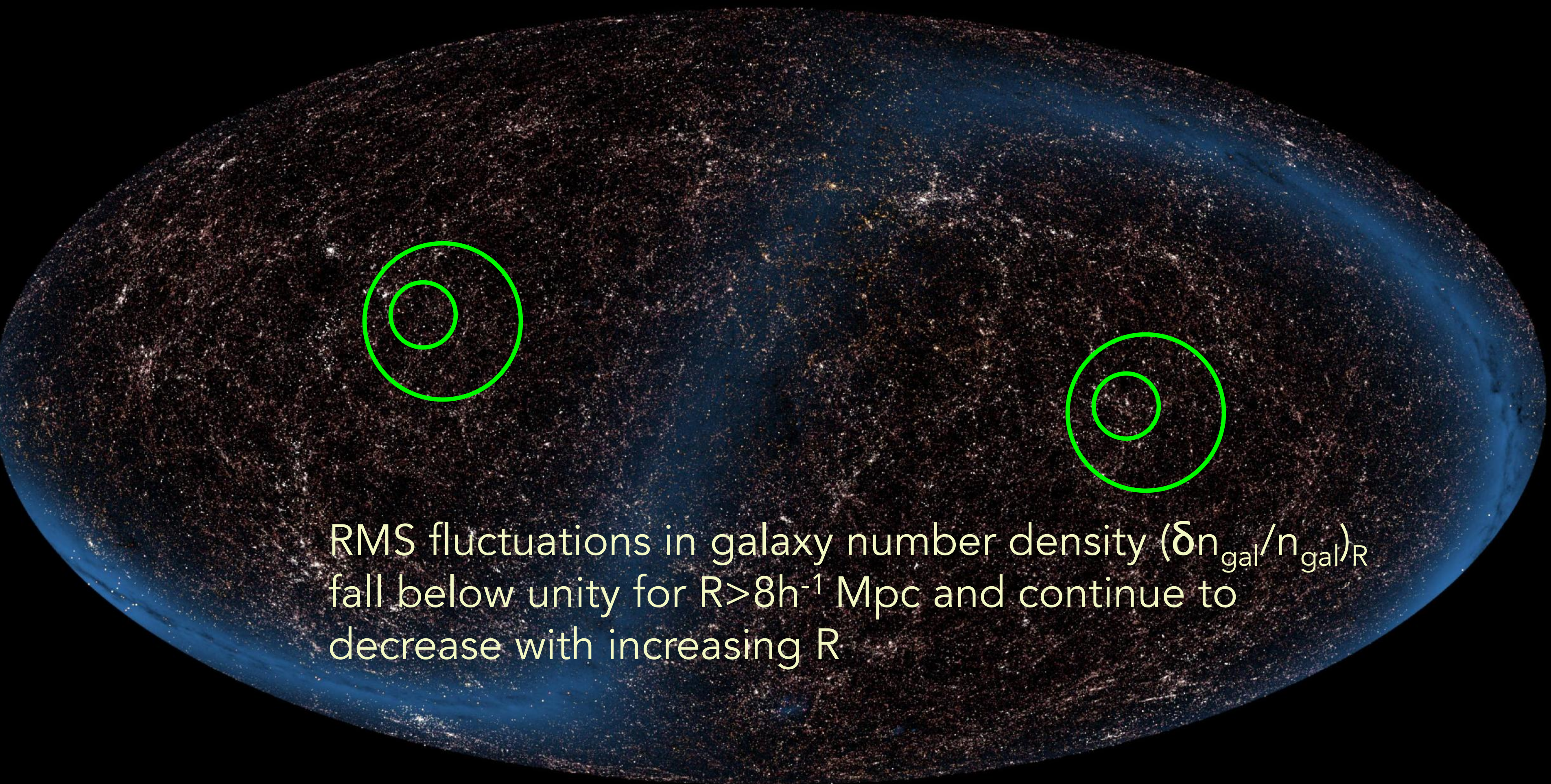


Snapshot of the Universe at 380,000 years (last scattering).
Temperature varies by only ~0.00001 deg across the sky.

The Cosmological Principle

- On large scales, the Universe appears (nearly) *isotropic* around us: looks on average the same in every direction on the sky.
- Assume we are not privileged observers: our Galaxy looks much like the others.
- Then the Universe should appear isotropic to *all* Fundamental Observers (those who define the local standard of rest and see no dipole).
- In that case, one can show the Universe must be *homogeneous:* have the same properties (density, etc) at every location, averaged over large scales.

Large-scale Map of Galaxies Today



RMS fluctuations in galaxy number density $(\delta n_{gal}/n_{gal})_R$ fall below unity for $R>8h^{-1}$ Mpc and continue to decrease with increasing R

2MASS Infrared Sky Survey: Universe much lumpier now, but it looks statistically homogeneous on large scales.

Homogeneity & Isotropy

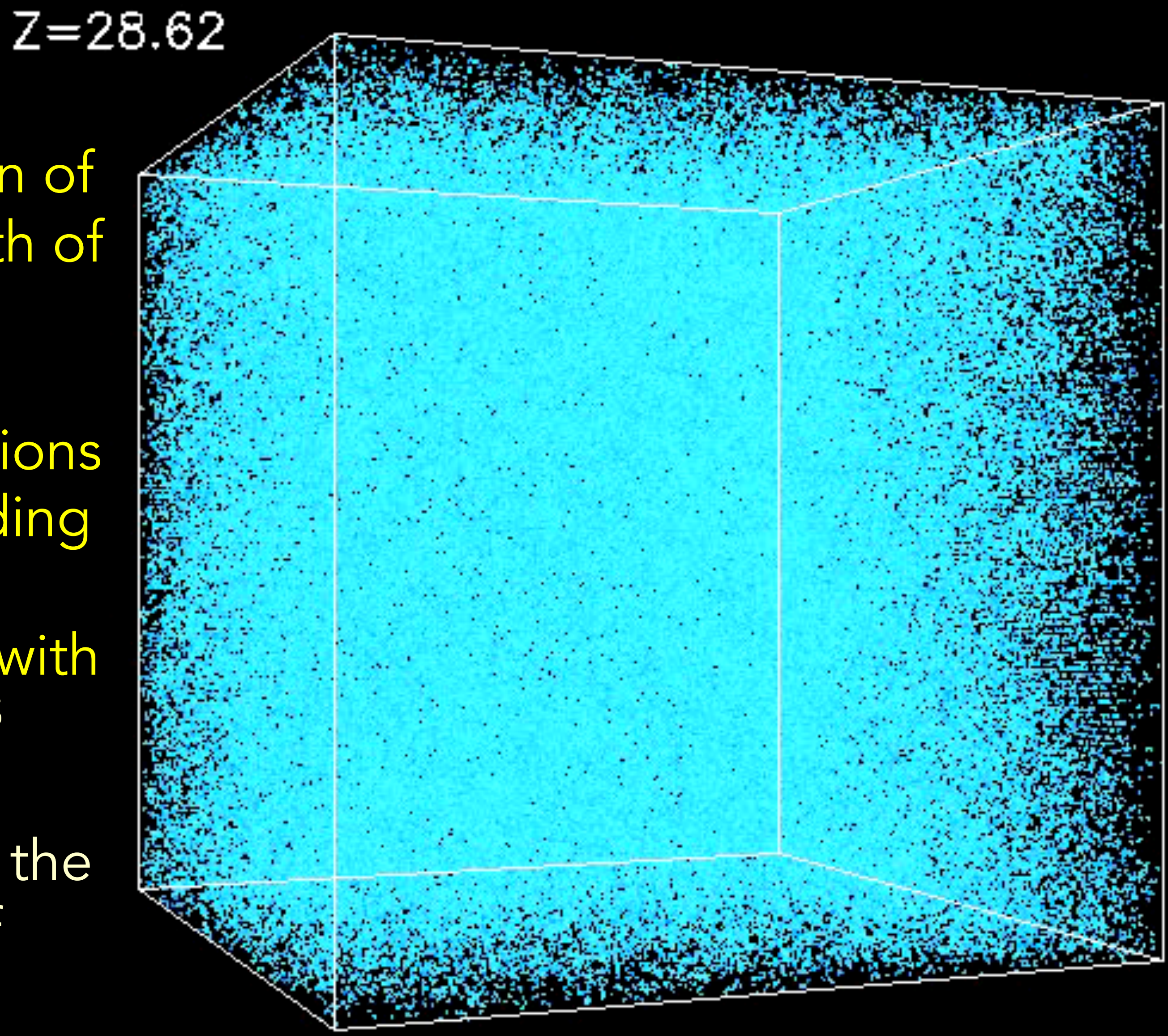
- CMB Temperature fluctuations $\delta T/T \sim 10^{-5}$ and galaxy density (~mass) fluctuations both consistent with a Universe with small fluctuation in gravitational potential (or curvature): $\delta\Phi_R \sim 10^{-5}$, which is approximately scale(R)-invariant (from inflation) and approx. constant in time (for matter-dominated universe):

$$\delta\Phi_R \sim \frac{G\delta M_R}{R} \sim \frac{GR^3\delta\rho_R}{R} \sim GR^2\bar{\rho}\left(\frac{\delta\rho}{\bar{\rho}}\right)_R \sim H^2R^2\left(\frac{\delta\rho}{\bar{\rho}}\right)_R \sim \left(\frac{R}{3000h^{-1}\mathrm{Mpc}}\right)^2\left(\frac{\delta\rho}{\bar{\rho}}\right)_R$$

$$\left(\frac{\delta\rho}{\bar{\rho}}\right)_R \sim 10^{-5}\left(\frac{R}{3000h^{-1}\mathrm{Mpc}}\right)^{-2} \sim 1 \text{ for } R \sim 8h^{-1}\mathrm{Mpc}$$

N-body Simulation of the growth of matter density perturbations in expanding ΛCDM Universe with $\delta\Phi_R \sim 10^{-5}$

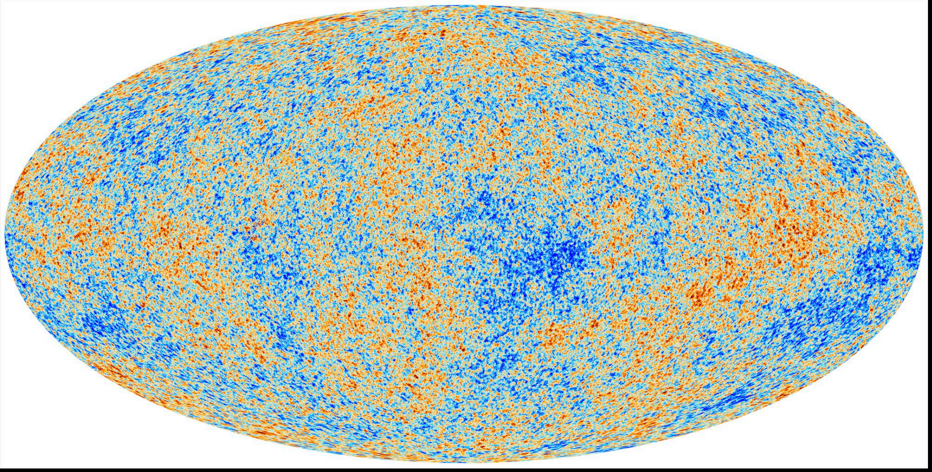
Gravity is the engine of structure formation

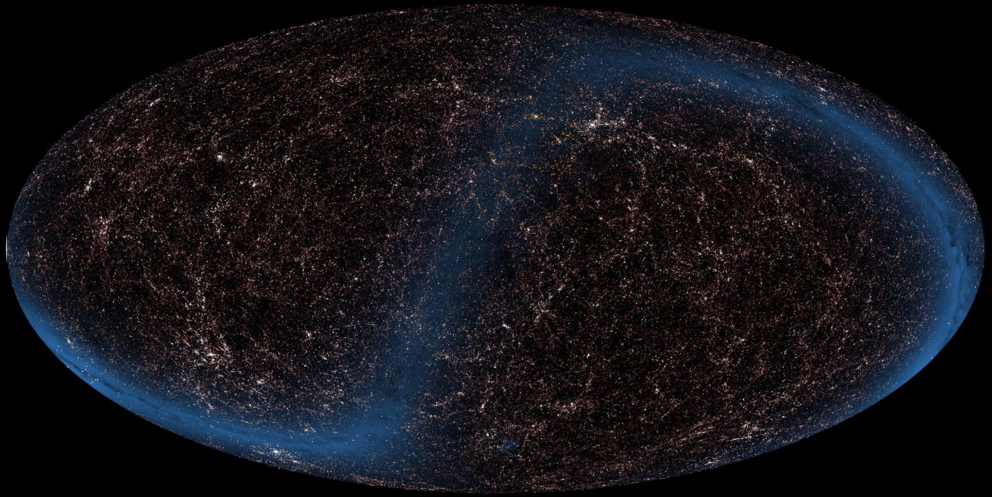


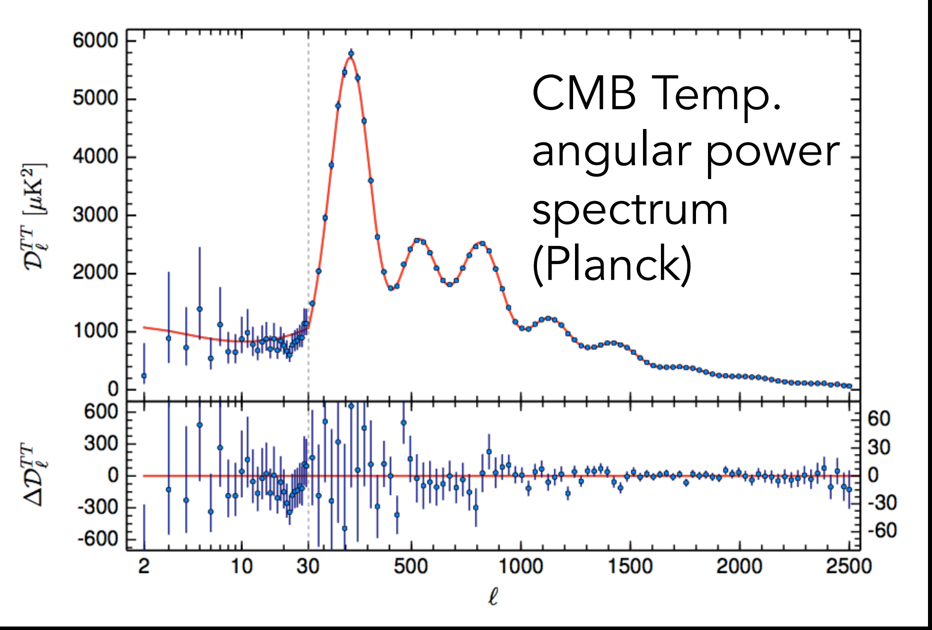
Galaxies form in and are biased tracers of collapsed halos of dark matter

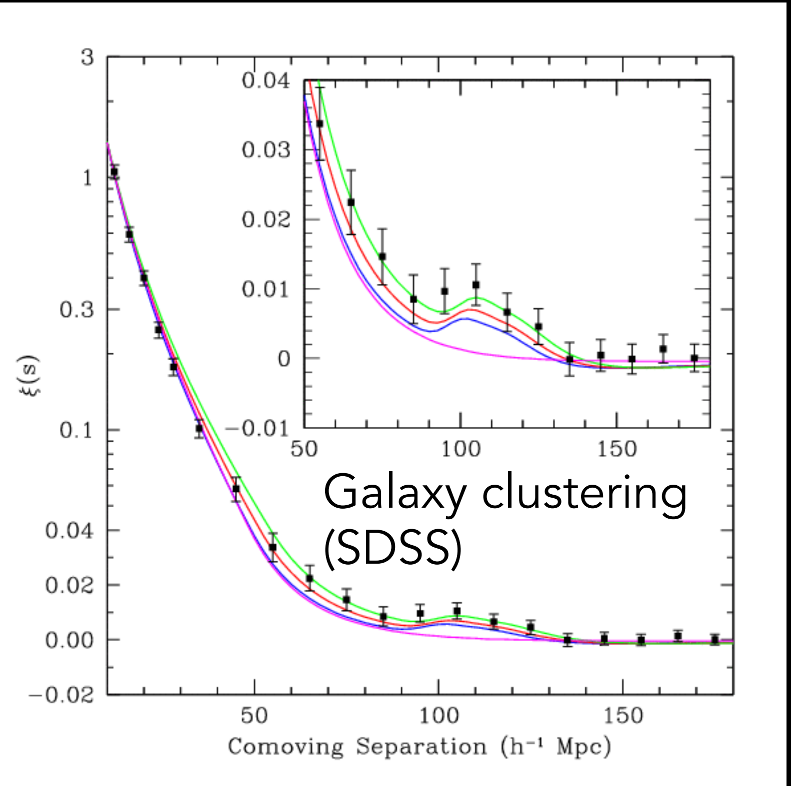
Kravtsov

Same ΛCDM Model fits Early & Late Structure



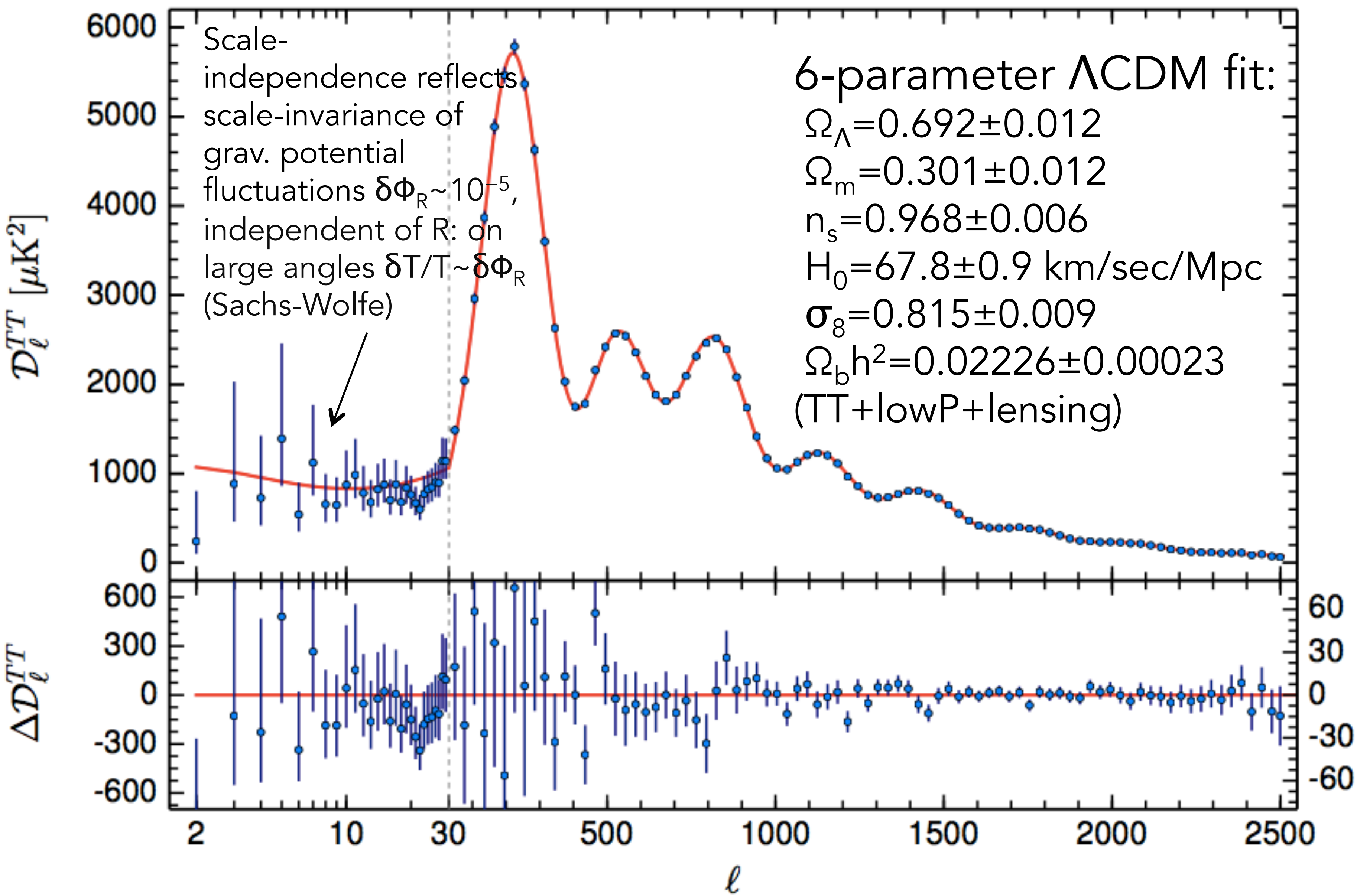






Compare CMB and clustering amplitudes

Planck 2015 Results



Homogeneity & Isotropy

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$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)dr_i dr^i$$

- Spacetime metric is thus close to that for the Friedmann-Lemaitre-Robertson-Walker model.

Friedmann–Lemaitre–Robertson–Walker (FLRW) model

Alexander Friedmann
Russian
1922-24 derivations
(died in 1925)

George Lemaitre
Belgian priest
1927 derivations

Howard Percy Robertson
American
+
Arthur Geoffrey Walker
English

1935 – proof that FLRW expression for spacetime interval is the only one for a universe that is both homogeneous and isotropic

Universe appears homogeneous & isotropic

Only mode that preserves those properties is expansion or contraction:

Cosmic scale factor $a(t)$

Model completely specified by *a(t)* and spatial curvature

On average, galaxies <u>at rest</u> in these expanding (comoving) coordinates

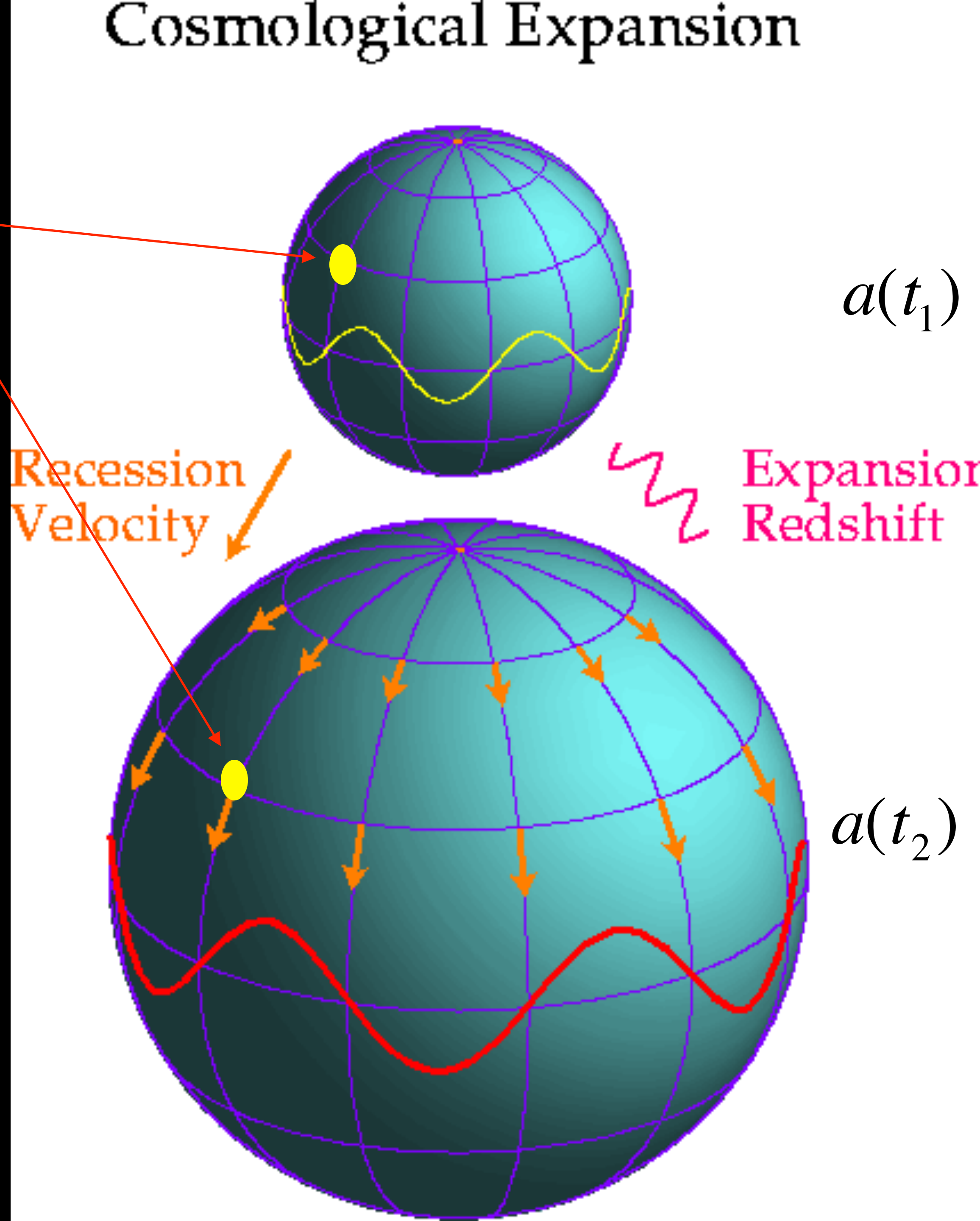
Wavelength of radiation scales with scale factor:

$$\lambda \sim a(t)$$

Redshift of light:

$$1 + z = \frac{\lambda(t_2)}{\lambda(t_1)} = \frac{a(t_2)}{a(t_1)}$$

emitted at t_1, observed at t_2 (for comoving observers); indicates relative size of Universe directly



Distance between galaxies:

$$d(t) = a(t)r$$

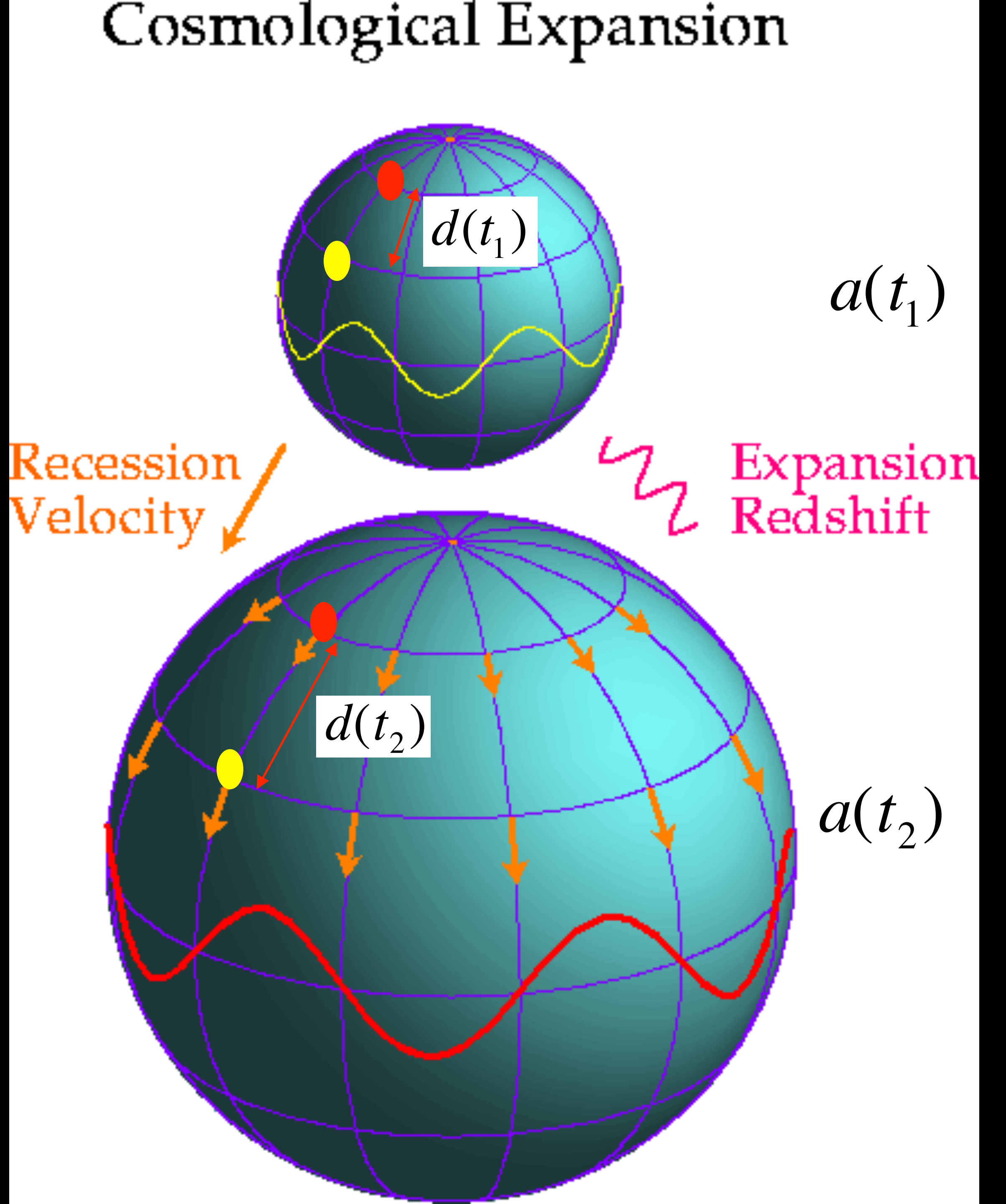
where

$r =$ fixed comoving distance

Recession speed:

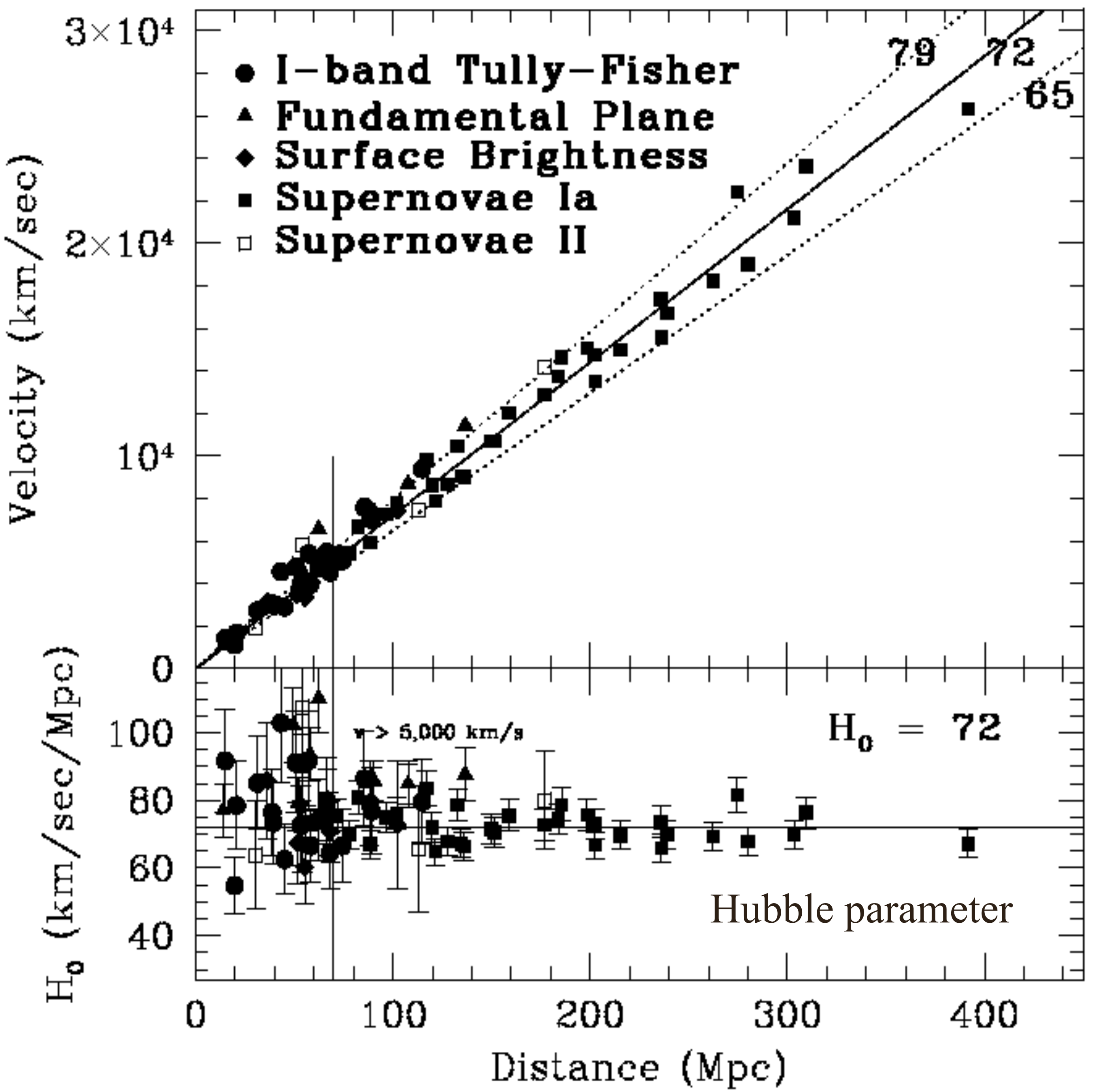
$$v = \frac{d(d(t))}{dt} = \frac{rd(a(t))}{dt} = \frac{d}{a}\frac{da}{dt} \equiv dH(t) \approx dH_0 \text{ for small } d << 1/H_0$$

Hubble Law (1929)



Modern Hubble Diagram

Hubble Space Telescope Key Project

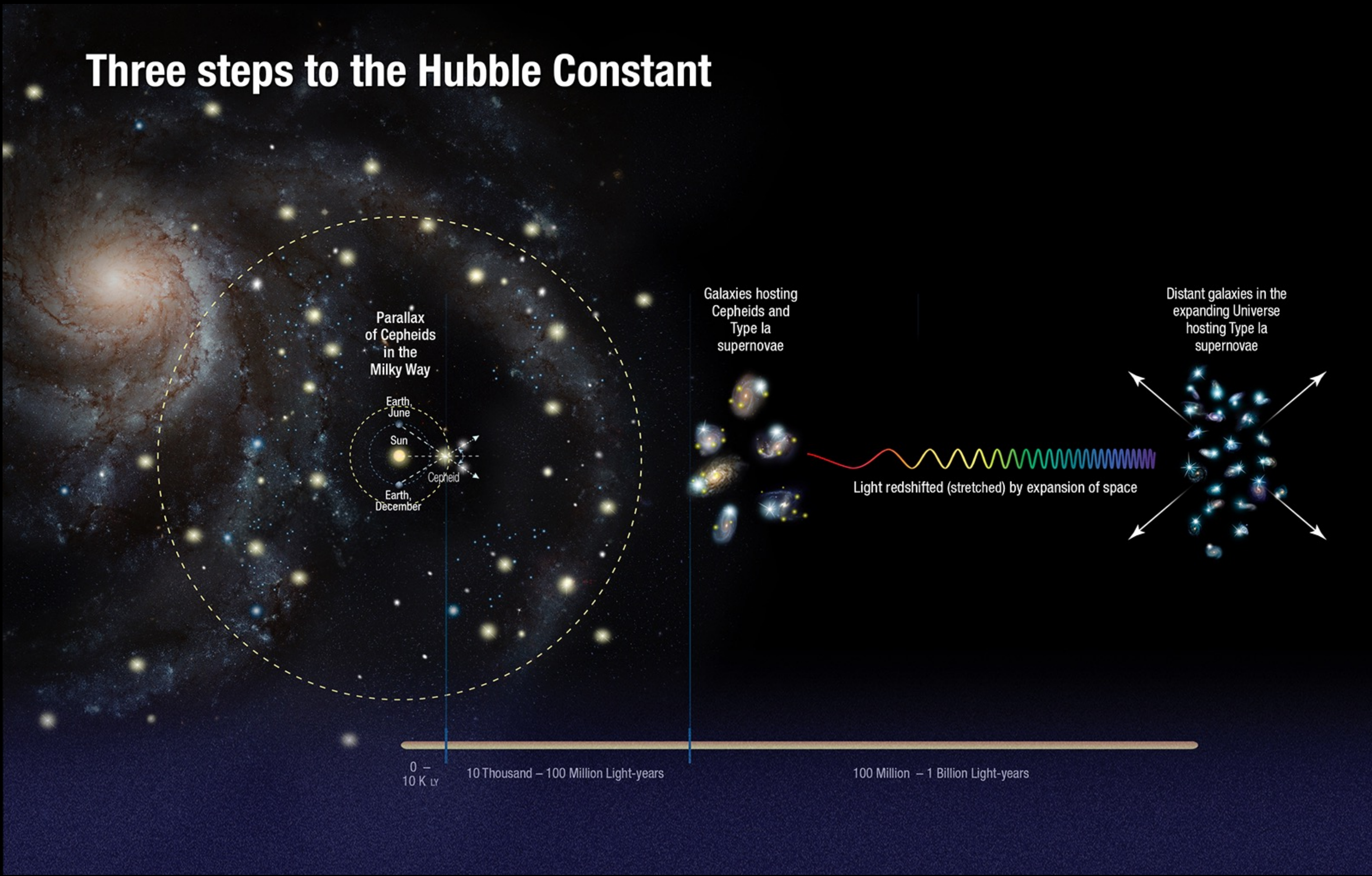


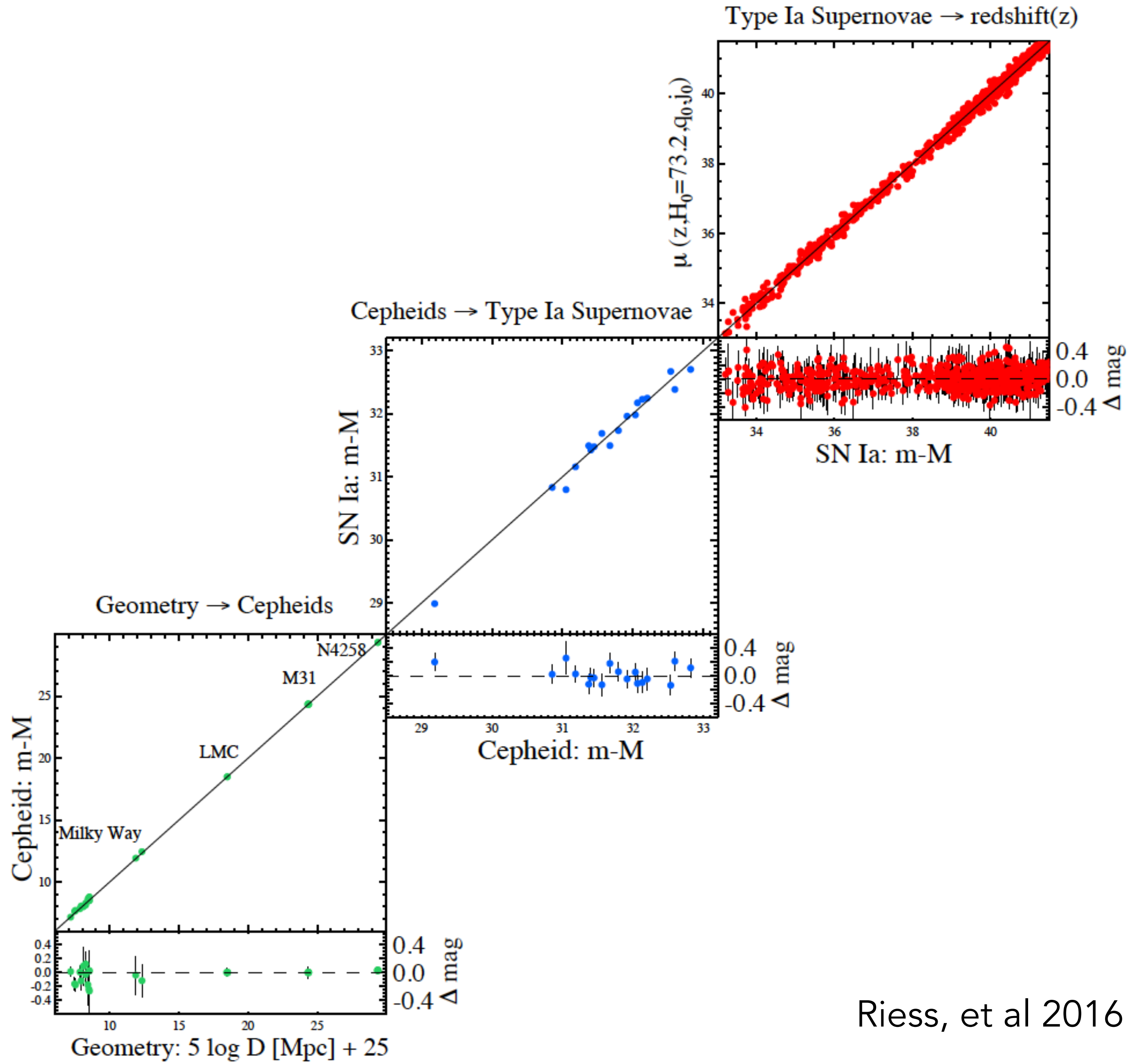
Freedman etal 2001

Recent "Local" Measurements of H_0

- Cepheids+SNe Ia: H_0=73.24±1.74 km/sec/Mpc (Riess, Macri, Hoffman, Scolnic, etal 2016), consistent with earlier distance-ladder measurements (Riess, etal 2011, Freedman, etal 2012)

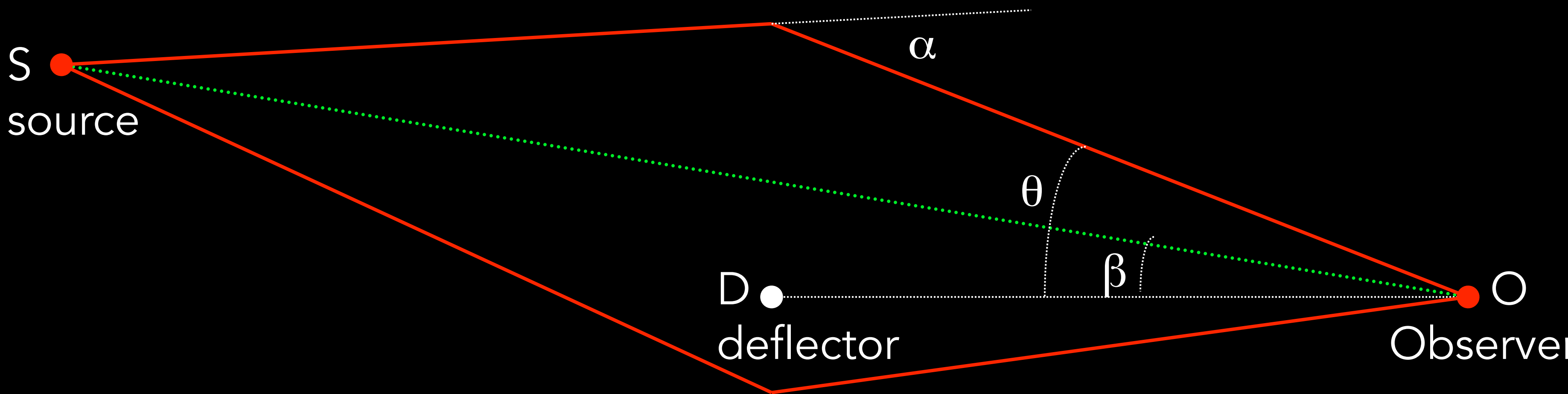
- Strong Lensing QSO Time Delays: H_0=72.8±2.4 for flat ΛCDM with Ω_m=0.32 (H0LiCOW: Bonvin, etal 2016) from 3 lens systems


Three steps to the Hubble Constant
Parallax of Cepheids in the Milky Way
Earth, June
Sun
Cepheid
Earth, December
Galaxies hosting Cepheids and Type Ia supernovae
Light redshifted (stretched) by expansion of space
Distant galaxies in the expanding Universe hosting Type Ia supernovae
0 – 10 K LY
10 Thousand – 100 Million Light-years
100 Million – 1 Billion Light-years



Riess, et al 2016

Strong Lensing Time Delays



Lens equation: $\vec{\beta} = \vec{\theta} - \frac{D_{DS}}{D_{OS}}\vec{\alpha}$

Time Delay between source and image:

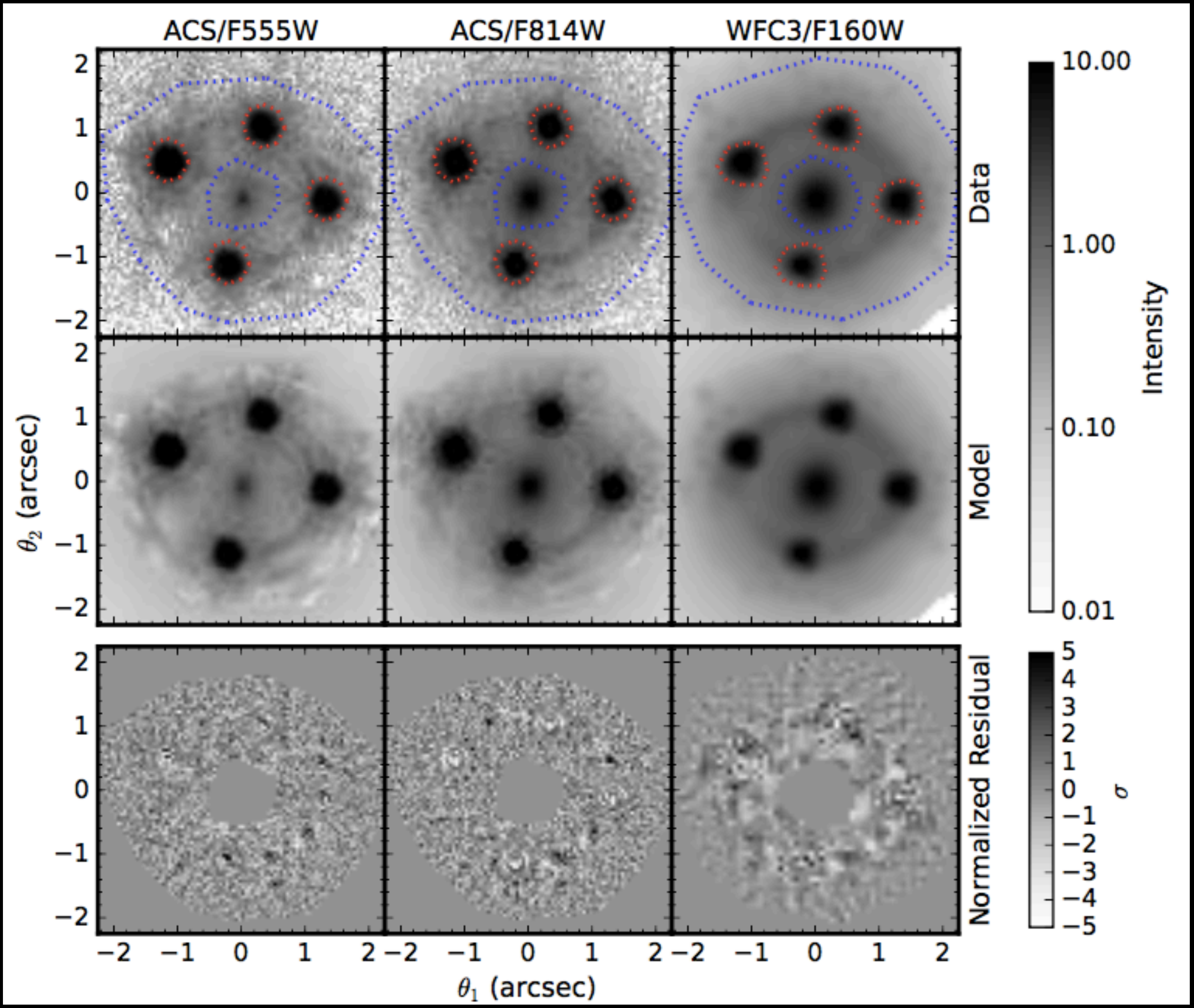
$$t(\vec{\theta},\vec{\beta}) = \frac{D_{\Delta t}}{c}\left[\frac{(\vec{\theta}-\vec{\beta})^2}{2} - \psi(\vec{\theta})\right] \quad \text{where: } D_{\Delta t} = (1+z_D)\frac{D_{OD}D_{OS}}{D_{DS}} \propto H_0^{-1}$$

Refsdal 1964; Review: Treu & Marshall 2016

HE 0435−1223 Lens Model

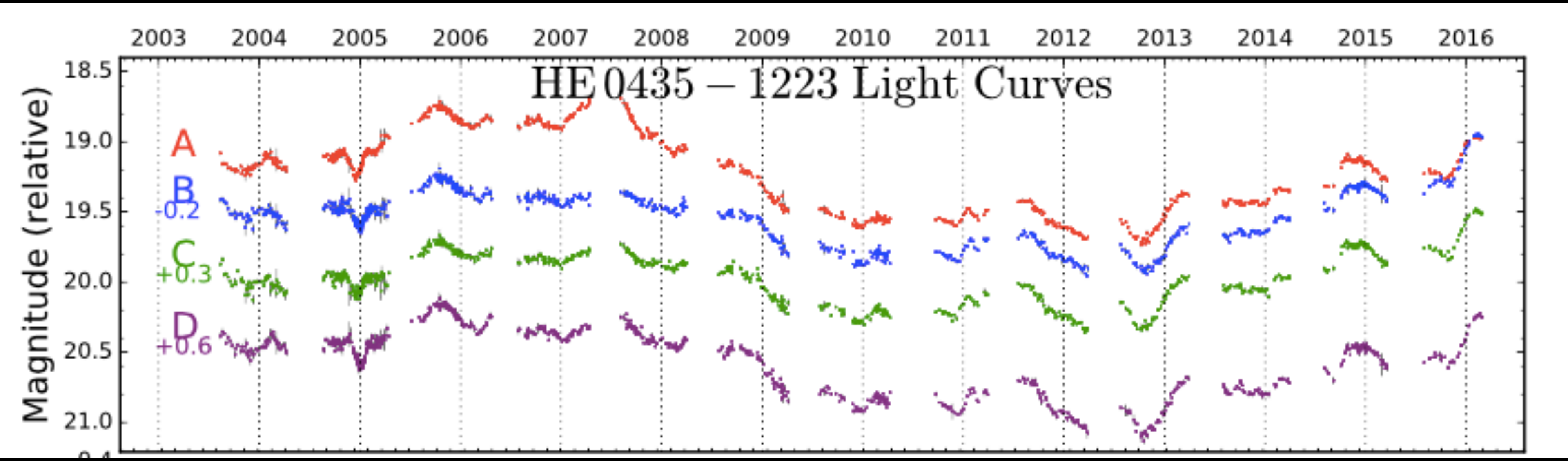
Multiple images of same QSO

z_D=0.45
z_S=1.69



Wong, etal 2016

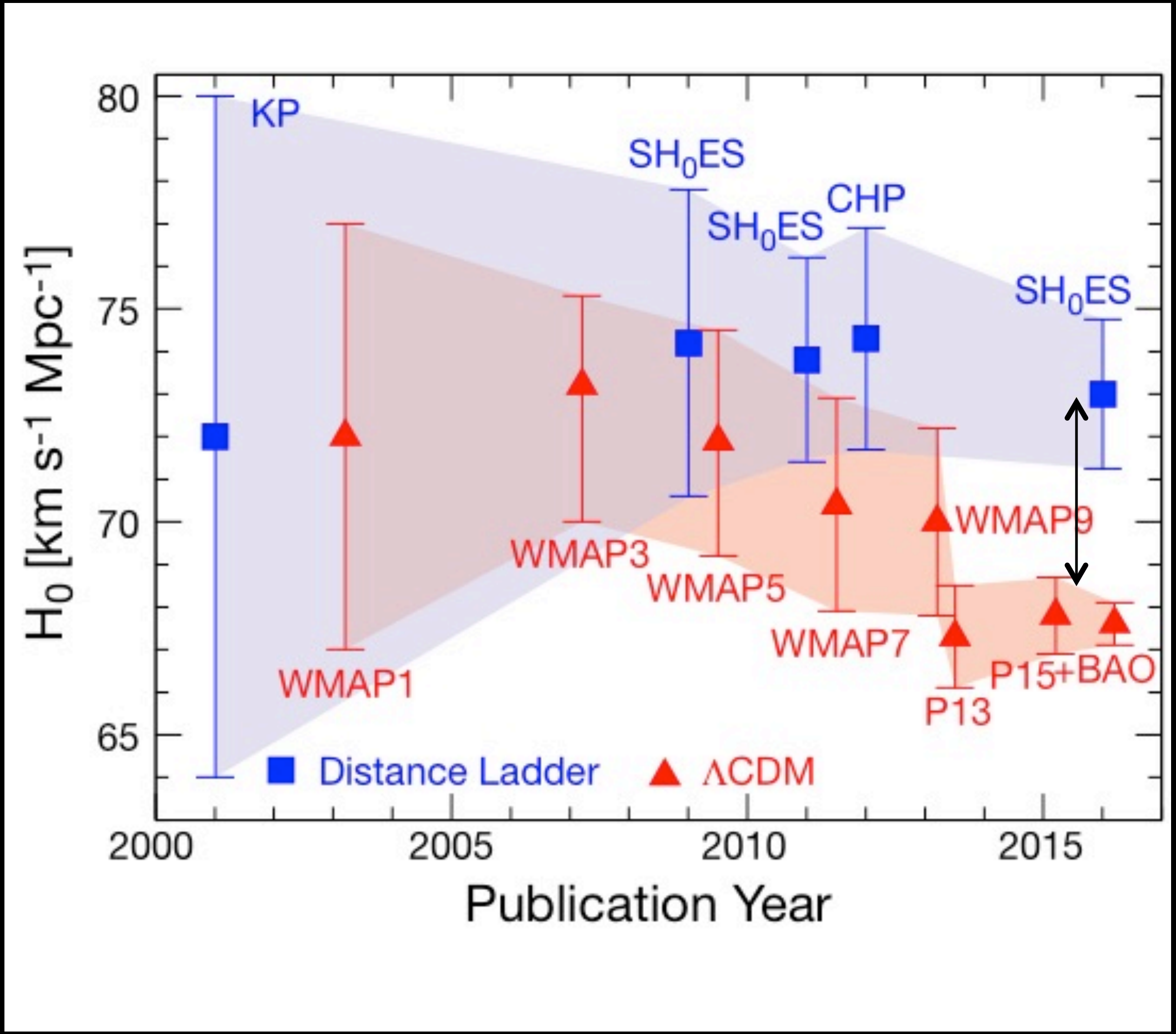
HE 0435−1223 Time Delay



Bonvin, etal 2016

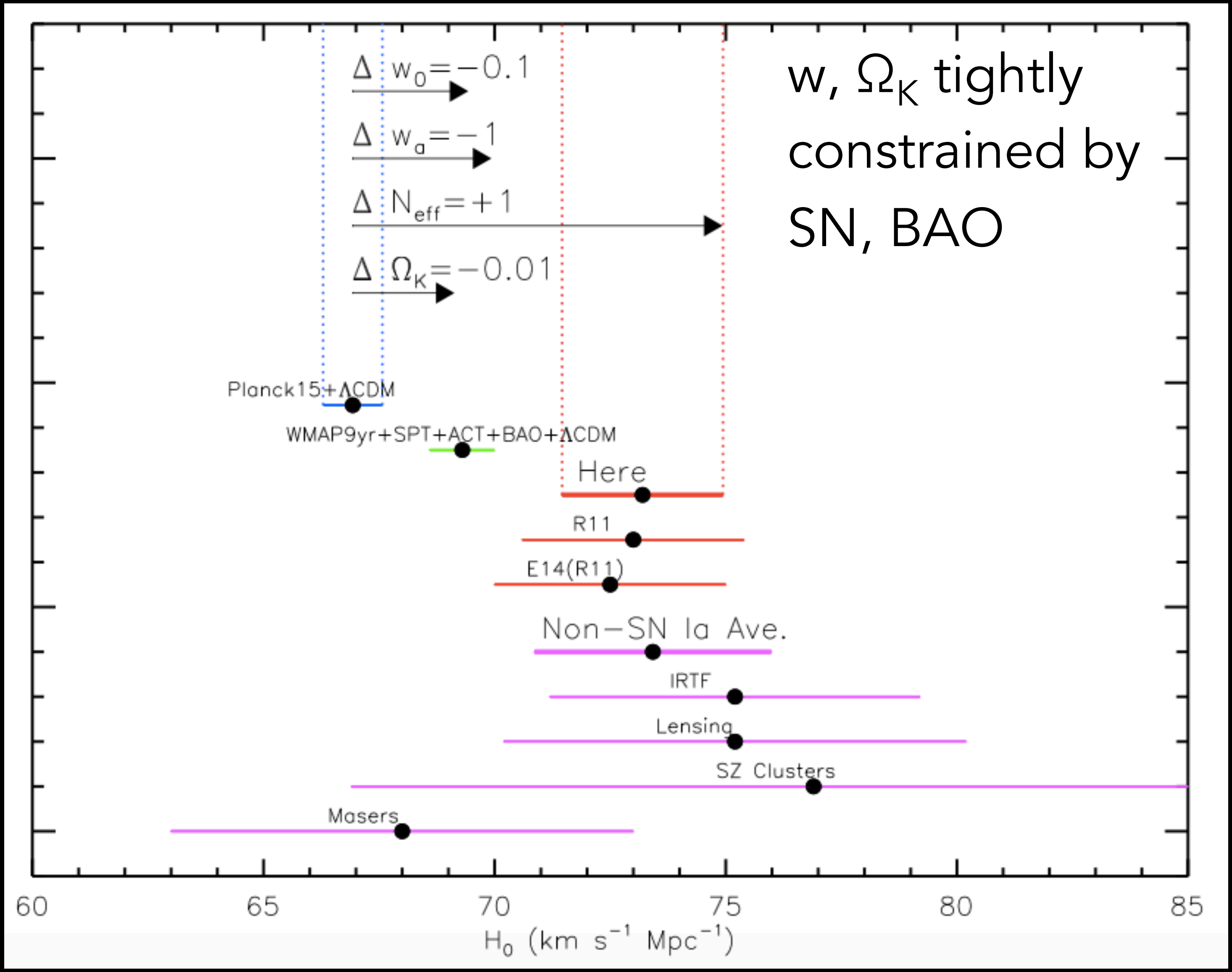
H_0: CMB vs. Local Measurements

CMB
results
assume
ΛCDM
model



3.4σ
discrepancy

Reconciling H_0: Physics beyond ΛCDM?

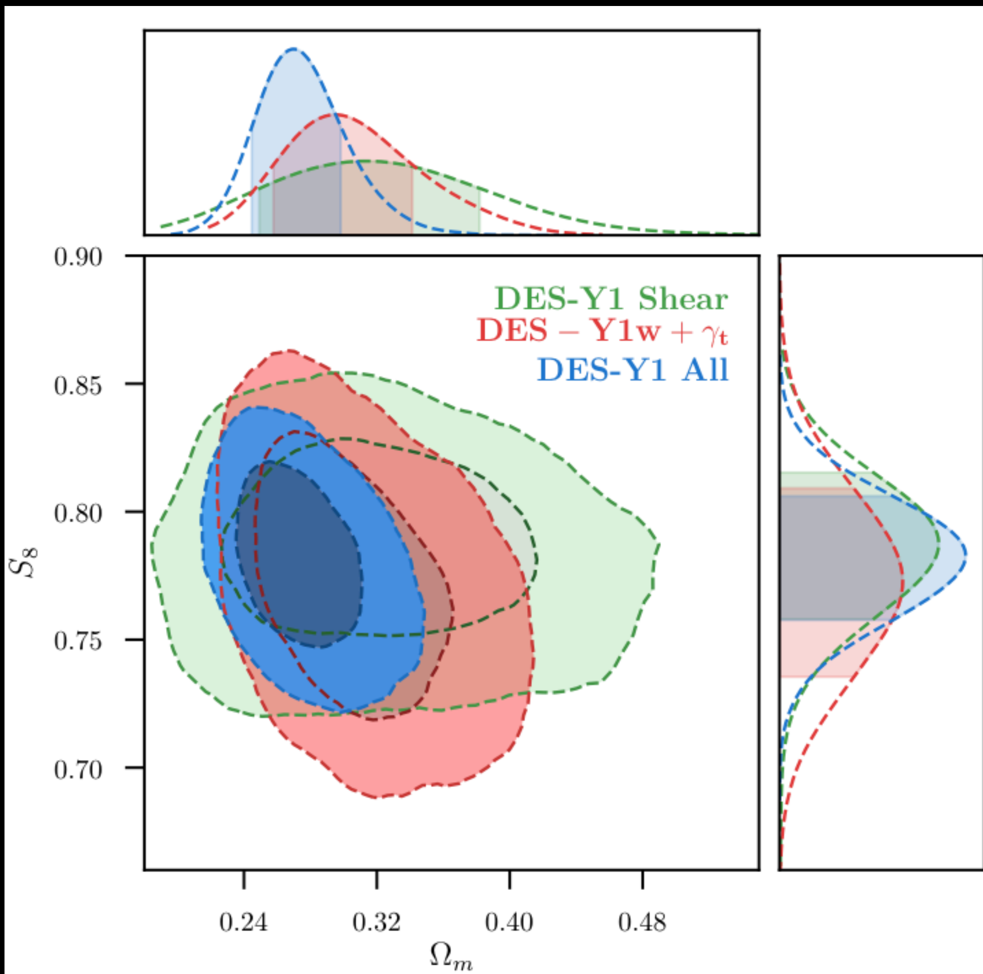


Riess, etal 2016

Tensions: Cracks in the Paradigm?

- "2 to 3-sigma" tensions: systematics or new physics?
  - Planck vs local H_0
  - Planck vs WL σ_8
  - Ly-α BAO
- Compare situation of fundamental physics in the 1890's?
- Some tensions come and go:
  - Planck vs SNLS (Ω_m)
  - Planck CMB vs SZ clusters? (σ_8)

Multi-Probe Constraints: ΛCDM



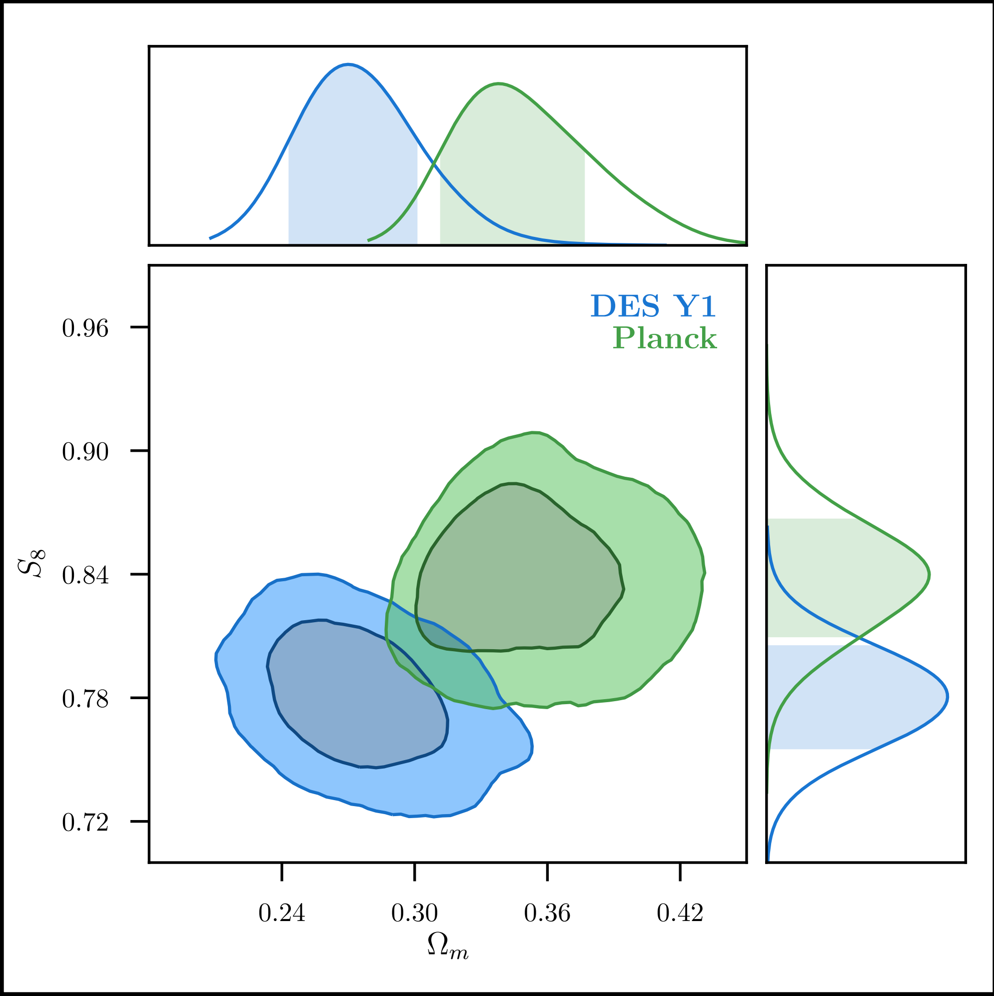
$S_8=\sigma_8(\Omega_m/0.3)^{0.5}$

- DES Year 1 results:
  - Weak Lensing Cosmic Shear
  - Galaxy-galaxy lensing+galaxy clustering

DES Collaboration 2017

Comparison of DES Y1 with Planck CMB: low-z vs high-z in ΛCDM

- DES and Planck constrain S_8 and Ω_m with comparable strength
- Differ in central values by >1σ, in same direction as for KIDS



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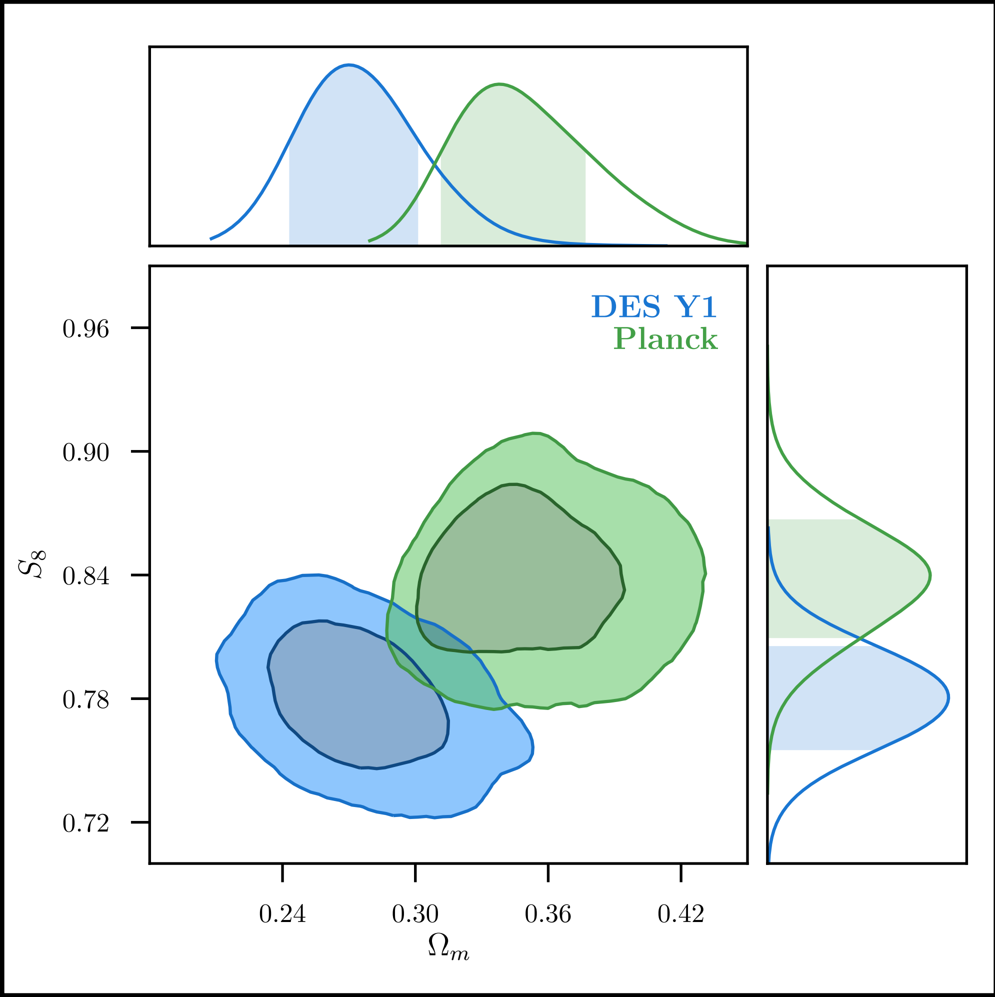
Comparison of DES Y1 with Planck CMB: low-z vs high-z in ΛCDM

- DES and Planck constrain S_8 and Ω_m with comparable strength
- Differ in central values by >1σ, in same direction as for KIDS
- Bayes factor (evidence ratio):
- $R = \frac{P(\text{DES,Planck} \mid \Lambda\text{CDM})}{P(\text{DES} \mid \Lambda\text{CDM})P(\text{Planck} \mid \Lambda\text{CDM})} = 4.2$
- "Substantial" evidence for consistency in ΛCDM



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Combination of DES Y1 with Planck CMB: low-z vs high-z in ΛCDM

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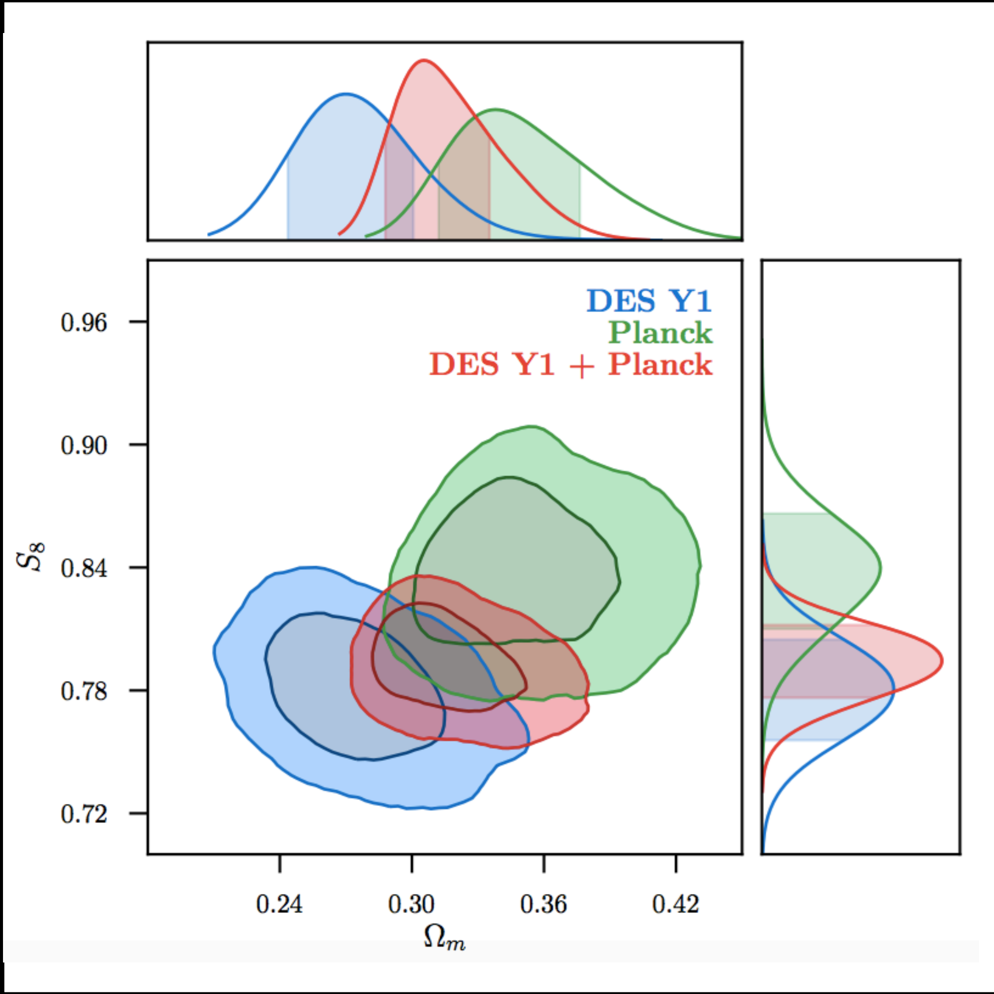
- "Substantial" evidence for consistency in ΛCDM
- Consistency even stronger comparing Planck to multiple low-z probes: DES+BAO +JLA (SN)

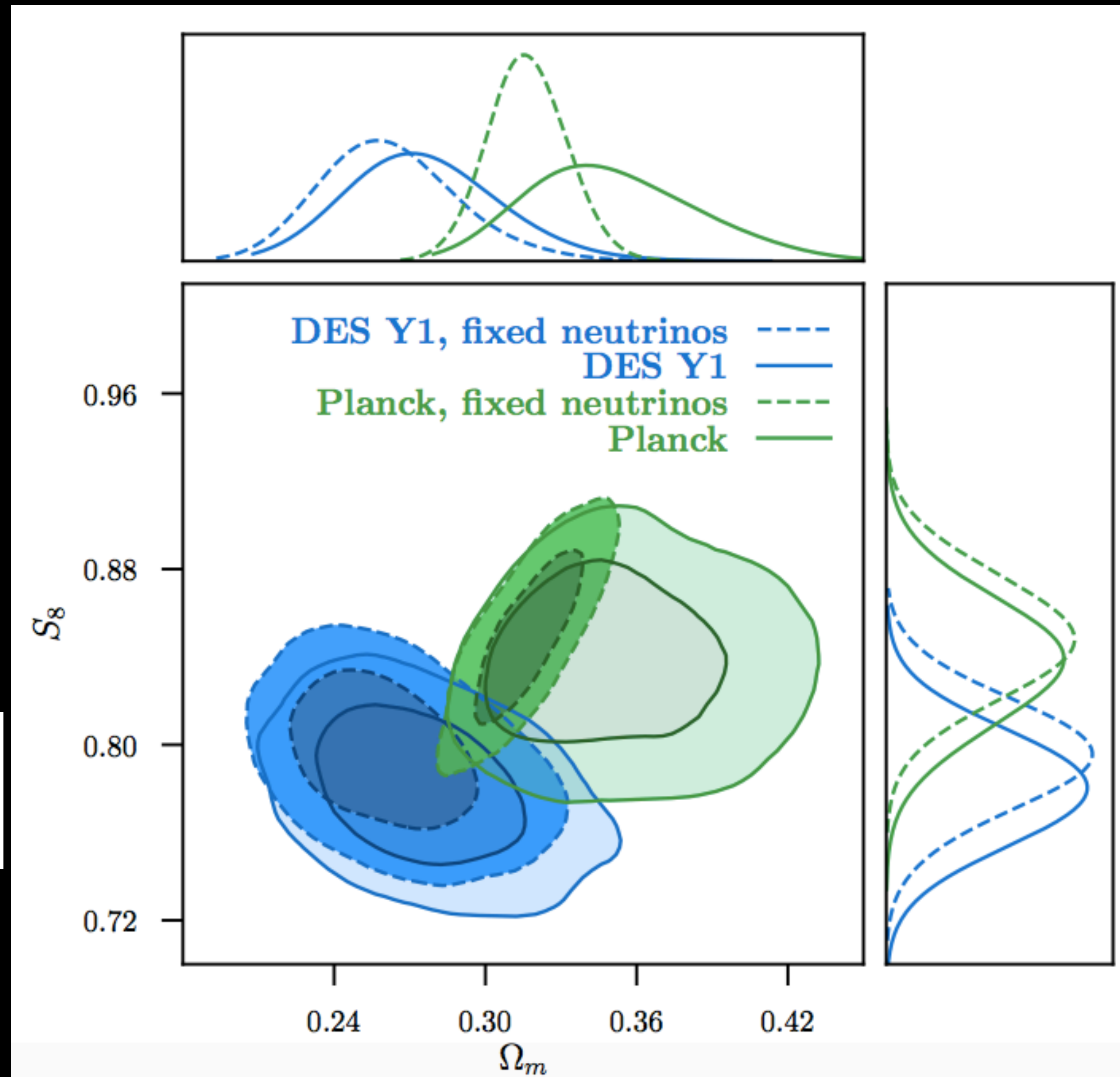


$S_8 = \sigma_8(\Omega_m/0.3)^{0.5}$

Beware parameter degeneracies

- Hold neutrino mass at 0.06 eV (lower limit from oscillation experiments)
- DES 3x2 still/more consistent with Planck in ΛCDM

S_8	$= 0.797 \pm 0.022$	DES Y1
	$= 0.801 \pm 0.032$	KiDS+GAMA [62]
	$= 0.742 \pm 0.035$	KiDS+2dFLenS+BOSS



Hubble Parameter and Expansion

- Hubble parameter: current expansion rate
  H_0=70 km/sec/Mpc=100 *h* km/sec/Mpc, *h*=0.7
- Hubble time: $t_H=1/H_0=9.8h^{-1}$ Gyr=14 billion years
- Hubble distance: $d_H=c/H_0=3000h^{-1}$ Mpc
- Distances: $d\approx v/H_0=cz/H_0=d_Hz$

- Hubble time ~ time it currently takes for the distance between a pair of galaxies to double
- Redshift z variously taken as indicator of scale factor, distance, or look-back time.

Expansion Kinematics

- Taylor expand about present epoch:

$$a(t) = a(t_0) + \dot{a}(t)|_0(t - t_0) + \frac{1}{2}\ddot{a}(t)|_0(t - t_0)^2 + ...$$

which implies to 2nd order in $t - t_0$:

$$\frac{a(t)}{a_0} = 1 + \left(\frac{\dot{a}}{a}\right)_0 (t - t_0) + \frac{1}{2}\left(\frac{\ddot{a}}{a}\right)_0 (t - t_0)^2 = 1 + H_0(t - t_0) - \frac{q_0 H_0^2}{2}(t - t_0)^2$$

where $H(t) = \dot{a}/a(t)$ $H_0 = (\dot{a}/a)\big|_{t=t_0}$ and $q_0 \equiv -(a\ddot{a}/\dot{a}^2)_0$

Recent expansion history completely determined by H_0 and q_0

Differentiating with respect to t and keeping terms linear in $t - t_0$,

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{\dot{a}(t)}{a_0}\frac{a_0}{a(t)} = \left(H_0 - q_0 H_0^2 t + q_0 H_0^2 t_0\right)\left(1 - H_0(t - t_0)\right)$$

$$= H_0\left[1 - (1 + q_0)H_0(t - t_0)\right]$$

The redshift is given generally by

$$1 + z = \frac{a_0}{a(t)} \ ,$$

so that to this order of approximation from Eqn. 7 we have

$$z = -H_0(t - t_0) + \mathcal{O}(t - t_0)^2 \ ,$$

and we therefore find to this order,

$$H(z) = H_0\left[1 + (1 + q_0)z\right] \ .$$

Not an accurate approx., but useful for seeing scaling with parameters

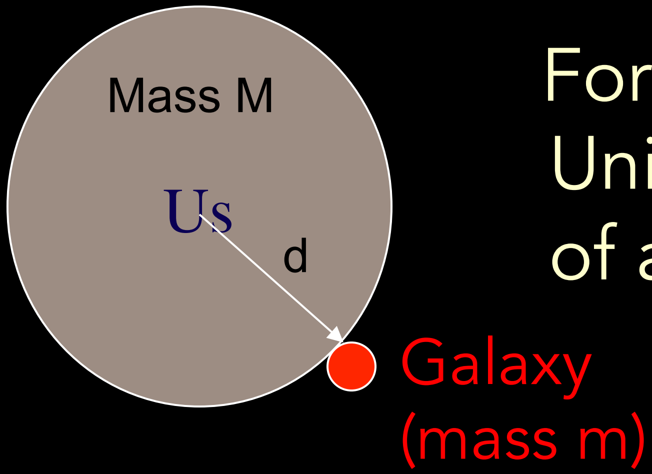
How does the expansion of the Universe change over time?

Gravity:

our Galaxy is pulling on all the receding galaxies

naively expect them to slow down: $v=Hd$, $d \sim a(t)$, hence $v \sim Ha = da/dt$ should decrease, hence expect $\ddot{a}<0$: expansion of the Universe should slow down over time

Expansion Dynamics

- Wait a minute: isn't that galaxy being pulled *away* from us by other galaxies on the other side of it?
- Yes, but it's also being pulled *toward* us by other galaxies on this side.
- 

For gravity, for a homogeneous Universe, we can ignore the effects of all bodies outside a sphere of radius d centered on us (Newton-Birkhoff theorem).
- Can consider galaxy moving in the gravitational field of a spherical body of mass $M=(4\pi/3)\rho d^3$

Orbits

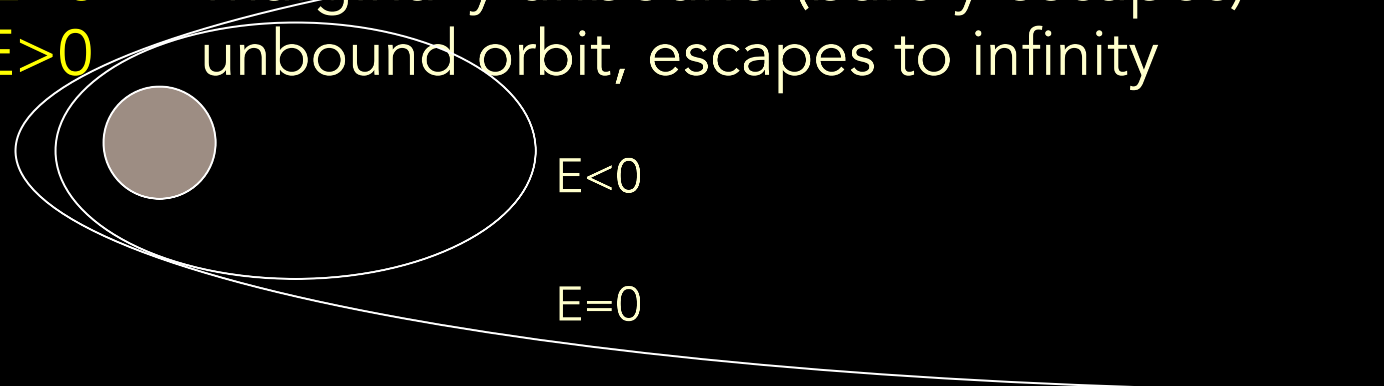
- Galaxy motion determined by same equation that governs orbits of satellites around Earth.
- Conservation of Energy:

Kinetic energy + Potential Energy = Total Energy E = constant

$$\frac{1}{2}m\mathrm{v}^2 - \frac{GMm}{d} = E$$

$$\mathrm{v}_{esc} = \sqrt{\frac{2GM}{d}} = d\sqrt{\frac{8\pi G\rho}{3}}$$

$v < v_{escape}$	E<0	bound elliptical orbit (e.g., Moon)
$v = v_{escape}$	E=0	marginally unbound (barely escapes)
$v > v_{escape}$	E>0	unbound orbit, escapes to infinity



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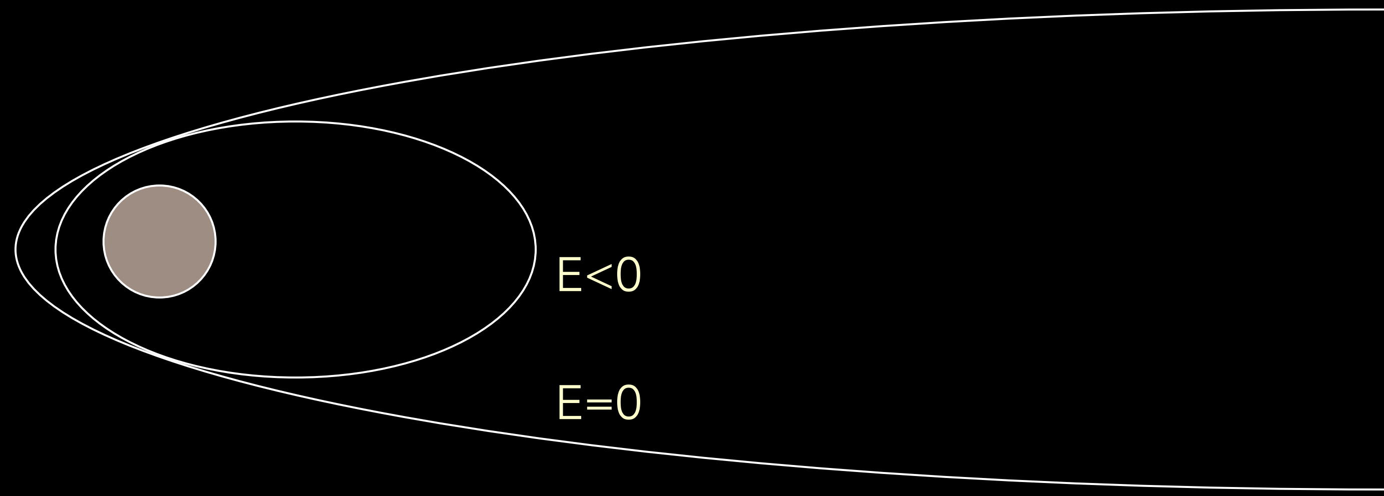
$$v = Hd$$

$$v_{esc} = \sqrt{\frac{2GM}{d}} = d\sqrt{\frac{8\pi G\rho}{3}}$$

- For $v = v_{escape}$, $E=0$, and $H^2(t) = \frac{8\pi G\rho(t)}{3}$



Orbits

- Galaxy motion determined by same equation that governs orbits of satellites around Earth.

$$\mathrm{v} = Hd$$

$$\mathrm{v}_{esc} = \sqrt{\frac{2GM}{d}} = d\sqrt{\frac{8\pi G\rho}{3}}$$

- In general,

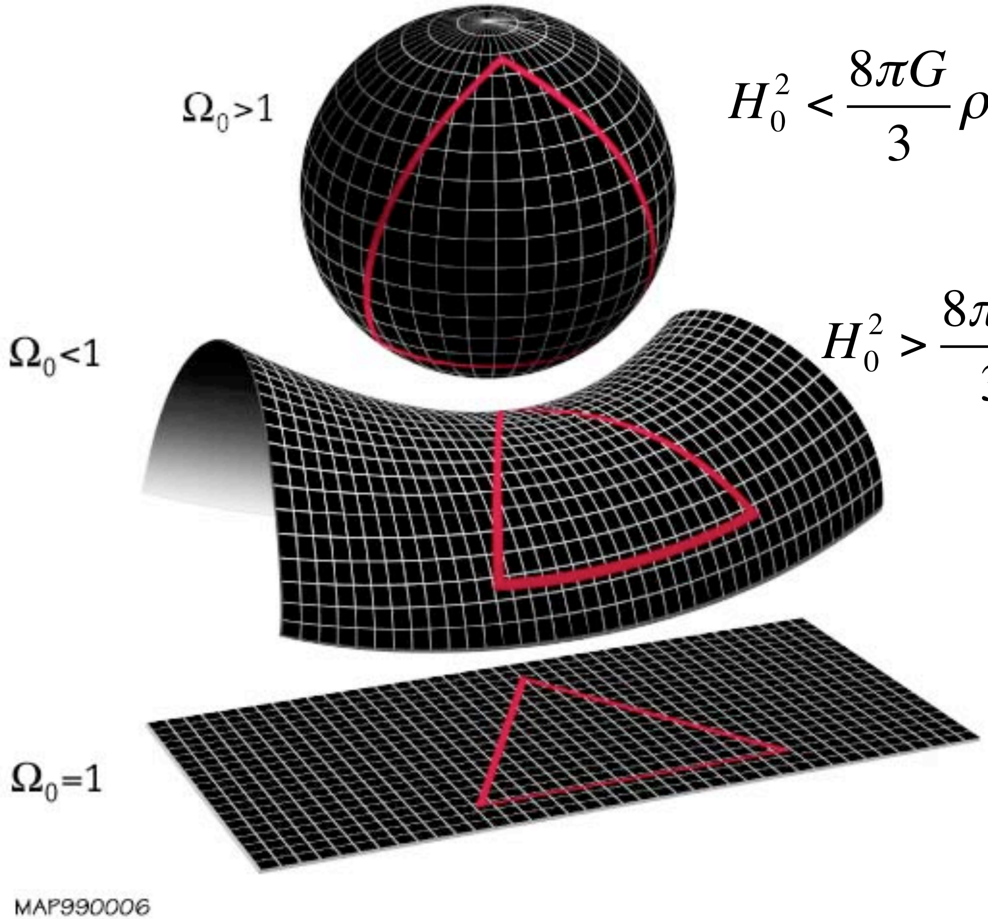
$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho(t)}{3} + \frac{2E}{md^2} = \frac{8\pi G\rho(t)}{3} - \frac{k}{a^2(t)}$$

1st order Friedmann equation

spatial curvature in GR

Spatial Curvature and Density in GR

Define $\Omega \equiv \dfrac{\rho}{\rho_{crit}}$, where $\rho_{crit} \equiv \dfrac{3H_0^2}{8\pi G} = 1.88h^2 \times 10^{-29}\,\mathrm{gm/cm^3}$

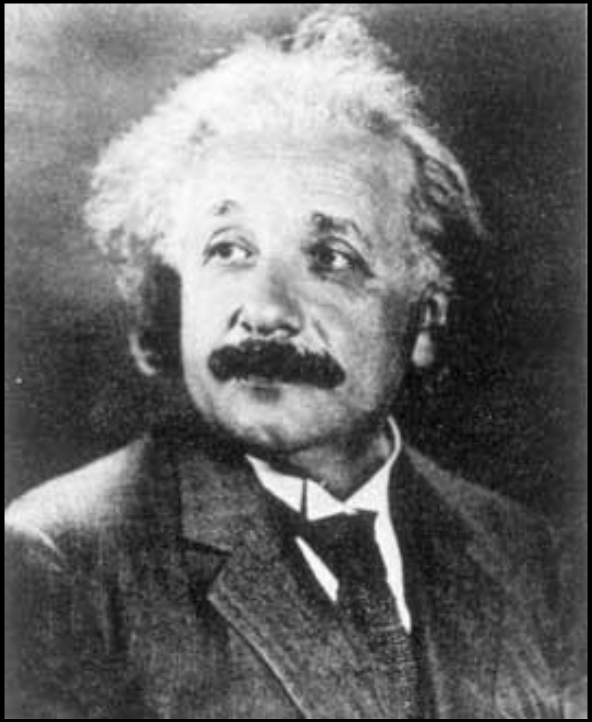


$$H_0^2 < \frac{8\pi G}{3}\rho_0 \iff k = +1,\ E < 0 \text{ positive curvature}$$

$$H_0^2 > \frac{8\pi G}{3}\rho_0 \iff k = -1,\ E > 0 \text{ negative curvature}$$

$$H_0^2 = \frac{8\pi G}{3}\rho_0 \iff k = 0,\ E = 0 \text{ flat}$$

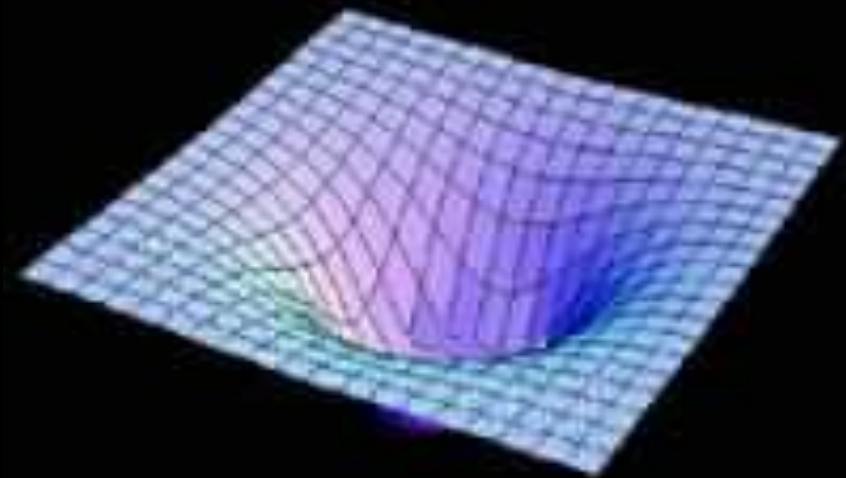
Einstein's Theory of Gravity: General Relativity



Matter and Energy curve Space-Time

Everything, including light, moves in this curved Space-time

A massive star attracts nearby objects by distorting spacetime



Space vs Spacetime Curvature

Curvature of 3-dimensional Space vs. Curvature of 4-dimensional Spacetime:

General Relativity: implies that 4d Spacetime is generally curved, determined by mass-energy.

Cosmology: mainly concerned with the curvature of 3-dimensional space (K) (i.e., of a slice through spacetime at fixed time) since it is related to the density and fate of the Universe.

Local Conservation of Energy-Momentum

First law of thermodynamics:

$$dE = -p\,dV$$

Energy:

$$E = \rho V \sim \rho a^3$$

First Law becomes:

$$\frac{d(\rho a^3)}{dt} = -p\frac{d(a^3)}{dt}$$

$$a^3\dot{\rho} + 3\rho a^2\dot{a} = -3pa^2\dot{a} \quad \Rightarrow$$

$$\boxed{\frac{d\rho_i}{dt} + 3H(t)(p_i + \rho_i) = 0}$$

2nd Order Friedmann Equation

First order Friedmann equation:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - k$$

Differentiate:

$$2\dot{a}\ddot{a} = \frac{8\pi G}{3}(a^2\dot{\rho} + 2a\dot{a}\rho) \Rightarrow$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left[\dot{\rho}\left(\frac{a}{\dot{a}}\right) + 2\rho\right]$$

Now use conservation of energy - momentum:

$$\frac{d\rho_i}{dt} + 3H(t)(p_i + \rho_i) = 0 \Rightarrow$$

2nd order Friedmann equation:

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}\left[-3(p+\rho) + 2\rho\right] = -\frac{4\pi G}{3}\left[\rho + 3p\right]$$

Cosmological Dynamics in GR

$$H^2(t) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i(t) - \frac{k}{a^2(t)}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \left(\rho_i + \frac{3p_i}{c^2}\right)$$

Friedmann Equations from General Relativity

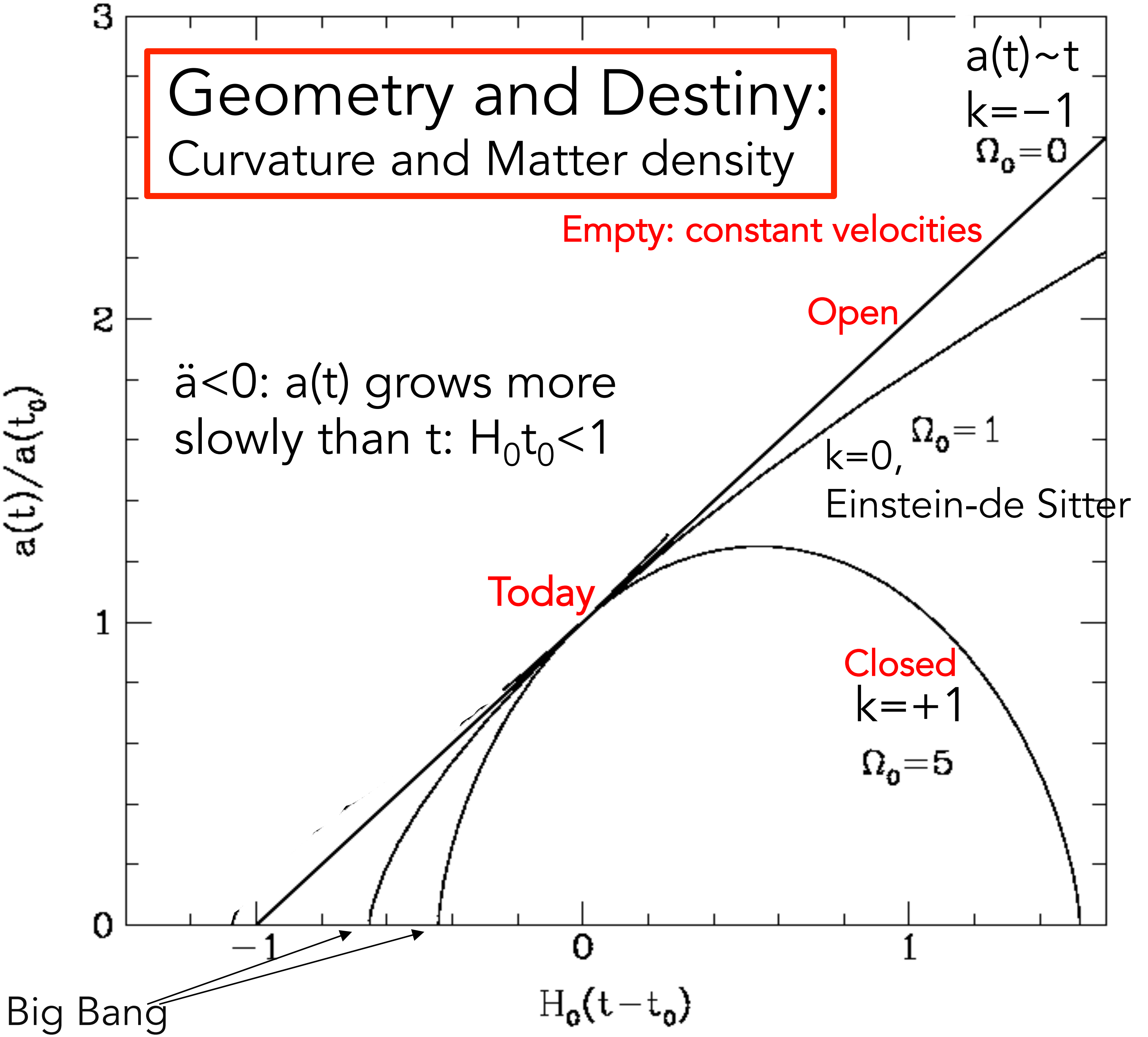
Non-relativistic matter: $p_m \sim \rho_m \mathrm{v}^2 \sim 0$

Relativistic particles: $p_r = \rho_r c^2 / 3$

In both cases, expansion decelerates: $\ddot{a} < 0$ due to attractive nature of gravity

Will the Universe expand forever or recollapse in a Big Crunch?

Is the gravity of matter enough to reverse expansion?



Deceleration and Age of the Universe

- Due to gravity, we expect scale factor $a(t)$ to grow more slowly than t.
- In that case, the age of the Universe t_0 would be less than the Hubble time:

$$t_0 < 1/H_0 = 14 \text{ billion years}$$

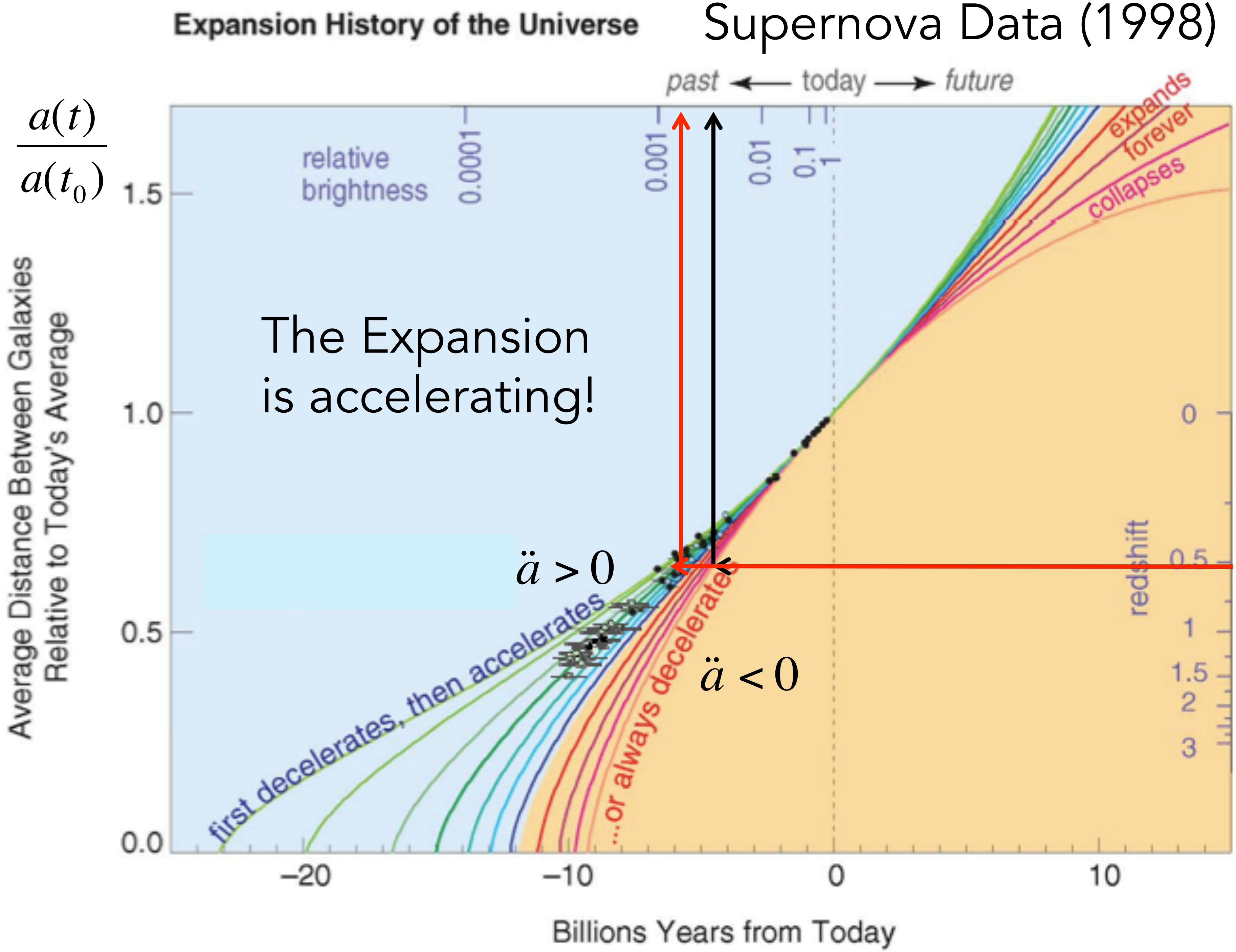
- Example:
  Einstein-de Sitter model:
  Flat, matter-dominated

$$\rho = \rho_{crit} \equiv \frac{3H^2}{8\pi G}, \quad \Omega \equiv \frac{\rho}{\rho_{crit}} = 1$$

$$\left(\frac{\dot{a}}{a}\right)^2 \sim \frac{1}{a^3} \Rightarrow a^{1/2} da \sim dt \Rightarrow a \sim t^{2/3}$$

$$H = \frac{2}{3t}$$

Supernova Data (1998)



Cosmological Dynamics and Dark Energy

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \rho_i\left(1+3w_i\right)$$

Friedmann Equation from GR

Equation of state parameter: $w_i = p_i / \rho_i c^2$

Non-relativistic matter: $w_m \approx 0$

Relativistic particles: $w_{rad} = 1/3$

Acceleration ($\ddot{a} > 0$) requires dominant component with negative pressure:

Dark Energy: $w_{DE} < -1/3$

or Replace GR dynamics with another gravity theory

or drop assumption of homogeneity & isotropy.

Cosmological Constant (Λ) as Vacuum Energy

Einstein:

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Lemaitre:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\equiv 8\pi G\left(T_{\mu\nu}(\text{matter}) + T_{\mu\nu}(\text{vacuum})\right)$$

Vacuum Energy:

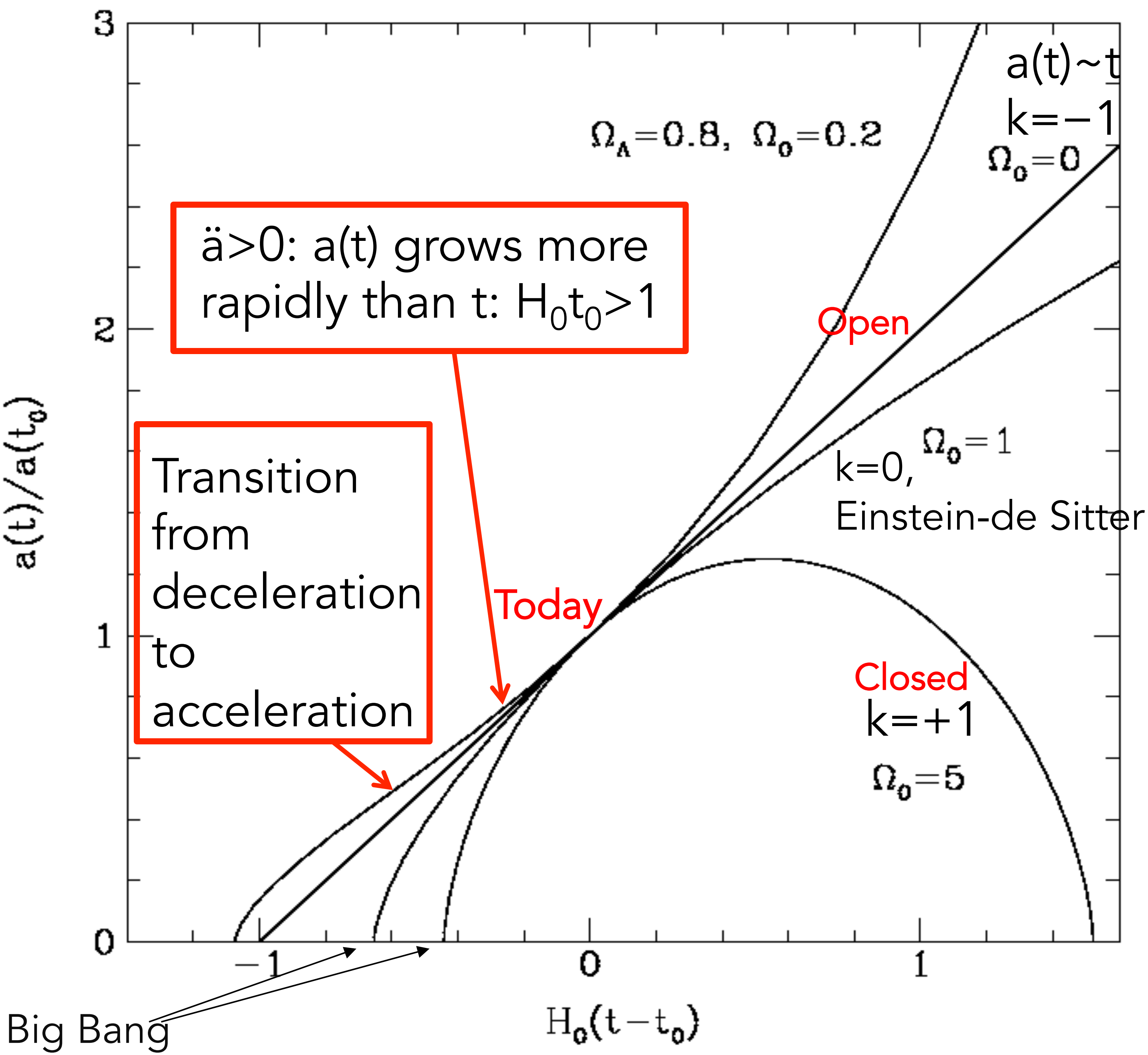
$$T_{\mu\nu}(\text{vac}) = \frac{\Lambda}{8\pi \text{G}} g_{\mu\nu}$$

$$\rho_{\text{vac}} = T_{00} = \frac{\Lambda}{8\pi \text{G}}, \quad p_{\text{vac}} = T_{ii} = -\frac{\Lambda}{8\pi \text{G}}$$

$$w_\Lambda = -1 \;\Rightarrow\; H \equiv \frac{\dot{a}}{a} = \text{constant} \;\Rightarrow\; a(t) \propto \exp(Ht)$$

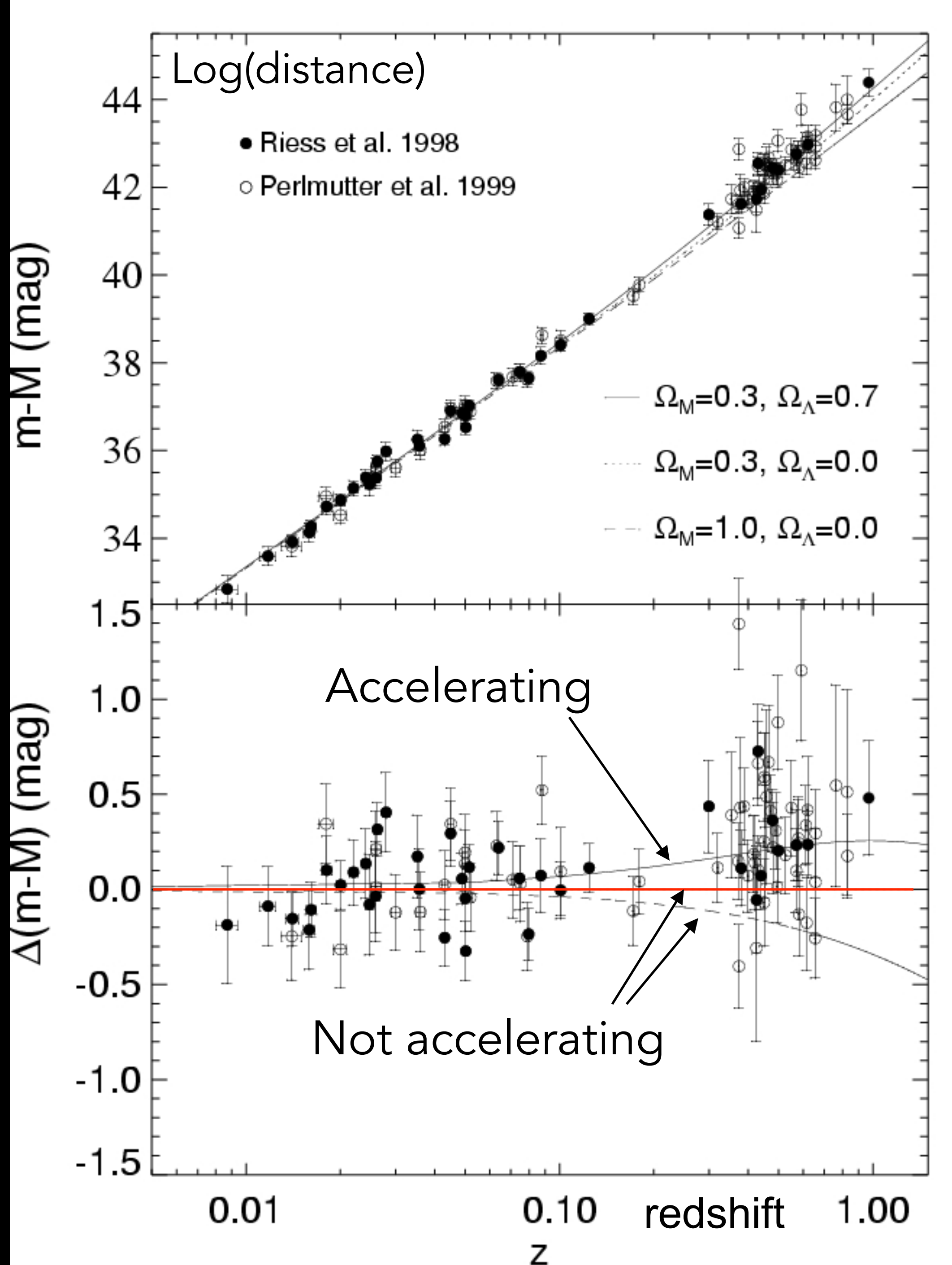
Will the Universe expand forever or recollapse in a Big Crunch?

Is the gravity of matter enough to reverse expansion?



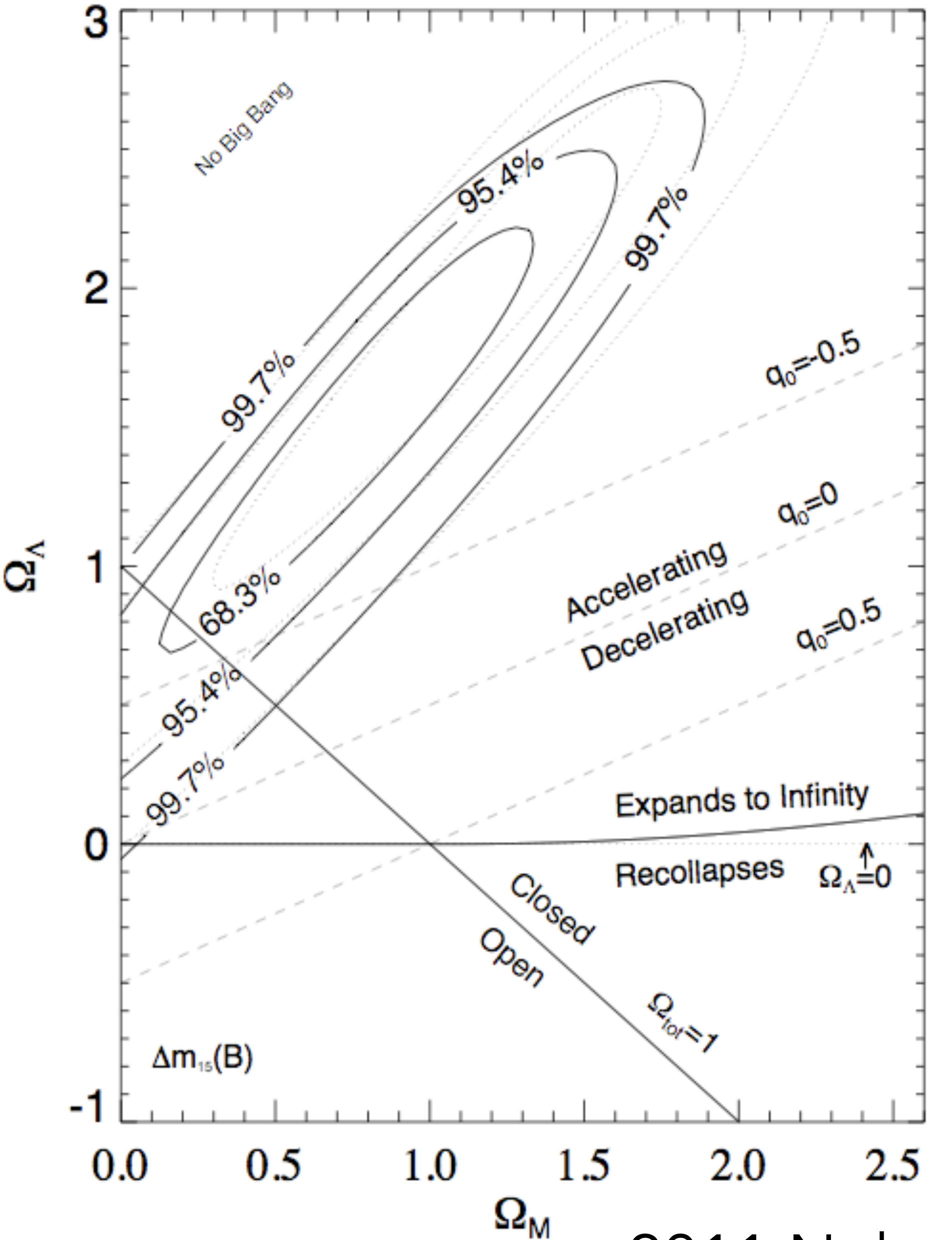
Discovery of Cosmic Acceleration from High-redshift Supernovae

Type Ia supernovae that exploded when the Universe was 2/3 its present size are ~25% fainter than expected

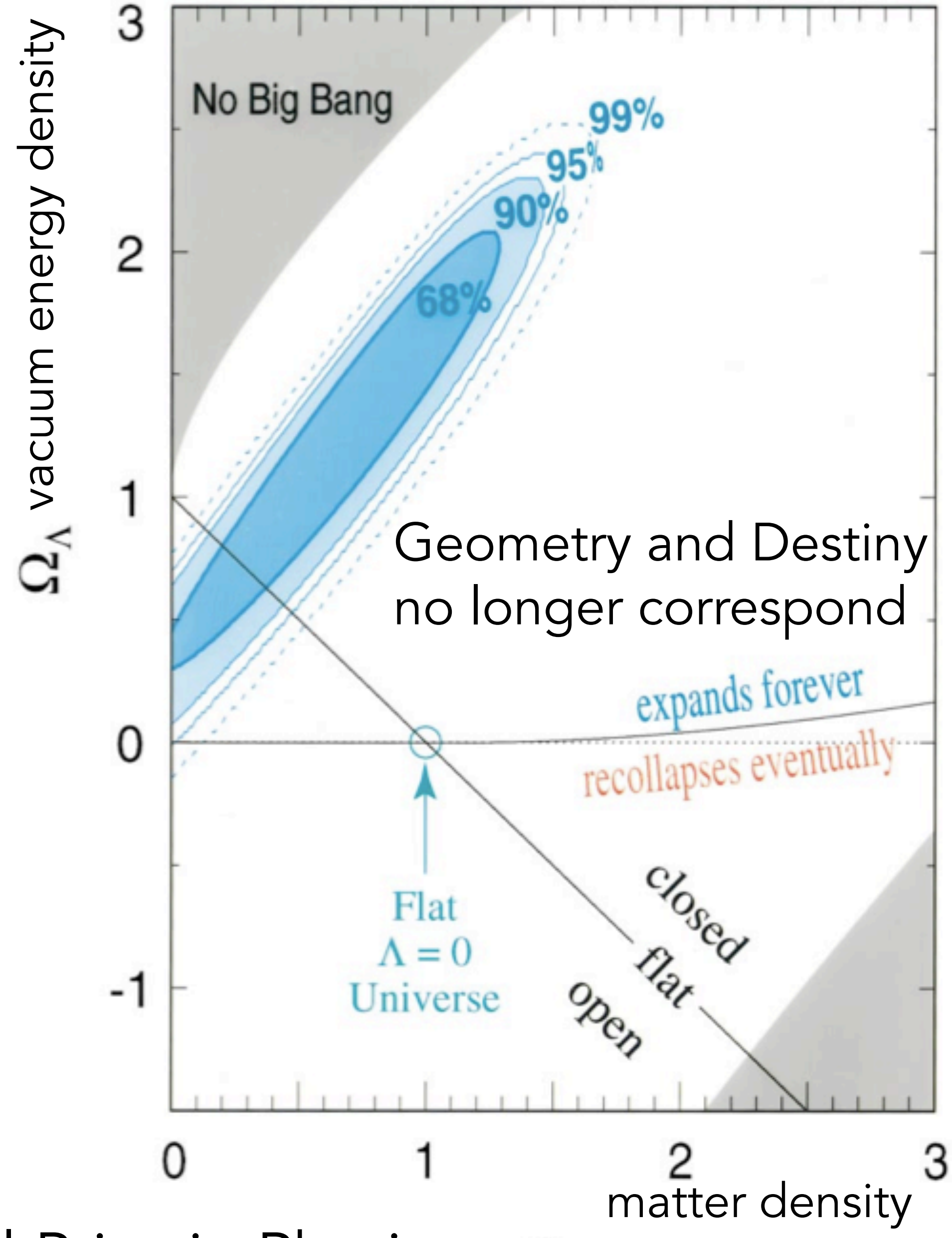


$\Omega_\Lambda = 0.7$
$\Omega_\Lambda = 0.$
$\Omega_m = 1.$

Riess et al. (1998, AJ)

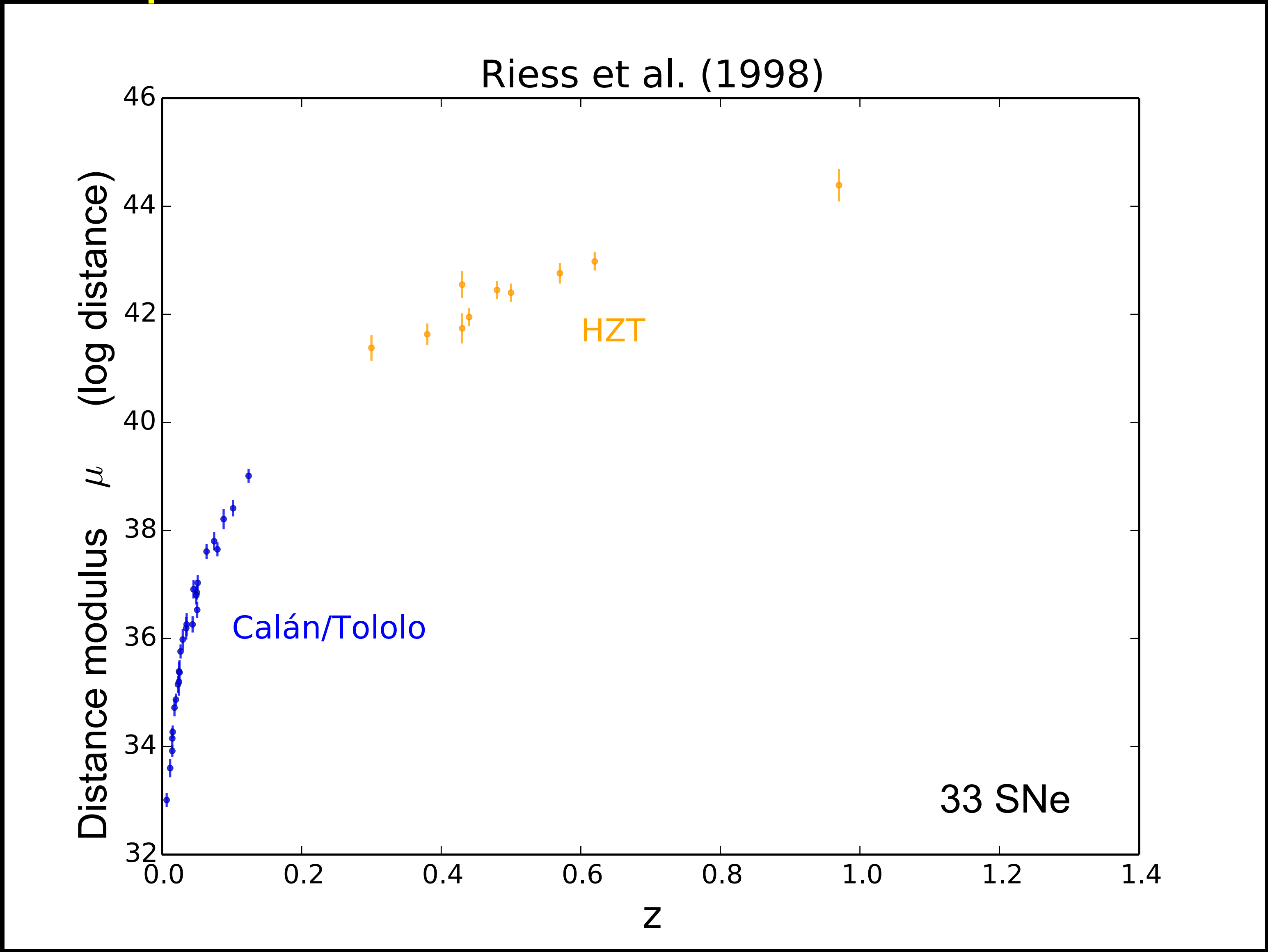


Perlmutter et al. (1999, ApJ)

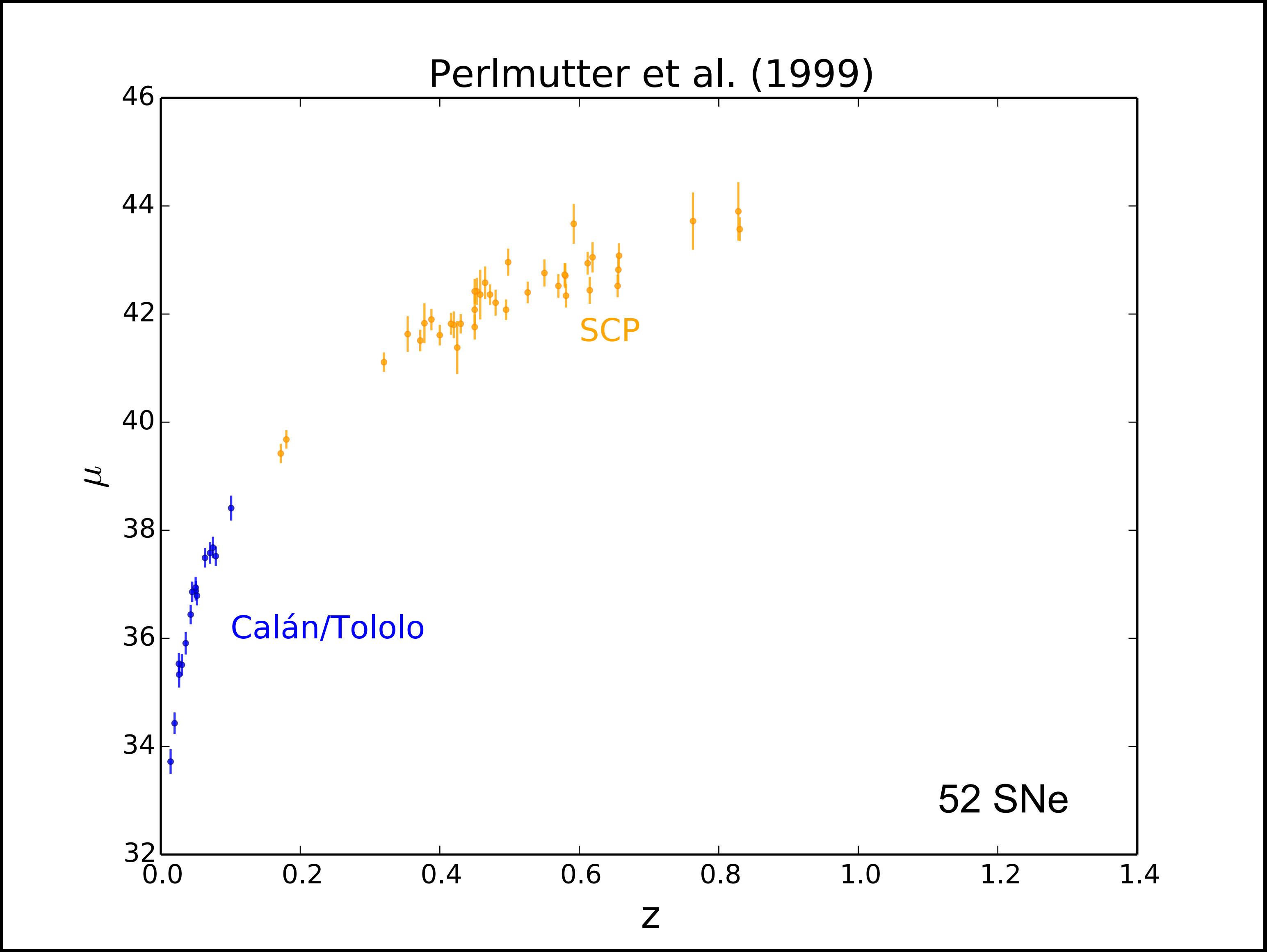


2011 Nobel Prize in Physics

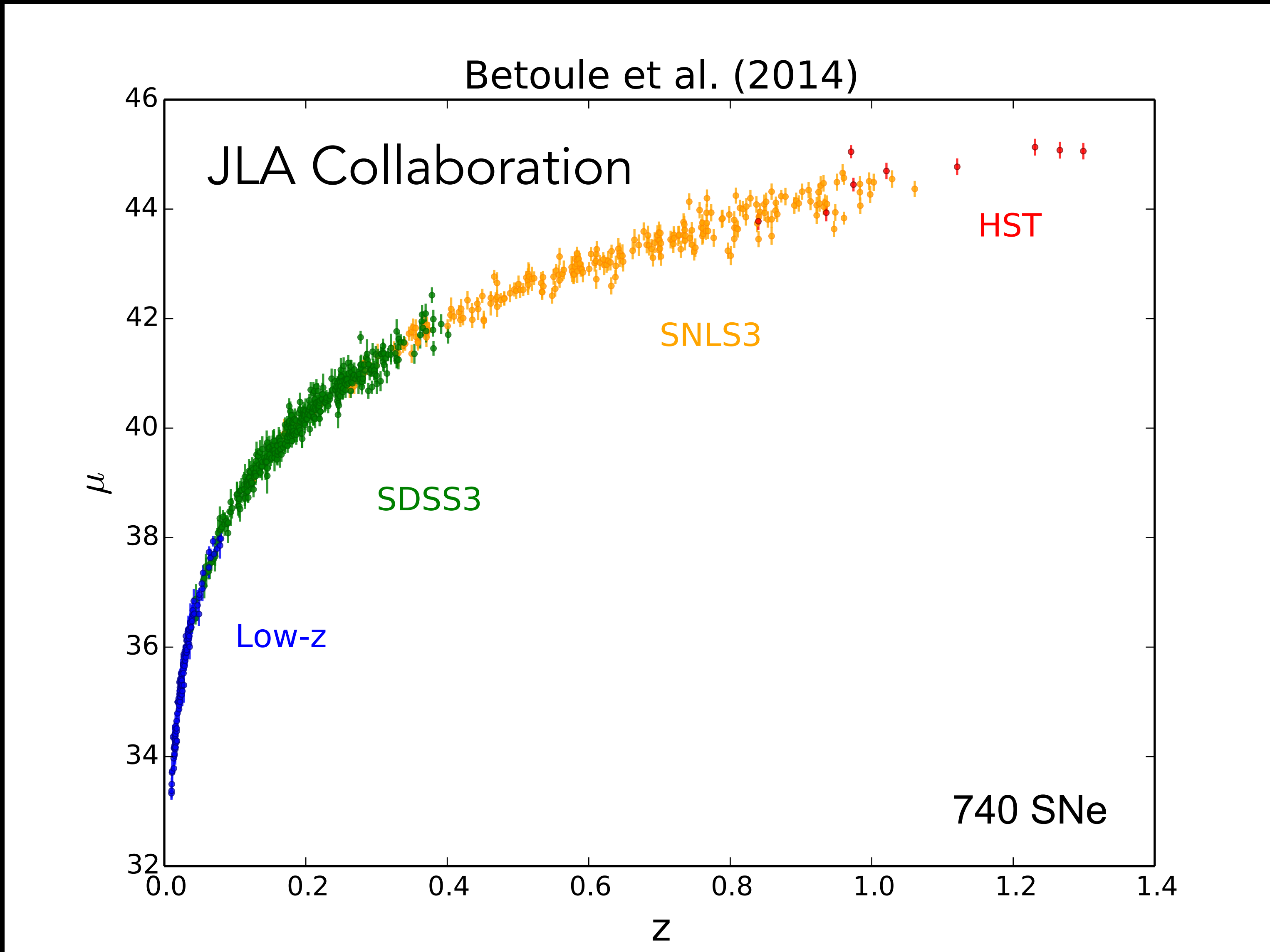
Supernova Ia Hubble Diagram



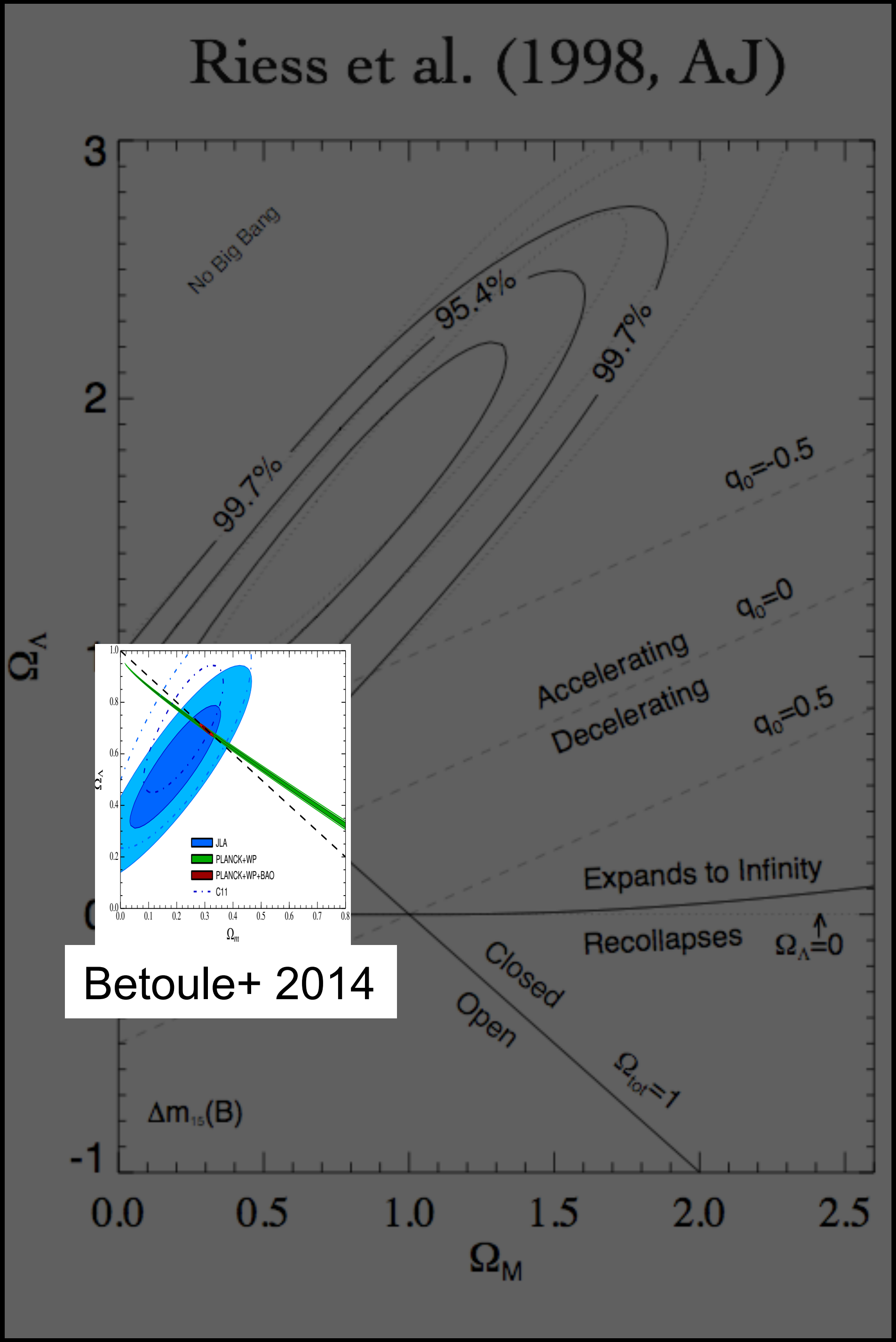
SN figures by A. Conley



Perlmutter et al. (1999)
46
44
42
40
38
36
34
32
μ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
z
SCP
Calán/Tololo
52 SNe


Betoule et al. (2014)
JLA Collaboration
HST
SNLS3
SDSS3
Low-z
740 SNe
μ
z
46
44
42
40
38
36
34
32
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Progress over the last 20 years



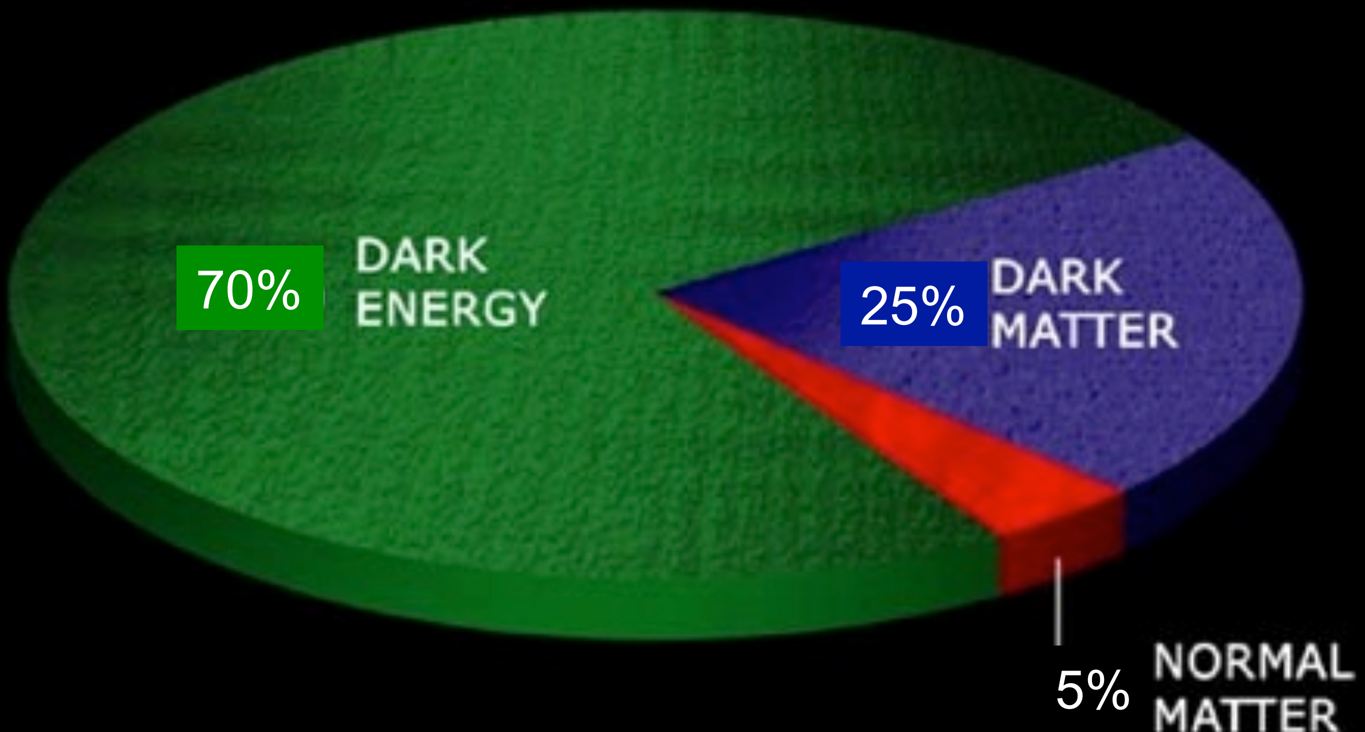
Supernovae

Cosmic Microwave Background (Planck, WMAP)

CMB+BAO

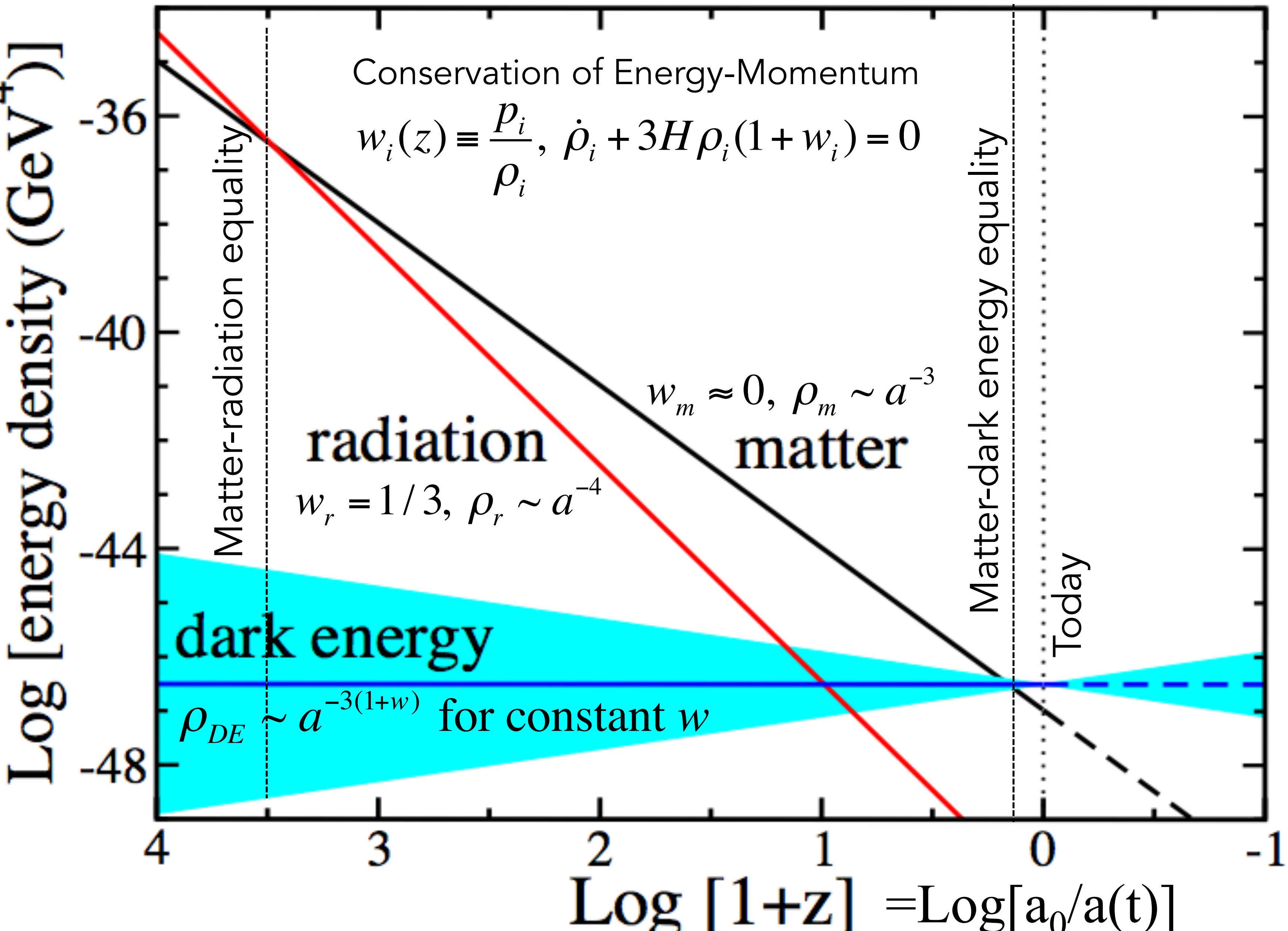
Here assuming $w_{DE}=-1$

ΛCDM Universe



Combining experiments now yields sub-percent precison on these values in context of ΛCDM.

Equation of State parameter *w* determines Cosmic Evolution



Three Epochs

- The evolution of the scale factor is determined by the dominant component *i*: for constant w_i, $a(t) \sim t^{2/3(1+w_i)}$.

- Radiation-dominated: $z>5000$: $a \sim t^{1/2}$, $T \sim 1/a \sim t^{-1/2}$
- Matter-dominated: $5000>z>0.3$: $a \sim t^{2/3}$
  - Note: CMB last-scattering $z_{LS}=1100 \rightarrow t_{LS}=380{,}000$ yr
- Dark energy-dominated: $z<0.3$: for $w_{DE}=-1$, $a \sim e^{Ht}$

- More generally, for matter+Λ,

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \left(\sinh\left[\frac{3\sqrt{\Omega_\Lambda} H_0 t}{2}\right]\right)^{2/3}$$

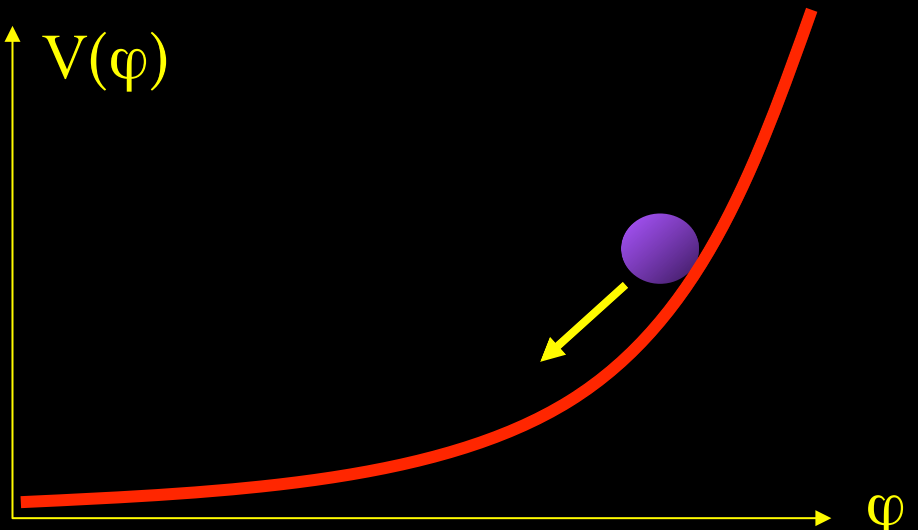
Scalar Field Dark Energy

- Dark Energy could be due to a very light scalar field **φ**, slowly evolving in a potential, V(**φ**):

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

- Density & pressure:

$$\rho = \tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)$$
$$P = \tfrac{1}{2}\dot{\varphi}^2 - V(\varphi)$$



- Slow roll:

$$\tfrac{1}{2}\dot{\varphi}^2 < V(\varphi) \Rightarrow P < 0 \Leftrightarrow w < 0 \text{ and time - dependent}$$

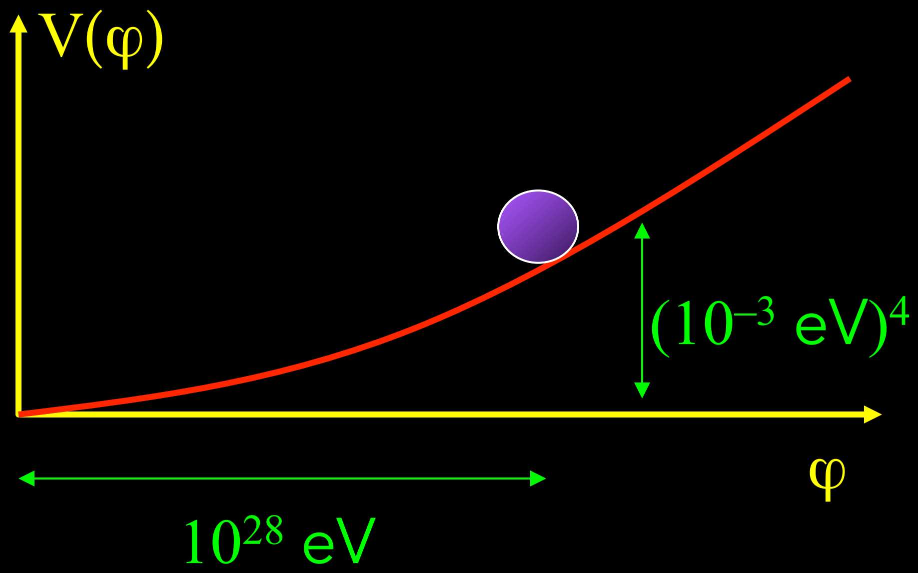
Scalar Field Dark Energy

General features:

$m < 3H_0 \sim 10^{-33}$ eV ($w < 0$)
(Potential > Kinetic Energy)

$V \sim m^2\varphi^2 \sim \rho_{crit} \sim 10^{-10}$ eV^4

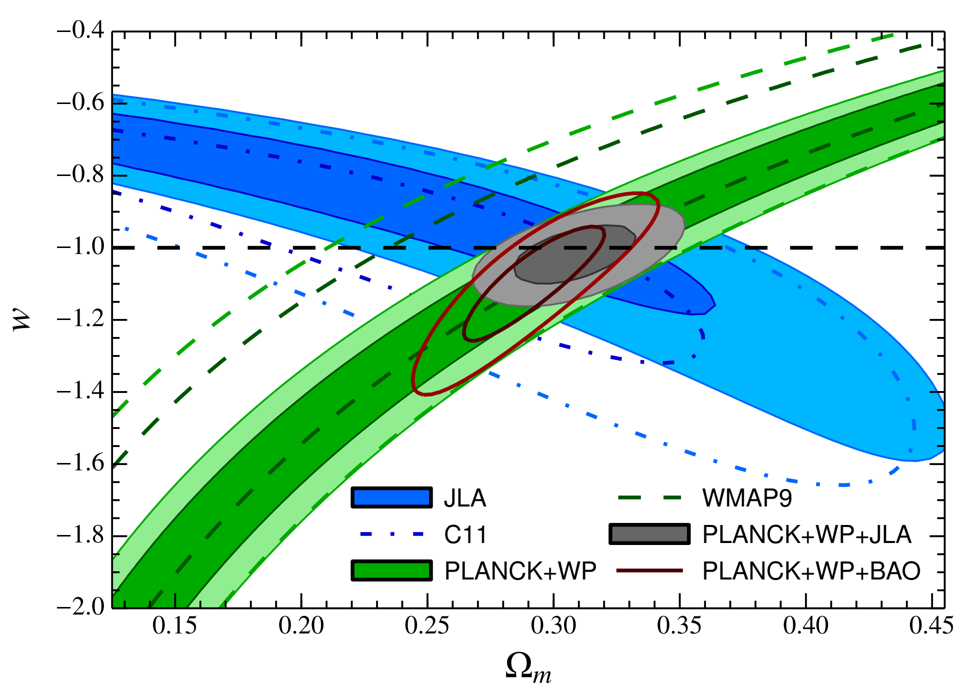
$\varphi \sim 10^{28}$ eV $\sim M_{Planck}$



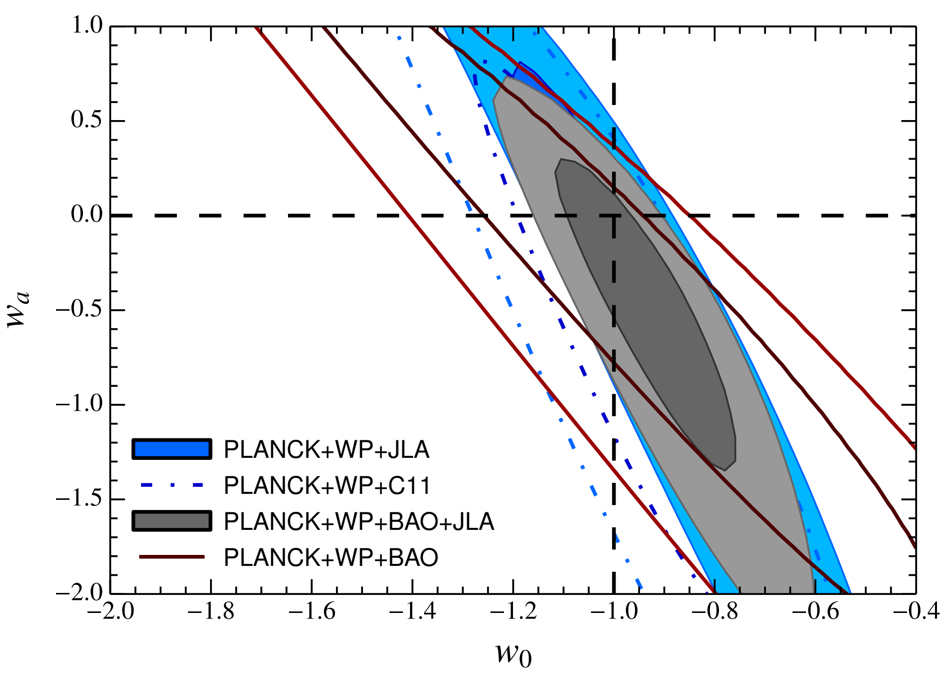
Ultra-light particle: Dark Energy hardly clusters, nearly smooth
Equation of state: $w > -1$ and evolves in time
Hierarchy problem: Why $m/\varphi \sim 10^{-61}$?
Weak coupling: Quartic self-coupling $\lambda_\phi < 10^{-122}$

Dark Energy Constraints from Supernovae, CMB, and Large-scale Structure

Assuming constant w



Assuming $w=w_0+w_a(1-a)$

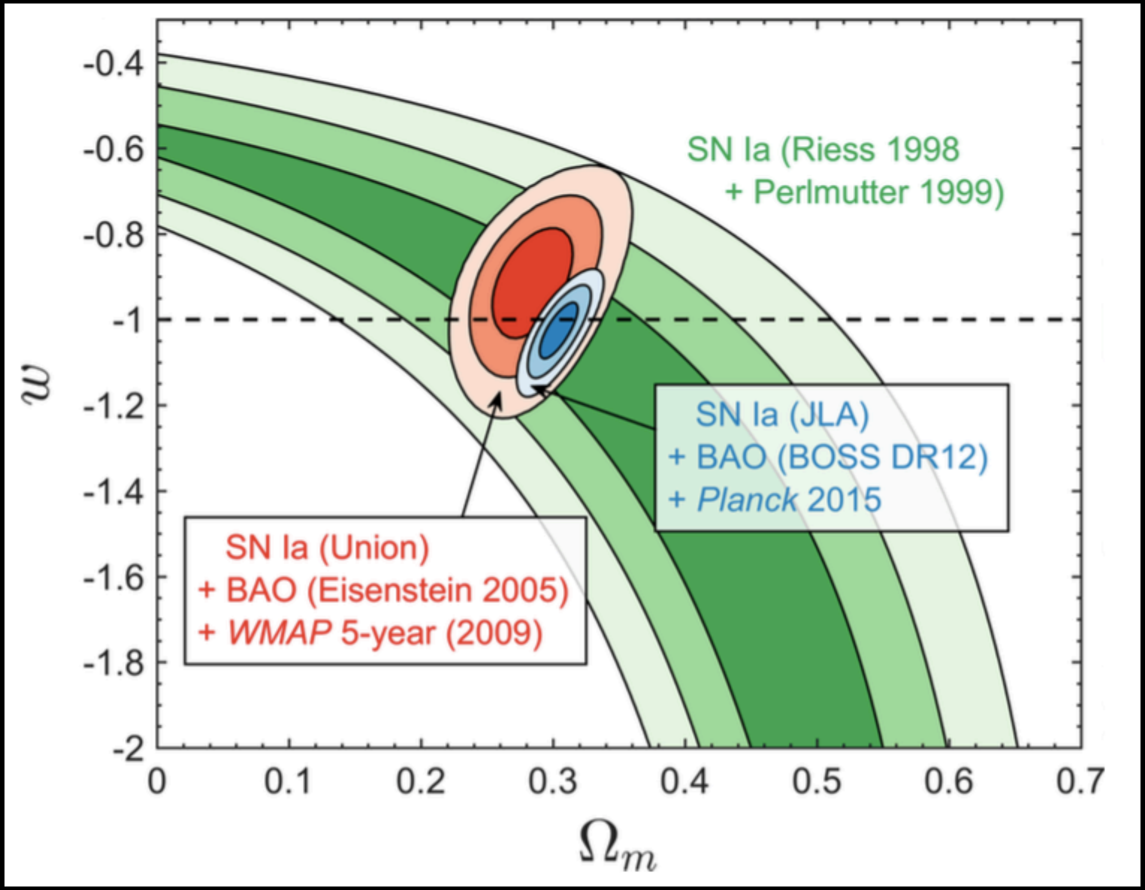


Betoule etal 2014

Consistent with vacuum energy (Λ): $w_0=-1$, $w_a=0$

Progress on w

assuming constant *w*



Huterer & Shafer in prep

Consistent with vacuum energy (Λ): $w = -1$

Early 1990's: Circumstantial Evidence for Λ

Primordial inflation successfully accounted for large-scale smoothness and structure of the Universe and predicted density of the Universe should be the critical density needed for the geometry to be flat: $\Omega_{tot}=1$.

Measurements of the amount of matter in galaxies and clusters indicated not enough dark matter for a flat Universe ($\Omega_m \sim 0.2$): there must be additional unseen, unclustered stuff to make up the difference, if inflation is correct.

Measurements of large-scale structure (APM survey) were consistent with primordial perturbations from inflation with Cold Dark Matter plus Λ.

Hubble parameter and globular cluster age measurements suggested $H_0 t_0 \geq 1$, requiring dark energy or Λ.

The 2nd order Friedmann equation for a single component Universe gives

$$\left(\frac{\ddot{a}}{a}\right)_0 = -\frac{4\pi G}{3}(\rho_0 + 3p_0) \ . \tag{18}$$

From the first order Friedmann equation, the density parameter is given by

$$\Omega_0 = \frac{\rho_0}{\rho_{crit}} = \frac{\rho_0}{3H_0^2/8\pi G} = \frac{8\pi G\rho_0}{3H_0^2} \ , \tag{19}$$

so that

$$H_0^2 = \frac{8\pi G}{3}\frac{\rho_0}{\Omega_0} \ . \tag{20}$$

Combining Eqns. 18 and 20 gives the deceleration parameter,

$$\begin{aligned} q_0 &= -\left(\frac{a\ddot{a}}{\dot{a}^2}\right)_0 = -\frac{\ddot{a}_0}{H_0^2 a_0} = \frac{4\pi G}{3}(\rho_0 + 3p_0)\frac{3\Omega_0}{8\pi G\rho_0} \\ &= \frac{\Omega_0}{2}\left(1 + \frac{3p_0}{\rho_0}\right) \ . \end{aligned} \tag{21}$$

For a multi-component Universe, this generalizes to

$$q_0 = \frac{1}{2}\sum_i \Omega_i(1 + 3w_i) \ , \tag{22}$$

where the equation of state parameter $w_i = p_i/\rho_i$. For non-relativistic matter plus dark energy, this becomes

$$q_0 = \frac{\Omega_m}{2} + \frac{\Omega_{DE}}{2}(1 + 3w) \ . \tag{23}$$

Exercises

1. If the Universe contains only non-relativistic matter and vacuum energy (Λ) and is spatially flat, calculate the value of the present matter density parameter, Ω_m, such that the Universe today is just marginally accelerating.

2. If Ω_m=0.3 and Ω_Λ=0.7, determine the redshift at which the Universe starts to accelerate and the redshift of matter-vacuum energy equality.

3. Suppose H_0=70 km/sec/Mpc and is constant in time. For a galaxy at a distance of 100 Mpc, calculate the increase in its recession speed (in km/sec) over a 10-year period. How might you nevertheless measure this "Hubble drift", which would be a direct measurement of cosmic acceleration?

How do we measure cosmological parameters?

- They impact the expansion history of the Universe:

$$E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_{DE}\exp\left[3\int(1+w(z))d\ln(1+z)\right] + \left(1-\Omega_m-\Omega_{DE}\right)\left(1+z\right)^2$$

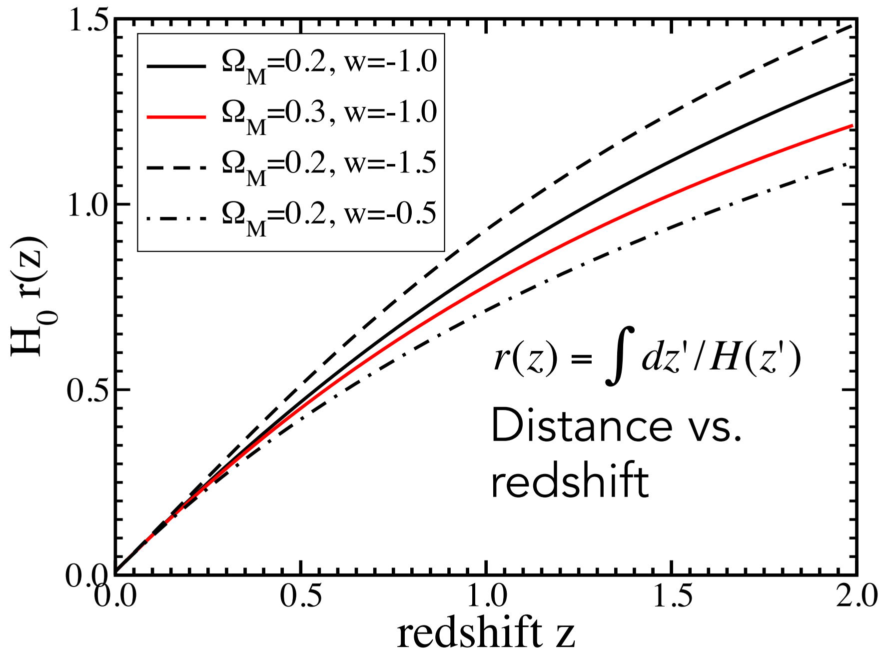
- and the growth & scale-dependence of large-scale density perturbations:

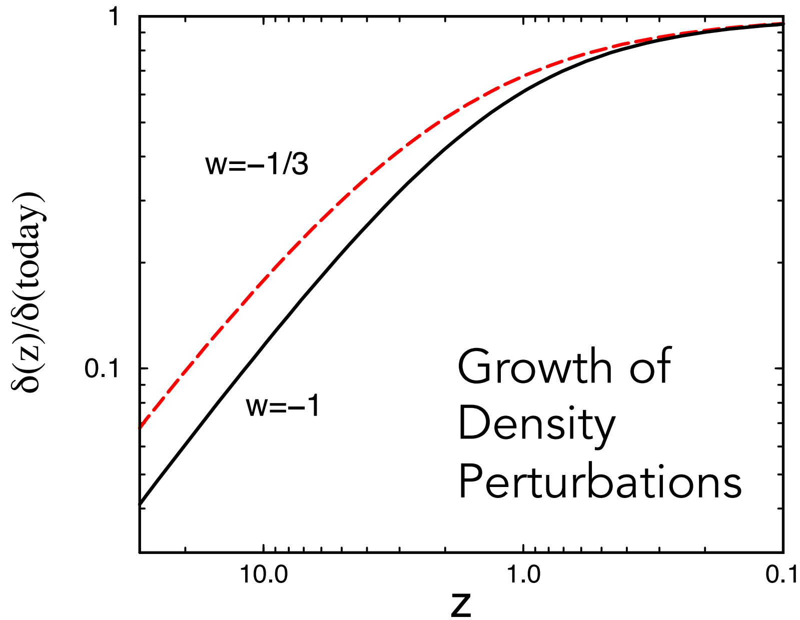
$$\frac{\delta\rho}{\rho}(k,z;\ \Omega_m,\Omega_{DE},w(z),n_s,H_0,\sigma_8,.\Omega_b,...)$$

See Lecture 3

- Find observables that are sensitive to these.

Geometry & Structure





• Weak Lensing	Distances+growth
• Supernovae	Distances
• Baryon Acoustic Oscillations	Distances and H(z)
• Cluster counts	Distances+growth
• Redshift Distortions	Growth

Cosmological Observables I: Geometry

Friedmann-Lemaitre-Robertson-Walker Metric:

$$ds^2 = c^2dt^2 - a^2(t)\left[d\chi^2 + S_k^2(\chi)\left\{d\theta^2 + \sin^2\theta\, d\phi^2\right\}\right]$$

$$= c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left\{d\theta^2 + \sin^2\theta\, d\phi^2\right\}\right]$$

where

$$r = S_k(\chi) = \sinh(\chi),\ \chi,\ \sin(\chi) \text{ for } k = -1,0,1$$

Comoving distance:

$$cdt = a\, d\chi \;\Rightarrow$$

$$\chi(a) = \int \frac{cdt}{a'} = \int \frac{cdt}{a'da'}da' = c\int_0^a \frac{da'}{a'^2 H(a')}$$

$$\chi(z) = c\int_0^z \frac{dz'}{H(z')} = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}$$

Coordinate Distance and q_0

$$H(z) = H_0 \left[1 + (1+q_0)z\right] \ . \tag{11}$$

The coordinate distance is

$$a_0\chi = a_0 \int \frac{dt}{a(t)} = a_0 \int \frac{dt}{da}\frac{da}{a} = a_0 \int \frac{da}{H(a)a^2} \ . \tag{12}$$

Using Eqn. 9, this can be written as

$$a_0\chi(z) = \int \frac{dz}{H(z)} \ . \tag{13}$$

Using Eqn. 11, this becomes

$$a_0\chi(z) = \int \frac{dz}{H_0[1+(1+q_0)z]} \simeq \frac{1}{H_0}\int dz[1-(1+q_0)z] = \frac{1}{H_0}\left[z-(1+q_0)\frac{z^2}{2}\right] \ . \tag{14}$$

The radial distance $r = \sin\chi, \chi, \sinh\chi$ for $k = +1, 0, -1$. For small distances, $\chi \ll 1$, this means $r = \chi \pm \mathcal{O}(\chi^3)$. Since, from Eqn. 14, $\chi \propto z + \mathcal{O}(z^2)$, the expression for $a_0 r(z)$ to $\mathcal{O}(z^2)$ is identical to the expression for $a_0\chi(z)$ to the same order, i.e., Eqn. 14.

Coordinate Distance and q_0

Recall $$q_0 = \frac{\Omega_m}{2} + \frac{\Omega_{DE}}{2}(1+3w)$$

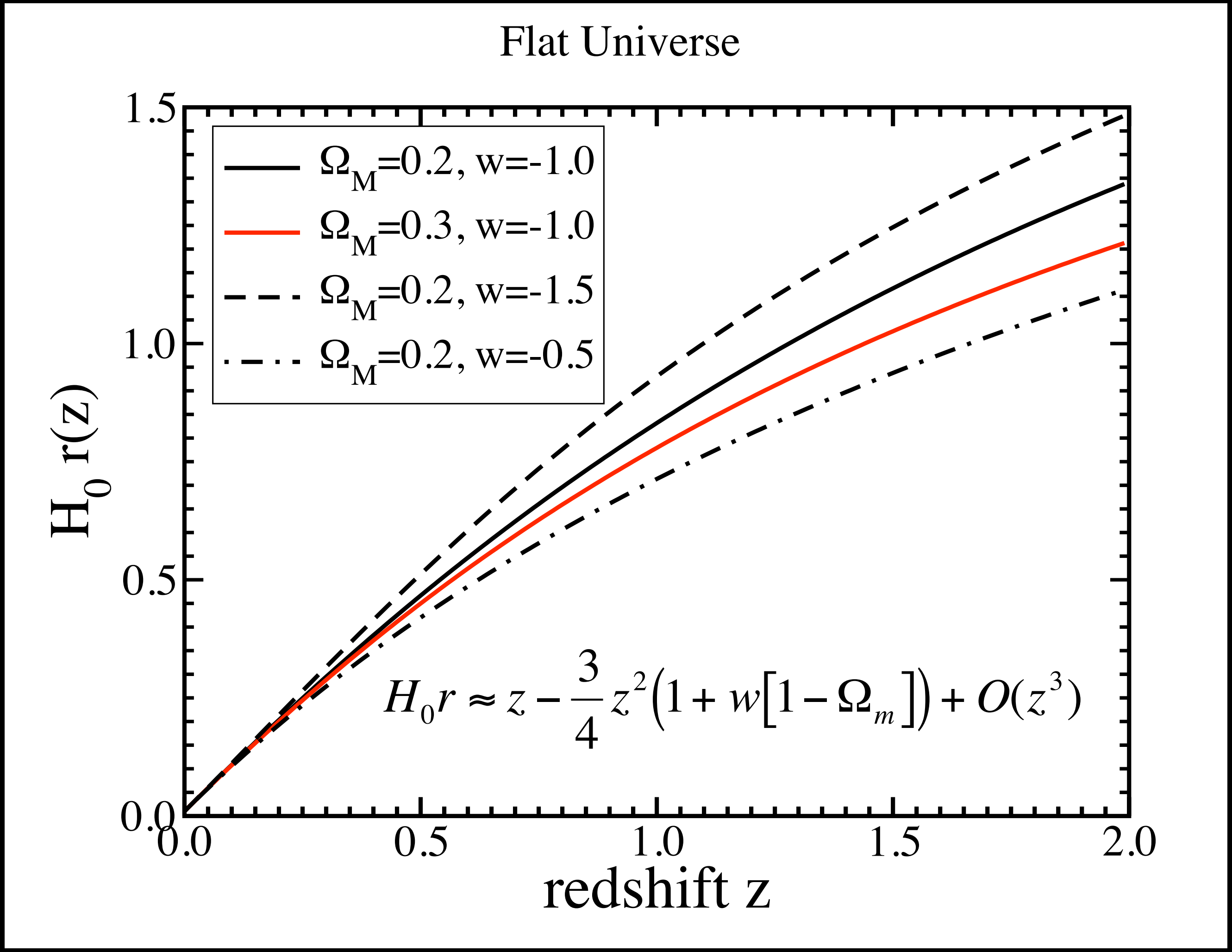
For a flat Universe, $\Omega_{DE} = 1 - \Omega_m$;

$$q_0 = \frac{\Omega_m}{2} + \frac{(1-\Omega_m)}{2}(1+3w) = \frac{1}{2} + \frac{3w}{2}(1-\Omega_m)\ , \qquad (25)$$

$$H_0 r \approx z - (1+q_0)\frac{z^2}{2} + O(z^3)$$
$$\approx z - \frac{3}{4}z^2\left(1 + w\left[1-\Omega_m\right]\right) + O(z^3)$$

Not accurate, but indicates scaling with parameters

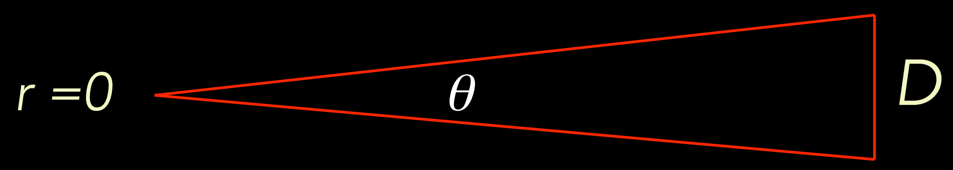
Coordinate Distance



Percent-level determination of *w* requires percent-level distance estimates

Angular Diameter Distance

- Observer at $r = 0$, t_0 sees source of proper diameter D at coordinate distance r which emitted light at time t:

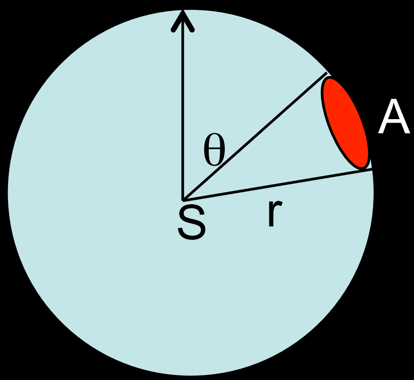


- From FLRW metric, proper distance across the source is $D = a(t) r\theta$ so the angular diameter of the source is $\theta = D / a(t) r$
- In Euclidean geometry, $d = D/\theta$ so we define the

Angular Diameter Distance: $d_A \equiv \dfrac{D}{\theta} = a(t)r = a(t)S_k(\chi) = \dfrac{r}{1+z}$

Luminosity Distance

- Source S at origin emits light at time t_1 into solid angle $d\Omega$, received by observer O at coordinate distance r at time t_0, with detector of area A: (by convention, choose $a_0=1$)



Proper area of detector given by the metric:

$$A = r\ d\theta\ r\sin\theta\ d\phi = r^2 d\Omega$$

Unit area detector at O subtends solid angle

$d\Omega = 1/r^2$ at S.

Power emitted into $d\Omega$ is $dP = L\,d\Omega/4\pi$

Energy flux received by O per unit area is

$$f = \frac{L\ d\Omega}{4\pi} = \frac{L}{4\pi r^2}$$

Include Expansion

- Expansion reduces received flux due to 2 effects:
  1. Photon energy redshifts: $E_\gamma(t_0) = E_\gamma(t_1)/(1+z)$
  2. Photons emitted at time intervals δt_1 arrive at time intervals δt_0:

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)}$$

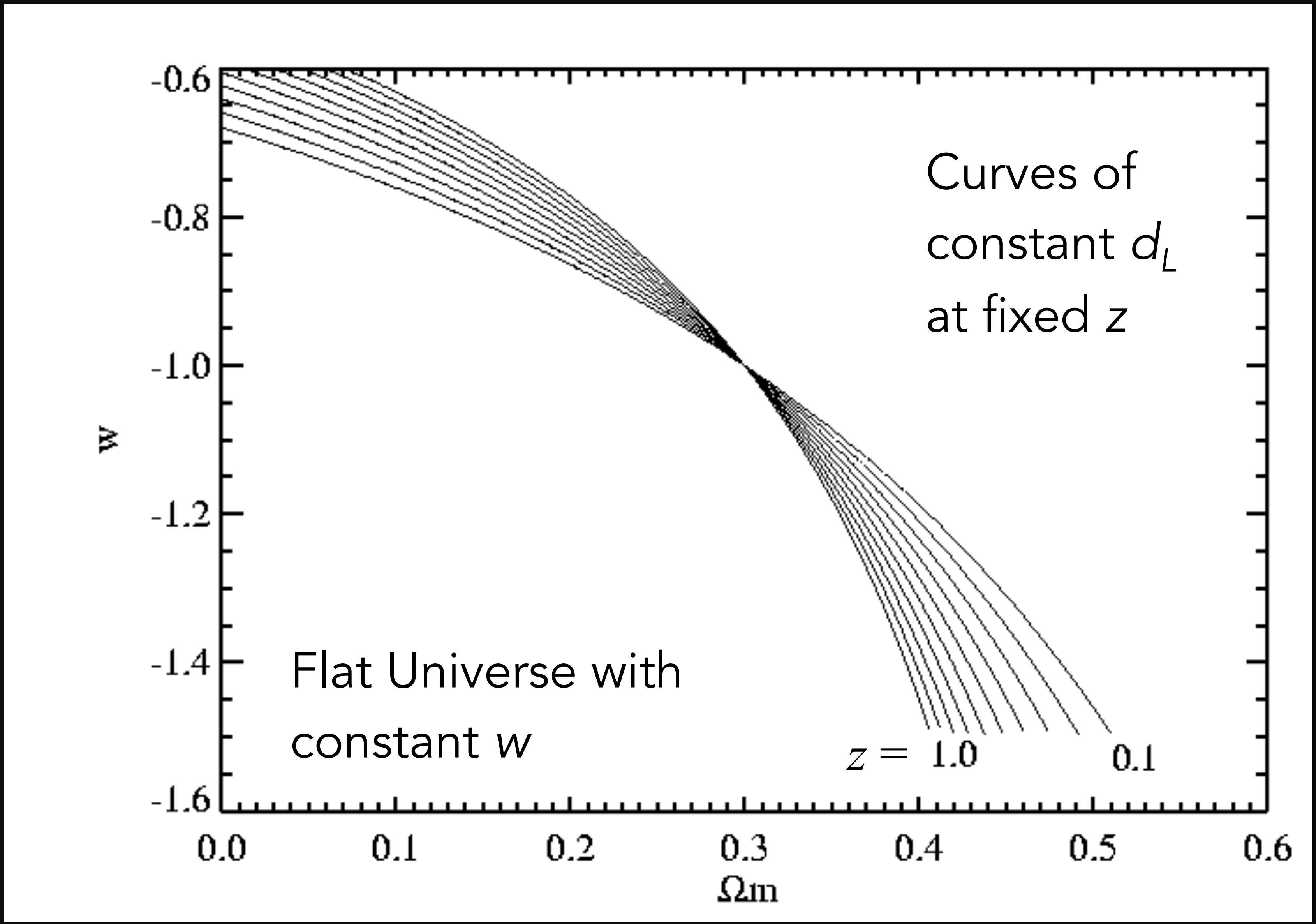
$$\int_{t_1}^{t_1+\delta t_1} \frac{dt}{a(t)} + \int_{t_1+\delta t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1+\delta t_1}^{t_0} \frac{dt}{a(t)} + \int_{t_0}^{t_0+\delta t_0} \frac{dt}{a(t)}$$

$$\frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)} \quad \Rightarrow \quad \frac{\delta t_0}{\delta t_1} = \frac{a(t_0)}{a(t_1)} = 1+z$$

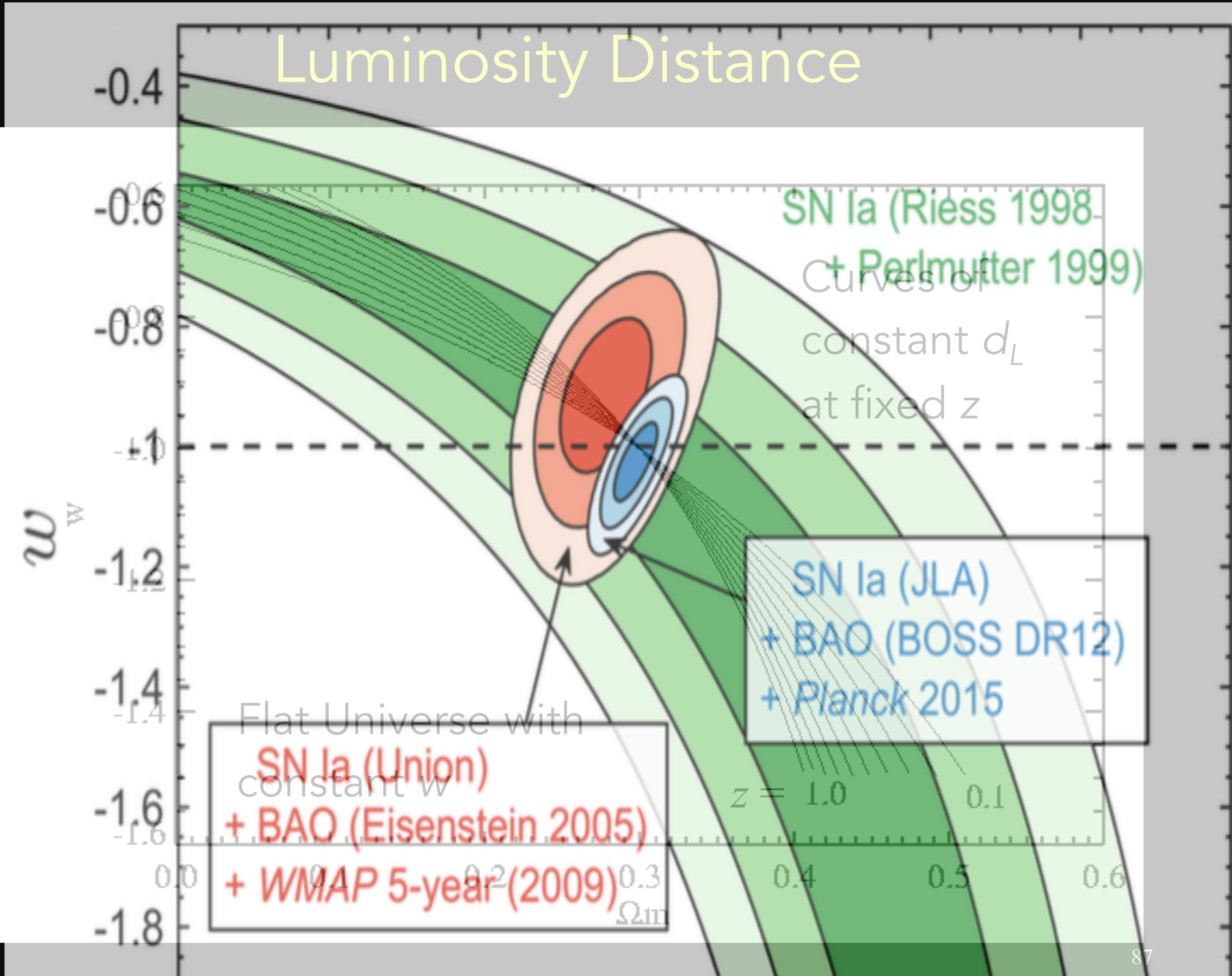
$$f = \frac{L\, d\Omega}{4\pi} = \frac{L}{4\pi r^2 (1+z)^2} \equiv \frac{L}{4\pi d_L^2} \quad \Rightarrow \quad d_L = r(1+z) = (1+z)^2 d_A$$

Luminosity Distance

Luminosity Distance



Luminosity Distance



Curves of constant d_L at fixed z

Flat Universe with constant w

Distance Modulus

- Consider logarithmic measures of luminosity and flux:

$$M = -2.5\log(L) + c_1, \quad m = -2.5\log(f) + c_2$$

- Define distance modulus:

flux — measure redshift from spectra

$$\mu \equiv m - M = 2.5\log(L/f) + c_3 = 2.5\log(4\pi d_L^2) + c_3$$
$$= 5\log[H_0 d_L(z;\Omega_m,\Omega_{DE},w(z))] - 5\log H_0 + c_4$$
$$= 5\log[d_L(z;\Omega_m,\Omega_{DE},w(z))/10\text{pc}]$$

- For a population of *standard candles* (fixed *M*), measurements of μ vs. *z*, aka the Hubble diagram, constrain cosmological parameters.

K corrections due to redshift

SN spectrum

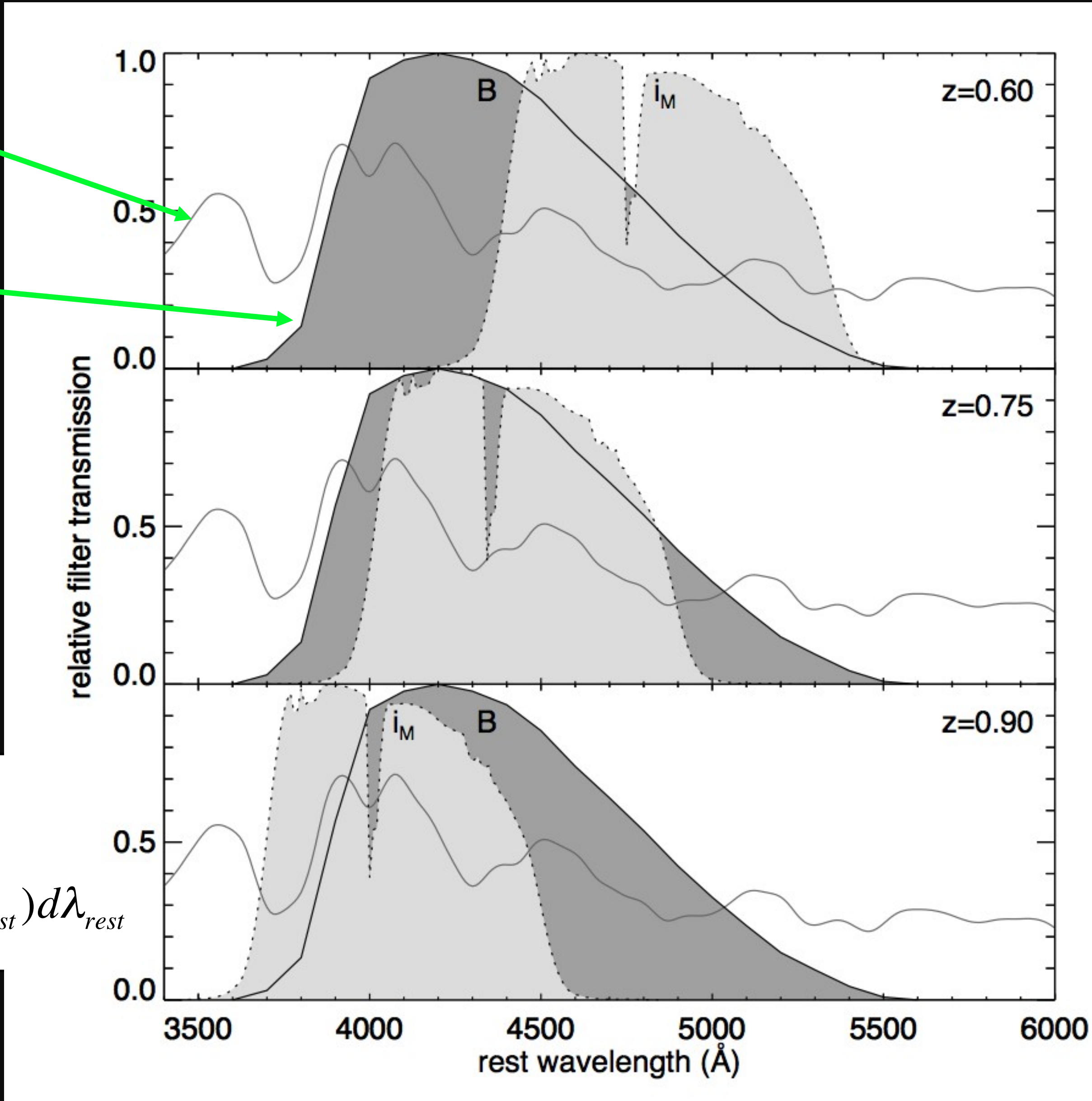
Rest-frame *B* band filter

Equivalent restframe *i* band filter at different redshifts

(i_{obs}=7000-8500 A)

$$f_i = \int S_i(\lambda) F_{obs}(\lambda) d\lambda$$

$$= (1+z)\int S_i[\lambda_{rest}(1+z)] F_{rest}(\lambda_{rest}) d\lambda_{rest}$$



Absolute vs. Relative Distances

- Recall logarithmic measures of luminosity and flux:

$$M_i = -2.5\log(L_i) + c_1, \quad m_i = -2.5\log(f_i) + c_2$$

$$m_i = 5\log[H_0 d_L] - 5\log H_0 + M_i + K(z) + c_4$$

i=passband; K corrects for flux redshifting out of passband

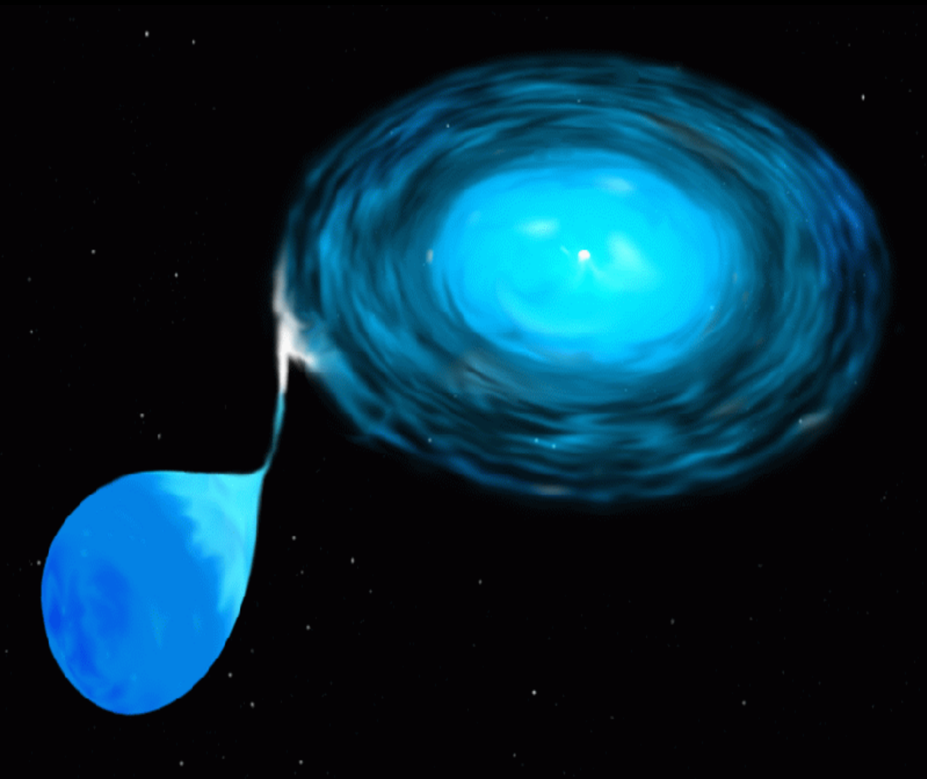
- If M_i is known, measurement of m_i → absolute distance to object at redshift z, thereby determine H_0 (for $z<<1$, $d_L=cz/H_0$)
- If M_i (and H_0) *un*known but constant, from measurement of m_i can infer distance to object at redshift z_1 *relative* to object at z_2:

$$m_1 - m_2 = 5\log\left(\frac{d_1}{d_2}\right) + K_1 - K_2$$

independent of H_0 .

- Use low-redshift SNe to vertically `anchor' the Hubble diagram, i.e., to determine $M - 5\log H_0$

Type Ia Supernovae

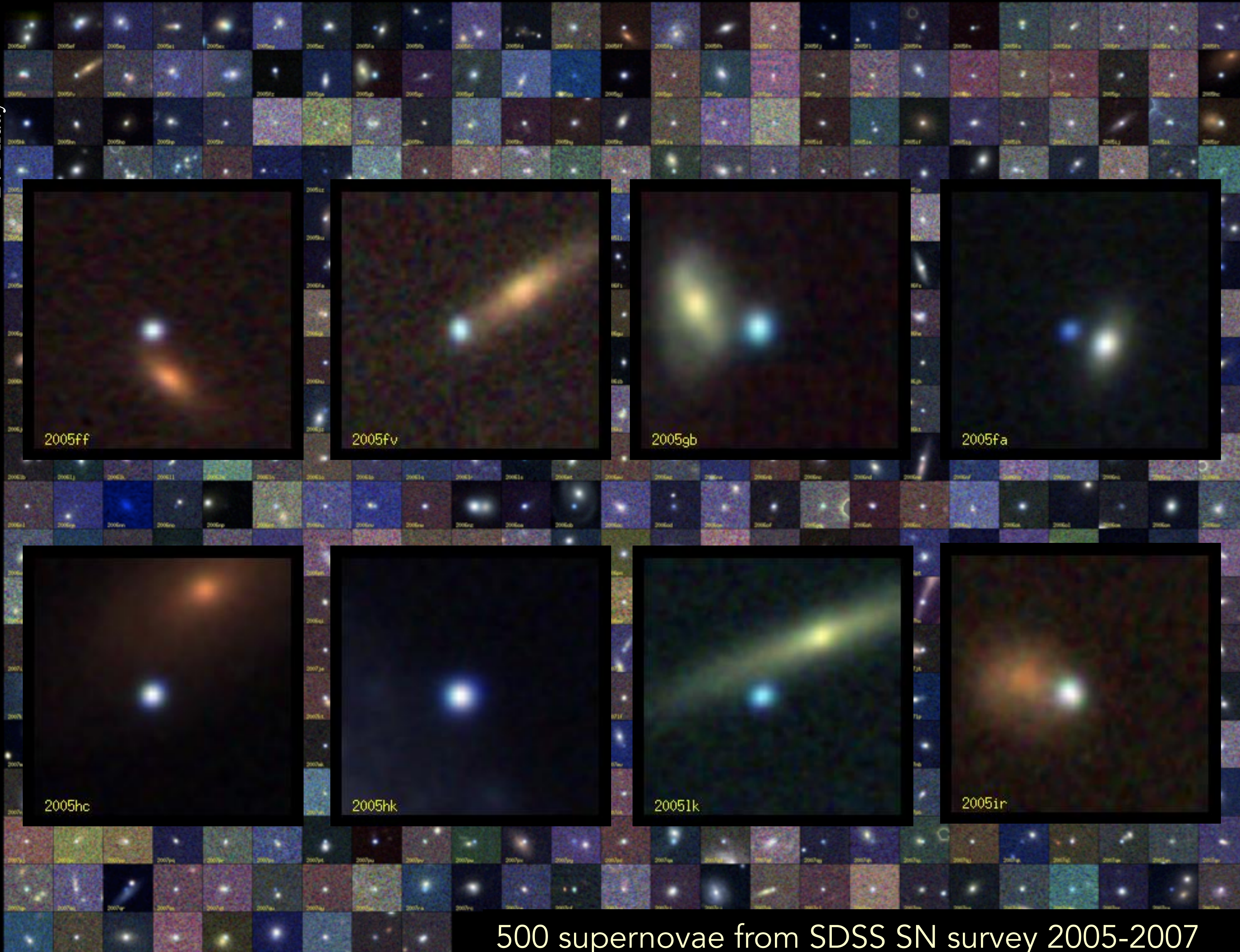


Thermonuclear explosions of Carbon-Oxygen White Dwarfs

White Dwarf accretes mass from or merges with a companion star, growing to a critical mass ~$1.4M_{sun}$ (Chandrasekhar)

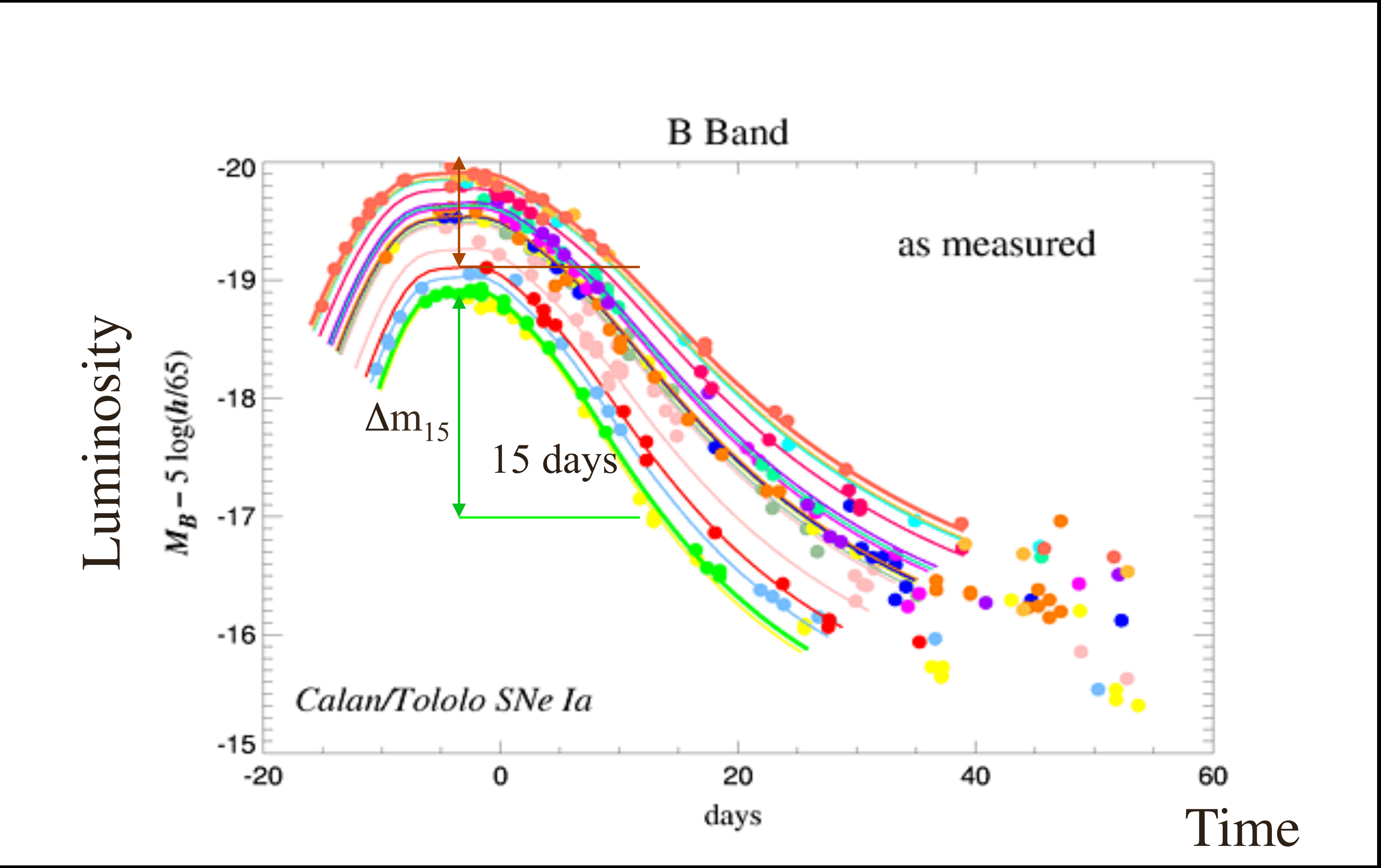
In the core of the star, light elements are burned in fusion reactions to form Nickel. Radioactive decay of Nickel and Cobalt powers light-curve for a couple of months.

B. Dilday



500 supernovae from SDSS SN survey 2005-2007

Type Ia Supernovae as Standardizable Candles



Empirical Correlation: Brighter SNe Ia decline more slowly and are bluer

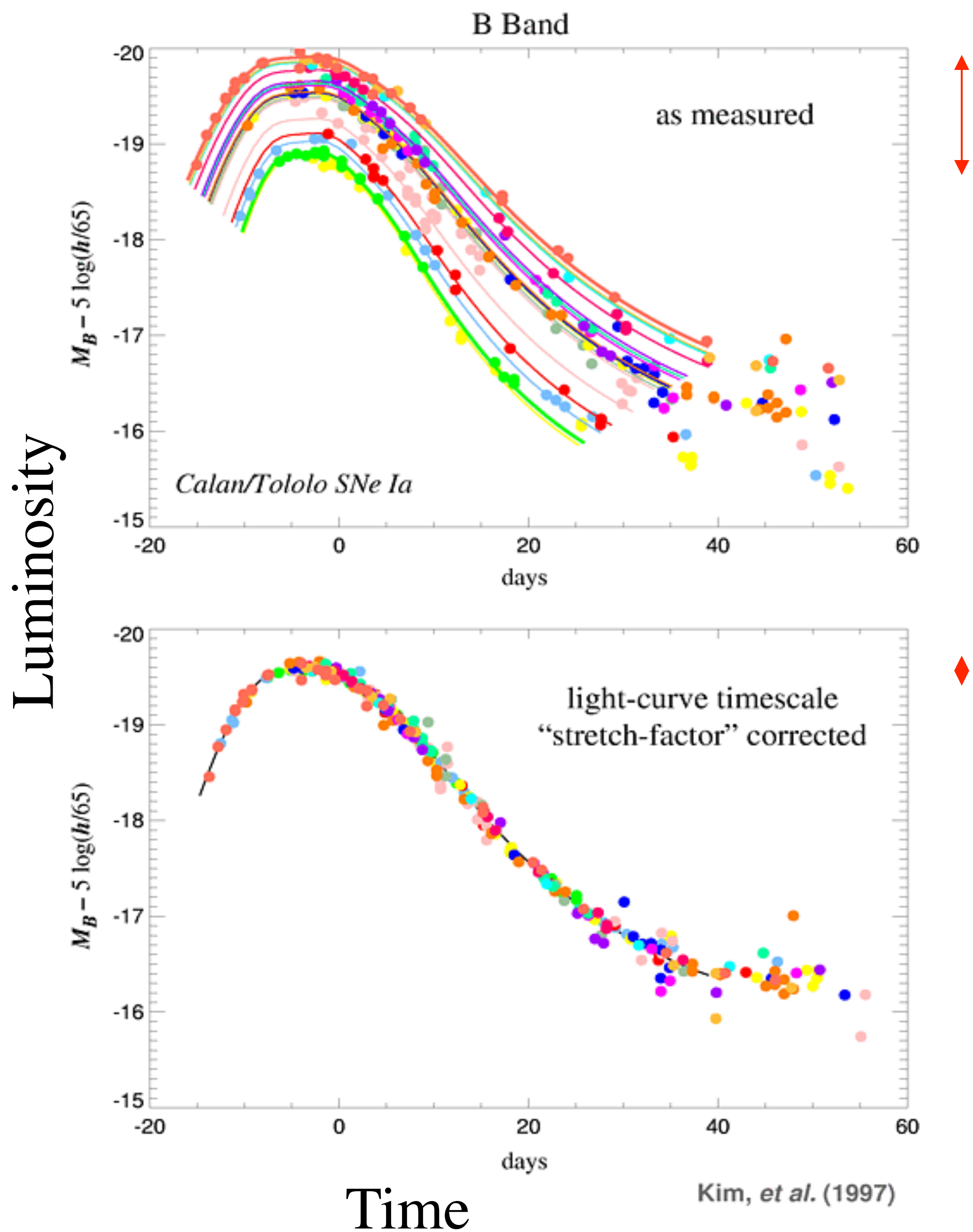
Phillips 1993

Type Ia SNe as calibrated Standard Candles

Peak brightness correlates with decline rate & color

Correct distance modulus for these correlations

After correction, $\sigma_\mu \sim 0.14$ mag (~7% relative distance error)

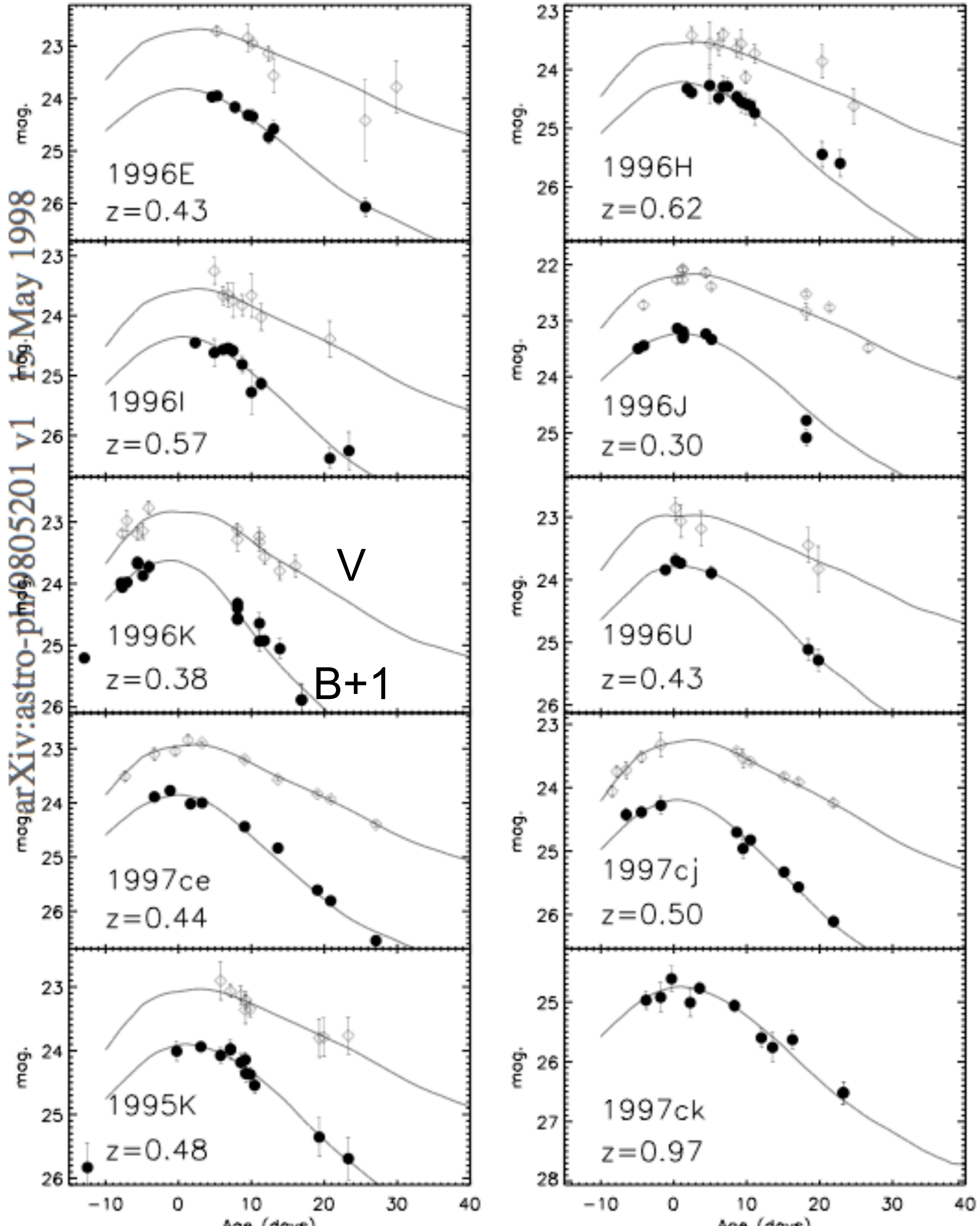


Acceleration Discovery Data: High-z SN Team

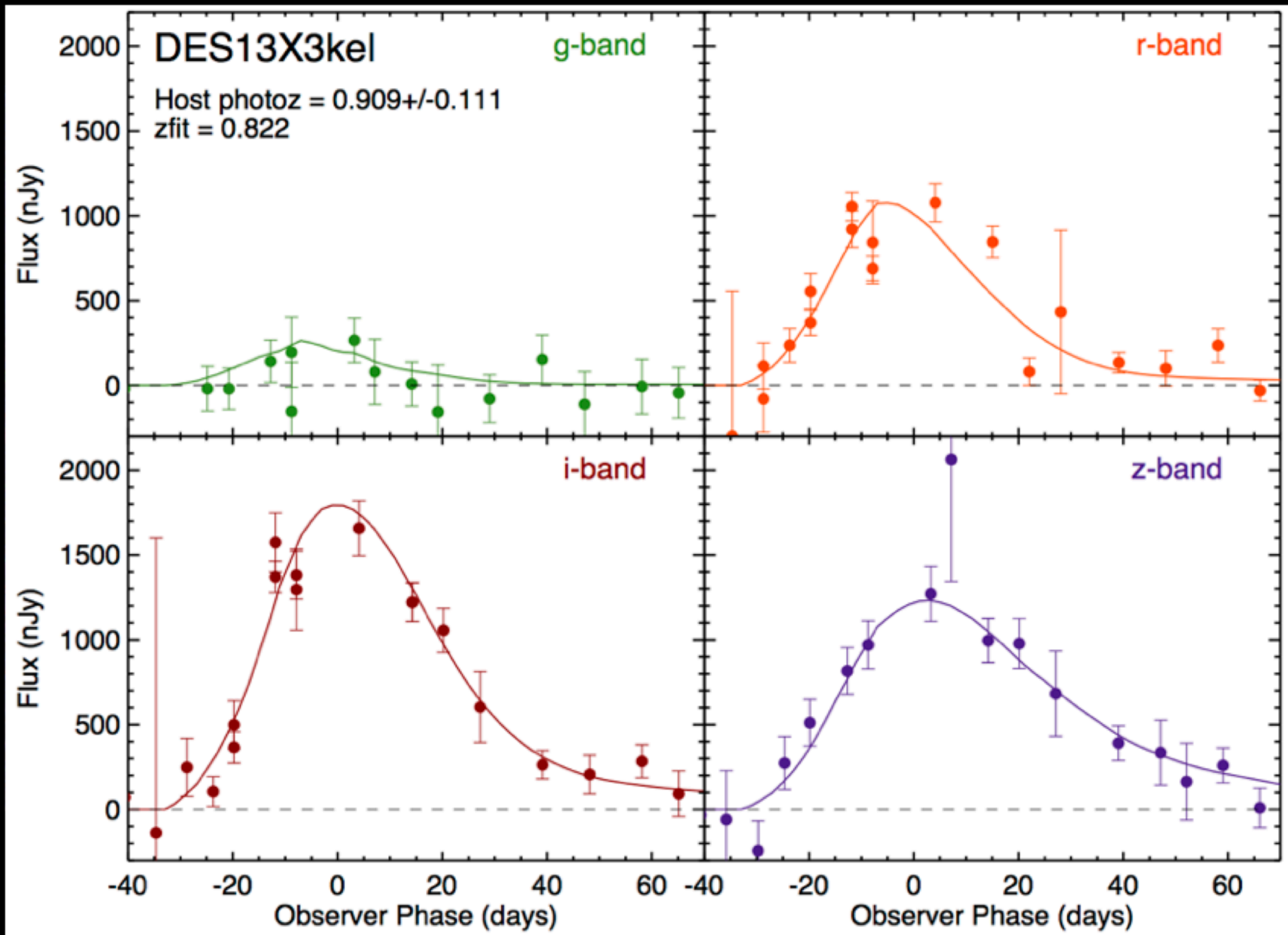
10 of 16 shown; transformed to SN rest-frame

Riess etal

Schmidt etal

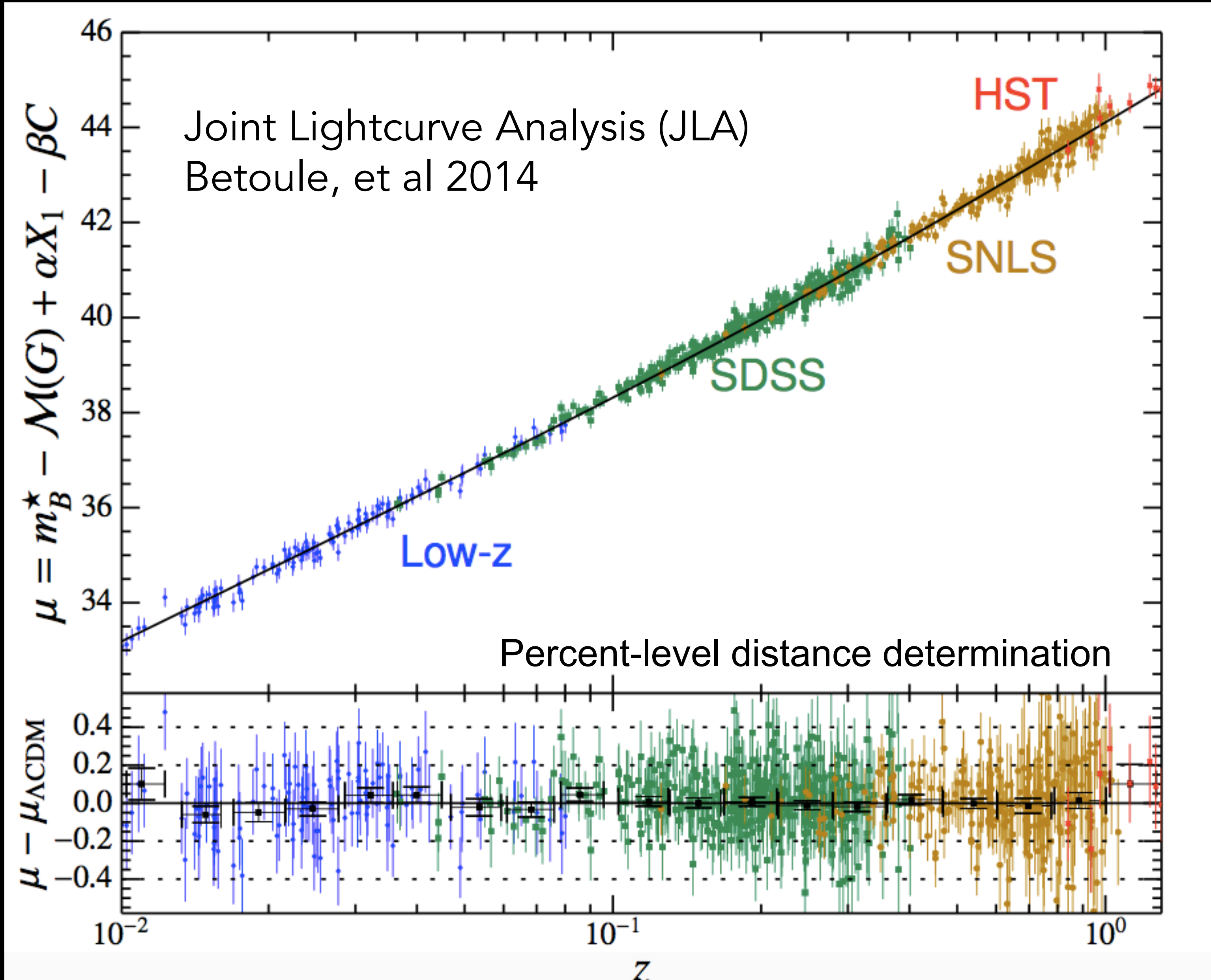


DES High-Redshift Supernovae



Slide from C. D'Andrea

Supernova Ia Hubble Diagram



Coordinate Distance and q_0

$$H(z) = H_0\left[1 + (1+q_0)z\right] \; . \tag{11}$$

The coordinate distance is

$$a_0\chi = a_0 \int \frac{dt}{a(t)} = a_0 \int \frac{dt}{da}\frac{da}{a} = a_0 \int \frac{da}{H(a)a^2} \; . \tag{12}$$

Using Eqn. 9, this can be written as

$$a_0\chi(z) = \int \frac{dz}{H(z)} \; . \tag{13}$$

Using Eqn. 11, this becomes

$$a_0\chi(z) = \int \frac{dz}{H_0[1+(1+q_0)z]} \simeq \frac{1}{H_0}\int dz[1-(1+q_0)z] = \frac{1}{H_0}\left[z - (1+q_0)\frac{z^2}{2}\right] \; . \tag{14}$$

The radial distance $r = \sin\chi, \chi, \sinh\chi$ for $k = +1, 0, -1$. For small distances, $\chi \ll 1$, this means $r = \chi \pm \mathcal{O}(\chi^3)$. Since, from Eqn. 14, $\chi \propto z + \mathcal{O}(z^2)$, the expression for $a_0 r(z)$ to $\mathcal{O}(z^2)$ is identical to the expression for $a_0\chi(z)$ to the same order, i.e., Eqn. 14.

Luminosity Distance and q_0

The luminosity distance is given by $d_L(z) = (1+z)a_0 r(z)$. Using Eqn. 14 and the result of part (d), to order z^2 this gives

$$\begin{aligned} d_L(z; H_0, q_0) &= \frac{z(1+z)}{H_0}\left[1-(1+q_0)\frac{z}{2}\right] = \frac{1}{H_0}\left[z+z^2-(1+q_0)\frac{z^2}{2}+\mathcal{O}(z^3)\right] \\ &= \frac{z}{H_0}\left[1+(1-q_0)\frac{z}{2}\right] . \end{aligned} \tag{15}$$

The distance modulus is given by

$$\begin{aligned} \mu(z; H_0, q_0) &= 5\log_{10}\left(\frac{d_L}{10\ \mathrm{pc}}\right) = 5\log_{10}\left[\frac{z}{H_0}\frac{1+(1-q_0)z/2}{10\ \mathrm{pc}}\right] \\ &= 5\log z - 5\log(H_0 \cdot 10\ \mathrm{pc}) + 5\log\left[1+\frac{z}{2}(1-q_0)\right] . \end{aligned} \tag{16}$$

The last term in Eqn. 16 can be massaged using Stirling's approximation: for $x \ll 1$, $\ln(1+x) \simeq x$. Exponentiating and taking the $\log_{10}$ gives $\log_{10}(1+x) \simeq \log_{10} e^x = x\log_{10} e$, so that

$$5\log_{10}\left[1+\frac{z}{2}(1-q_0)\right] \simeq \frac{5z}{2}(1-q_0)\log_{10} e = 1.086z(1-q_0) . \tag{17}$$

Distance Modulus and q_0

Recall $q_0 = \frac{\Omega_m}{2} + \frac{\Omega_{DE}}{2}(1+3w)$

For a flat Universe, $\Omega_{DE} = 1 - \Omega_m$; from Eqn. 22,

$$q_0 = \frac{\Omega_m}{2} + \frac{(1-\Omega_m)}{2}(1+3w) = \frac{1}{2} + \frac{3w}{2}(1-\Omega_m) \ , \qquad (25)$$

so the difference in distance modulus between two flat models with fixed H_0 and Ω_m is

$$\Delta\mu = \frac{3}{2}(1-\Omega_m)(1.086z)\Delta w = 0.6\Delta w \ , \qquad (26)$$

where the last expression is evaluated using $\Omega_m = 0.25$ and $z = 0.5$. Since $\sigma_\mu = 0.15$ mag, to determine w to a precision of $\Delta w = 0.1$ requires roughly $\Delta\mu = 0.06 > \sigma_\mu/\sqrt{N} = 0.15/\sqrt{N}$, or $N > 6$ supernovae. For a precision $\Delta w = 0.01$, we have $\Delta\mu = 0.006$, and we need $N > 600$ supernovae at $z \sim 0.5$. If Ω_m isn't exactly known and in the presence of systematic errors, this number of course would be larger.

Cosmic Volume Element

- Counting a set of objects, e.g., galaxy clusters, with known or predictable number density, provides a cosmological test
- Proper area dA at redshift z and radial coordinate r subtends solid angle dΩ at the origin given by

$$dA = a(t_e) r d\theta a(t_e) r \sin\theta d\varphi = a_e^2 r^2 d\Omega = \frac{r^2 d\Omega}{(1+z_e)^2}$$

- Rate of proper displacement with z along light ray is

$$d\ell = c dt = \frac{dz}{(1+z)H(z)}$$

=linear depth of sample in redshift interval (z,z+dz)

- Proper volume element of sample is then

$$d^2 V_p = dA d\ell = \frac{r^2(z)}{H(z)(1+z)^3} d\Omega dz$$

Volume Element

- For proper number density of objects $n_p(z)$, the number counts per unit redshift and solid angle are then

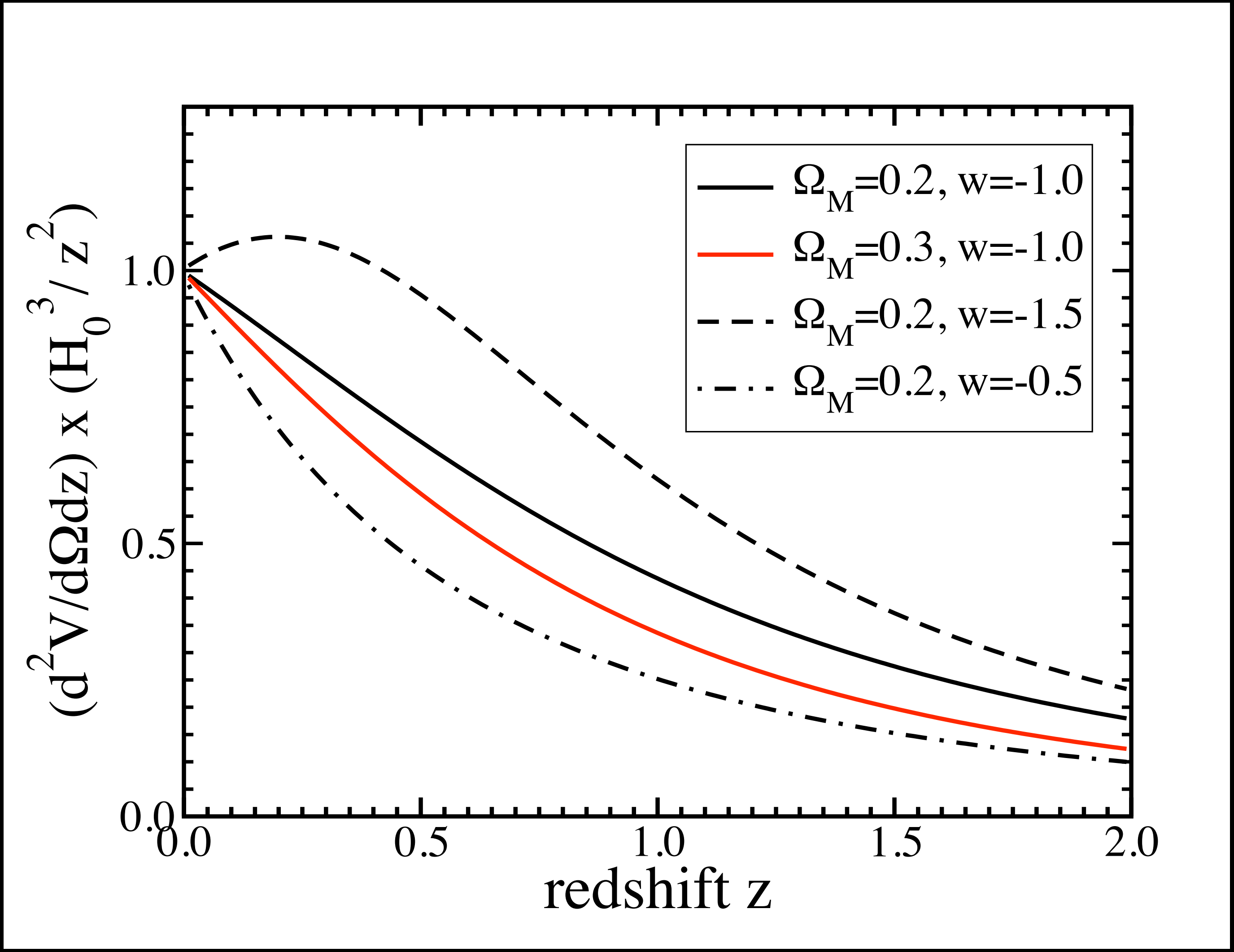
$$\frac{d^2N}{dz\,d\Omega} = n_p(z)\frac{d^2V_p}{dz\,d\Omega} = \frac{n_p(z)r^2(z)}{H(z)(1+z)^3}$$

- Define the comoving number density $n_c(z)=n_p(z)/(1+z)^3$, which is constant if objects are conserved, and comoving volume element $d^2V_c=d^2V_p(1+z)^3$, in which case

$$\frac{d^2N}{dz\,d\Omega} = n_c(z)\frac{d^2V_c}{dz\,d\Omega} = \frac{n_c(z)r^2(z)}{H(z)}$$

- For dark matter halos, structure formation theory predicts $n_c(M,z)$ as a function of cosmological parameters: primarily sensitive to rate of growth of linear density perturbations, $\delta(z)$.

Volume Element

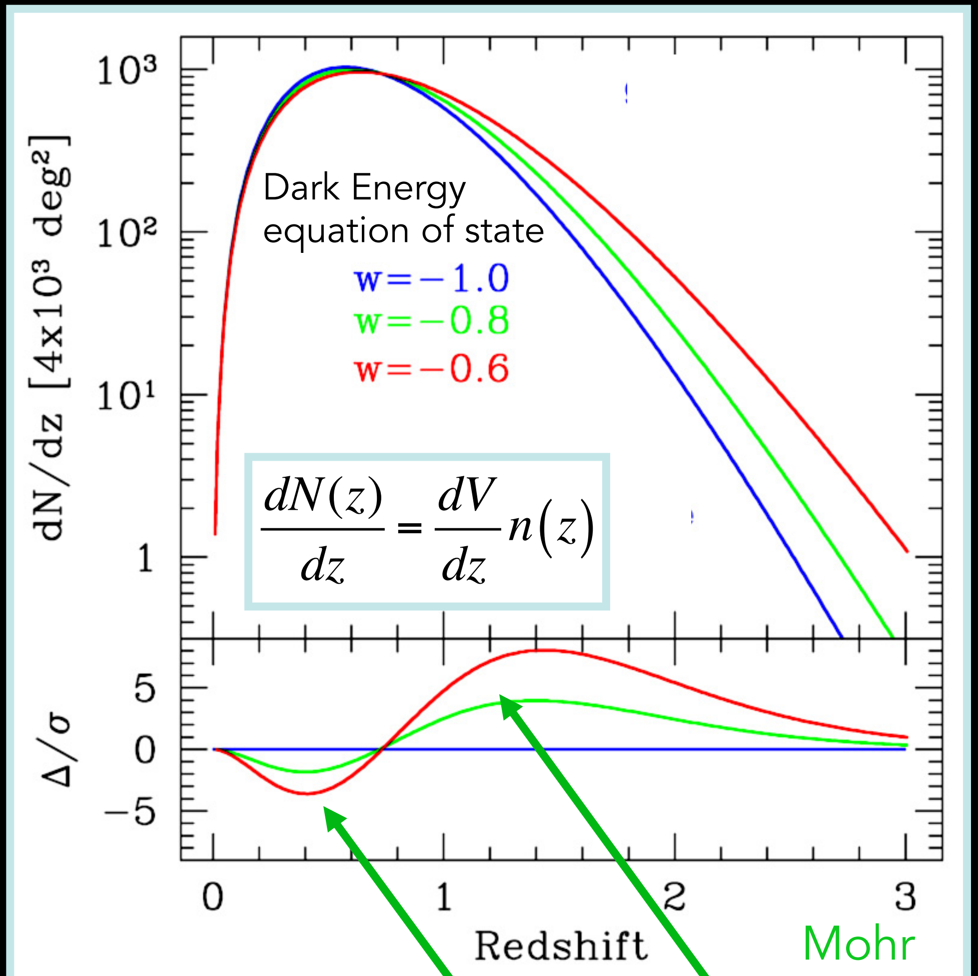


Raising *w* at fixed Ω_m decreases volume

Cluster Counts

- Clusters are proxies for massive dark matter halos and can be identified to z>1
- N-body simulations predict halo density $n(M,z; w,\Omega_m,\ldots)$
- Challenge: determine halo mass vs. cluster observable O relation $\underline{p(O|M,z)}$ with sufficient precision
- Multiple observable proxies O for cluster mass: optical richness, SZ flux, weak lensing mass, X-ray flux, velocity dispersion,…

Number of halos above mass threshold



$$\frac{dN(z)}{dz} = \frac{dV}{dz} n(z)$$

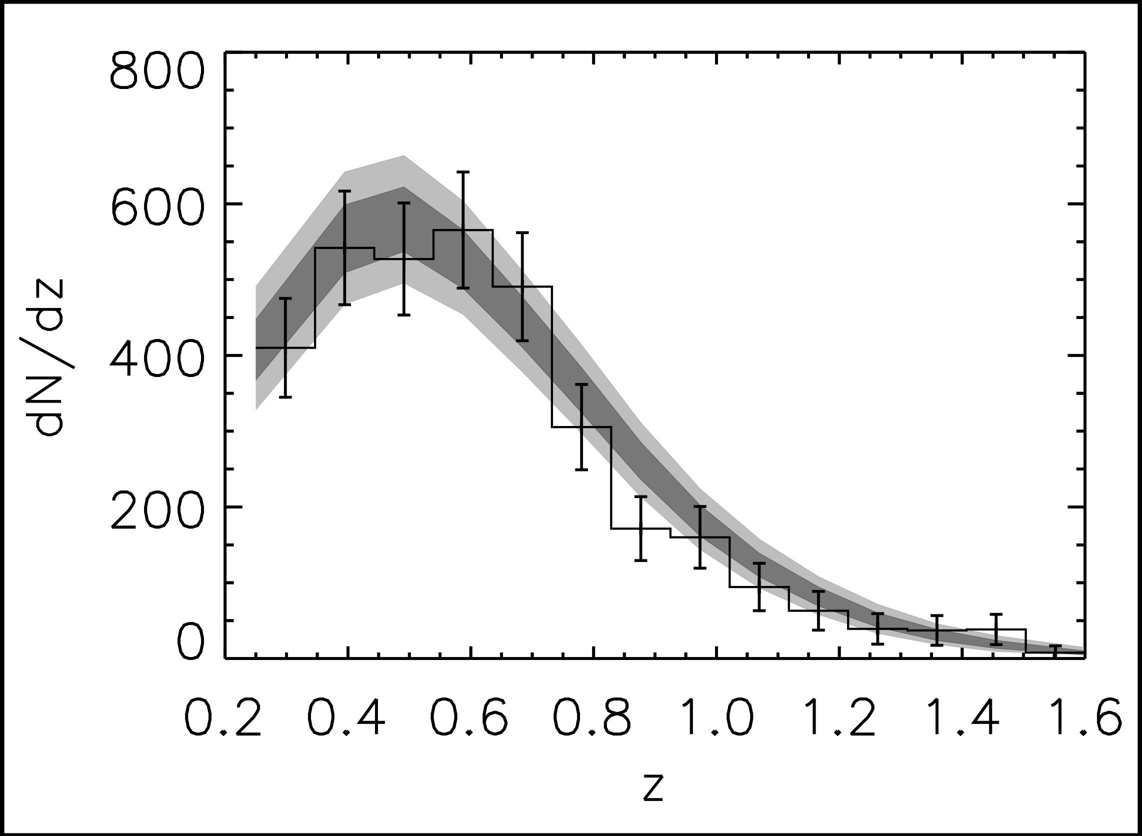
$$\frac{d^2N}{dz d\Omega} = \frac{r^2(z)}{H(z)} \int f(O,z) dO \int \underline{p(O \mid M,z)} \frac{dn(z)}{dM} dM$$

Volume

Growth

SPT-SZ Cluster Counts

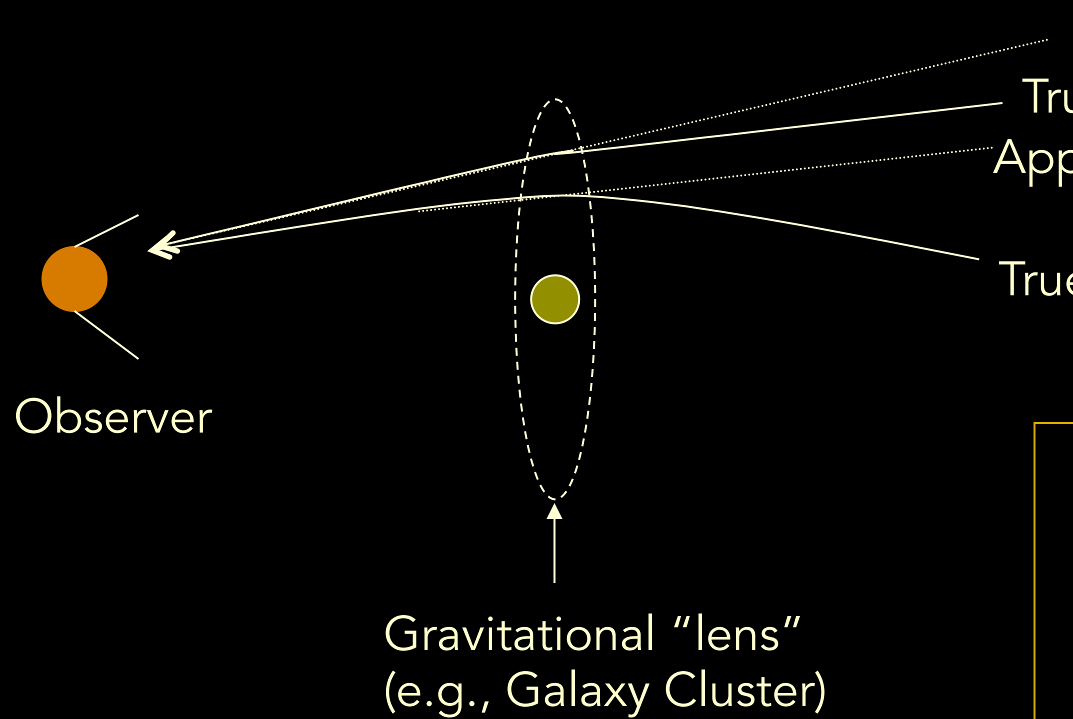
- Sunyaev-Zel'dovich effect: detect clusters as decrements in CMB intensity at long wavelengths (photons Compton-upscattered by hot electrons in cluster potential well)
- SZ flux decrement correlates tightly with halo mass.



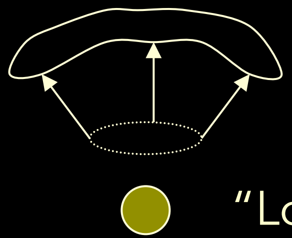
377 clusters at z>0.25 over 2500 sq. deg.

de Haan, et al

Gravitational Lensing of Extended Sources



Objects farther from the line of sight are distorted less.

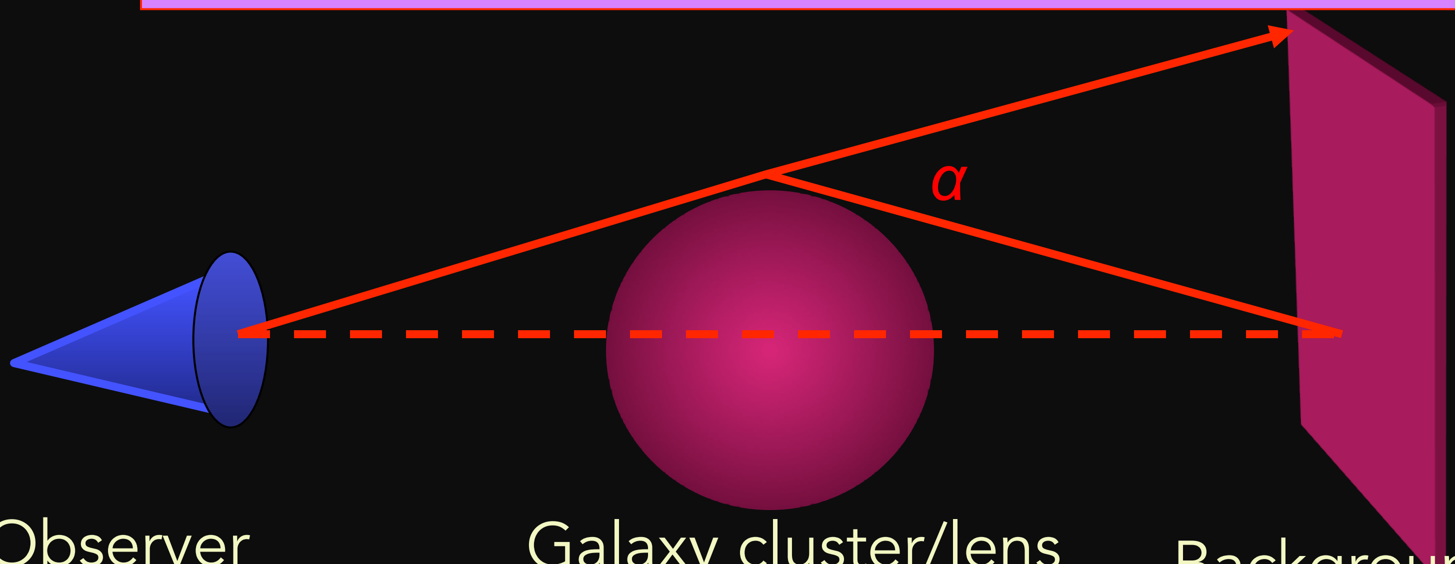


"Looking into" the lens: extended objects are tangentially distorted...

Gravitational Lensing

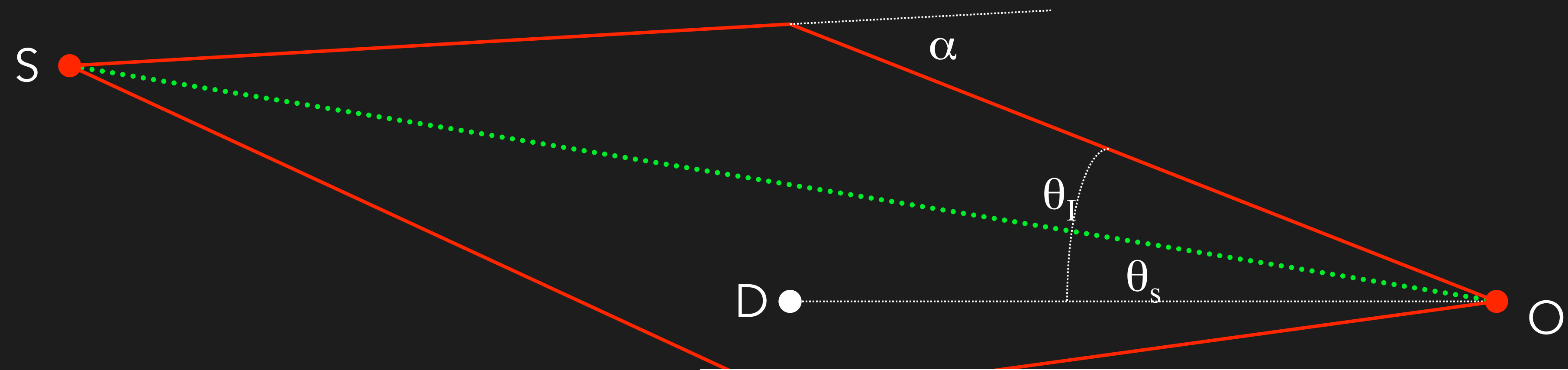
- Photon trajectory in curved spacetime:

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)dr_i dr^i$$



$$\alpha = \int 2\nabla_\perp \Phi d\chi$$

Gravitational Lensing



Lens equation:

$$\vec{\theta}_S = \vec{\theta}_I - \frac{D^A_{DS}}{D^A_{OS}}\vec{\alpha}\ , \ \ \vec{\alpha} = \nabla\Psi, \ \ \nabla^2\Psi = 2\frac{\Sigma}{\Sigma_{crit}} \equiv 2\kappa$$

Amplification Matrix: convergence & shear:

$$\frac{\partial\theta^i_S}{\partial\theta^j_I} = A_{ij} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix}$$

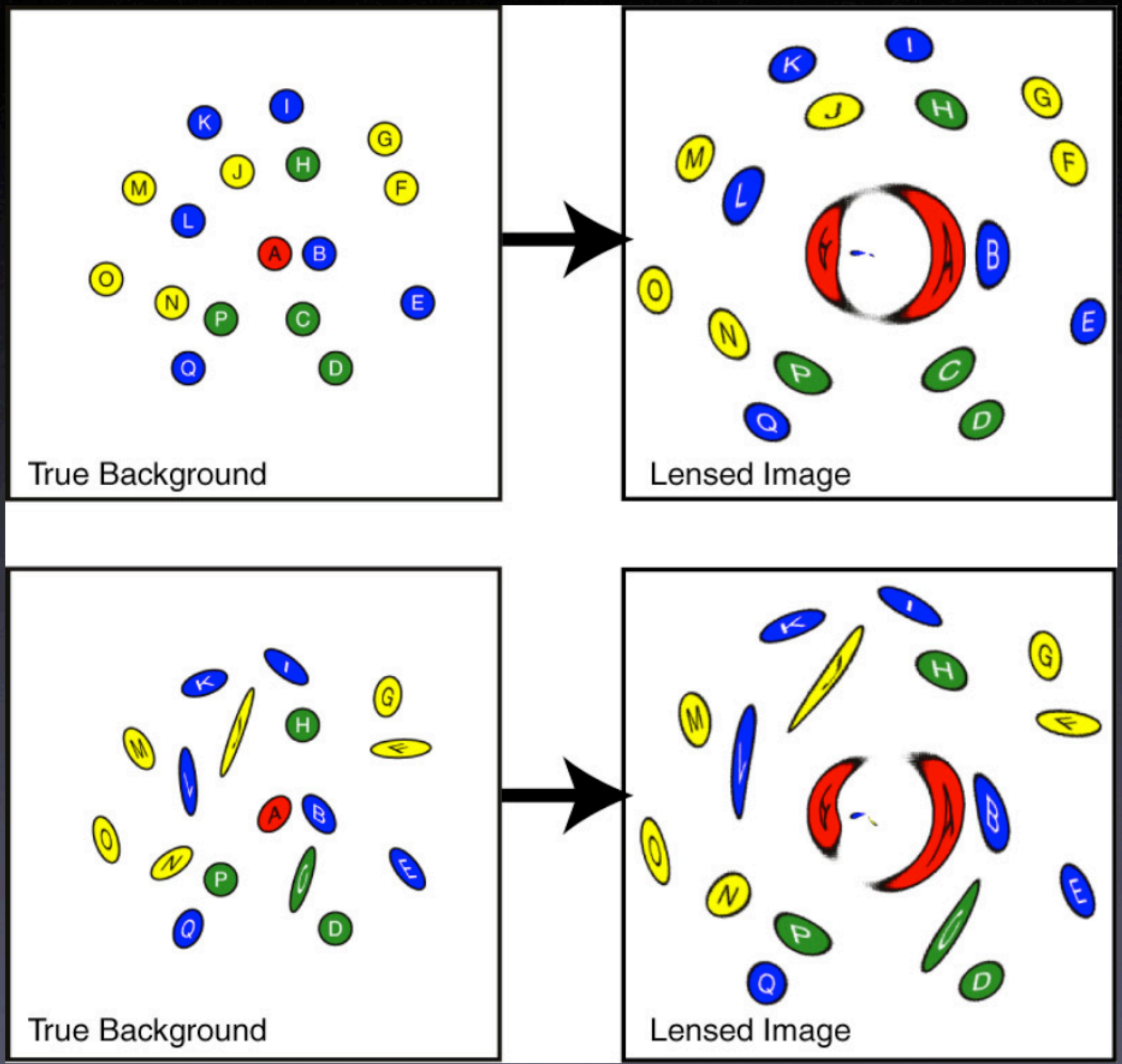
$$\gamma_1 = \partial^2\Psi / \partial\theta_1^2 - \partial^2\Psi / \partial\theta_2^2\ , \ \ \gamma_2 = \partial_{12}\Psi$$

Amplification: $A = (\det A_{ij})^{-1}$

Shear: $\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$ estimate from galaxy shapes

The deflection α is sensitive to *all* mass, luminous or dark: lensing probes dark matter halos of galaxies and clusters.

Weak gravitational lensing



- Deflection angles are not generally observable since lensing mass cannot be removed!
- In weak gravitational lensing, we instead measure the gradients of the deflection angle as distortions to the shapes of galaxies.
- The intrinsic variation of galaxy shapes then becomes a source of noise which averages away as $\sqrt{N}$
- Cosmic signal is ~0.02; shape noise is $0.25/\sqrt{N}$; N~1e9!

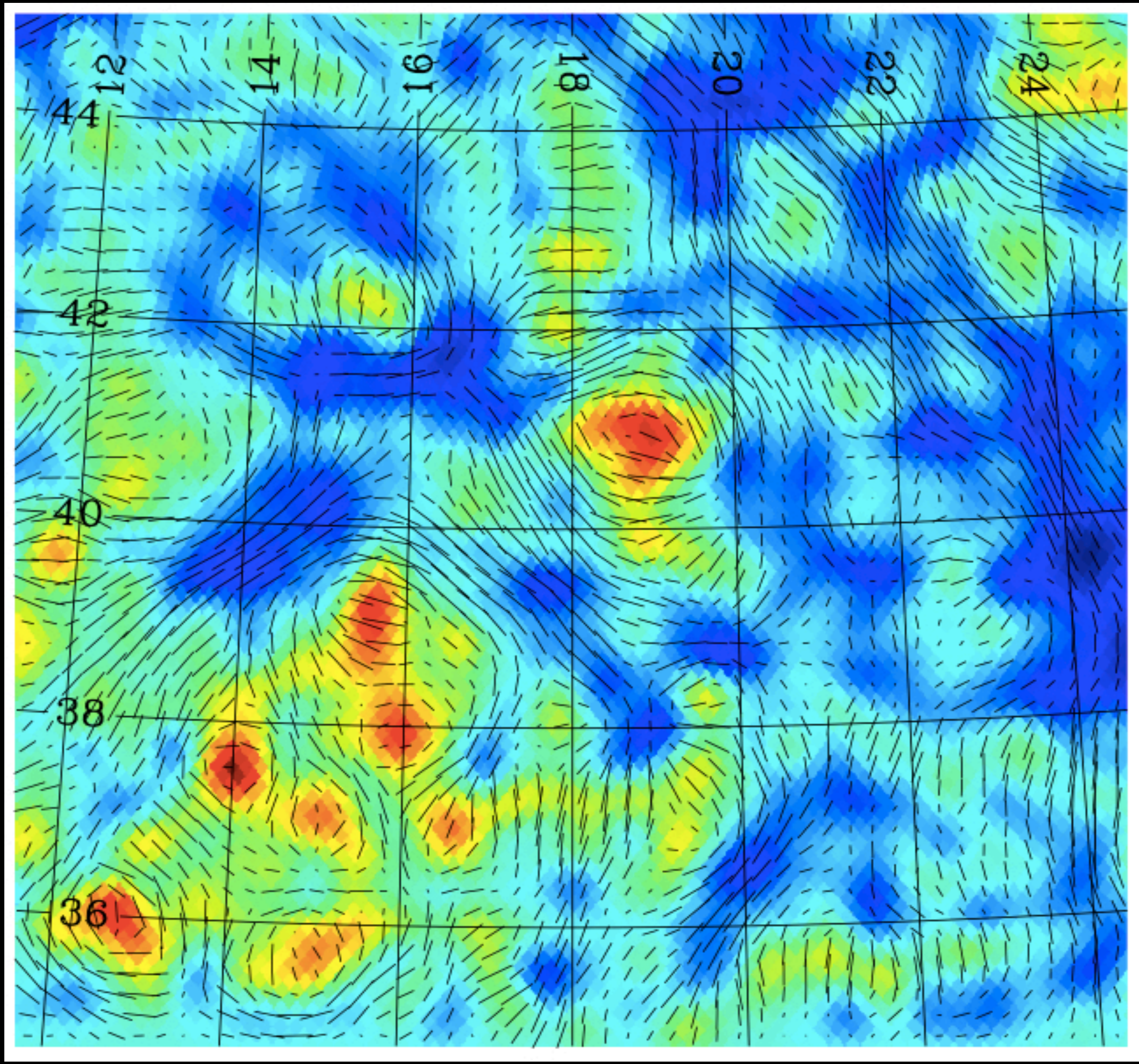
Weak Lensing Mass and Shear

N-body Simulation

Tick marks: induced shear

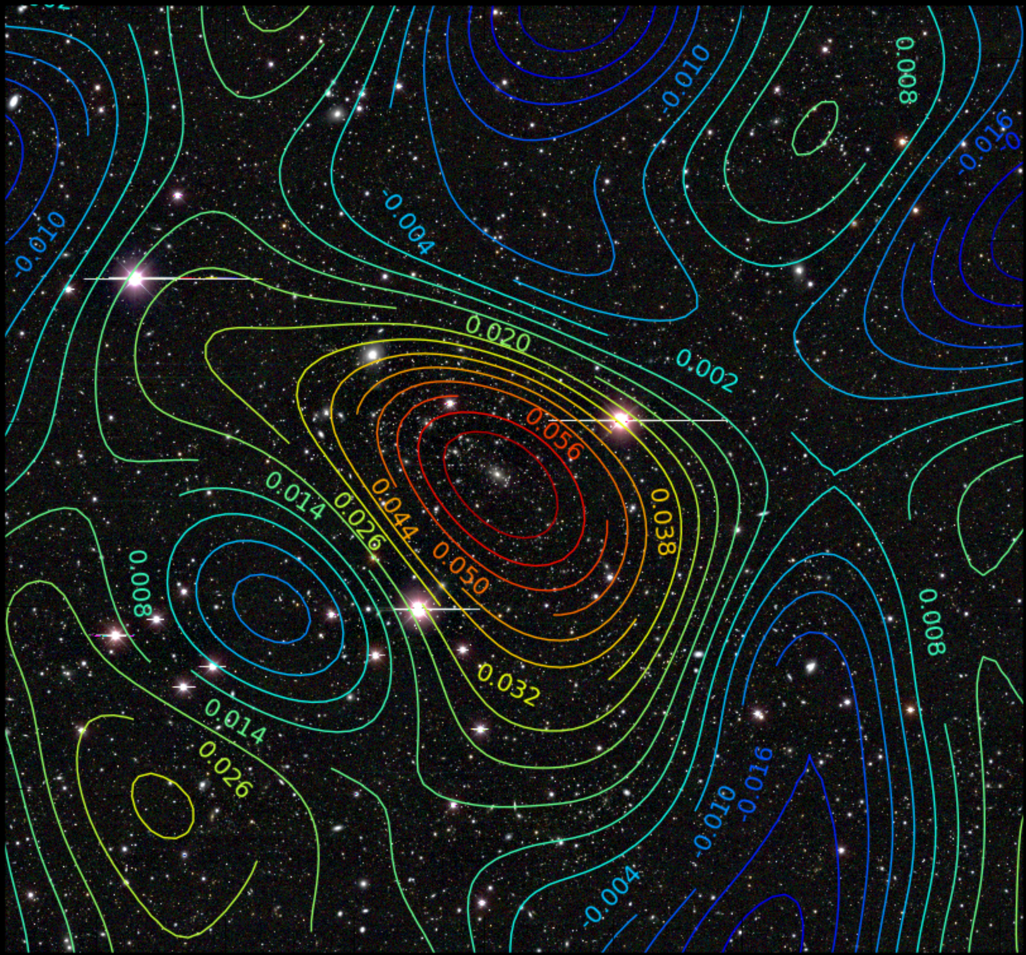
Colors: projected mass density

Becker, Kravtsov, etal



Cluster Weak Lensing

- Image: light from a cluster of galaxies
- Contours: inferred mass distribution in cluster from weak lensing of background galaxies
- DES Science Verification data
- Use WL to calibrate cluster masses

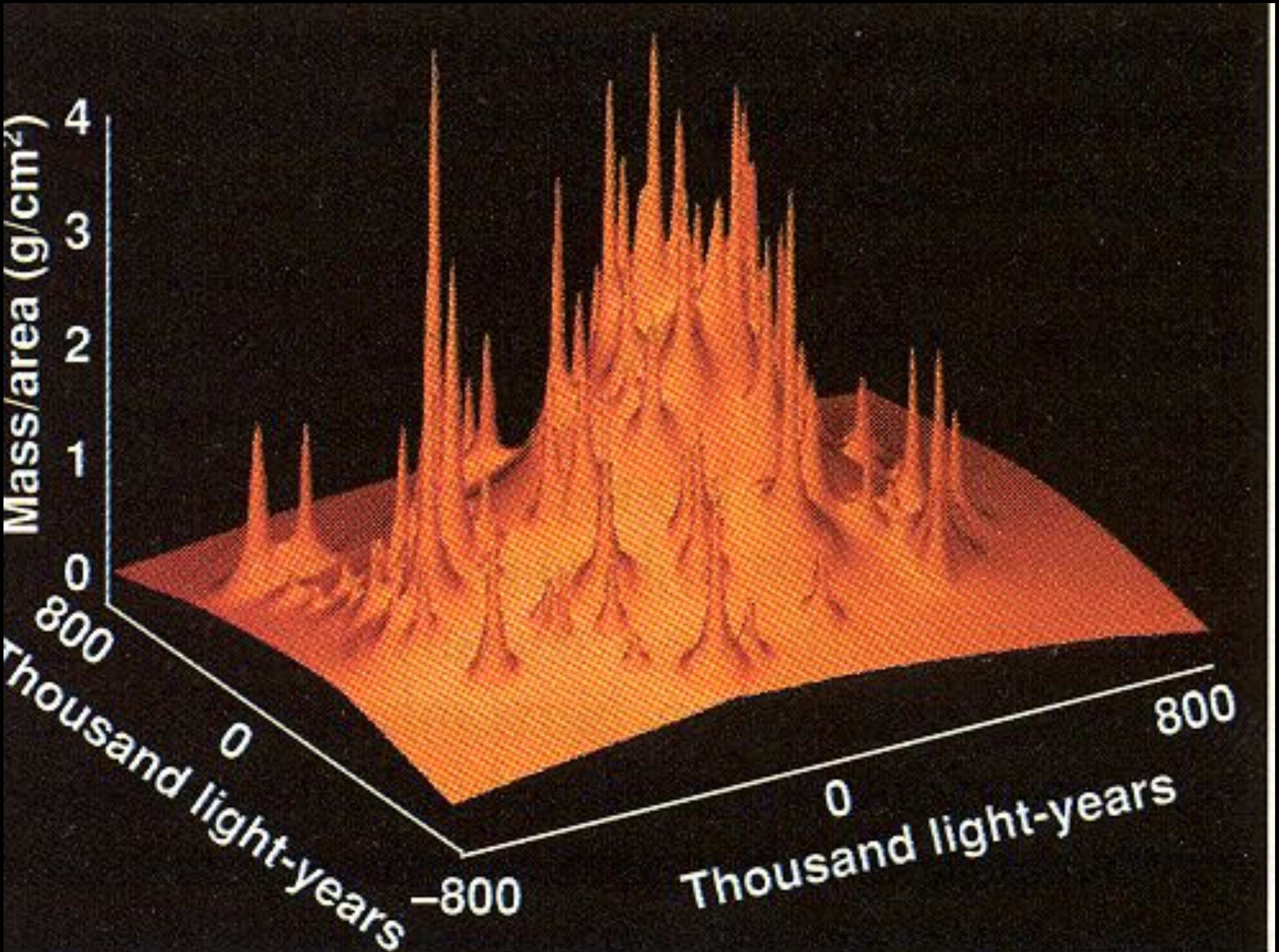


Melchior, etal

Strong & Weak Lensing by a Cluster

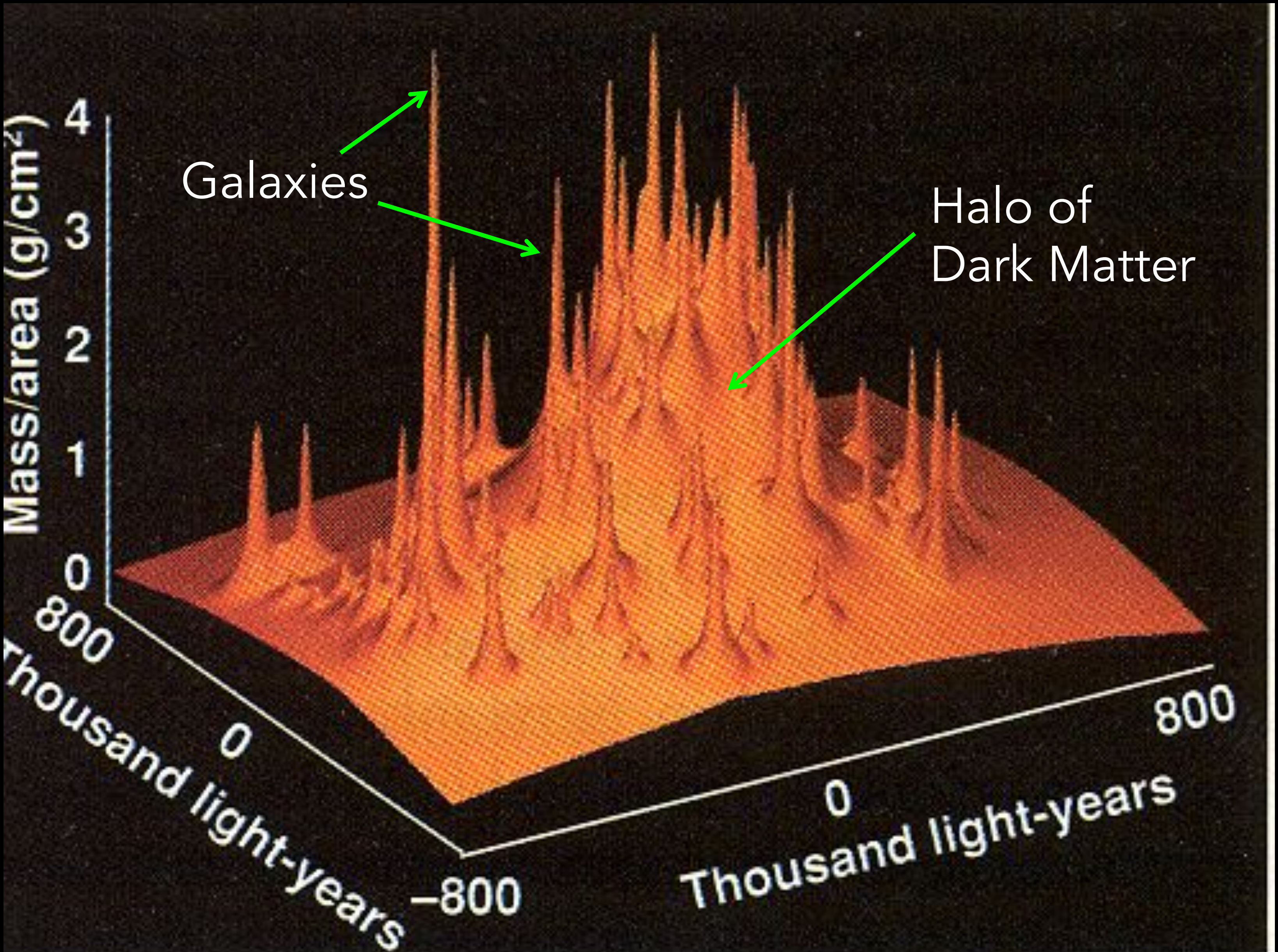
Mass Distribution in a Cluster of Galaxies

inferred from gravitational lensing

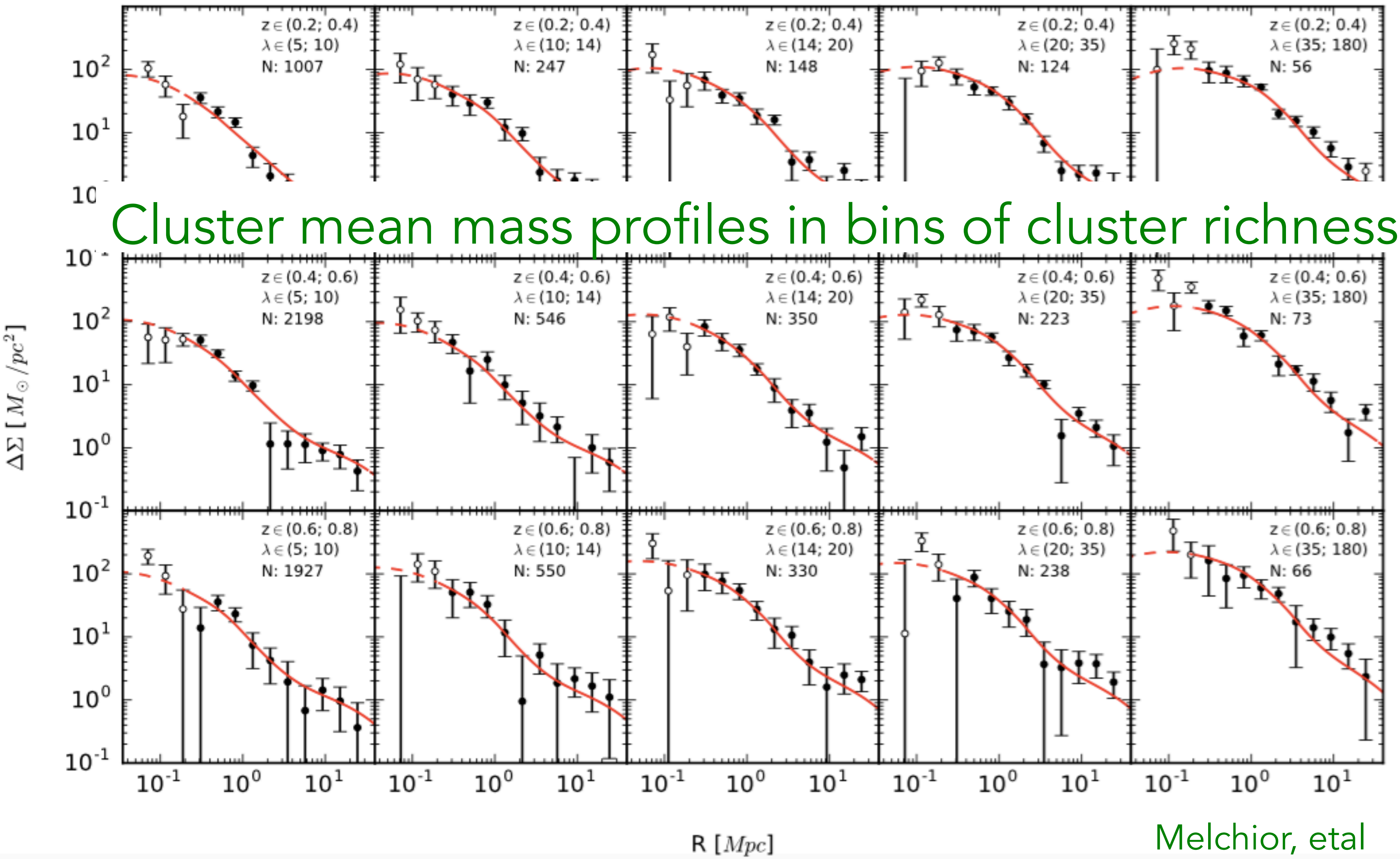


Mass Distribution in a Cluster of Galaxies

inferred from gravitational lensing



Cluster Statistical Weak Lensing



Cluster mean mass profiles in bins of cluster richness

Melchior, etal

Dark Matter

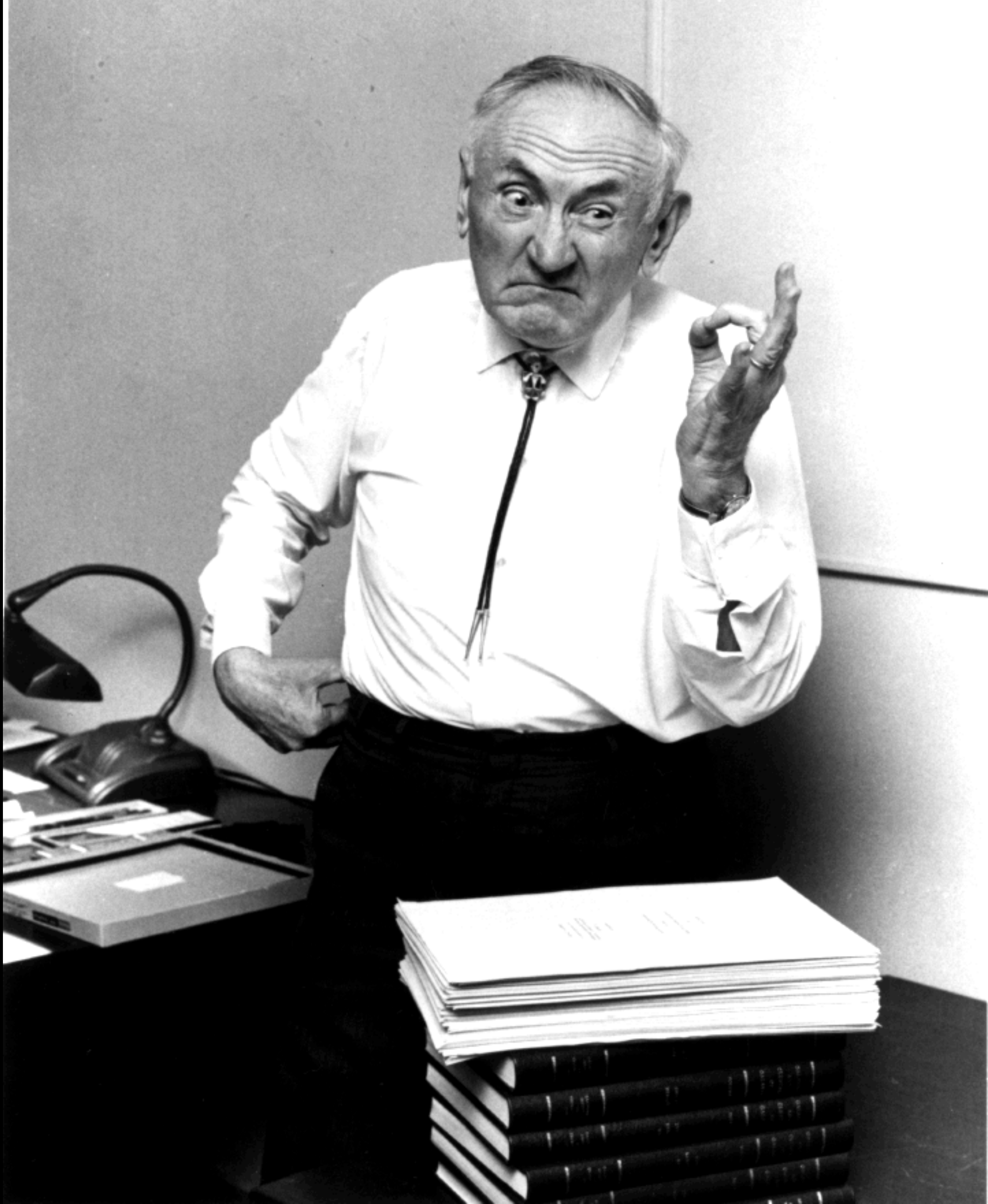
- A component that does not interact with light but the presence of which is inferred from its gravitational effect on luminous matter or light.
- 1930's: initial evidence from velocity dispersion of galaxies in Coma cluster (Zwicky)
- 1970's-80's: mounting evidence from spiral galaxy rotation curves (Rubin, etal)
- 1990's-2000's: support from gravitational lensing and cosmological measurements
- Alternative: modification of gravity (Cf. General Relativity vs. planet Vulcan)

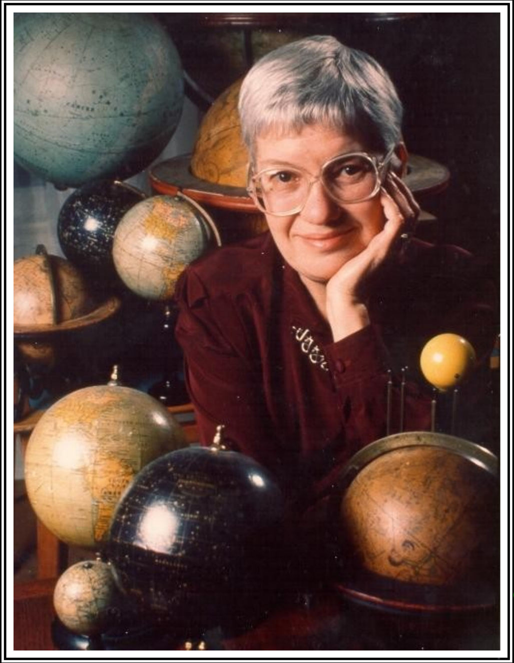
Coma Cluster of Galaxies

velocity dispersion $\sigma(v)$~1000 km/sec > $(GM_{gal}/r)^{1/2}$ virial theorem

Fritz Zwicky
(1898-1974)

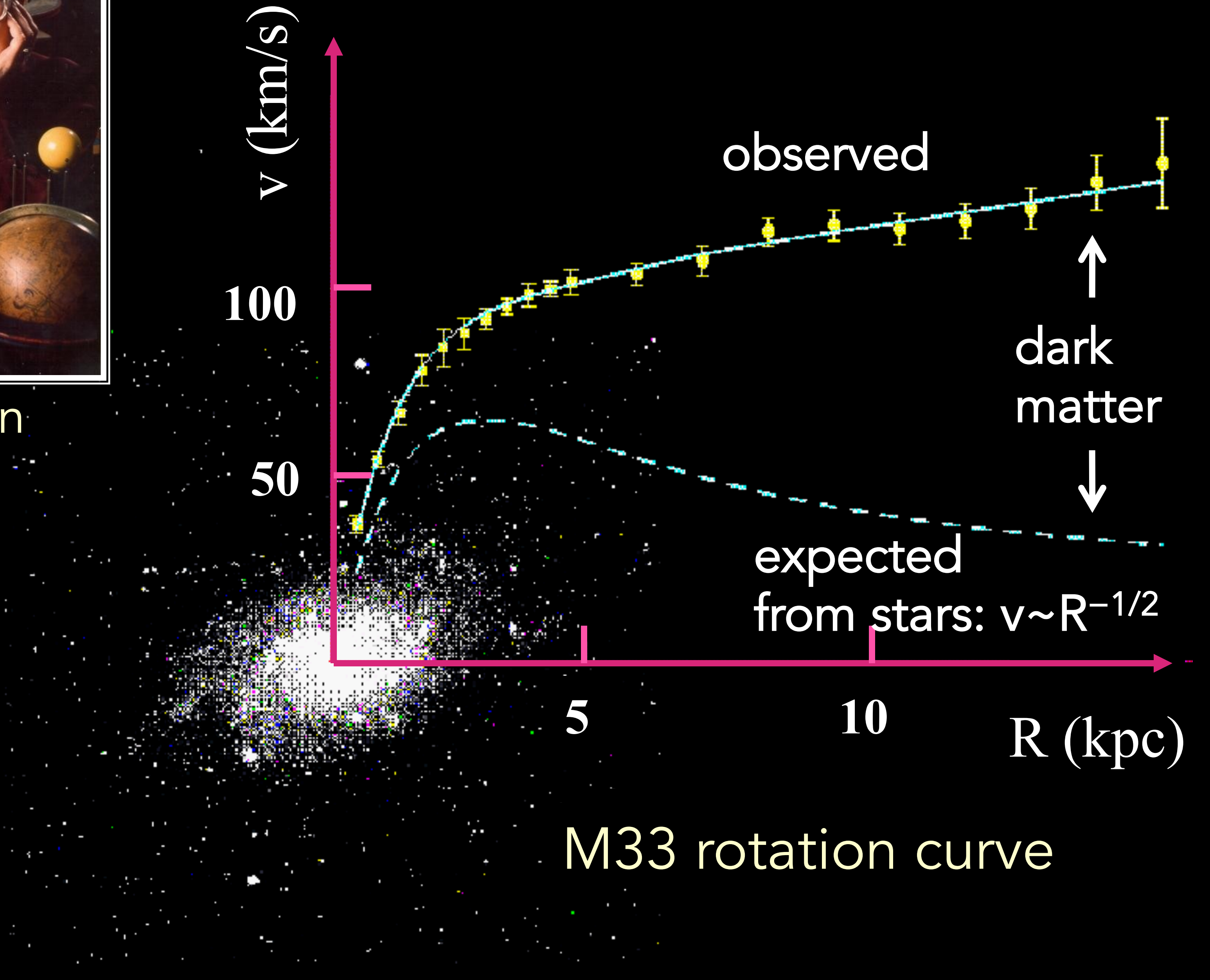
Postulated dark matter to explain large velocity dispersion of Coma galaxies.



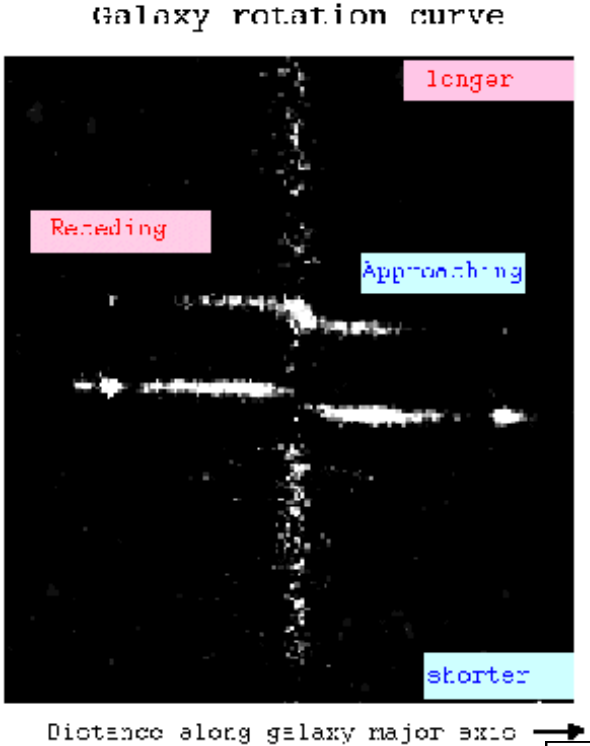


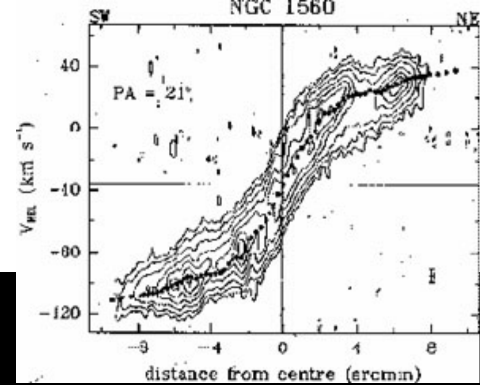
Vera Rubin
(1970's)

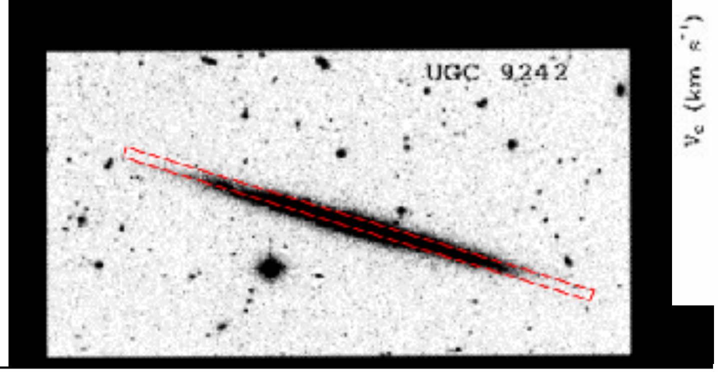
Rotation of Spiral Galaxies

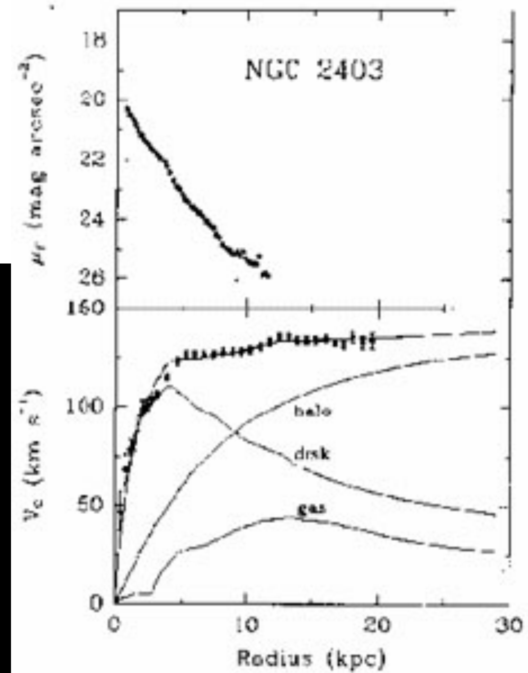


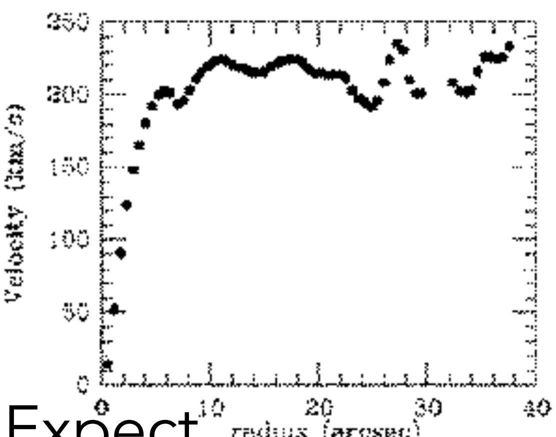
M33 rotation curve

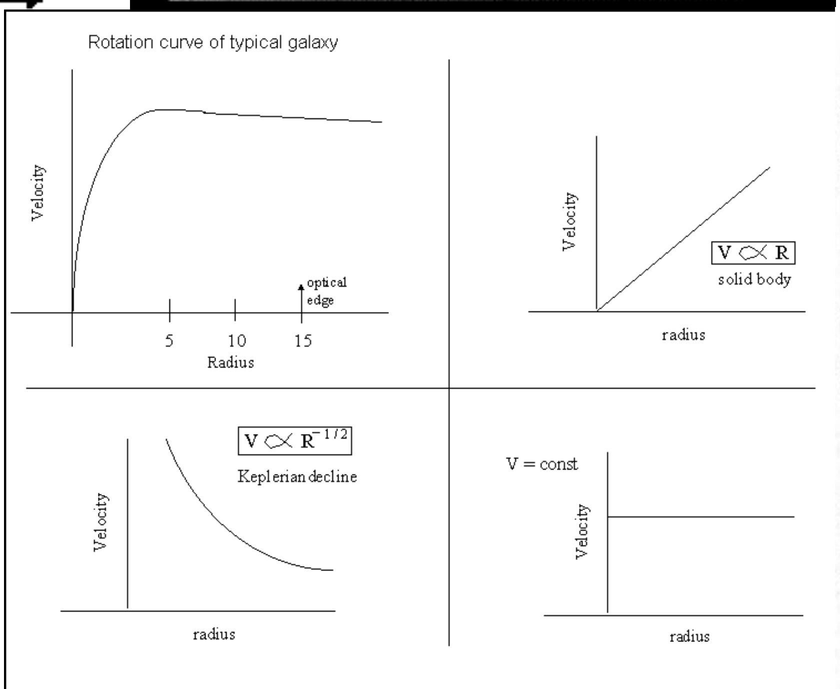


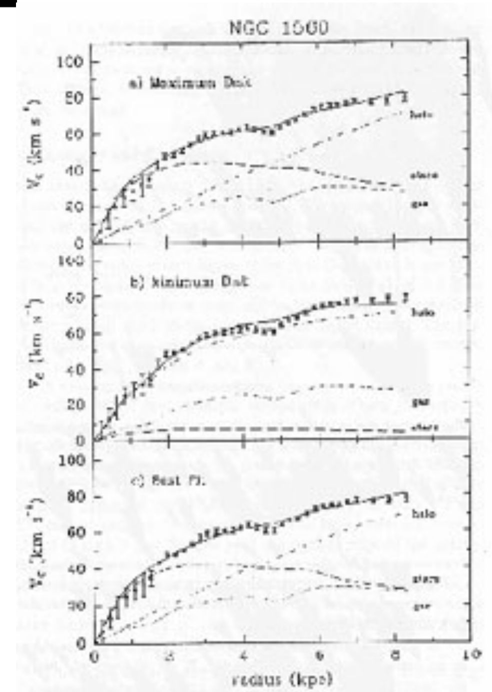








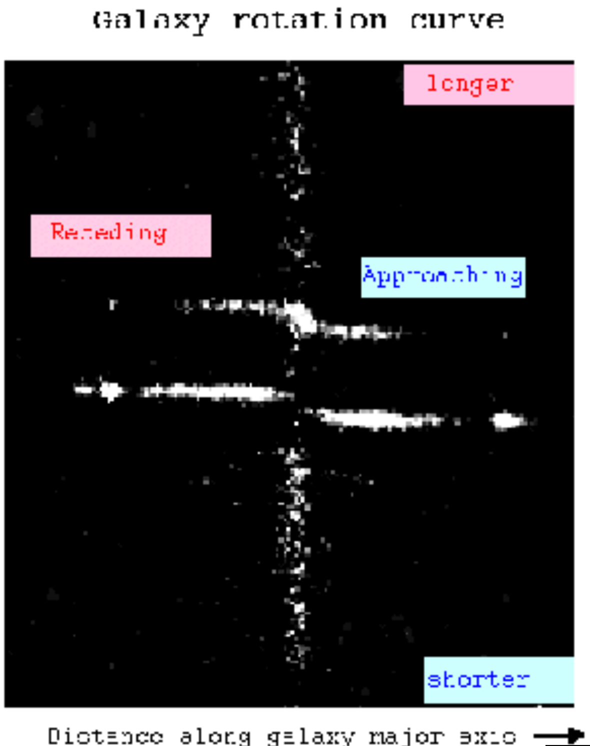


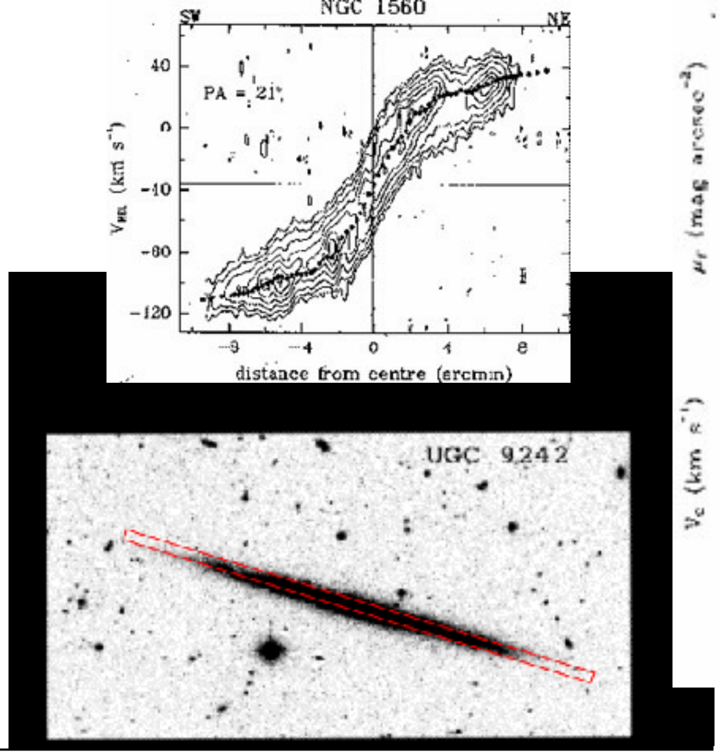


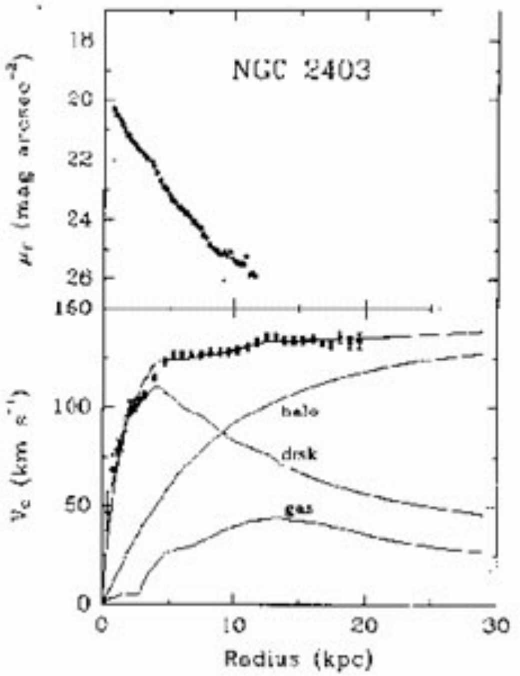
Expect

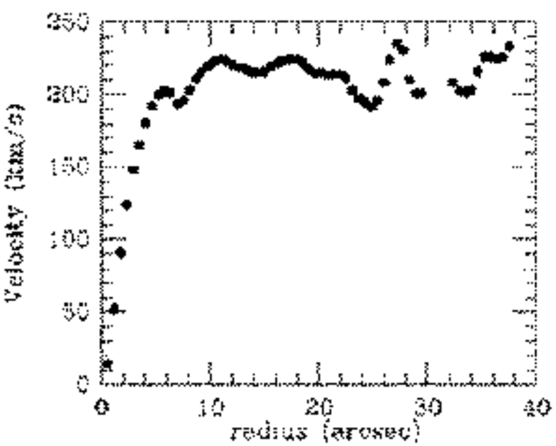
$v_{rot}^2 = GM(r)/r \sim r^{-1}$

in outer part of disk

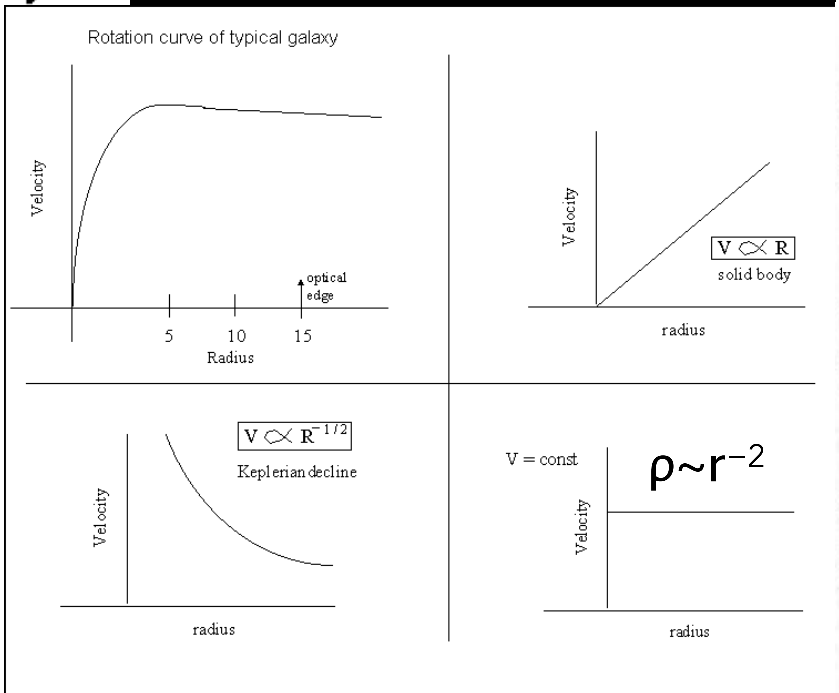


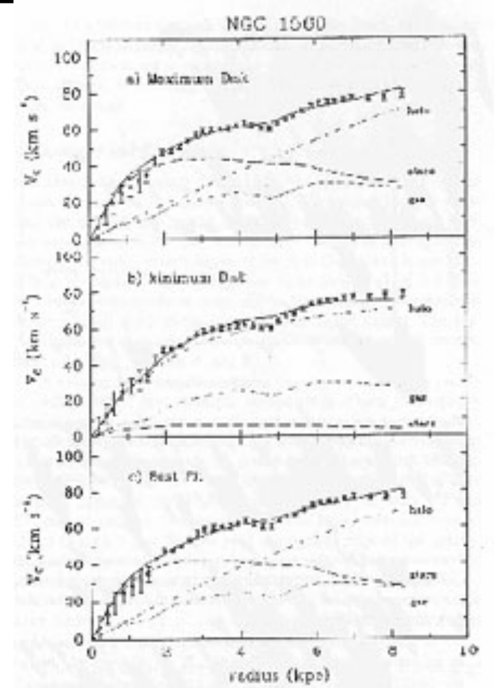




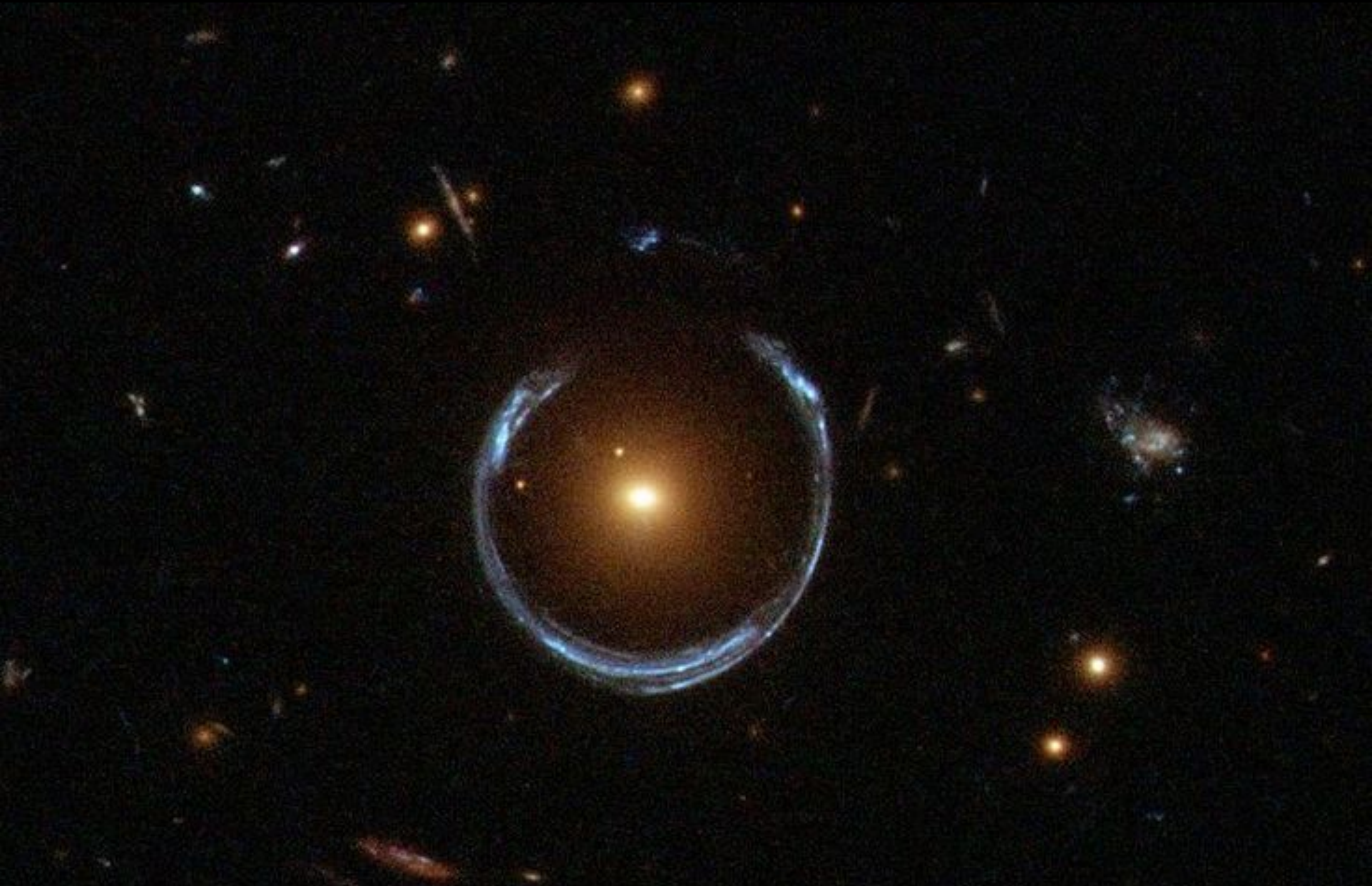


$v_{rot}^2 = GM(r)/r \sim r^{2-\alpha}$
for $\rho \sim r^{-\alpha}$





Gravitational Lensing by Dark Matter in Galaxies



Weak Lensing by Galaxies

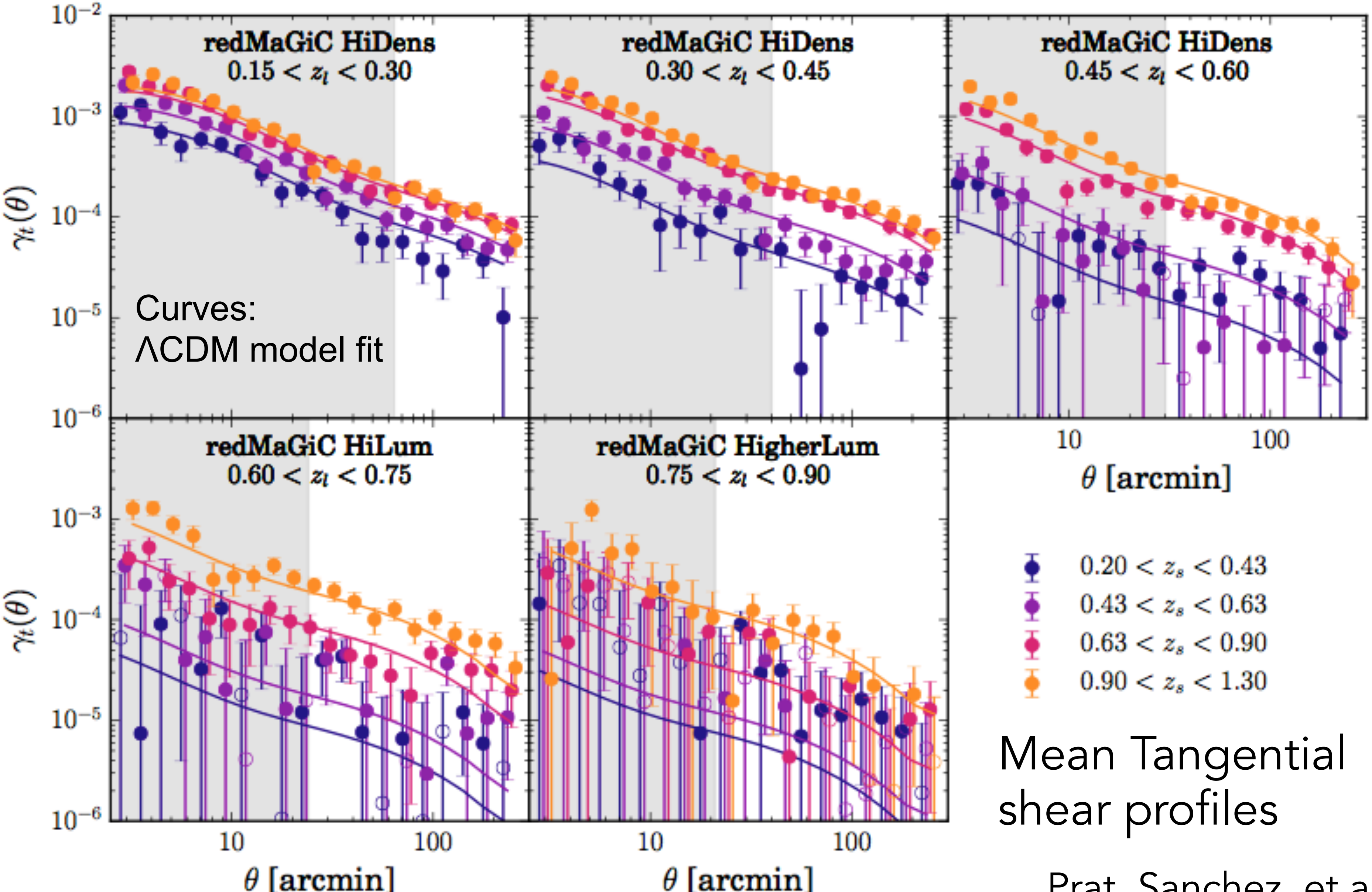
Galaxy-galaxy lensing (galaxy-shear correlation):

- Correlate the shapes of distant `background' galaxies with the positions of foreground lens galaxies, whose dark halos deflect and weakly shear the passing light bundles.

- As with cluster statistical lensing, this method probes the *average* mass profile of a population of lens galaxies and supports extended dark matter halos.

- Extend measurement to larger scales to probe cosmology.

DES Galaxy-galaxy lensing: Y1 Results



Mean Tangential shear profiles

Prat, Sanchez, et al

The Inflationary Scenario

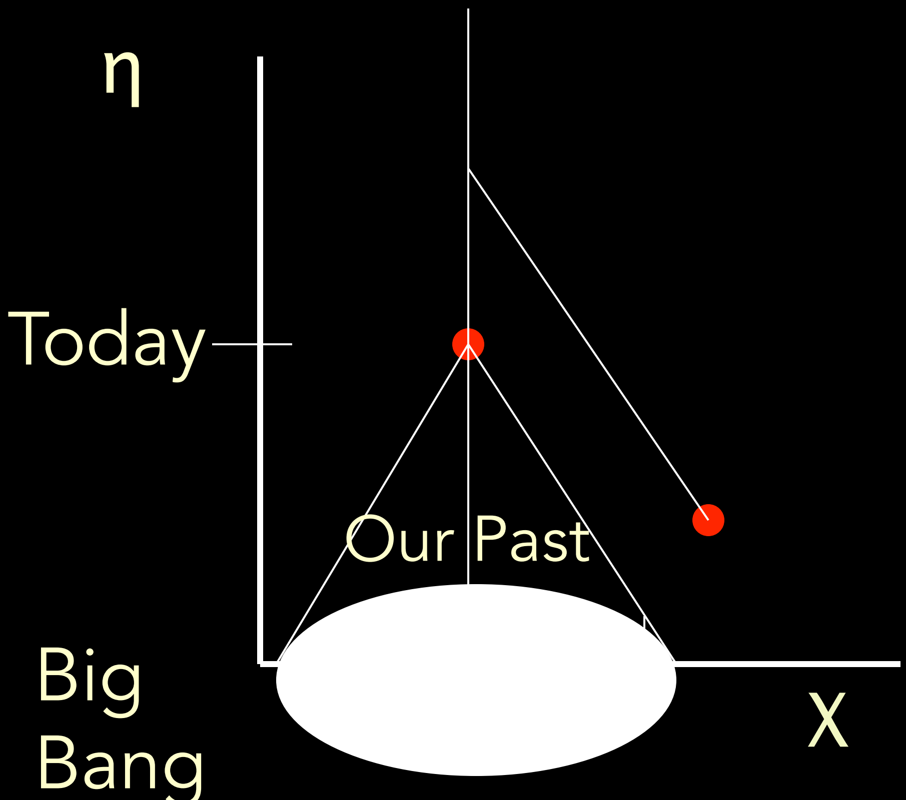
Alan Guth (1980): postdoc at SLAC who was thinking about the cosmological consequences of symmetry-breaking Phase Transitions in the early Universe. He realized that if a transition proceeded very slowly, it could have profound implications for cosmic evolution. He was motivated by several cosmological conundrums:

Horizon/homogeneity, flatness, and structure problems

Why is the Universe homogeneous, isotropic, and nearly flat? These are not *stable* features of the standard Big Bang cosmology.

How can large-scale structure form without violating causality?

Causal Structure of Spacetime

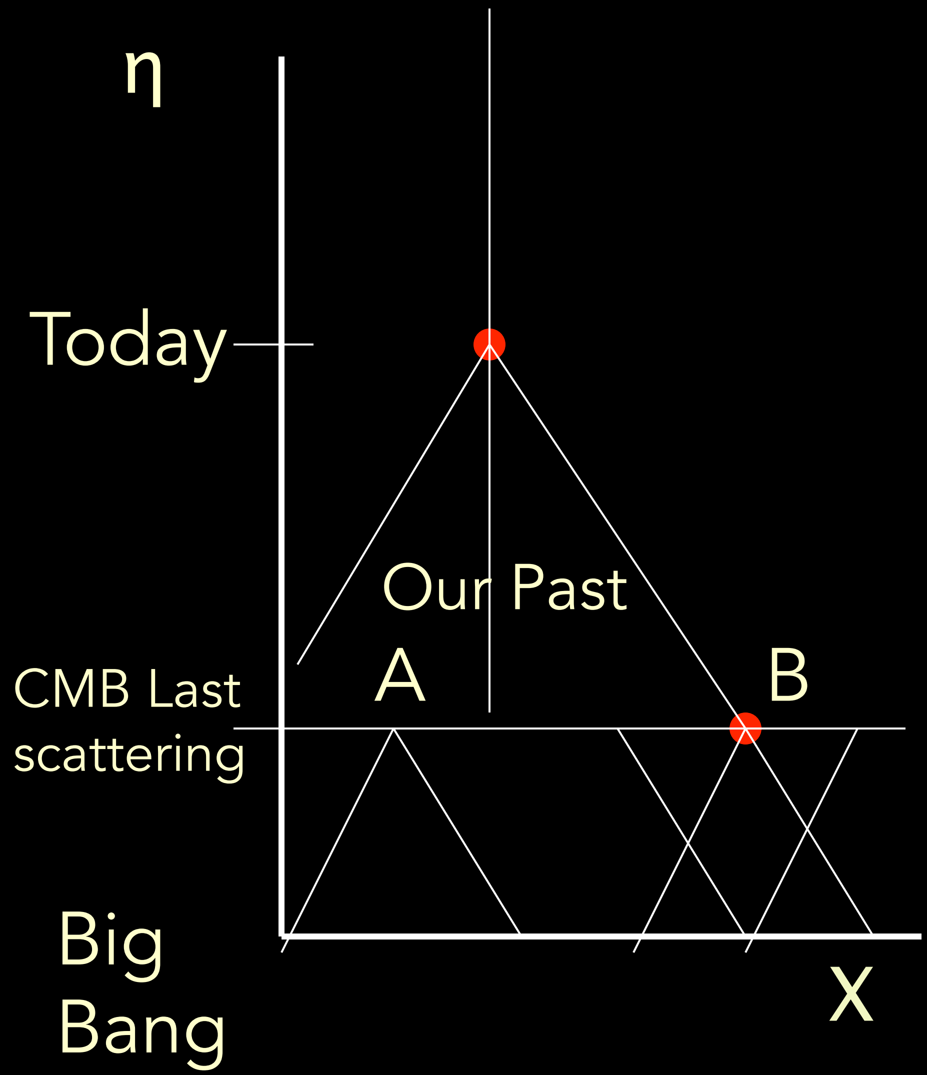


The causal past of an event lies inside the past light cone. It defines our *horizon*: spacetime volume of events we can be influenced by.

Light cones at 45 deg in comoving, conformal coordinates:

$$ds^2=dt^2-a^2dX^2$$
$$=a^2(d\eta^2-dX^2)$$

Horizon Problem



If A and B are separated by more than ~2 deg on the CMB sky, they were not yet in causal contact: outside each other's past light cones. Yet their temperatures agree to 1 part in 10^5. Why?

Structure/Causality Problem

Another symptom of the Horizon problem:

Large-scale structures we see today in galaxy surveys were, at early times, larger than the horizon. The seeds for structure (density perturbations) could not have been produced causally unless you wait until very late times (and we have no theory of how to form such seeds at late times):

Perturbation scale $\lambda_{pert} \sim a(t)$.
Horizon scale $d_H \sim 1/H \sim ct \sim a^{3/2}$ (matter-domination).
At early times, $\lambda_{pert} > d_H$. Perturbations cross inside the horizon when $\lambda = H^{-1}$.

Flatness Problem

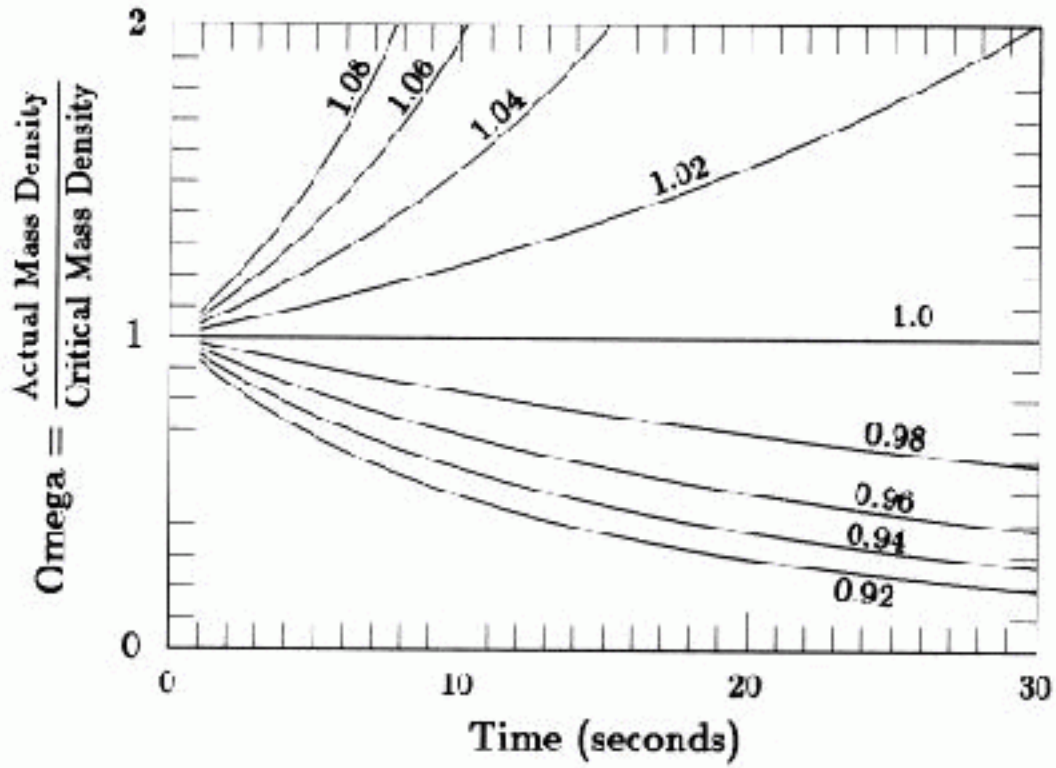
CMB indicates that the observable Universe (within our present horizon) is remarkably flat: $\Omega_{tot} = 1$

As the Universe evolves, spatial curvature contribution to expansion rate becomes <u>more</u> important with time: k/a^2 vs $1/a^3$, $1/a^4$ for matter, radiation.

Negative curvature universe (K<0) becomes empty.
Positive curvature universe (K>0) rapidly recollapses.

Natural timescale for this evolution is $t_{Planck} = L_{Planck}/c \sim 10^{-43}$ sec. But Universe still appears flat at 10^{17} sec $\sim 10^{60}$ Planck times. Universe must have been \`fine tuned' to be very precisely flat at t_{Planck} for it still to be nearly flat today.

Flatness or Ω Problem



Near-flatness is an unstable property of the Universe

Problems of Initial Conditions

Flatness and homogeneity are unstable conditions. If the early Universe had been slightly more curved or inhomogeneous, it would look much different today.

The present state of the observable Universe appears to depend sensitively on the initial state. If we consider an `ensemble' of Universes at the Planck time with varying curvature and inhomogeneity, only a tiny fraction of them would evolve to a state that looks like our Universe today. Our observed Universe is in some (hard to quantify) sense very improbable.

Possible Solutions

1. That's the way it is: we're just lucky. Or invoke anthropic selection: if it wasn't this way, we wouldn't be here to talk about it.

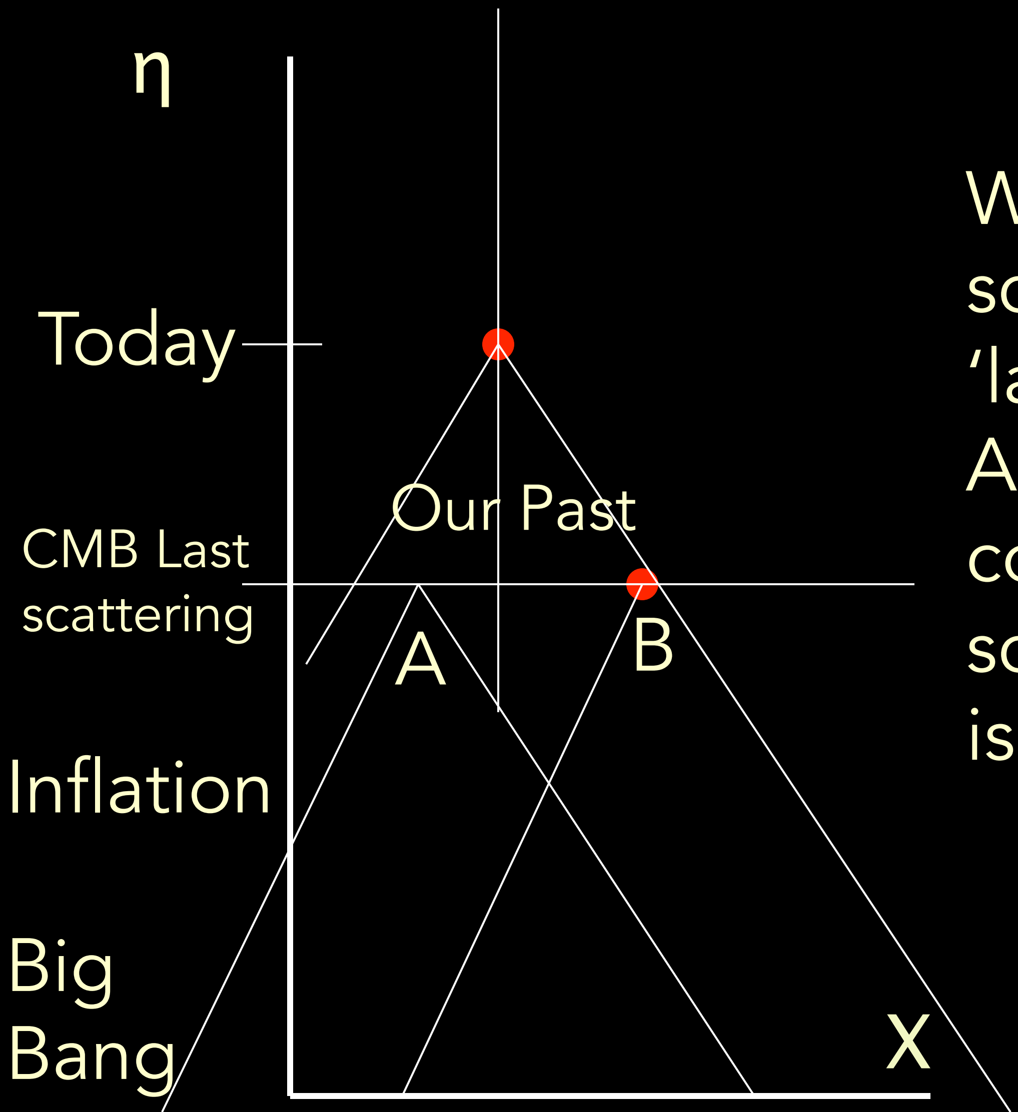
2. Fundamental Theory might constrain the possible conditions at the Planck time to be flat and nearly homogeneous and with the small-amplitude density perturbations needed to form large-scale structure.

3. Dynamical solution: perhaps the very early Universe evolved in a different way: INFLATION

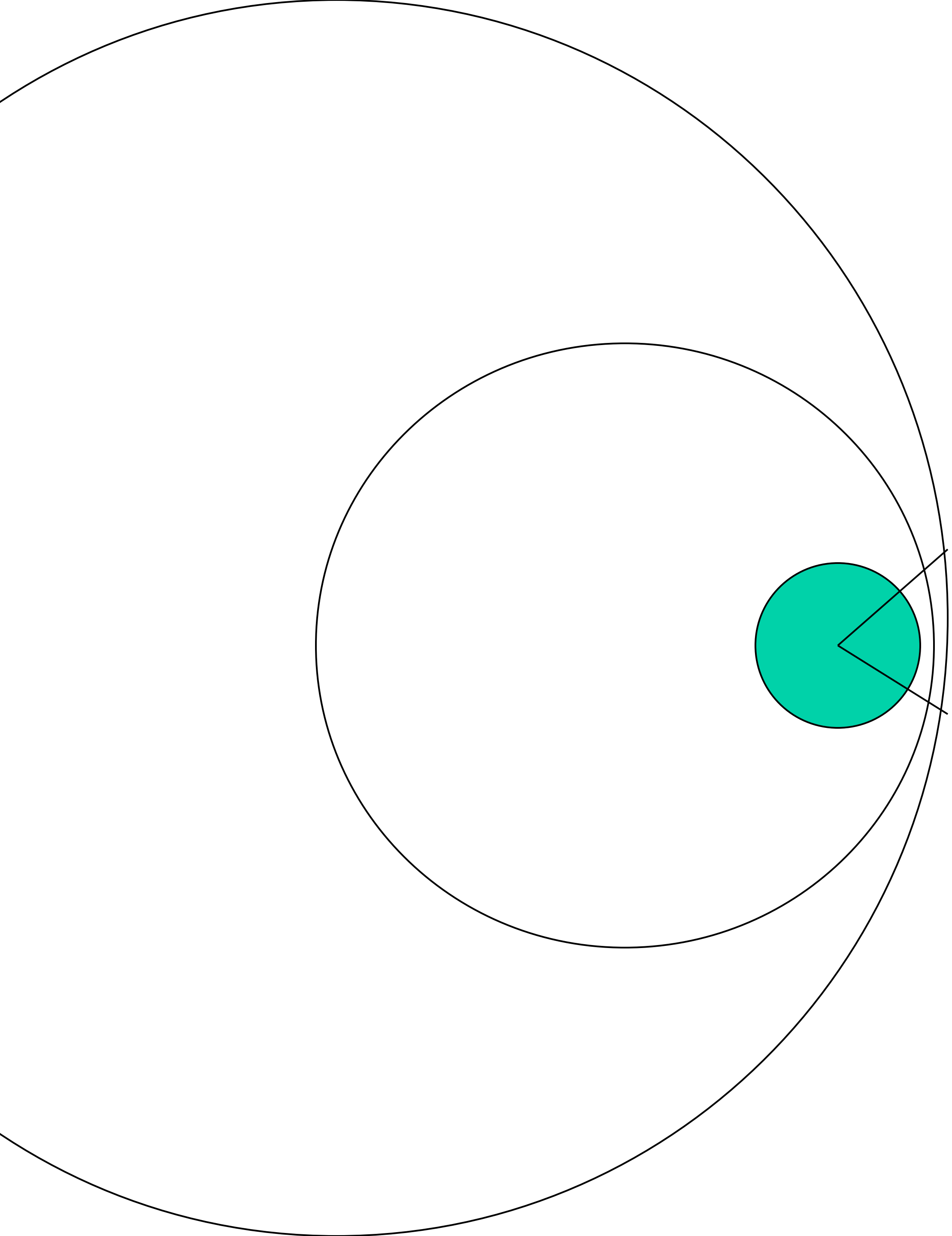
Inflation in the Early Universe

- Hypothetical epoch of *accelerated* expansion in the very early Universe, tiny fraction of a second after Big Bang.
- If this period lasts long enough, it effectively stretches inhomogeneity and spatial curvature to unobservably large scales, "solving" horizon and flatness problems.
- In this model, a Universe with our observed properties becomes an 'attractor' of cosmic evolution, rather than an unstable point: our Universe appears "more likely".
- Bonus: causal origin for density perturbation seeds for large-scale structure.

Causal Structure with Inflation



With inflation, CMB last scattering occurs much ‘later’ in conformal time: A, B now in causal contact at time of last scattering. Can explain isotropy.



Solving the Flatness problem:

Since the Universe after inflation is much larger, the part we can see looks much flatter.

If inflation lasts longer than a minimal amount, observable Universe should be indistinguishable from flat, in accord with CMB anisotropy measurements.

Minimal Duration of Inflation

How long should inflation last in order to solve the horizon and flatness problems?

Can show this requires Universe to grow at least as much during inflation as it has since then:

$$\frac{a_{end}}{a_{begin}} > \frac{a_0}{a_{end}} = \frac{T_{end}}{T_0} = \frac{10^{15}\,GeV}{10^{-4}\,eV} = 10^{28}$$

for inflation occurring around the Grand Unification epoch.
For exponential inflation, this can happen rather quickly:

$a(t) \sim e^{Ht} \sim e^{60} \sim 10^{28}$

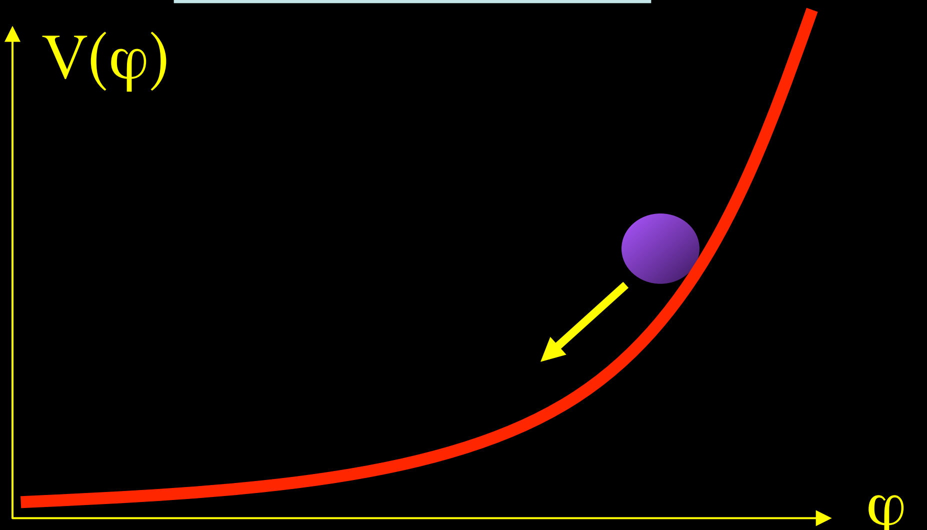
so this growth only requires 60 `expansion times':
e.g., from $t \sim 10^{-35}$ seconds to $t \sim 10^{-33}$ seconds

Scalar Field Inflation: Slow Roll

- Inflation *could* be due to a very light scalar field **φ**, slowly evolving in a potential, V(**φ**):

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

$$H_{\inf}^2 \approx \frac{8\pi G}{3} V(\varphi)$$



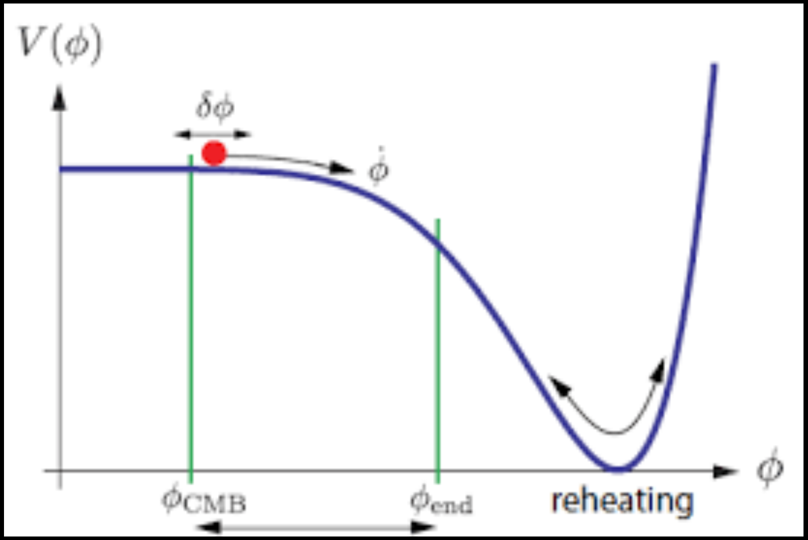
- Density & pressure:

$$\rho = \tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)$$
$$P = \tfrac{1}{2}\dot{\varphi}^2 - V(\varphi)$$

- Slow roll:

$$\tfrac{1}{2}\dot{\varphi}^2 << V(\varphi) \Rightarrow P < 0 \Leftrightarrow w < 0 \text{ accelerated expansion}$$

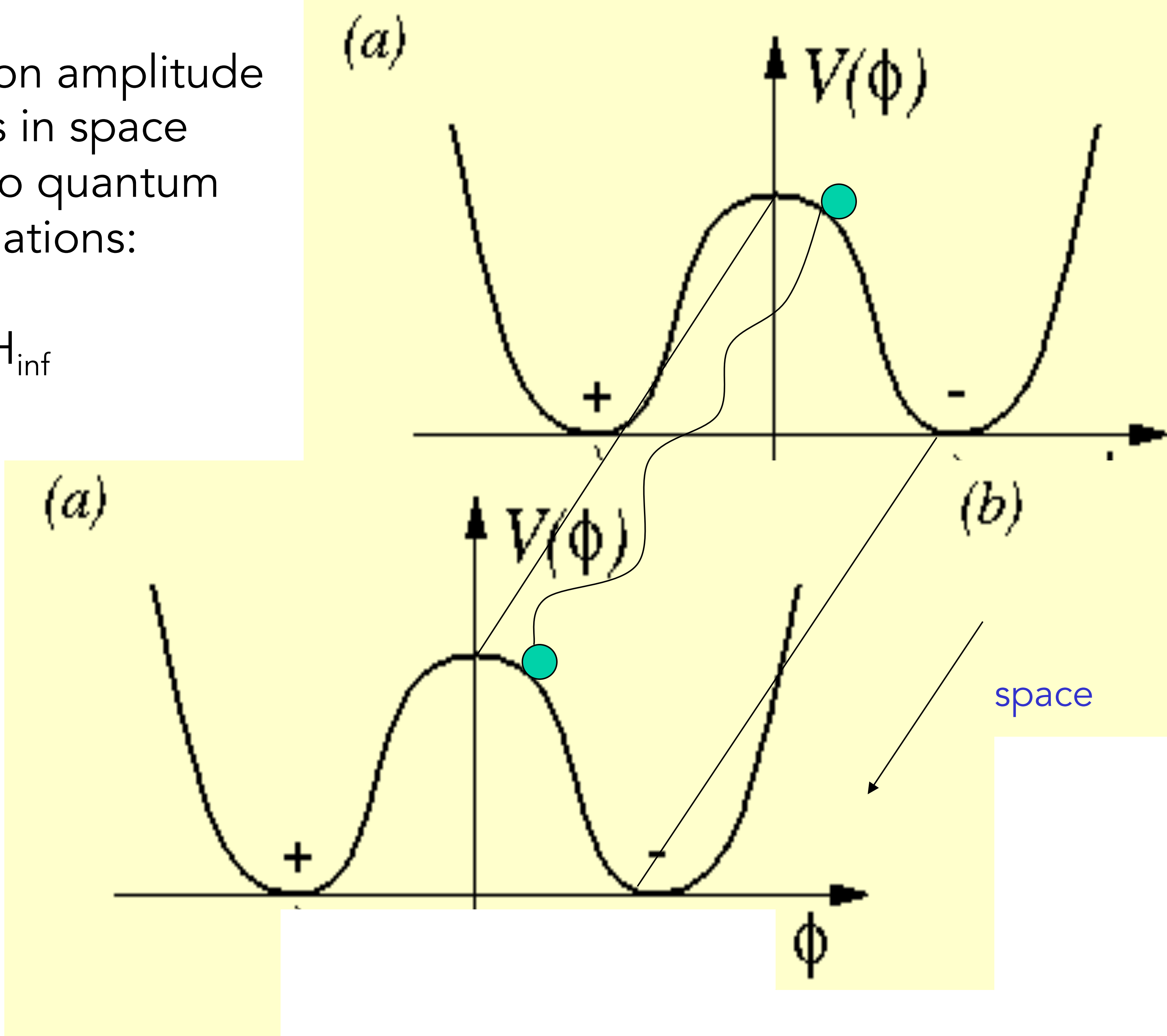
The End of Inflation: Reheating



During inflation, temperature and density decay exponentially: Universe becomes cold and empty. When scalar field approaches the minimum of its potential, it speeds up and starts oscillating. These oscillations lead to decay of the field into lighter particles, reheating the Universe to a hot, dense state. This process must be efficient enough so baryogenesis, particle dark matter, and nucleosynthesis can occur.

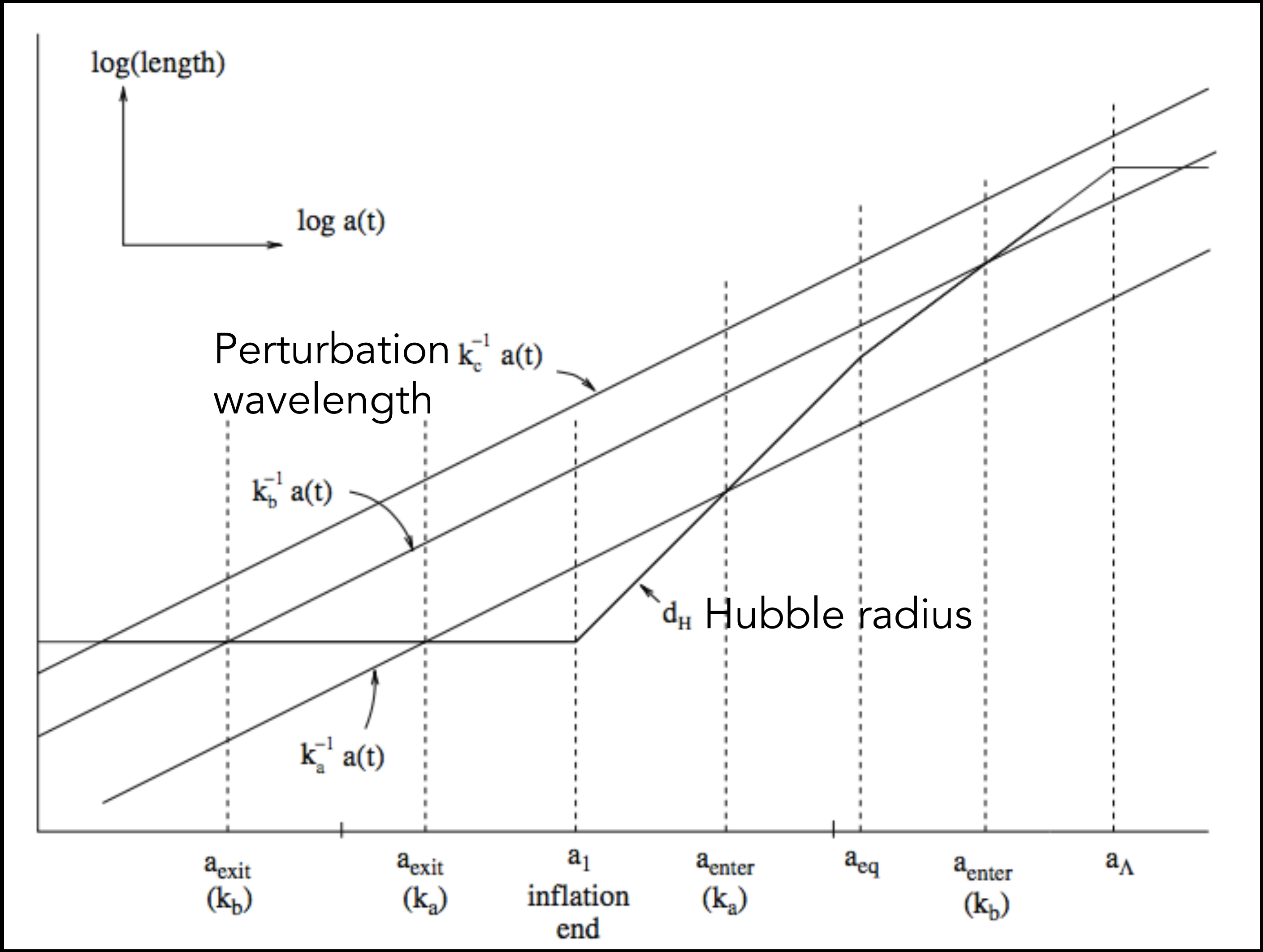
Inflaton amplitude varies in space due to quantum fluctuations:

$\delta\varphi \sim H_{inf}$



Causal Origin of Perturbations

During inflation, $d_H = 1/H$ ≈constant for exponential expansion. Perturbations start inside the horizon as quantum fluctuations and get stretched outside.



Quantum Fluctuations & Density Perturbations

Curvature Perturbation at horizon crossing:

$$\delta R \sim \delta\Phi_{grav} \sim \frac{\delta\rho}{p+\rho} \sim \frac{V'(\varphi)\delta\varphi}{\dot{\varphi}^2}$$

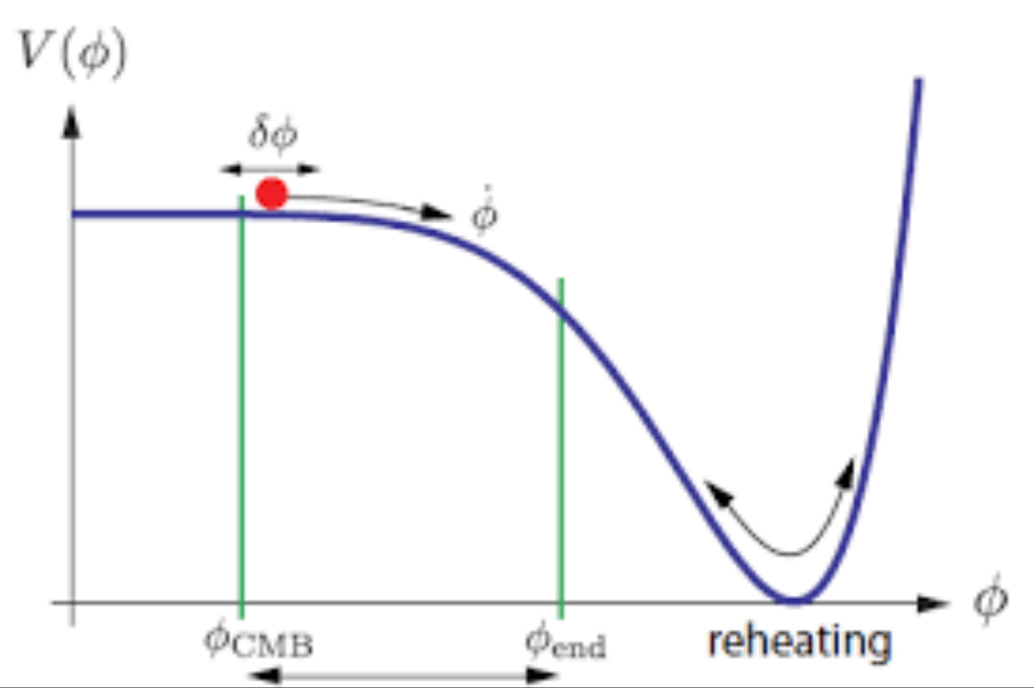
$$\sim \frac{V'(\varphi)H_{\inf}}{\dot{\varphi}^2} \sim \frac{(8\pi GV)^{3/2}}{V'}$$

$$\sim 10^{-5}\left(\frac{k}{H_0}\right)^{(n_s-1)/2}$$

where

$$n_s = 1 + M_{Pl}^2\left[3\left(\frac{V'}{V}\right)^2 - 2\left(\frac{V''}{V}\right)\right] \approx 1$$

nearly scale-invariant for slowly rolling field.



Also produce tensor perturbations (gravity waves), with relative amplitude

$$r = 8M_{Pl}^2\left(\frac{V'}{V}\right)^2$$

Inflation Spectrum in more detail

$$\delta R \sim \delta\Phi_{grav} \sim \frac{V'(\varphi)\delta\varphi}{\dot{\varphi}^2} \sim \frac{V'(\varphi)H_{\inf}}{\dot{\varphi}^2} \sim \frac{H^2\dot{\varphi}}{\dot{\varphi}^2} \sim \frac{H^2}{\dot{\varphi}}$$

where we used slow-roll equation of motion $3H\dot{\varphi} = V'$

Differentiating $H^2 \sim V / M_{Pl}^2 \Rightarrow \dot{H} \sim \frac{\dot{\varphi}^2}{M_{Pl}^2} \Rightarrow$

$$\delta R \sim \frac{H^2}{M_{Pl}\sqrt{\dot{H}}} \sim \left(\frac{k}{H_0}\right)^{(n_s-1)/2}$$

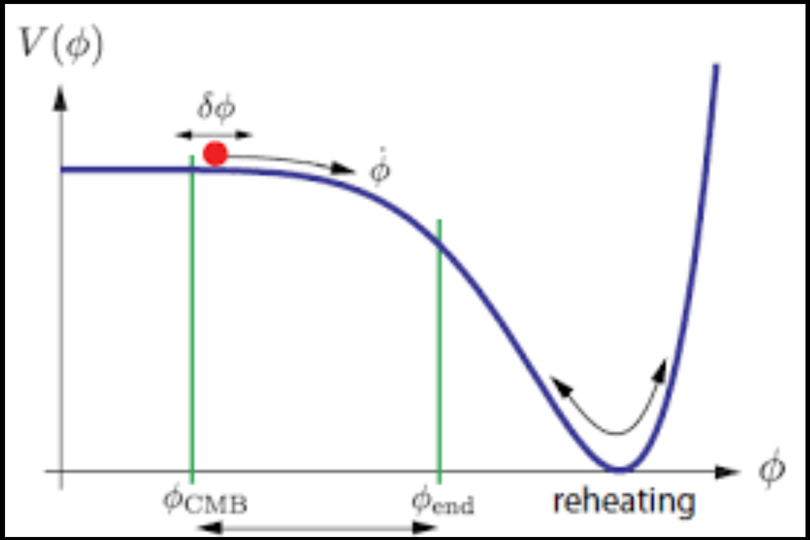
Then

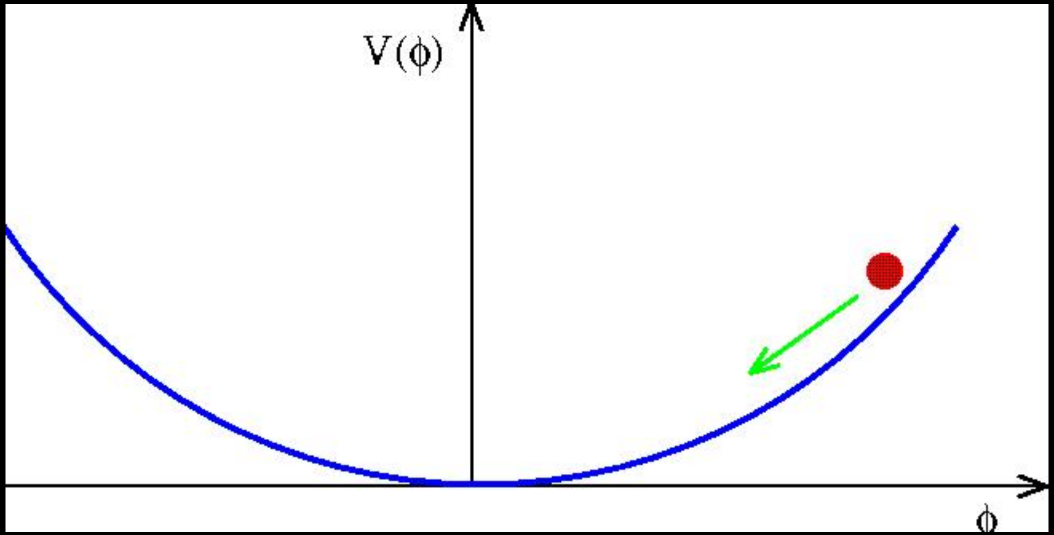
$$n_s - 1 = \left(\frac{d\ln(\delta R)^2}{d\ln k}\right)_{k=aH} = -2\varepsilon - \frac{\dot{\varepsilon}}{H\varepsilon}$$

where

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad n_s = 1 + M_{Pl}^2\left[3\left(\frac{V'}{V}\right)^2 - 2\left(\frac{V''}{V}\right)\right] \approx 1$$

Gravity Waves can test models of Inflation

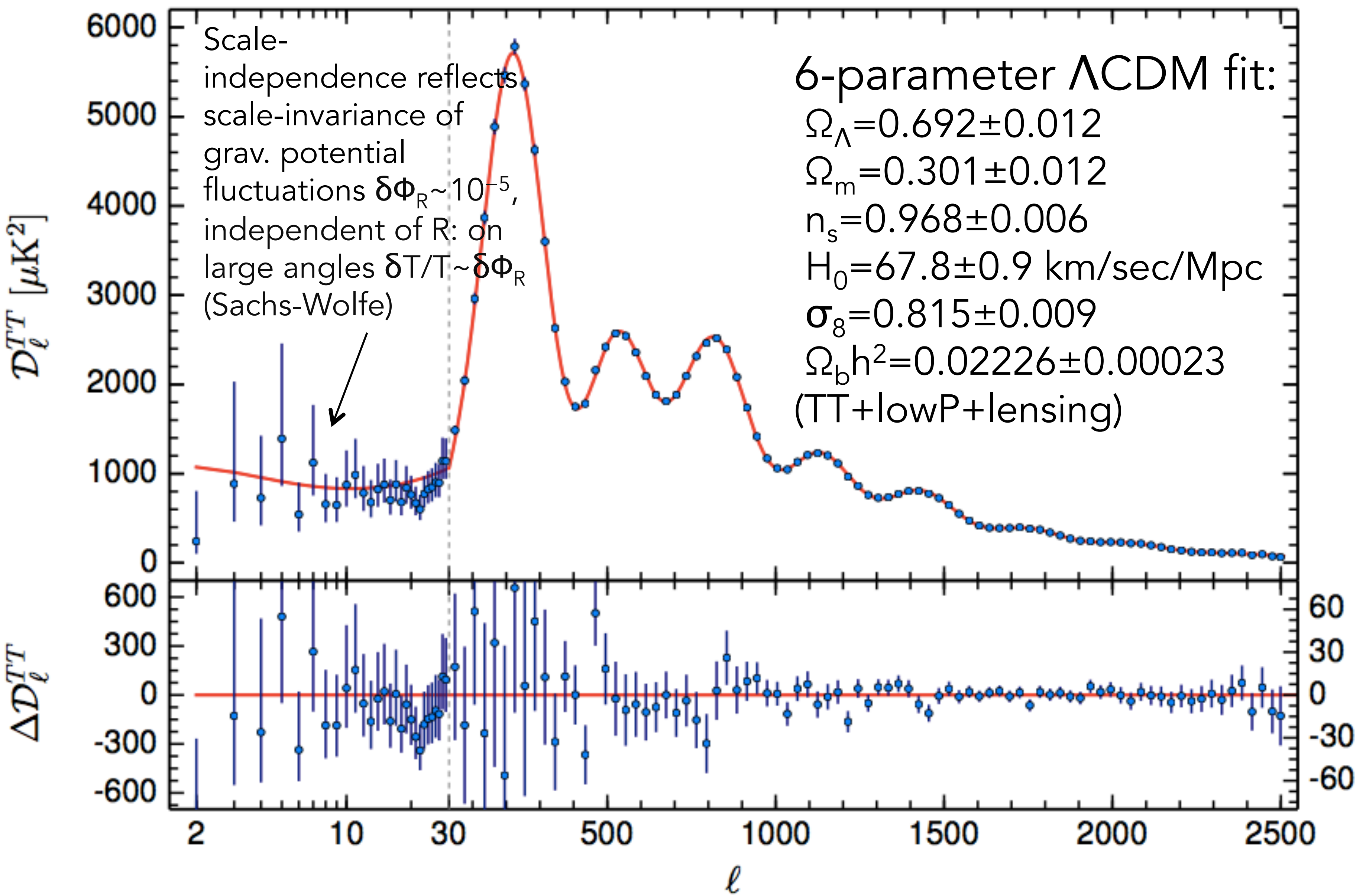




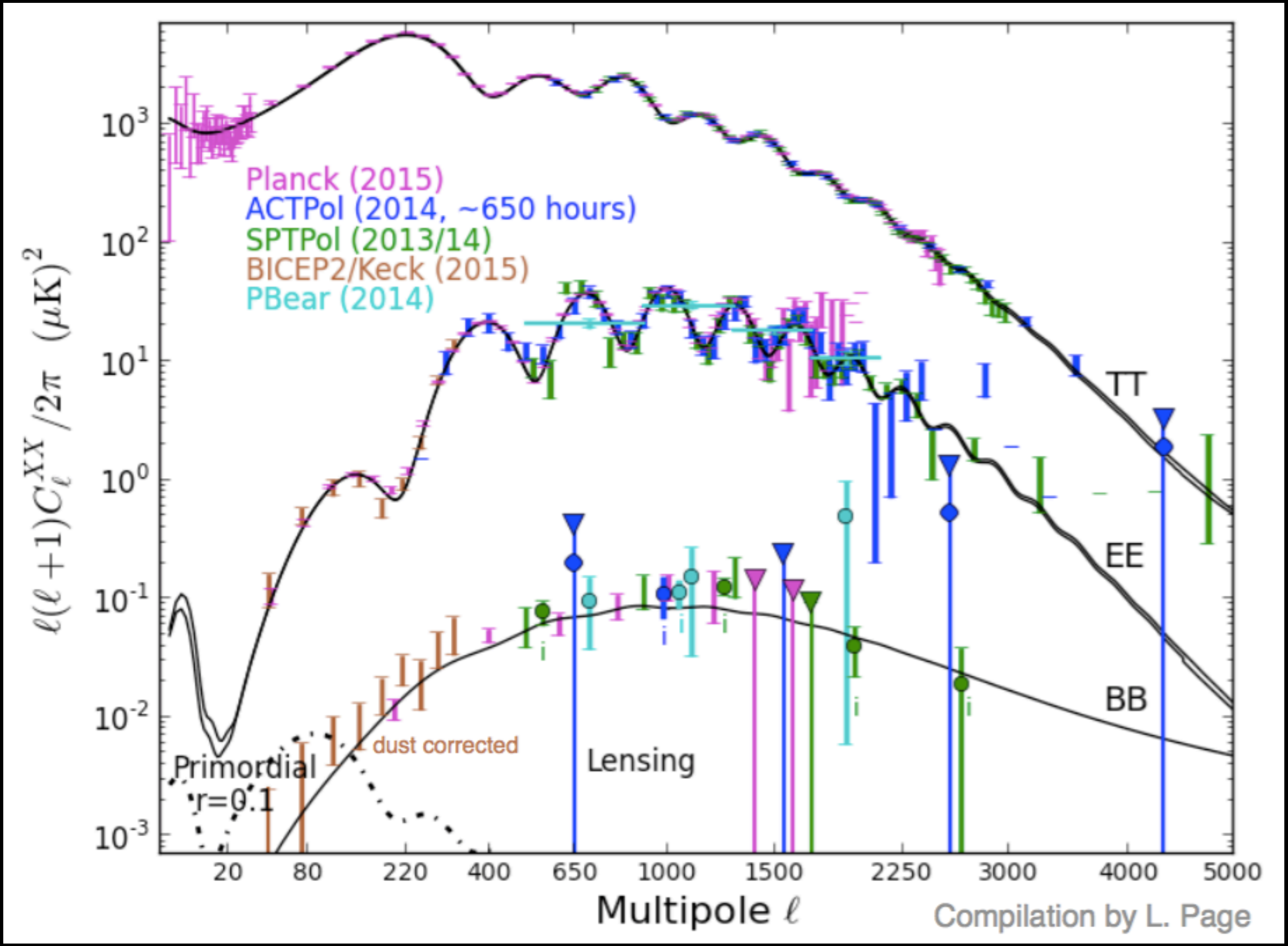
Models inspired by Symmetry breaking: Field evolves from small to large value. Expect little to no gravity wave signal.

'Large field' Models: typically expect detectable gravity wave signal in the CMB.

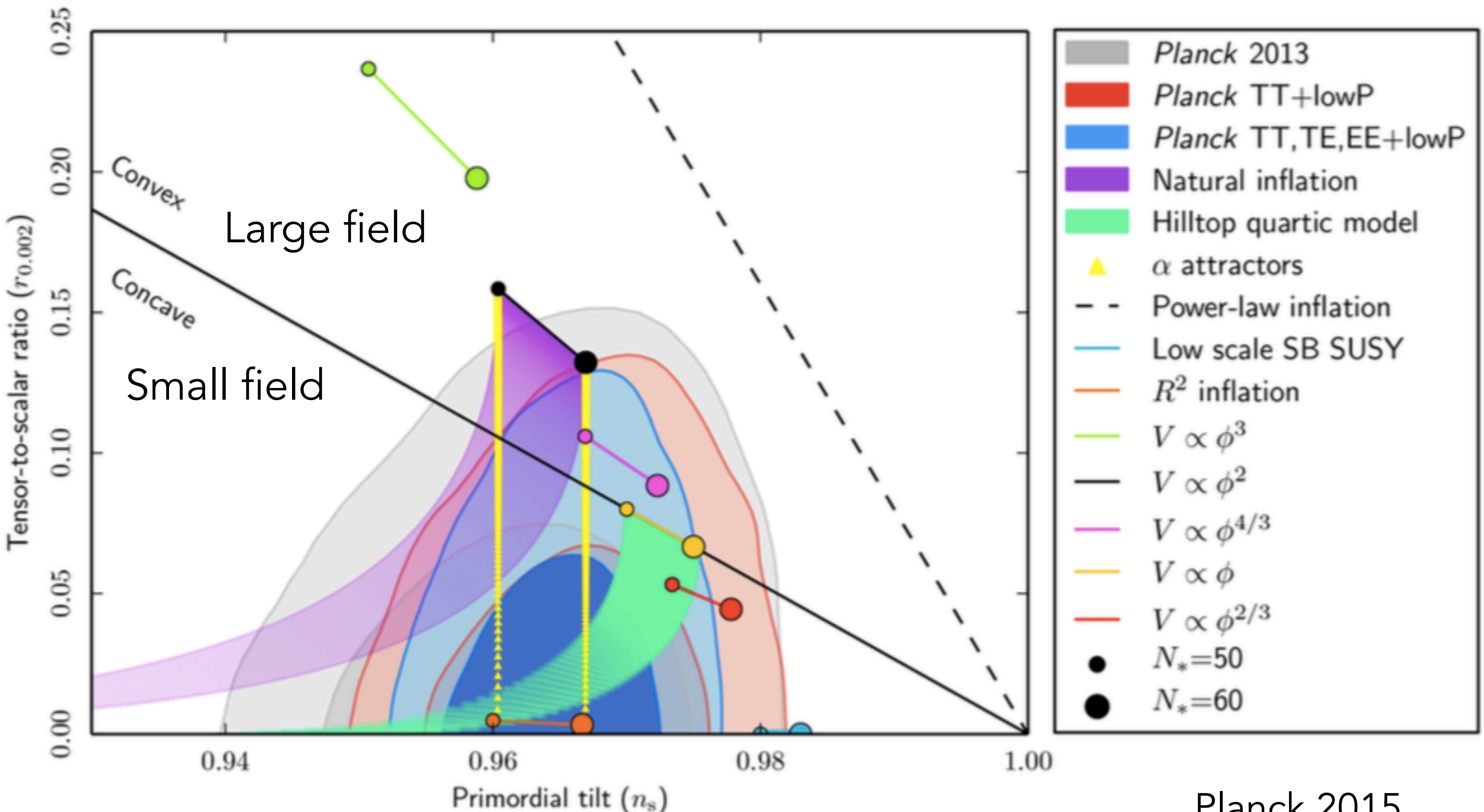
Planck 2015 Results



CMB Results: Temperature+Polarization



Constraints on Inflation



Planck 2015

Shape of the Matter Power Spectrum

From Inflation:

$$\delta\Phi_{grav} \sim 10^{-5}\left(\frac{k}{H_0}\right)^{(n_s-1)/2}$$

$n_s \approx 1$, nearly scale-invariant

Recall:

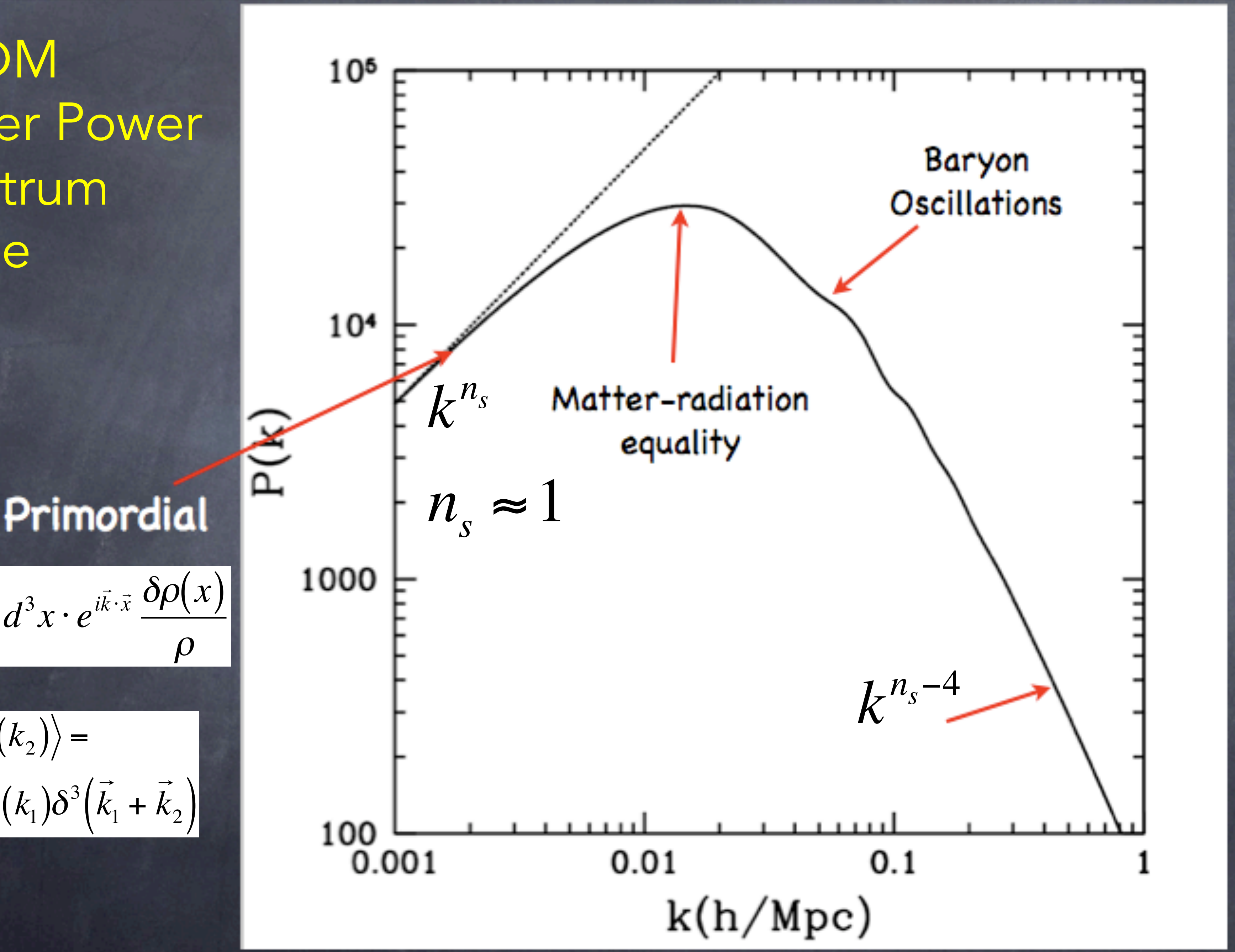
$$\delta\Phi_R \sim \frac{G\delta M_R}{R} \sim H^2 R^2 \left(\frac{\delta\rho}{\bar{\rho}}\right)_R$$

Fourier transform:

$$\delta\Phi_k \sim k^{-2}(\delta\rho / \rho)_k \sim k^{-2}(k^3 P(k))^{1/2} \sim k^{(n_s-1)/2}$$

$$P(k) \sim k^{n_s}$$

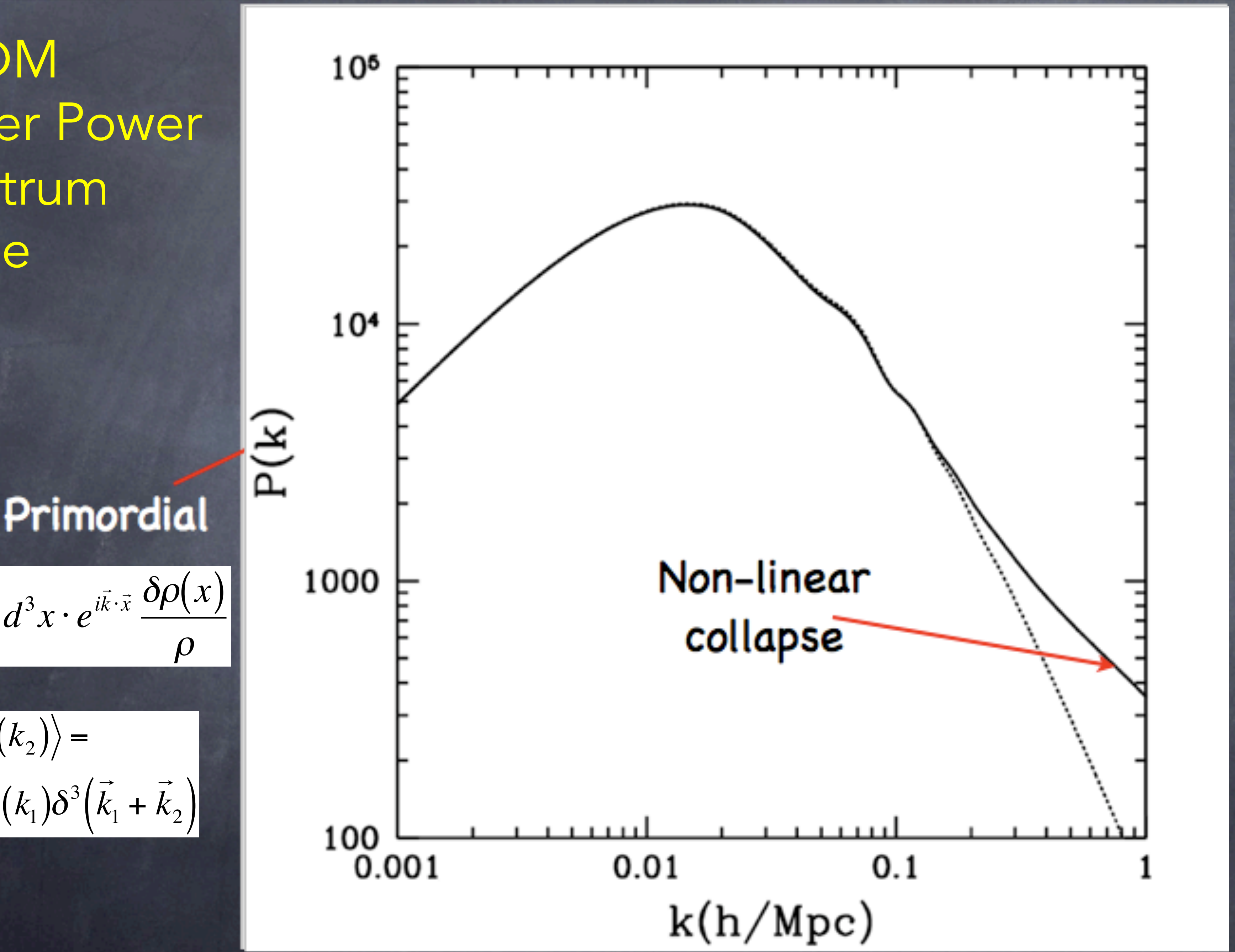
ΛCDM Matter Power Spectrum Shape



$$\delta(k) = \int d^3x \cdot e^{i\vec{k}\cdot\vec{x}} \frac{\delta\rho(x)}{\rho}$$

$$\langle \delta(k_1)\delta(k_2) \rangle = (2\pi)^3 P(k_1)\delta^3\left(\vec{k}_1 + \vec{k}_2\right)$$

ΛCDM Matter Power Spectrum Shape



$$\delta(k) = \int d^3x \cdot e^{i\vec{k}\cdot\vec{x}} \frac{\delta\rho(x)}{\rho}$$

$$\langle \delta(k_1)\delta(k_2) \rangle = (2\pi)^3 P(k_1)\delta^3(\vec{k}_1 + \vec{k}_2)$$

Power Spectrum Transfer Function

Power Spectrum Evolution:

$$P(k,z) \sim k^{n_s} T^2(k,z;\Omega_m,h)$$

Linear Perturbation Theory:

$$\delta_m(x,t) \equiv \frac{\rho_m(x,t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}$$

$$\ddot{\delta}_m + 2H(t)\dot{\delta}_m - 4\pi G\bar{\rho}\delta_m = \ddot{\delta}_m + 2H(t)\dot{\delta}_m - \frac{3}{2}\Omega_m(t)H^2(t)\delta_m = 0$$

Perturbations on small scales, $k_c > k_{eq} = a(t_{eq})H(t_{eq}) \approx 0.07/\mathrm{Mpc}$, enter Hubble radius when Universe still radiation-dominated: $\Omega_m << 1$, amplitude frozen until matter-radiation equality:

$$T^2(k) \rightarrow k^{-4} \text{ for } k >> k_{eq}$$

Late-time Perturbation Evolution slowed by Dark Energy

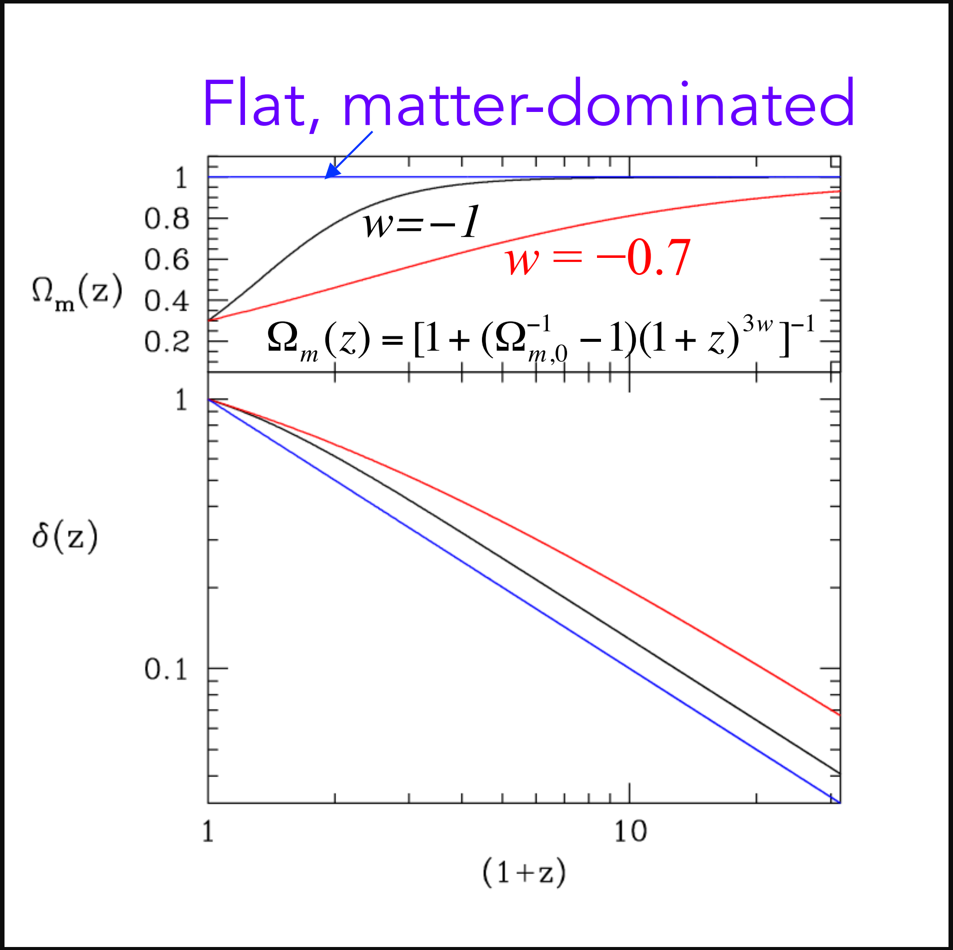
Linear growth rate:

$$\delta_m(x,t) \equiv \frac{\rho_m(x,t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}$$

$$\ddot{\delta}_m + 2H(t)\dot{\delta}_m - \frac{3}{2}\Omega_m(t)H^2(t)\delta_m = 0$$

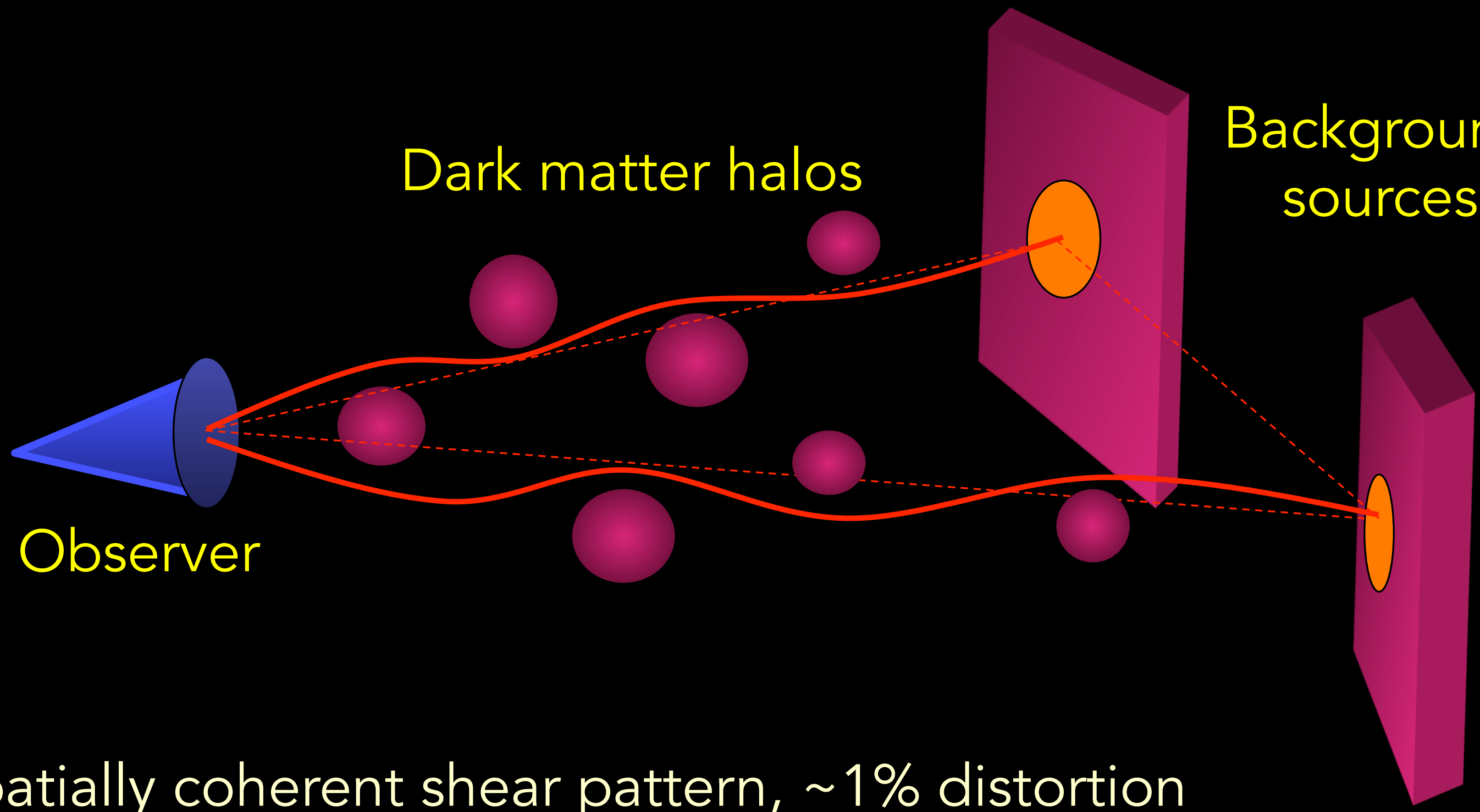
Damping due to expansion

Growth due to gravitational instability



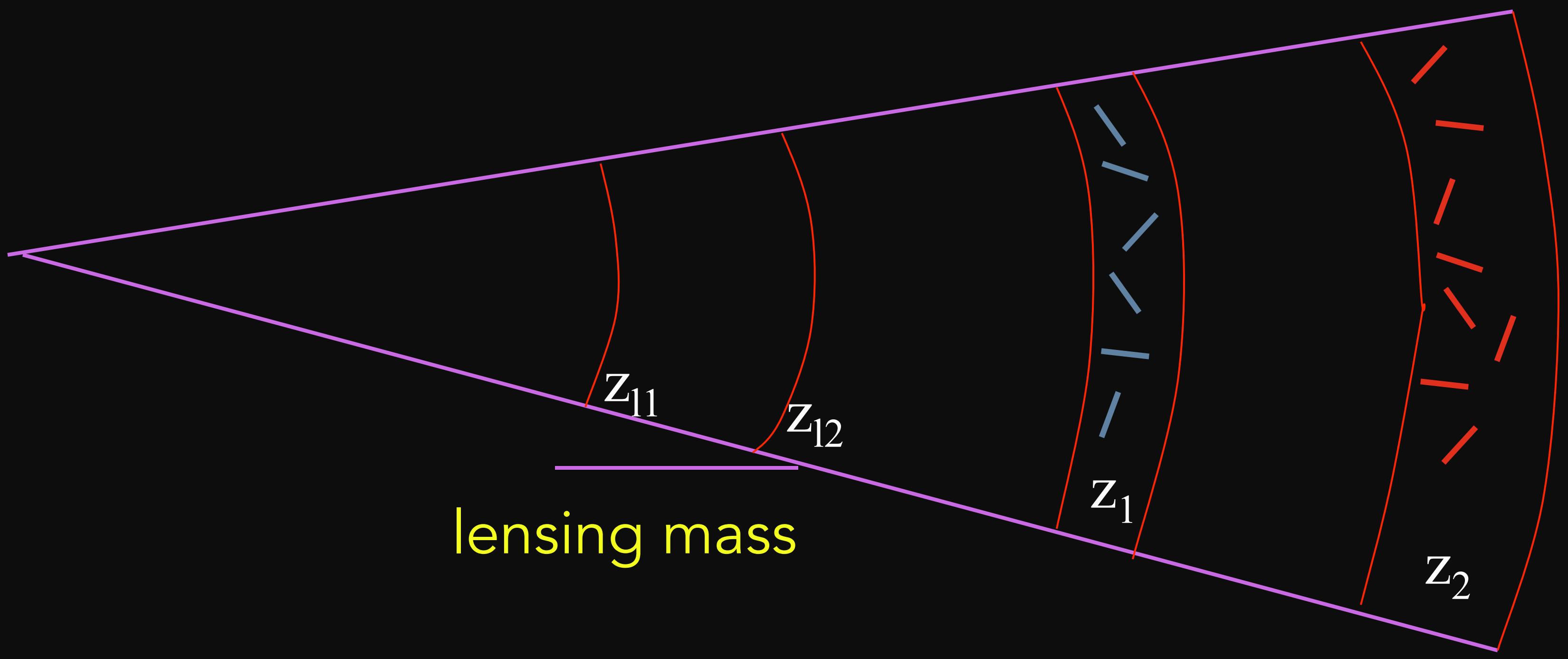
Raising w at fixed Ω_{DE}: decreases net growth of density perturbations, requires higher amplitude of structure at early times

Weak Lensing Cosmic Shear



- Spatially coherent shear pattern, ~1% distortion
- Radial distances depend on *expansion history* of Universe
- Foreground mass distribution depends on *growth* of structure

Lensing Tomography



Shear at z_1 and z_2 given by integral of growth function & distances over lensing mass distribution.

DARK ENERGY SURVEY

Cosmic Shear

Shear-shear correlation function:

$$\hat{\xi}_{\pm}^{ij}(\theta) = \frac{1}{2\pi}\int d\ell\,\ell\,J_{0/4}(\theta\ell)\,P_{\kappa}^{ij}(\ell)$$

Convergence power spectrum

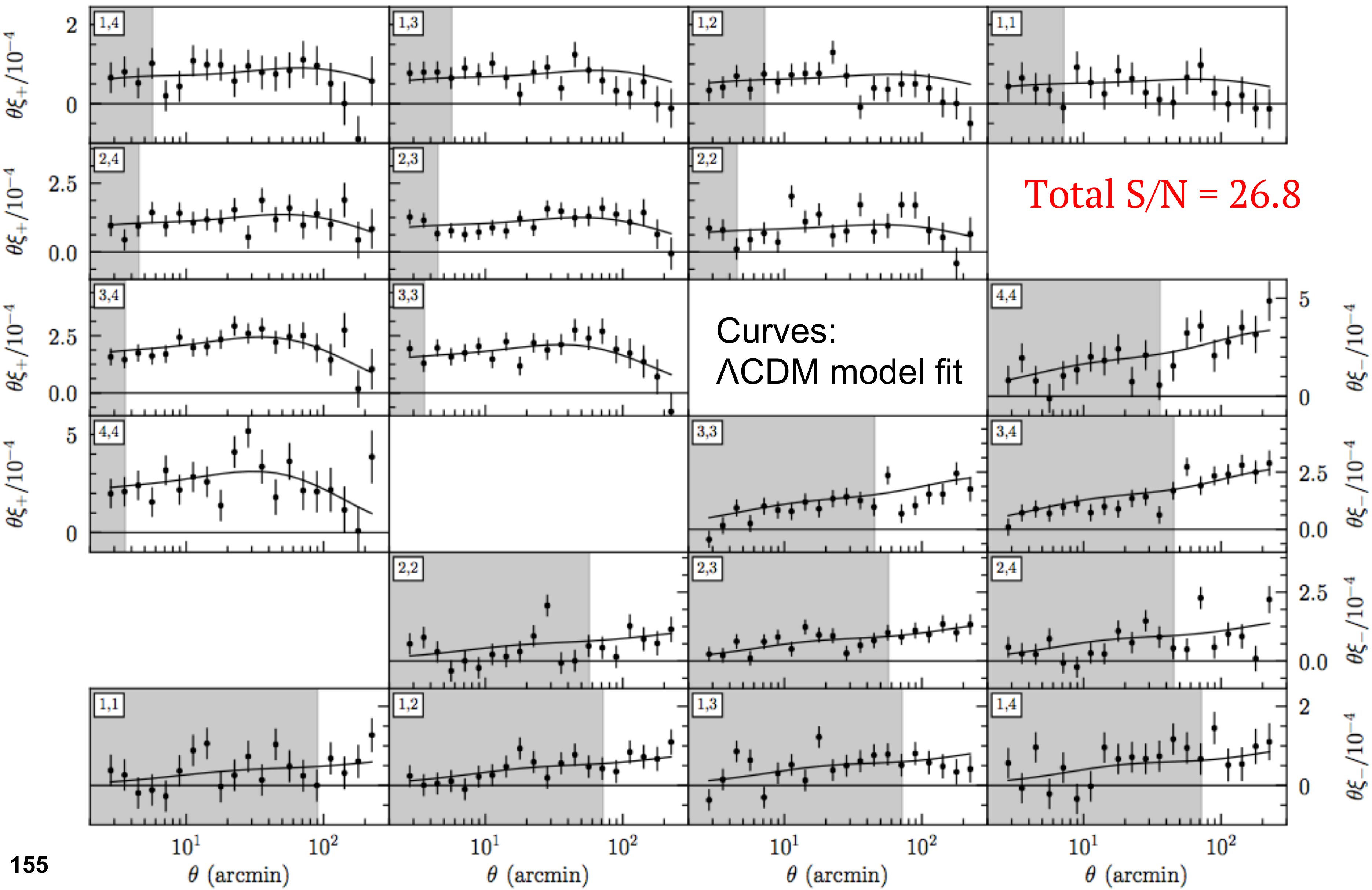
$$P_{\kappa}^{ij}(\ell) = \int_0^{\chi_H} d\chi \frac{q^i(\chi)q^j(\chi)}{\chi^2} P_{\rm NL}\left(\frac{\ell+1/2}{\chi},\chi\right)$$

ΛCDM Mass power spectrum: amplitude & growth of structure

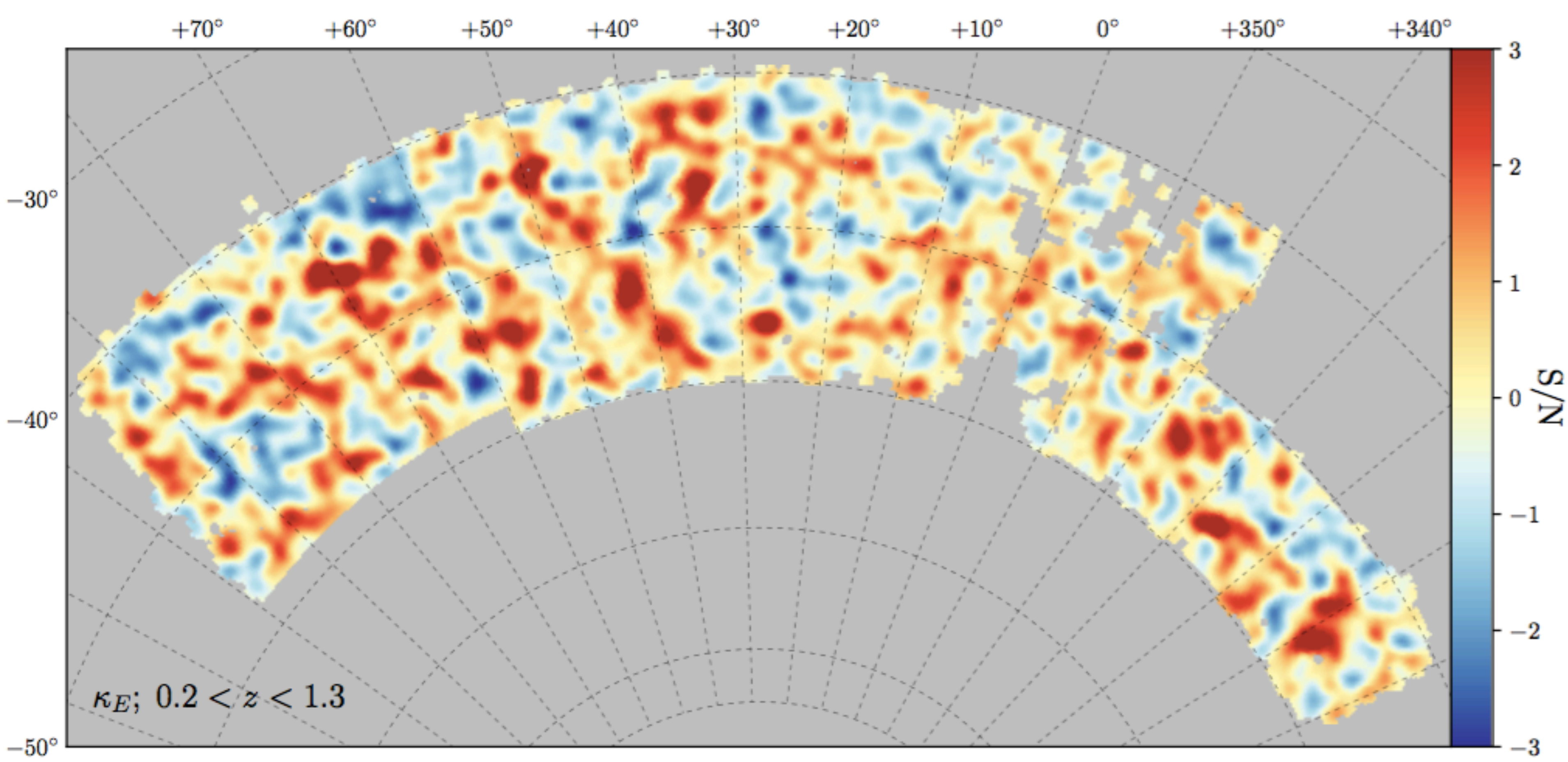
$$q^i(\chi) = \frac{3}{2}\Omega_m\left(\frac{H_0}{c}\right)^2 \frac{\chi}{a(\chi)}\int_{\chi}^{\chi_H} d\chi' n^i(\chi')\frac{dz}{d\chi'}\frac{\chi'-\chi}{\chi'}$$

Geometry (distances or expansion)

DES Y1 Cosmic Shear Results Troxel, et al



Weak Lensing Mass Map of LSS



Reconstruction based on 26 million source galaxy shape measurements Chang, et al

Concluding Remarks

Cosmology has come a long way since 1980-era questions:

Why is the Universe nearly flat and homogeneous?
What is the origin of large-scale structure and how can it form causally?
Is the dark matter baryonic?
How much dark matter is there and how does it impact structure formation?
Where are the anisotropies in the CMB?
What is the large-scale distribution of galaxies and mass in the Universe?
How fast is the Universe expanding? 50 vs 100?

But the cosmological physics questions remain.

Cosmology 2017

Model	Data Sets	Ω_m	S_8	n_s	Ω_b	h	$\sum m_\nu$ (eV) (95% CL)	w
ΛCDM	DES Y1 $\xi_\pm(\theta)$	$0.323^{+0.048}_{-0.069}$	$0.791^{+0.019}_{-0.029}$	...	...	...	...	...
ΛCDM	DES Y1 $w(\theta) + \gamma_t$	$0.293^{+0.043}_{-0.033}$	$0.770^{+0.035}_{-0.030}$	...	...	...	...	...
ΛCDM	DES Y1 3x2	$0.264^{+0.032}_{-0.019}$	$0.783^{+0.021}_{-0.025}$	...	...	...	...	...
ΛCDM	Planck (No Lensing)	$0.334^{+0.037}_{-0.020}$	$0.840^{+0.024}_{-0.028}$	$0.960^{+0.006}_{-0.008}$	$0.0512^{+0.0036}_{-0.0022}$	$0.656^{+0.015}_{-0.026}$	...	...
ΛCDM	DES Y1 + Planck (No Lensing)	$0.303^{+0.029}_{-0.013}$	$0.793^{+0.018}_{-0.014}$	$0.971^{+0.006}_{-0.005}$	$0.0481^{+0.0040}_{-0.0010}$	$0.681^{+0.010}_{-0.025}$	< 0.62	...
ΛCDM	DES Y1 + JLA + BAO	$0.301^{+0.013}_{-0.018}$	$0.775^{+0.016}_{-0.027}$	$1.05^{+0.02}_{-0.08}$	$0.0493^{+0.006}_{-0.007}$	$0.680^{+0.042}_{-0.045}$	...	...
ΛCDM	Planck + JLA + BAO	$0.306^{+0.007}_{-0.007}$	$0.815^{+0.013}_{-0.015}$	$0.969^{+0.005}_{-0.005}$	$0.0485^{+0.0007}_{-0.0008}$	$0.679^{+0.005}_{-0.007}$	< 0.25	...
ΛCDM	DES Y1 + Planck + JLA + BAO	$0.301^{+0.006}_{-0.008}$	$0.799^{+0.014}_{-0.009}$	$0.973^{+0.005}_{-0.004}$	$0.0480^{+0.0009}_{-0.0006}$	$0.682^{+0.006}_{-0.006}$	< 0.29	...
wCDM	DES Y1 $\xi_\pm(\theta)$	$0.317^{+0.074}_{-0.054}$	$0.789^{+0.036}_{-0.038}$	...	...	...	...	$-0.82^{+0.26}_{-0.47}$
wCDM	DES Y1 $w(\theta) + \gamma_t$	$0.317^{+0.045}_{-0.041}$	$0.788^{+0.039}_{-0.067}$	...	...	...	...	$-0.76^{+0.19}_{-0.45}$
wCDM	DES Y1 3x2	$0.279^{+0.043}_{-0.022}$	$0.794^{+0.029}_{-0.027}$	...	...	...	...	$-0.80^{+0.20}_{-0.22}$
wCDM	Planck (No Lensing)	$0.220^{+0.064}_{-0.025}$	$0.798^{+0.035}_{-0.035}$	$0.960^{+0.008}_{-0.006}$	$0.0329^{+0.0100}_{-0.0030}$	$0.800^{+0.050}_{-0.090}$	...	$-1.50^{+0.34}_{-0.18}$
wCDM	DES Y1 + Planck (No Lensing)	$0.230^{+0.023}_{-0.015}$	$0.780^{+0.013}_{-0.023}$	$0.967^{+0.005}_{-0.004}$	$0.0359^{+0.0037}_{-0.0021}$	$0.785^{+0.023}_{-0.037}$	< 0.56	$-1.34^{+0.08}_{-0.15}$
wCDM	Planck + JLA + BAO	$0.304^{+0.008}_{-0.011}$	$0.814^{+0.013}_{-0.016}$	$0.968^{+0.005}_{-0.005}$	$0.0480^{+0.0010}_{-0.0020}$	$0.681^{+0.010}_{-0.009}$	< 0.29	$-1.03^{+0.05}_{-0.05}$
wCDM	DES Y1 + Planck + JLA + BAO	$0.299^{+0.009}_{-0.007}$	$0.798^{+0.012}_{-0.011}$	$0.973^{+0.005}_{-0.004}$	$0.0479^{+0.0015}_{-0.0012}$	$0.683^{+0.009}_{-0.010}$	< 0.35	$-1.00^{+0.04}_{-0.05}$

Much more to come!

DES Collaboration