CMB & Light Degrees of Freedom

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Image Credits: Planck, ANL
Light Relics – What and Why?

- **Light** – Particles which were relativistic at recombination ($m < 1$ eV)
- **Relics** – Left over from the early universe (non-negligible energy density at early times)

- Cosmology in general, and the CMB in particular, provides a window into very high energy physics through sensitivity to light relics

Image Credits: Planck, CMS
The “effective number of neutrino species” $N_{\text{eff}}$ measures the total energy density in radiation excluding photons. Because it receives contributions from all light relics, $N_{\text{eff}}$ need not have anything to do with neutrinos. $N_{\text{eff}}$ is a cosmological observable due to the gravitational influence of the radiation in the early universe.
Relativistic Particles in Equilibrium

• The distribution function for relativistic particles in thermal equilibrium and vanishing chemical potential is given by

\[ f(p) = \frac{1}{\exp \left( \frac{p}{T} \right) + 1} \]

• The energy density of a thermal bath of massless particles is

\[ \rho(T) = \int dp \frac{4\pi p^3 g}{\exp \left( \frac{p}{T} \right) + 1} = \begin{cases} 
\frac{g\pi^2}{30} T^4 & \text{Bosons} \\
\frac{7}{8} \frac{g\pi^2}{30} T^4 & \text{Fermions}
\end{cases} \]
Thermodynamics

• Let us define the quantity $g_\ast(T)$ which counts the number of spin states for all particles and antiparticles in equilibrium, including a factor $7/8$ for fermions, which then gives

$$\rho(T) = g_\ast(T) \frac{\pi^2}{30} T^4$$

• The pressure and entropy density for massless particles in thermal equilibrium are

$$P(T) = \frac{\rho(T)}{3} \quad s(T) = \frac{\rho(T) + P(T)}{T} = \frac{4\rho(T)}{3T}$$
Entropy Conservation

• The comoving entropy is conserved in thermal equilibrium, and so
  \[ \frac{d}{dt}(a^3 s(T)) = 0 \]

\[ T \propto g_*(T)^{-1/3} a^{-1} \]

• Away from mass thresholds, \( g_*(T) \) is constant, and the temperature falls as the inverse of the scale factor \( T \propto a^{-1} \)

• When particles fall out of equilibrium and annihilate, the particles remaining in equilibrium are heated relative to a free expansion
Neutrino Decoupling

• Let us see how this works for neutrinos in the instant decoupling limit
• Above about 1 MeV, weak interactions kept neutrinos in thermal equilibrium with the plasma of photons, electrons, and positrons
• Below 1 MeV (about 1 second after inflation) neutrinos decoupled and began a free expansion
• From this point on, we must separately keep track of the neutrino and photon temperatures
Neutrino Decoupling

• Shortly after neutrinos decoupled, the photon temperature fell below the electron mass, and electrons and positrons annihilated

\[
g_\ast_{\text{before}} = 2 + \frac{7}{8}(2 + 2) = \frac{11}{2} \quad g_\ast_{\text{after}} = 2
\]

• Noting that photons and neutrinos had a common temperature before decoupling, and neutrinos underwent free expansion after decoupling, we find

\[
\frac{g_\ast_{\text{before}} T_\gamma,\text{before}^3}{T_\nu,\text{before}^3} = \frac{g_\ast_{\text{after}} T_\gamma,\text{after}^3}{T_\nu,\text{after}^3} = \left(\frac{4}{11}\right)^{1/3}
\]
Neutrino Decoupling

• After neutrinos decoupled, photons were heated due to the annihilation of electrons and positrons

• Entropy conservation allows us to calculate the relative temperatures simply by counting states

\[
\frac{T_{\gamma, \text{after}}}{T_{\nu, \text{after}}} = \left( \frac{11}{4} \right)^{1/3} = 1.401
\]

\[
T_{\gamma,0} = 2.725 \text{ K}
\]

\[
T_{\nu,0} = 1.945 \text{ K}
\]
\[ \rho_r = \rho_\gamma \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right) \]

Fermions  (Temperature Ratio)$^4$

- We can now see that the definition of $N_{\text{eff}}$ was chosen to count the number of species of neutrinos (assuming one massless helicity state per species and instantaneous decoupling).
- Non-instantaneous decoupling and small non-equilibrium corrections slightly increase the prediction in the standard model.

\[ N_{\text{eff}}^{\text{SM}} = 3.046 \]

Mangano, et al. (2005)
BSM Contributions to $N_{\text{eff}}$

- We can apply the same techniques to calculate the contribution of any light thermal relic to $N_{\text{eff}}$ by comparing the relic temperature to the neutrino temperature

$$\left(\frac{T_{\text{relic}}}{T_{\nu}}\right)^3 = \frac{g_\star(T_{\nu-\text{decoupling}})}{g_\star(T_F)} = \frac{43/4}{g_\star(T_F)}$$

$$g_\star(T_{\nu-\text{decoupling}}) = 2 + \frac{7}{8} (2 + 2 + 3 \times 2) = 43/4$$

- For light thermal relics, the contributions are from:
  - Photons
  - 3 neutrinos
  - 3 antineutrinos
  - $e^-$ and $e^+$
BSM Contributions to $N_{\text{eff}}$

• As a result, we find the contribution of a light thermal relic to $N_{\text{eff}}$ is determined only by the number of spin states $g$ and its decoupling temperature $T_F$

$$
\Delta N_{\text{eff}} = \begin{cases} 
\frac{4g}{7} \left( \frac{43/4}{g^* (T_F)} \right)^{4/3} & \text{Boson} \\
\frac{g}{2} \left( \frac{43/4}{g^* (T_F)} \right)^{4/3} & \text{Fermion}
\end{cases}
$$
Standard Model Degrees of Freedom

- Since the masses and spin states of the particles of the Standard Model are known, we can easily calculate $g_*(T)$.

- At early times and high temperatures $T \gtrsim 100$ GeV,

\[ g_* = g_b + \frac{7}{8} g_f = 106.75 \]

\[ g_b = 28 \quad \text{photons (2), } W^\pm \text{ and } Z^0 (3 \cdot 3), \text{ gluons (8 \cdot 2), and Higgs (1)} \]

\[ g_f = 90 \quad \text{quarks (6 \cdot 12), charged leptons (3 \cdot 4), and neutrinos (3 \cdot 2)} \]
Standard Model Thermal History

- As the temperature drops below particle masses, those particles fall out of equilibrium and $g_*(T)$ decreases.
- There is a large drop in $g_*(T)$ at the QCD phase transition, when the degrees of freedom change from quarks and gluons to mesons and baryons.

$$g_*(T > \Lambda_{\text{QCD}}) = 2 + 2 \times 8 + \frac{7}{8} (3 \times 12 + 4 + 4 + 3 \times 2) = 61.75$$

$$g_*(T < \Lambda_{\text{QCD}}) = 2 + 3 + \frac{7}{8} (4 + 4 + 3 \times 2) = 17.25$$

Image Credit: Baumann
Natural Targets for $N_{\text{eff}}$

- This analysis shows that there is a minimum contribution to $N_{\text{eff}}$ for any light thermal relic (assuming only Standard Model states)

\[
\Delta N_{\text{eff}} \geq \begin{cases} 
0.027 & \text{Real Scalar} \\
0.047 & \text{Weyl Fermion} \\
0.054 & \text{Vector}
\end{cases}
\]

- These lower bounds apply if the relic was in thermal equilibrium at any time after inflation
Cosmology: Low Energy Probe of High Energy Physics

• CMB temperature today: 2.725 K (= 2*10^{-4} eV)
• Photon decoupling: 3000 K (=0.25 eV)
• Neutrino decoupling: 10^{10} K (=1 MeV)
• QCD phase transition: 10^{12} K (=150 MeV)
• EW phase transition: 10^{15} K (= 100 GeV)
• Reheating: As large as 10^{15} GeV

• Constraints on $N_{\text{eff}}$ probe physics all the way up to the reheating scale
Application: Axion Constraints

- An observation of $N_{\text{eff}}$ ruling out scalar thermal relics would put extremely strong essentially mass independent constraints on axion-like particles.
- These constraints do not require that the axions are the dark matter (in contrast to e.g. ADMX).

Baumann, Green, Wallisch (2016)
• The harmonic transform of the two-point correlation function of CMB fluctuations is known as the angular power spectrum.

• Most of the statistical information of CMB maps is described by the angular power spectrum.
Effects of Light Relics on the CMB

- Increased radiation density leads to increased damping (when holding the scale of matter-radiation equality fixed)
- Anisotropic stress due to radiation free streaming has two effects:
  - Shift in amplitude at small scales
  - Phase shift of acoustic peaks at small scales

Bashinsky, Seljak (2004); Hou, Keisler, Knox, Millea, Reichardt (2012); Follin, Knox, Millea, Pan (2015); Baumann, Green, JM, Wallisch (2015);
Diffusion Damping of the CMB

• The free streaming of photons during recombination smooths out fluctuations on scales smaller than the damping scale

\[ \Delta(k) \propto \exp \left( -\left( \frac{k}{k_D} \right)^2 \right) \]

\[ k_D^{-2} \propto (n_e H)^{-1} \]

• \( N_{\text{eff}} \) affects the expansion rate during radiation domination, and thus the damping scale

\[ 3 M_{\text{pl}}^2 H^2 \simeq \rho_\gamma \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}} \right) \]

• The change to the damping scale is the most prominent effect of \( N_{\text{eff}} \) on the CMB, but it is degenerate with changes to the free electron fraction (due to e.g. changes to the primordial helium abundance)
The Special Role of the Phase Shift

- Fluctuations in free-streaming radiation travel faster than the sound speed of the photon-baryon plasma.
- These fluctuations lead to a characteristic phase shift (and amplitude shift) of the acoustic peaks of the CMB (and LSS) power spectrum at small angular scales.
- The phase shift is particularly important for several reasons:
  - It is difficult to reproduce in the absence of free-streaming radiation.
  - The phase shifts break degeneracies which would otherwise be present.
  - Various forms of dark radiation can be distinguished by the phase shift.
  - Future constraints will be driven by the phase shift.

\[ N_{\text{eff}} = 3.046 \]
\[ N_{\text{fluid}} = 0 \]
\[ N_{\text{eff}} = 2.046 \]
\[ N_{\text{fluid}} = 1 \]

Unlensed
Lensed

Bashinsky, Seljak (2004); Baumann, Green, JM, Wallisch (2015); Baumann, Green, Zaldarriaga (2017)
Current Constraints on Thermal Relics

\[ N_{\text{eff}}^{\text{CMB}} = 3.04 \pm 0.18 \]

\[ N_{\text{eff}}^{\text{SM}} = 3.046 \]

\[ \Delta N_{\text{eff}} \geq \begin{cases} 0.027 & \text{Real Scalar} \\ 0.047 & \text{Weyl Fermion} \\ 0.054 & \text{Vector} \end{cases} \]

- Current constraints on \( N_{\text{eff}} \) agree with the Standard Model prediction and disfavor light relics which decoupled after the QCD phase transition

Planck (2015); CMB-S4 Science Book (2016)
Next Generation CMB Experiments

**SIMONS OBSERVATORY**

- Observing 2021-2022
- Telescope array in Atacama Desert in Chile
- Funded, construction starting soon

**CMB-S4**

- Observing mid- to late-2020s
- Multiple telescopes in Atacama Desert, at South Pole, and perhaps a Northern Site
- Strong support from DOE and NSF

**Science Goals:**
- **Fundamental Physics:** Inflation, **Light Relics**, Neutrino Properties, ...
- **Astrophysics:** Mass Maps, Cluster Science, Cross-Correlations, ...
CMB-S4 $N_{\text{eff}}$ Forecasts

- For plausible design parameters, the projected $1\sigma$ CMB-S4 constraint on $N_{\text{eff}}$ is very near the important theoretical thresholds for light thermal relics

CMB-S4 Specs:

$\ell \geq 30$

$f_{\text{sky}} = 0.4$

+ BBN Consistency

CMB-S4 Science Book (2016)
Non-Thermal Light Relics

• Light states need not have been in thermal equilibrium with the Standard Model to contribute to the radiation energy density

• Measurements of $N_{\text{eff}}$ give constraints on all forms of decoupled radiation, including:
  – Sterile neutrinos
  – Gravitational waves
  – Dark photons
  – Many others

Chu, Cirelli (2006); Boyle, Buonanno (2007); Ackerman, et al. (2008); Steigman (2012); Meerburg, Hadzhiyska, Hlozek, JM (2015); Bridle, et al. (2016); …
Big Bang Nucleosynthesis

- Measurements of primordial light element abundances (Helium-4 and Deuterium) put a constraint on $N_{\text{eff}}$ at around 3 minutes after the end of inflation.
- BBN is weakly sensitive to the neutrino energy spectrum as well as the total radiation energy density.

$$N_{\text{eff}}^{\text{BBN}} = 3.28 \pm 0.28$$

PDG (2013); Cooke, et al. (2014); Cyburt, et al. (2015); Bond, Fuller, Grohs, JM, Wilson (In prep.)
**$N_{\text{eff}}$ Impact on BBN**

- Big Bang Nucleosynthesis is the process by which light elements were formed in the early universe.
- After weak decoupling, protons and neutrons only interconverted by free neutron decay which occurred until neutrons became bound into stable nuclei (mostly helium-4) when the temperature was low enough for deuterium to form.
- Increasing $N_{\text{eff}}$ increases the expansion rate, thereby decreasing the time for free neutron decay, increasing the neutron survival fraction, thus increasing the helium-4 abundance.

\[
\begin{align*}
&\uparrow N_{\text{eff}} & \uparrow H & \uparrow \text{Time for free neutron decay} & \uparrow n_n(T_D) & \uparrow Y_p
\end{align*}
\]
Time-Dependent Density

Primordial Abundances

\[ N_{\text{eff}}^{\text{BBN}} = 3.28 \pm 0.28 \]

CMB Measurements

\[ N_{\text{eff}}^{\text{CMB}} = 3.04 \pm 0.18 \]

- Current constraints agree with one another and with the SM prediction
- Combining these constraints gives insight into time-dependent changes in \( N_{\text{eff}} \)

Testing BBN

- Upcoming CMB experiments will provide joint constraints on $N_{\text{eff}}$ and the primordial helium abundance allowing sensitivity to BBN physics
- Delensing T and E spectra helps to break the degeneracy between $N_{\text{eff}}$ and $Y_p$

Green, JM, van Engelen (2016); CMB-S4 Science Book (2016)
Standard Model Light Relics – The CvB

- Neutrinos were abundantly produced in the early universe and decoupled at a temperature of about 1 MeV
- Relic neutrinos remain very abundant today, with a mean density of 112 cm\(^{-3}\) per species

\[ T_{\nu,0} = 1.945 \text{ K} = 0.17 \text{ meV} \quad \Omega_{\nu} h^2 \sim \frac{\sum m_{\nu}}{94 \text{ eV}} \]

- The non-negligible energy density of the cosmic neutrino background provides cosmology a unique opportunity to study neutrinos in the low energy regime
Neutrino Mass

- Neutrino oscillation experiments give evidence for neutrino mass and provide clear theoretical thresholds for mass scale measurements
- The overall mass scale of neutrinos is very difficult to observe in laboratory experiments, but is accessible with cosmological observations

Super-Kamiokande (1999); Sudbury Neutrino Observatory (2001); CMB-S4 Science Book (2016)
Effect of Neutrino Mass

- At least two species of neutrinos are non-relativistic at late times and contribute to the matter power spectrum.
- On small scales, neutrinos free stream out of potential wells and suppress the growth of structure.
- The matter power suppression is observable in CMB lensing.

Eisenstein, Hu (1997); Kaplinghat, Knox, Song (2003); Park, et al. (2016); CMB-S4 Science Book (2016)
Gravitational Lensing

- CMB photons are deflected by the gravitational potential of structures which intervene along our line of sight
- Gravitational lensing results in statistical anisotropy of CMB maps, thereby allowing for reconstruction of the lenses

Hu, Okamoto (2002); Image Credit: Planck
CMB-S4 Lensing Power Spectrum

- CMB-S4 will make a very precise measurement of the lensing power spectrum
- The neutrino mass measurement depends on the ability to break degeneracies
  - Optical Depth $\tau$
  - Matter Density $\Omega_m h^2$

CMB-S4 Science Book (2016)
CMB-S4 + DESI BAO Forecasts

CMB-S4 Specs:
\[
\ell \geq 30 \\
f_{\text{sky}} = 0.4 \\
\tau = 0.06 \pm 0.01
\]
(Current Planck) + DESI BAO

CMB-S4 Science Book (2016)
Forecasted cosmological constraints on neutrino mass are limited by measurements of the optical depth. Significant improvements could be achieved with satellite-based CMB observations to improve $\tau$ measurement.
Sterile and Exotic Neutrinos

• Sterile neutrinos hinted by short baseline anomalies would be detected at high significance with CMB-S4

• Modified thermal histories could also dilute neutrinos compared to standard expectations
Conclusions

• Next generation CMB observations will provide deep insights into cosmology and fundamental physics
• $N_{\text{eff}}$ in particular holds a great deal of promise for constraining physics beyond the standard model, and has a well motivated theoretical target at $\Delta N_{\text{eff}} = 0.027$ within reach of CMB-S4
• We stand to learn a huge amount by measuring $N_{\text{eff}}$ even without a significant deviation from the standard model prediction
• Joint constraints on Neff and primordial helium abundance will allow probes of big bang nucleosynthesis and neutrino physics
• Cosmology will make a detection of the sum of neutrino masses and may distinguish the mass hierarchy