

Atomic Precision Frontier

Probing light force-mediators by isotope shift spectroscopy

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Outline

- ◆ Introduction: brief motivation for light physics + exp' frontier.

See also Graham's talk.

- ◆ Main idea, pheno. (geometrical picture).

Berengut, Budker, Delaunay, Flambaum, Frugueule, Fuchs, Grojean, Harnik, Ozeri, GP & Soreq (17)
Frugueule, Fuchs, GP & Schlaffer; Delaunay, Ozeri, GP & Soreq (16)

- ◆ Bounds (with minimal theory).

- ◆ Conclusions.

Introduction

- ◆ Flavor-mass hierarchies, strong-CP problem, compactifications: solid reasons to expect light weakly interacting new fields.

. . . See e.g.: Kim (87); Feng, Moroi, Murayama & Schnapka (98); Wilczek (82); Gelmini, Nussinov & Yanagida (83)

- ◆ Solutions typically lead to pseudo-scalar couplings to matter, axions, familons, etc.; with active exp' effort.

Recent reviews: Graham, Irastorza, Lamoreaux, Lindner & Bibber (2015) Jaeckel & Ringwald (10); Kim & Carosi (08)

- ◆ However, light gauge bosons; CP violation; relaxions (new solution to Higgs-hierarchy problem) => vector/scalar interactions.

Graham, Kaplan & Rajendran (15)

Search strategies ?

- ◆ Interaction \w density: Direct DM (collision) or time dependent.

Talk by: Feng; Monzani; Pyle; Schuster; Bibber.

Talks: Bibber; Graham.

- ◆ Produce on shell in the sky (low mass) **or** in the lab (largish-coupling).

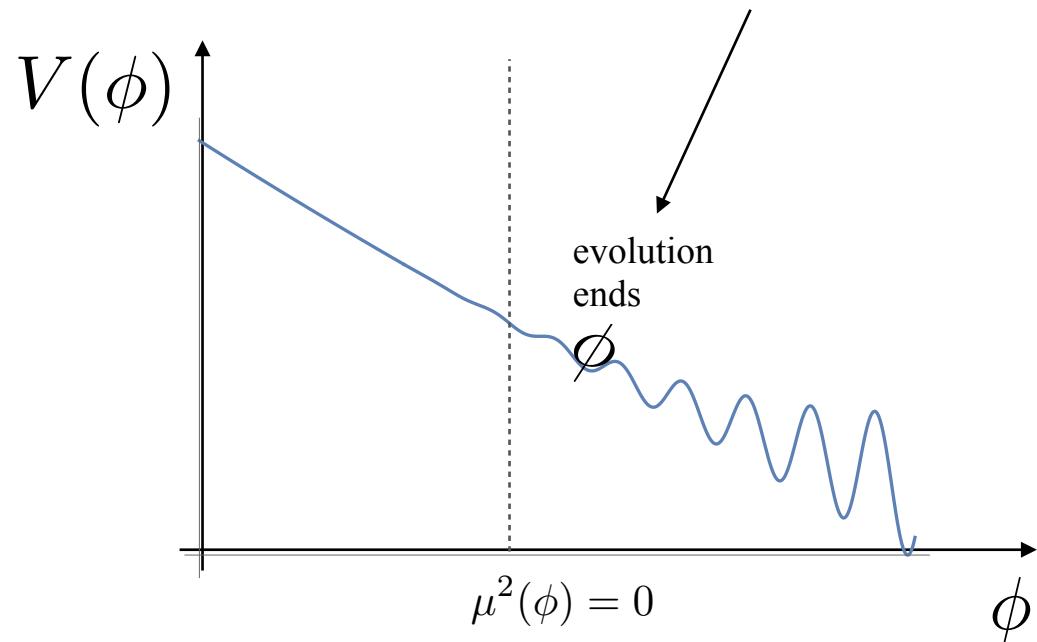
Talks by: Albert; Toro.

- ◆ Virtually (ex. Peskin-Takeuchi) via induced potential/force.

Ex.: The relaxion-axion CP problem

- ♦ Let us go back to consider the relaxion properties.

The relaxion halt at generic point, \backslashw O(1) phase, CP is badly broken!



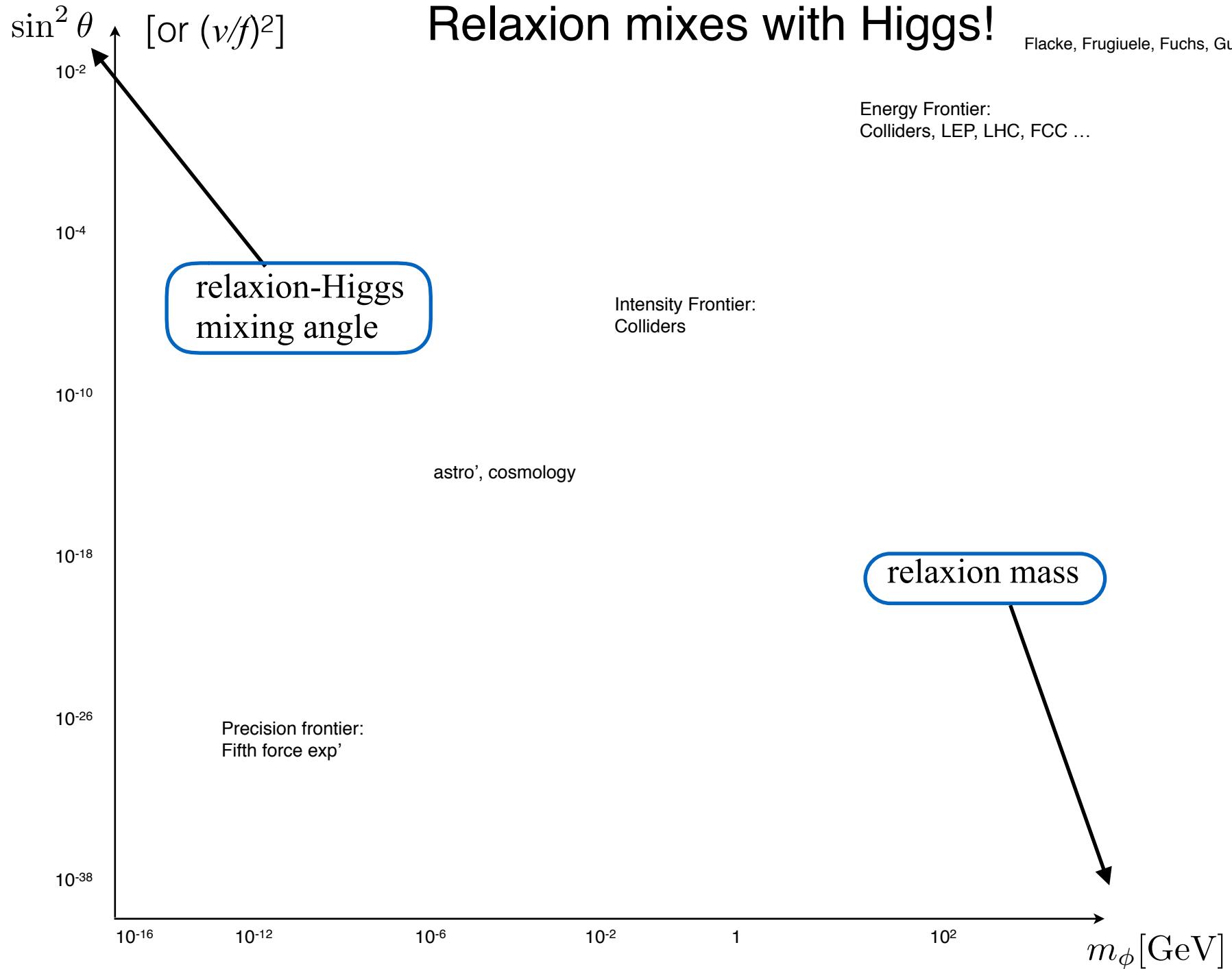
- ♦ Generically failed to play the role of axion \Rightarrow CP problem remains.

Graham, Kaplan & Rajendran (15)

- ♦ Leads to interesting implications such as mixing \backslashw Higgs.

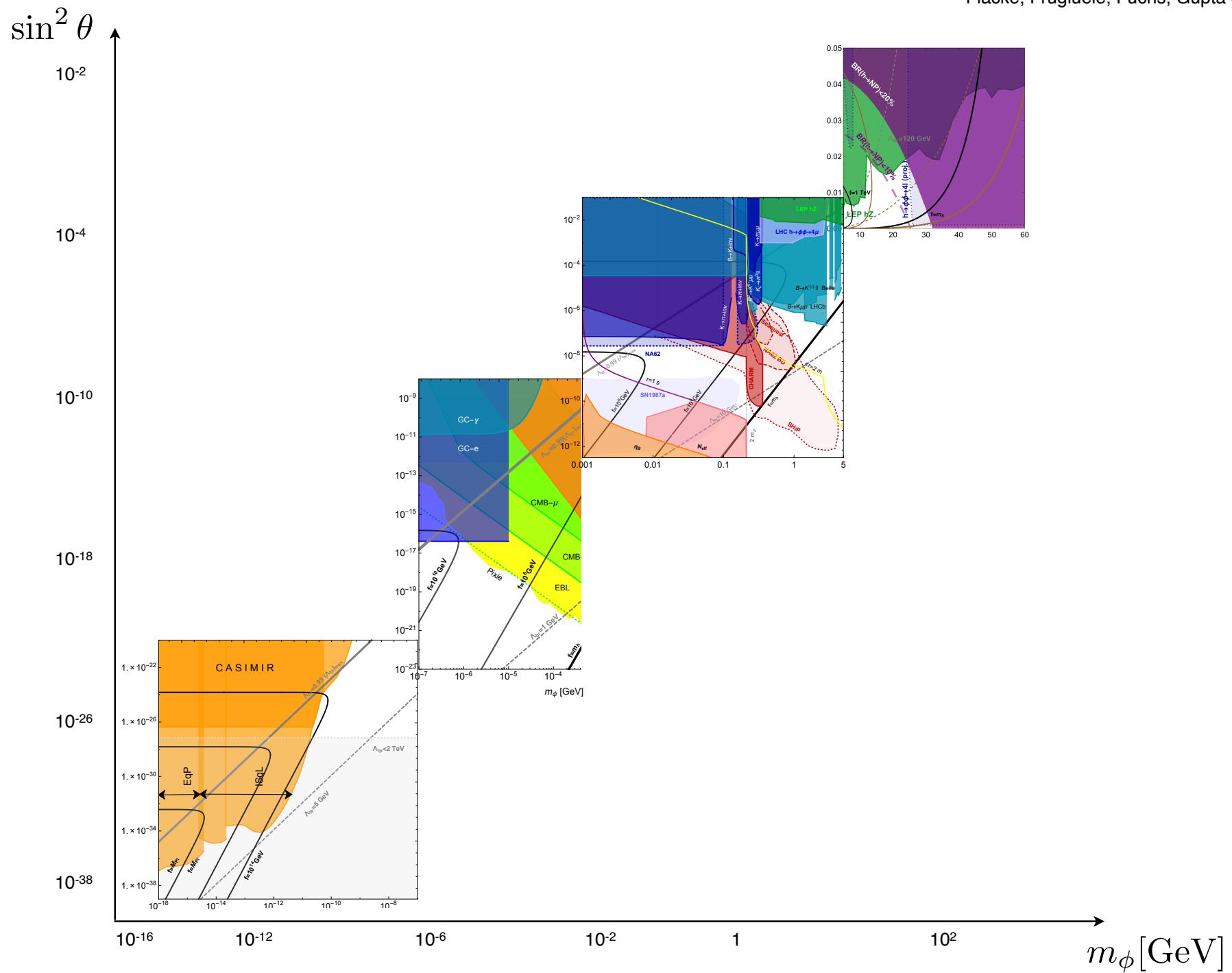
Flacke, Frugueule, Fuchs, Gupta & GP (16)

Illustration: how to search for a relaxion?



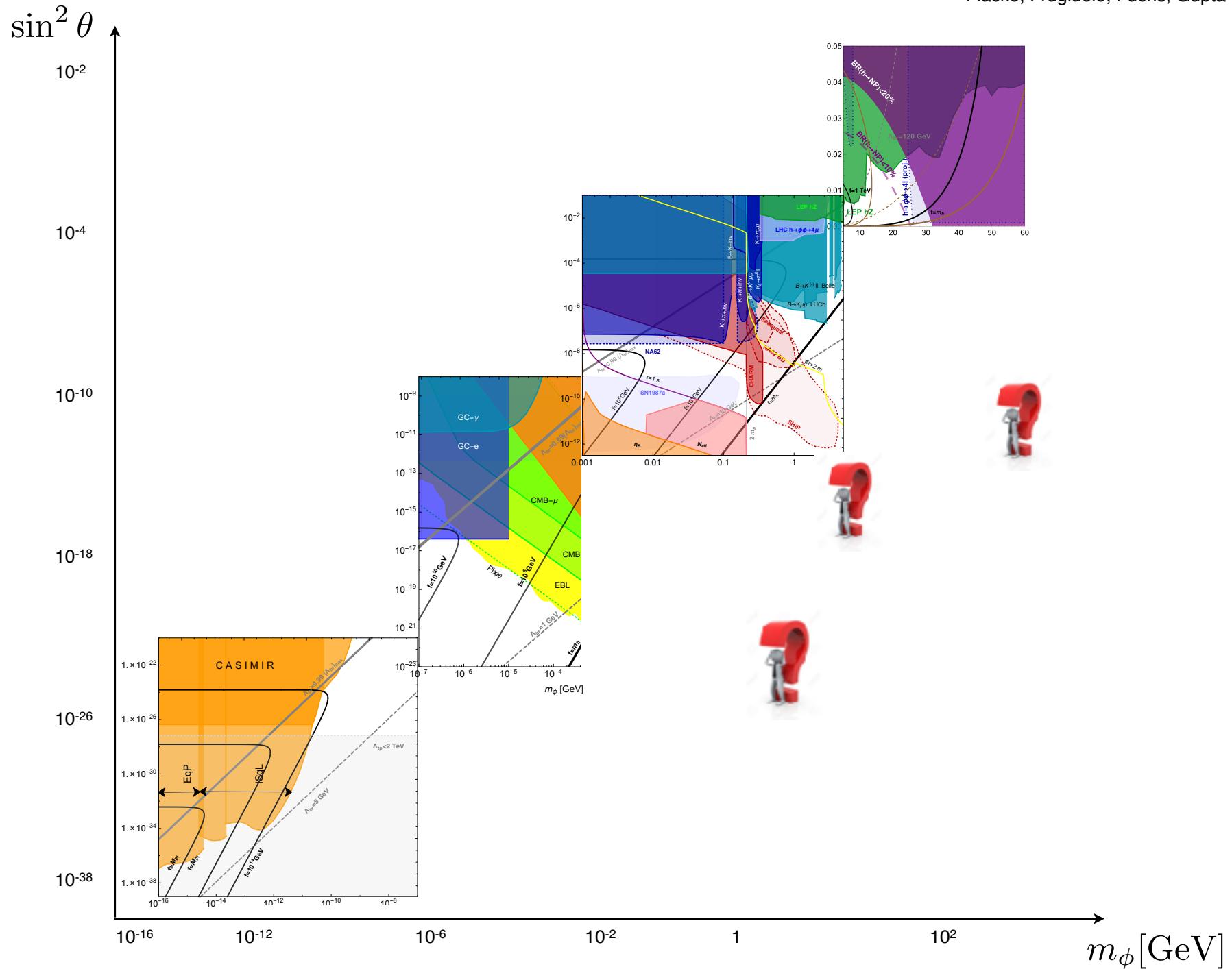
Current-near-future constraints

Flacke, Frugueule, Fuchs, Gupta & GP (16)



Current-near-future constraints

Flacke, Frigiuele, Fuchs, Gupta & GP (16)



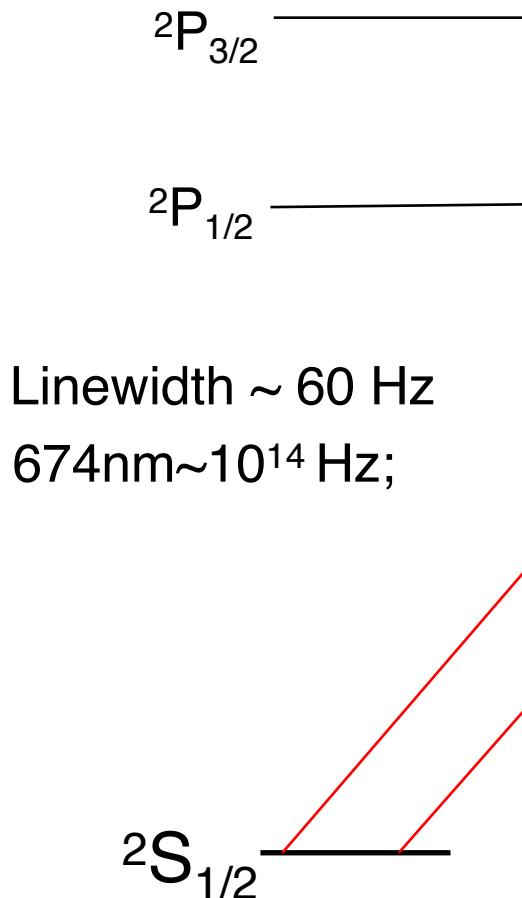
Change gear: the precision frontier of atomic clocks spectroscopy

Current state-of-the-art frequency measurements:

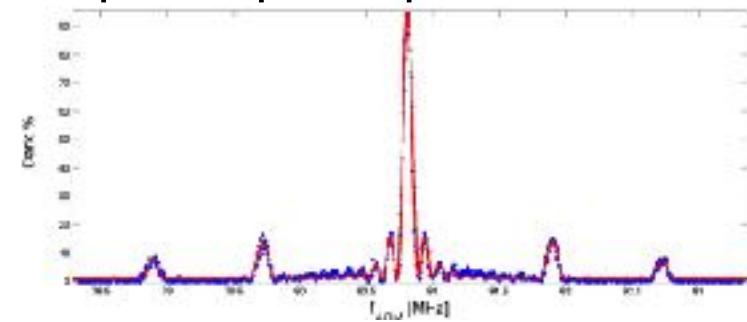
- Optical transitions in laser-cooled atoms/ions
- Relative accuracy: few 10^{-18} ; few mHz on top of 10^{15} Hz

Huntemann, Sanner, Lipphardt, Tamm & Peik (16); Bloom, Nicholson, Williams, Campbell, Bishof, Zhang, Zhang, Bromley & Ye (14)

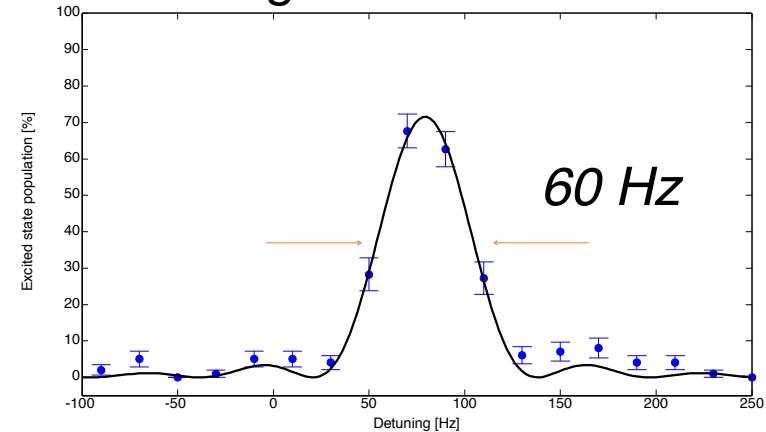
Sr⁺ ion-clock (Ozeri's lab @ Weizmann)



Optical quadrupole transition: wide scan



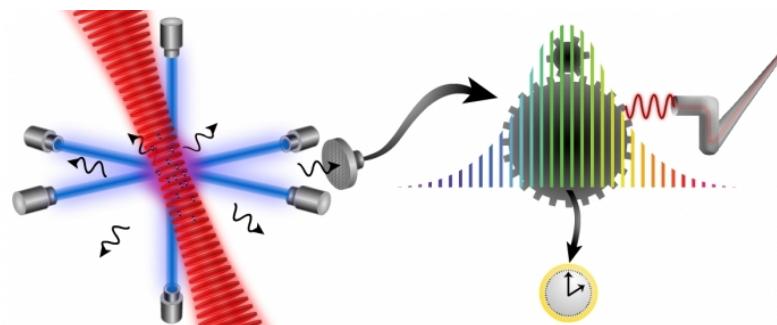
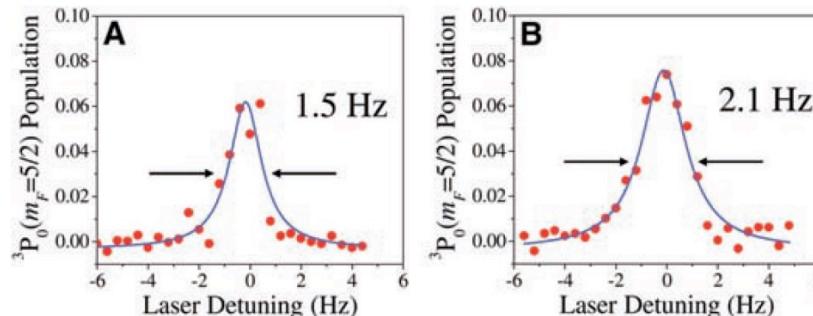
High resolution Carrier



Akerman, N; Navon, N; Kotler, S; Glickman, Y; Ozeri (2015)

Ion – neutral comparison

Neutral Sr bosons in optical lattices



J. Ye group JILA, Science, 314, 1430 (2006)

Can we use fantastic precision to bound
new interactions of above type ?

Main idea & Phenomenology (geometry) of Isotope-shifts (ISs) in atomic-clock spectroscopy

Basic concept: precision isotope shift spectroscopy

- ◆ New forces acts on electron & quarks leads to change of energy levels.

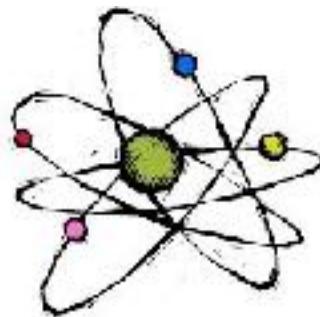
$$+ \quad \mathcal{L}_h = h (g_e \bar{e}e + g_q \bar{q}q) \quad \rightarrow$$
$$\Delta\nu_h \sim 10^2 \text{ kHz} \times \frac{g_e g_n}{10^{-10}} \times \left(\frac{1 \text{ MeV}}{M_h} \right)^2,$$

Delaunay, Ozeri, GP & Soreq (16)

- ◆ We cannot switch on and off these light Higgs-like couplings.
- ◆ Use different isotopes to effectively compare force mass dependence.
- ◆ Suppress nuclear effects via 2 transition comparison => King Linearity.

King (1963)

Consider the following systems



$\text{Ca}^{(+)}, \text{Sr}^{(+)}, \text{Yb}^{(+)}$

1. Two narrow electronic transitions (< 10 Hz possibility) with the same nucleus;
2. At least four stable (even) isotopes without nuclear spin for three independent IS comparisons.

The observables

- ◆ We have 3 isotope shifts ($AA'_{1,2,3}$) for 2 transitions ($i=1,2$):

$$\overrightarrow{m\nu}_i \equiv (m\nu_i^{AA'_1}, m\nu_i^{AA'_2}, m\nu_i^{AA'_3})$$

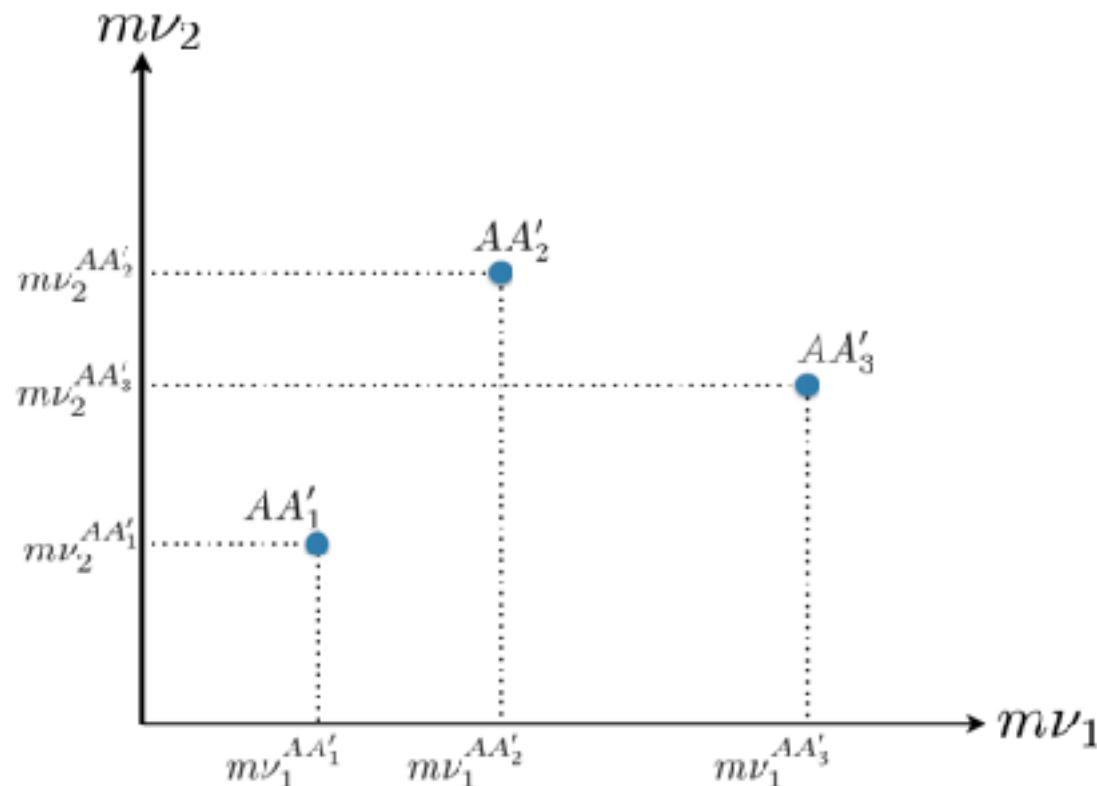
$$\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} . \quad m\nu_i^{AA'} \equiv \nu_i^{AA'}/\mu_{AA'}$$

$$\mu_{AA'} \equiv m_A^{-1} - m_{A'}^{-1}$$

Target accuracy: $\Delta m\nu_i^{AA'}/m\nu_i^{AA'} \lesssim 10^{-6}$.
(currently: 10^{-4} , projected $< 10^{-9}$)

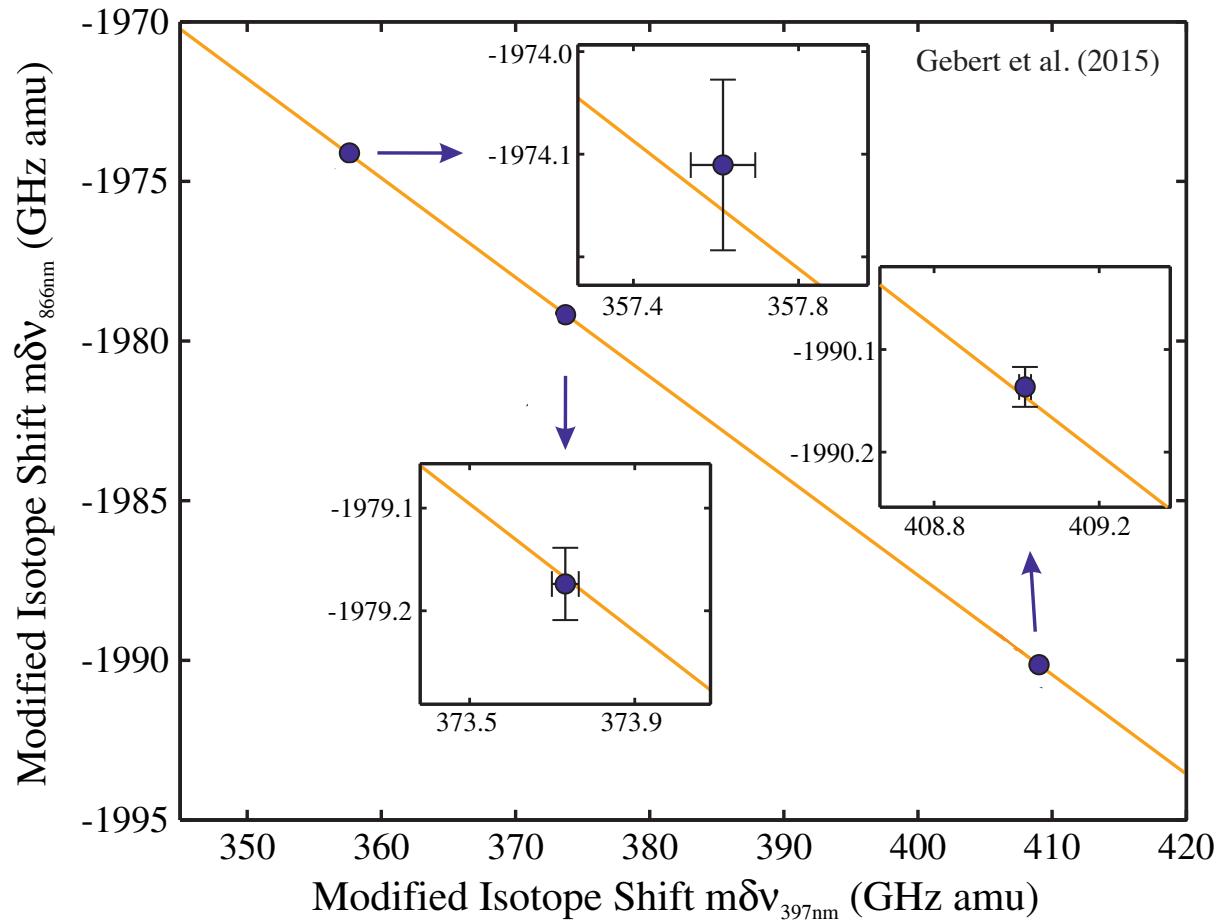
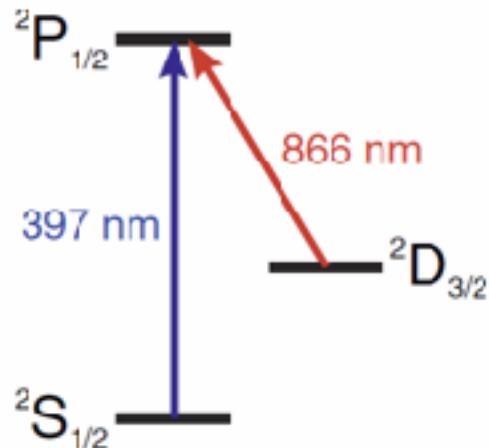
The observable: King comparison (1964)

- ◆ What would be the generic form of $\overrightarrow{m\nu}_2$ vs. $\overrightarrow{m\nu}_1$?
- ◆ 3 ISSs - $m\nu_2 = am\nu_1^2 + bm\nu_1 + c$:



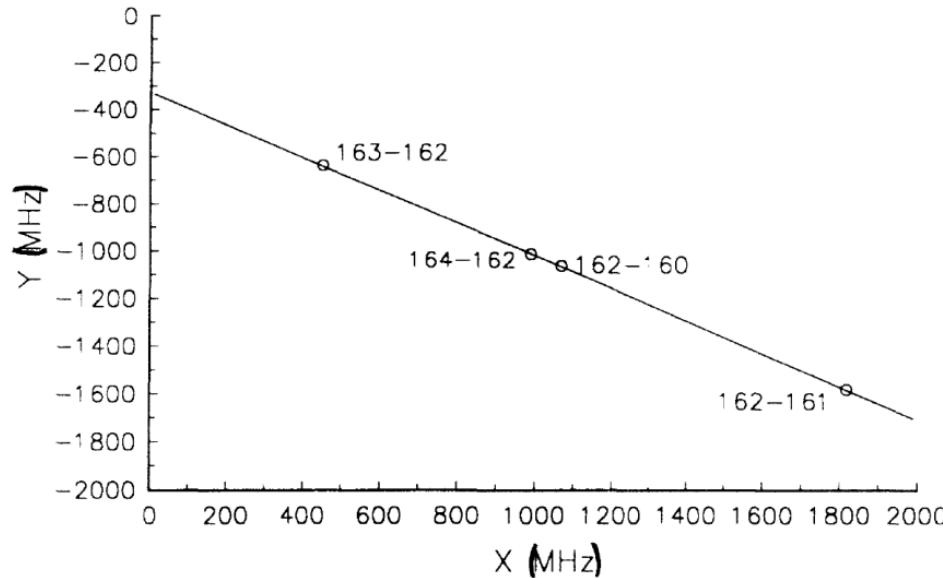
What about existing data ?

State of the art: King Linearity holds in Ca^+ ($1:10^4$)



No indication for King-linearity-violation (KLV), down to 100 kHz.

King Linearity in Dy



Dy King plot. Budker et al 1994

King-linearity-violation (KLV) seem to be suppressed in variety of systems.

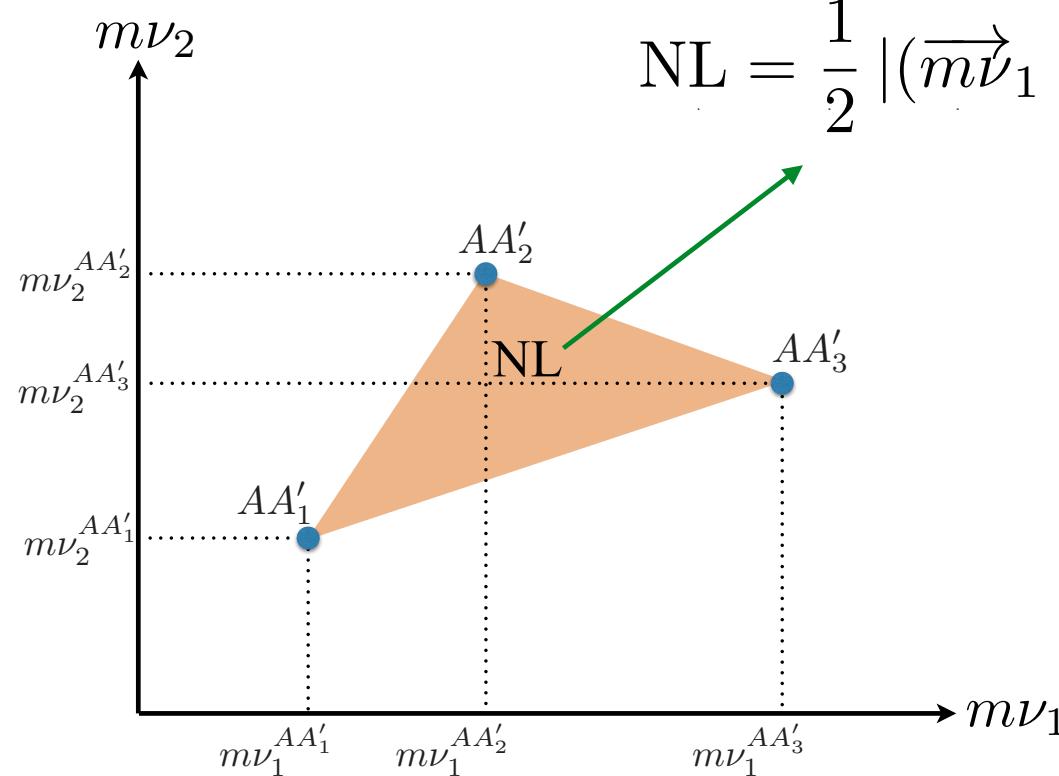
$$m\nu_2 = am\nu_1^2 + bm\nu_1 + c \quad \rightarrow \quad a \text{ is very small!}$$

King comparison

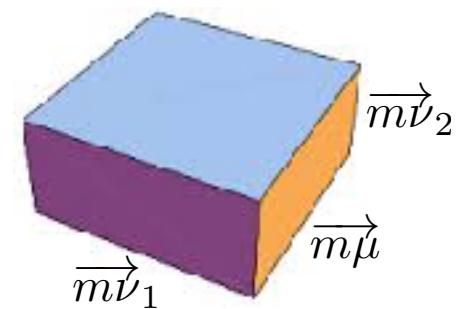
- ◆ Level of linearity can be quantified by comparing area of triangle to that of a cube: $NL / |\overrightarrow{m\nu}_2| |\overrightarrow{m\nu}_1| \ll 1$.

$$\overrightarrow{m\mu} \equiv (1, 1, 1).$$

$$NL = \frac{1}{2} |(\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu}|.$$



Or volume of parallelepiped:



King linearity implications

- ◆ Linearity implies that $\overrightarrow{m\nu}_2$ & $\overrightarrow{m\nu}_1$ must be linearly dependent:

$$\overrightarrow{m\nu}_2 = K_2 \overrightarrow{m\mu} + F_2 \vec{v} + \mathcal{O}(10^{-4})$$

$$\overrightarrow{m\nu}_1 = K_1 \overrightarrow{m\mu} + F_1 \vec{v} + \mathcal{O}(10^{-4})$$

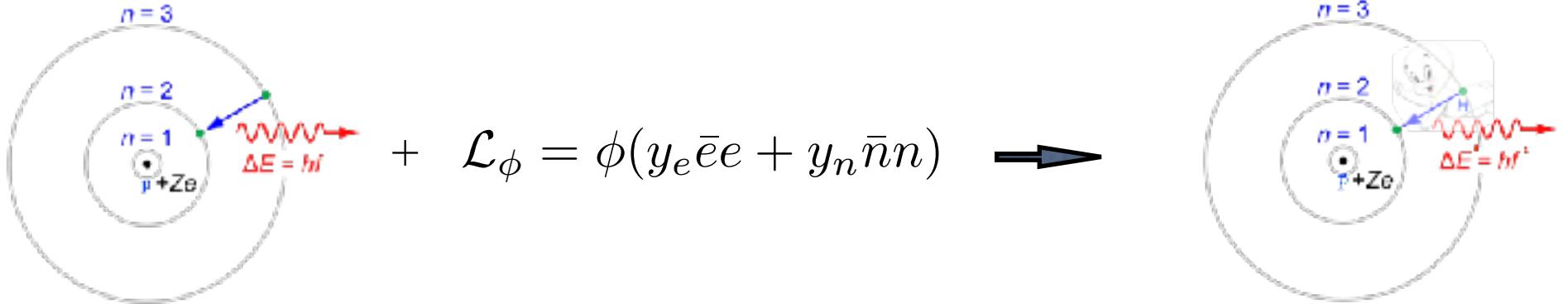
$$\overrightarrow{m\nu}_2 \cong K_{21} \overrightarrow{m\mu} + F_{21} \overrightarrow{m\nu}_1,$$

with $F_{21} \equiv F_2/F_1$ and $K_{21} \equiv K_2 - F_{21}K_1$.

F_i & \vec{v} are unknown but F_{21} & K_{21} can be measured precisely.

Adding light new physics (NP)

New forces acts on electron & quarks leads to change of energy levels.



◆ New physics part known, precisely calculated:

CI+MBPT: Dzuba, Flambaum & Kozlov (96) Berengut, Flambaum & Kozlov (06);

GRASP2K: Jonsson, Gaigalas, Biero, Fischer & Grant (2013)

(Combination of the many-body perturbation theory with the configuration-interaction method)

$$\overrightarrow{m\nu}_i = K_i \overrightarrow{m_\mu} + F_i \vec{v} + \boxed{y_e y_n X_i \vec{h}},$$

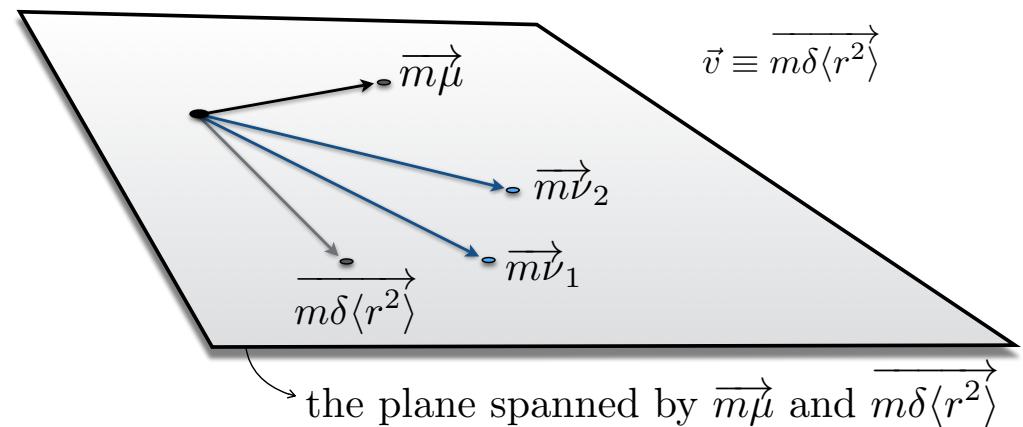
↓

Delaunay, Ozeri, GP & Soreq (16)

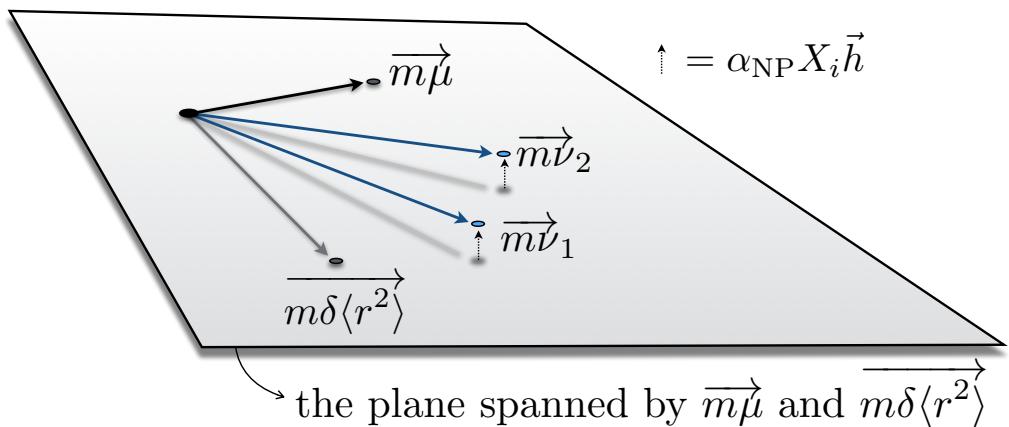
$$\overrightarrow{m\nu}_2 = K_{21} \overrightarrow{m\mu} + F_{21} \overrightarrow{m\nu}_1 + \alpha_{\text{NP}} \vec{h} X_1 (X_{21} - F_{21}),$$

and $X_{21} \equiv X_2/X_1$.

Illustration: adding light new physics (NP)



the plane spanned by $\overrightarrow{m\mu}$ and $\overrightarrow{m\delta\langle r^2 \rangle}$



the plane spanned by $\overrightarrow{m\mu}$ and $\overrightarrow{m\delta\langle r^2 \rangle}$

Limitation of method

$$\alpha_{\text{NP}} = \frac{(\overrightarrow{m\nu}_1 \times \overrightarrow{m\nu}_2) \cdot \overrightarrow{m\mu}}{(\overrightarrow{m\mu} \times \vec{h}) \cdot (X_1 \overrightarrow{m\nu}_2 - X_2 \overrightarrow{m\nu}_1)}$$

Berengut, Budker, Delaunay, Flambaum, Frugueule, Fuchs, Grojean, Harnik, Ozeri, GP & Soreq (17)

- ◆ Only useful to bound new physics (barring cancellation).
- ◆ Short range NP: $X_i \propto F_i \Rightarrow \vec{v}$ is redefined to absorb NP; requires extra carefulness when approaching this limit.
- ◆ As long as linearity holds bounds are limited by exp' accuracy:

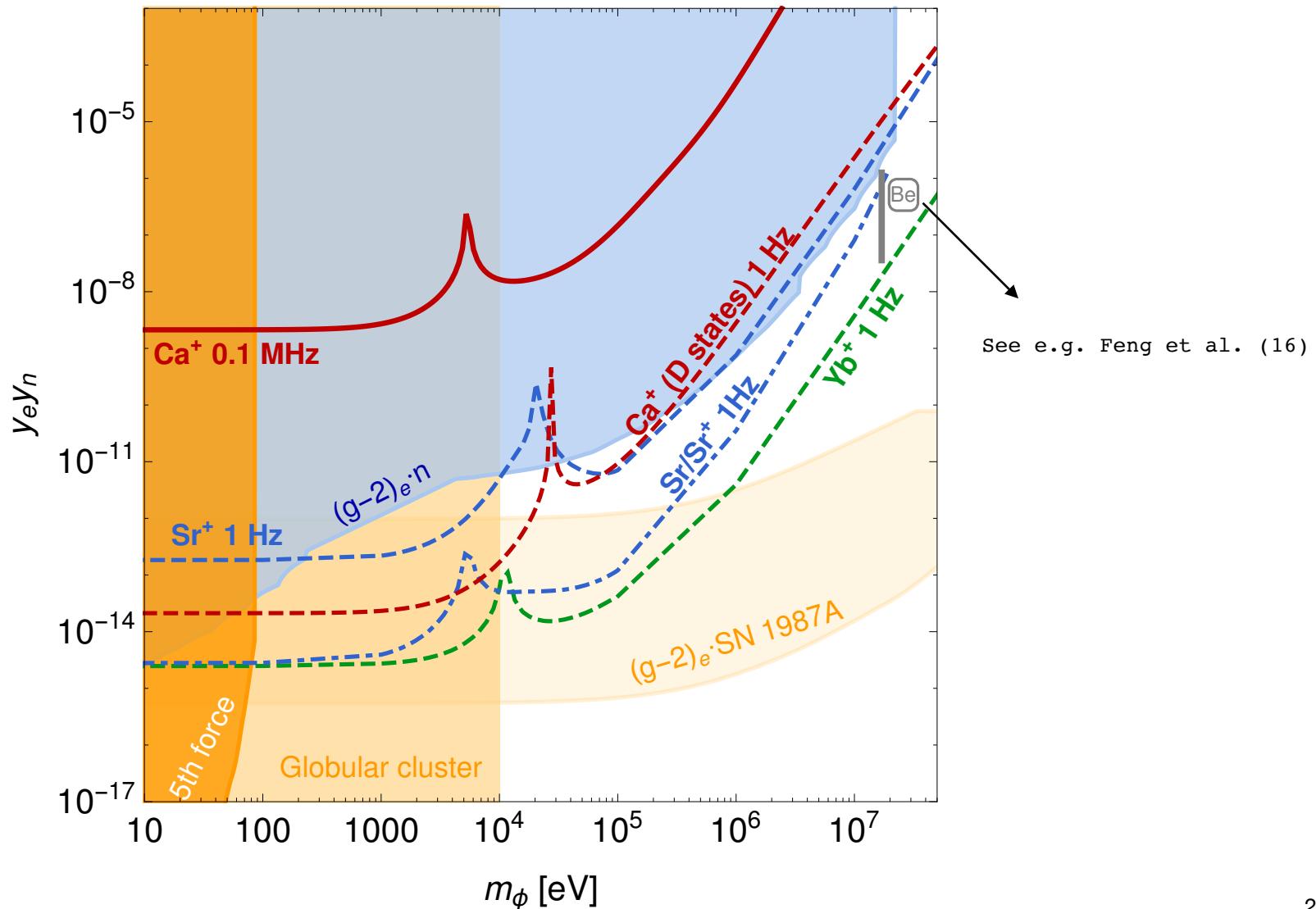
$$\alpha_{\text{NP}} \lesssim \sigma_{\alpha_{\text{NP}}} = \sqrt{\sum_k (\partial \alpha_{\text{NP}} / \partial O_k)^2 \sigma_k^2},$$

(O_K various exp' observables.)

- ◆ Once non-linearity observed bound will be set by observation.

Bounds & sensitivity

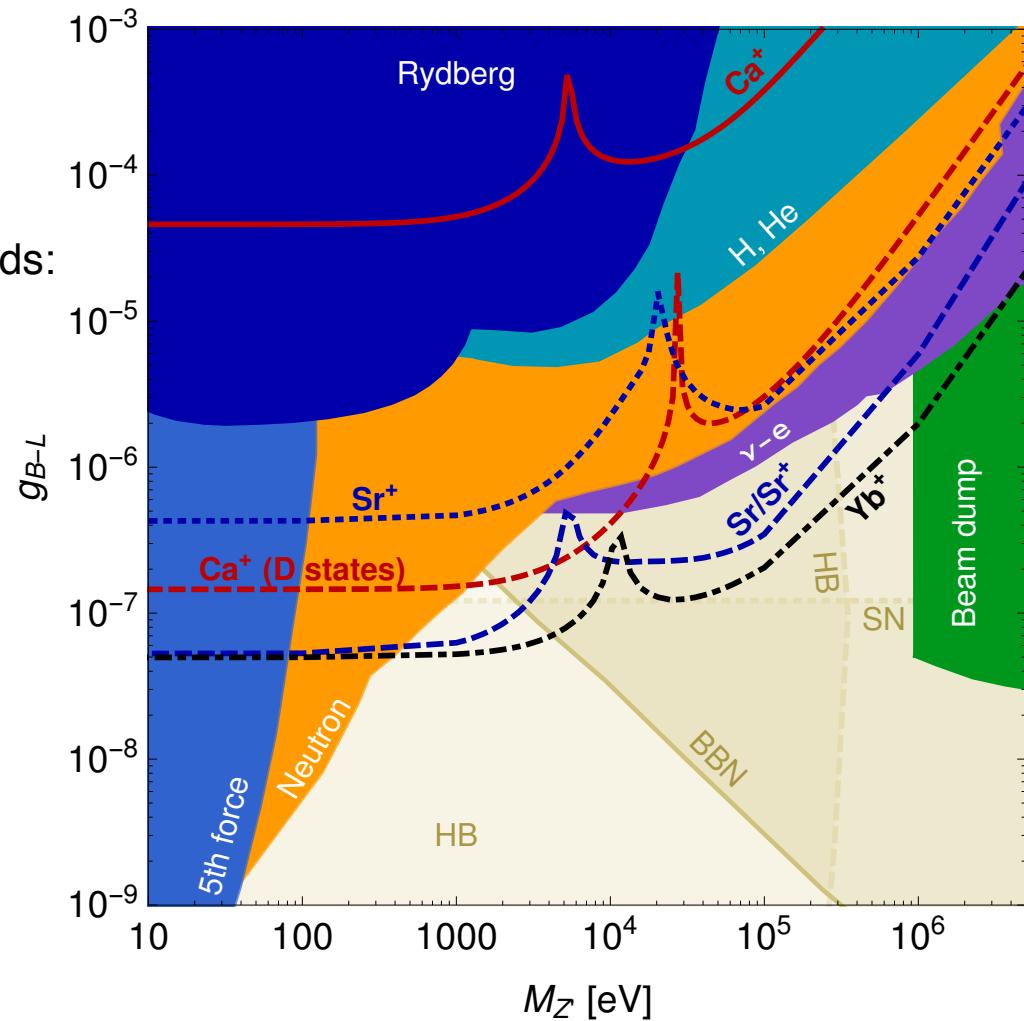
Berengut, Budker, Delaunay, Flambaum, Frugueule, Fuchs, Grojean, Harnik, Ozeri, GP & Soreq (17)



U(I)B-L

Frugiuele, Fuchs, GP & Schlaffer (16)

Complementarity with astro/cosmo' bounds:



non-univ. Higgs portal

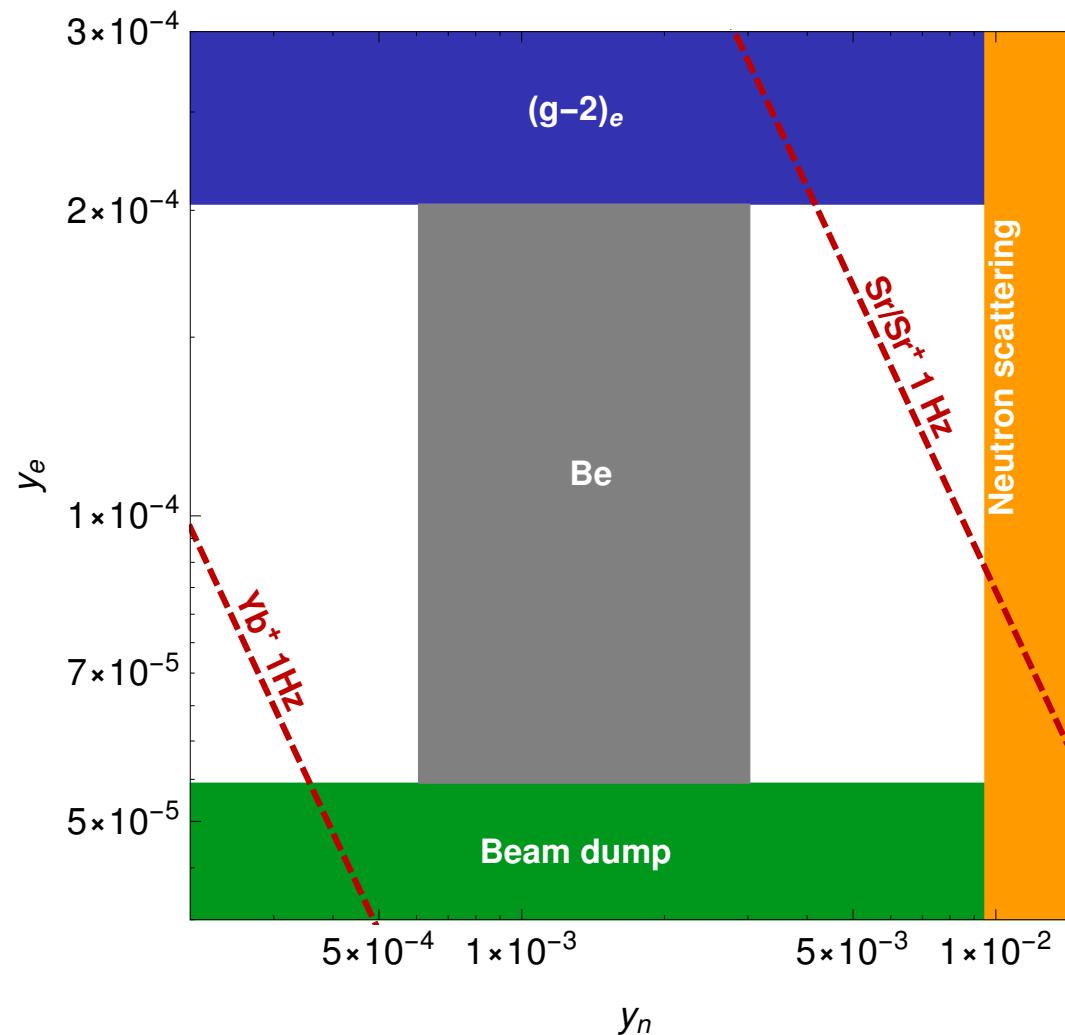
Frugiuele, Fuchs, GP & Schlaffer v2 (16)

$$\sin^2 \theta_{h\phi} \lesssim 2 \cdot 10^{-6} \cdot \left[\frac{4 \cdot 10^{-9}}{y_e y_n} \right] \frac{\sigma}{\text{Hz}} ,$$

Factor of 10 away from relaxion/Higgs portal bound.

Be 17 MeV anomaly

Frugiuele, Fuchs, GP & Schlaffer v2 (16)



Conclusions

- ◆ Proposal to probe light forces via precision spectroscopy.
- ◆ Sensitivity seems to probe interesting ranges.
- ◆ Experimental measurements: Mainz, PTB, Weizmann ...
- ◆ Precision will be improved by 5 orders of mag.
- ◆ Current theory close to predict non-linearities.

Backups

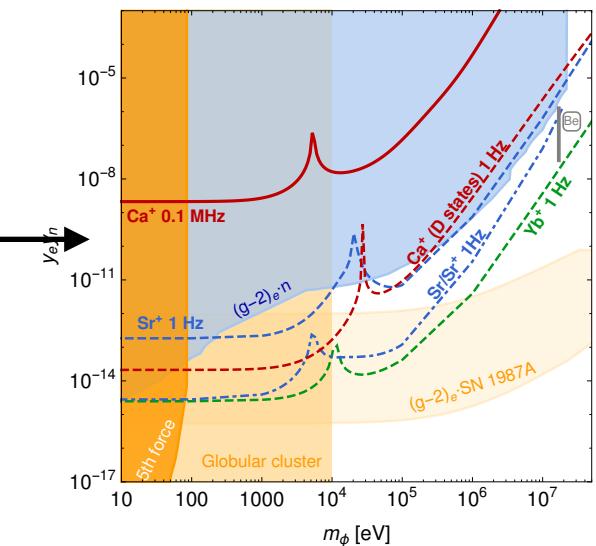
Light mediators

If mediator's mass, m_X , is smaller than inverse of outer electrons than the potential is Coulombic.

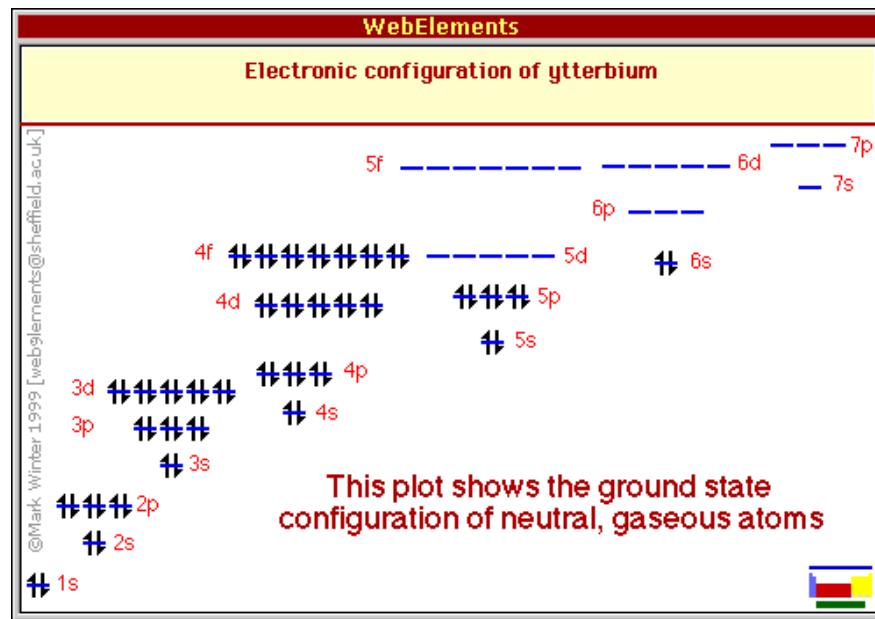
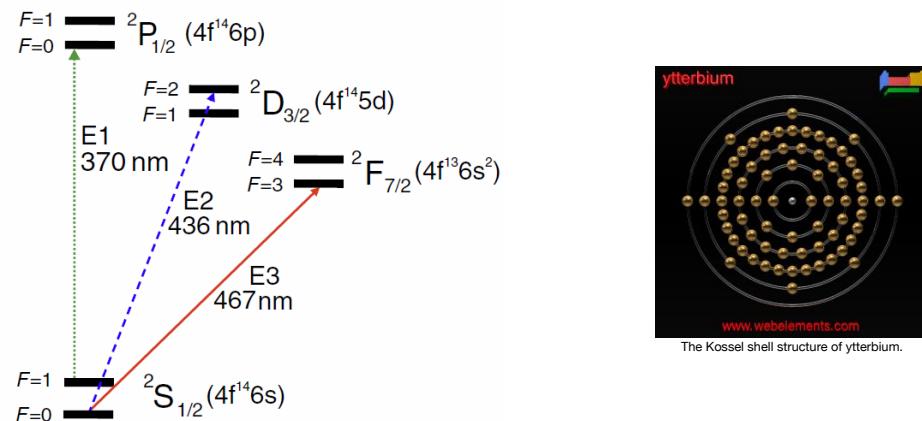
If mediator's mass is smaller than inverse distance of most inner electron from the nucleus then the full Yukawa potential is required.

Otherwise the potential is described via a delta function.

$$V(r) = \begin{cases} \frac{1}{r} & \text{for } m_X \lesssim \alpha m_e, \\ \frac{e^{-rm_X}}{r} & \text{for } \alpha m_e \lesssim m_X \lesssim \alpha m_e Z, \\ \frac{1}{m_X^2} \delta^3(r) & \text{otherwise.} \end{cases}$$



Ex.: Yb⁺ with Z=70, n=6 and A=168(4)-174(6).



The electronic configuration of ytterbium.

isotope symbol	Z (p)	N (n)	isotopic mass (u)	excitation energy	half-life	decay mode(s) ⁽²⁾	daughter isotope(s) ⁽²⁾	isobaric spin	representative isotopic composition (atom fraction)	range or natural variation (atom fraction)
^{168}Yb	70	98	167.94742(54)	250 ms	β^+	^{167}Yb	0+			
^{169}Yb	70	99	168.94844(54)	∞ (99%)	β^+	^{168}Yb	(1/2) ⁺			
^{170}Yb	70	100	169.95642(43)	7000 ms (20 ns)	β^+	^{169}Yb	0+			
^{171}Yb	70	101	170.96640(32)	1.68(1) s	β^+ , β (trans)	^{170}Yb	(1/2) ⁺			
^{172}Yb	70	102	171.97640(30)	1.68(1) s	β^+ , β (trans)	^{171}Yb	(1/2) ⁺			
^{173}Yb	70	103	172.98640(28)	2.67(1) μ s						
^{174}Yb	70	104	173.99640(26)	20(1) μ s						
^{175}Yb	70	105	174.99640(25)	9.68(1) μ s	β^+ , β (trans)	^{174}Yb	0+			
^{176}Yb	70	106	175.99640(24)	4.20(1) s	β (50%)	^{175}Yb	(1/2) ⁺			
^{177}Yb	70	107	176.99640(23)	127(1) μ s	β (99.9%)	^{176}Yb	(1/2) ⁺			
^{178}Yb	70	108	177.99640(22)	0.499(2) s	β (99.9%)	^{176}Yb	0+			
^{179}Yb	70	109	178.99640(21)	1.71(1) μ s	β (11%)	^{178}Yb	(1/2) ⁺			
^{180}Yb	70	110	179.99640(21)	26(1) μ s	β (99%)	^{178}Yb	0+			
^{181}Yb	70	111	180.99640(21)	38(6) 10 ⁻³ s	β (99.9%)	^{179}Yb	(1/2) ⁺			
^{182}Yb	70	112	181.99640(21)	1.48(13) min	β (0.01%)	^{179}Yb	0+			
^{183}Yb	70	113	182.99640(21)	1.81(2) min	β^+	^{182}Yb	(1/2) ⁻			
^{184}Yb	70	114	183.99640(21)	4.69(1) min	β^+	^{182}Yb	0+			
^{185}Yb	70	115	184.99640(21)	4.69(1) min	β^+	^{183}Yb	(1/2) ⁻			
^{186}Yb	70	116	185.99640(21)	26(1) μ s	β^+	^{184}Yb	0+			
^{187}Yb	70	117	186.99640(21)	38(6) 10 ⁻³ s	β^+	^{185}Yb	(1/2) ⁻			
^{188}Yb	70	118	187.99640(21)	1.48(13) min	β^+	^{185}Yb	0+			
^{189}Yb	70	119	188.99640(21)	1.81(2) min	β^+	^{186}Yb	(1/2) ⁻			
^{190}Yb	70	120	189.99640(21)	4.69(1) min	β^+	^{187}Yb	0+			
^{191}Yb	70	121	190.99640(21)	4.69(1) min	β^+	^{188}Yb	(1/2) ⁻			
^{192}Yb	70	122	191.99640(21)	11.56(28) min	β^+	^{189}Yb	(1/2) ⁻			
^{193}Yb	70	123	192.99640(21)	∞ (99%)	β^+	^{190}Yb	(1/2) ⁻			
^{194}Yb	70	124	193.99640(21)	2.98(1) s	β^+	^{191}Yb	(1/2) ⁻			
^{195}Yb	70	125	194.99640(21)	30(1) μ s	β^+	^{192}Yb	0+			
^{196}Yb	70	126	195.99640(21)	4.69(1) min	β^+	^{193}Yb	(1/2) ⁻			
^{197}Yb	70	127	196.99640(21)	4.69(1) min	β^+	^{194}Yb	(1/2) ⁻			
^{198}Yb	70	128	197.99640(21)	1.48(13) min	β^+	^{195}Yb	(1/2) ⁻			
^{199}Yb	70	129	198.99640(21)	1.81(2) min	β^+	^{196}Yb	(1/2) ⁻			
^{200}Yb	70	130	199.99640(21)	4.69(1) min	β^+	^{197}Yb	(1/2) ⁻			
^{201}Yb	70	131	200.99640(21)	4.69(1) min	β^+	^{198}Yb	(1/2) ⁻			
^{202}Yb	70	132	201.99640(21)	1.48(13) min	β^+	^{199}Yb	(1/2) ⁻			
^{203}Yb	70	133	202.99640(21)	1.81(2) min	β^+	^{200}Yb	(1/2) ⁻			
^{204}Yb	70	134	203.99640(21)	4.69(1) min	β^+	^{201}Yb	(1/2) ⁻			
^{205}Yb	70	135	204.99640(21)	4.69(1) min	β^+	^{202}Yb	(1/2) ⁻			
^{206}Yb	70	136	205.99640(21)	1.48(13) min	β^+	^{203}Yb	(1/2) ⁻			
^{207}Yb	70	137	206.99640(21)	1.81(2) min	β^+	^{204}Yb	(1/2) ⁻			
^{208}Yb	70	138	207.99640(21)	4.69(1) min	β^+	^{205}Yb	(1/2) ⁻			
^{209}Yb	70	139	208.99640(21)	4.69(1) min	β^+	^{206}Yb	(1/2) ⁻			
^{210}Yb	70	140	209.99640(21)	1.48(13) min	β^+	^{207}Yb	(1/2) ⁻			
^{211}Yb	70	141	210.99640(21)	1.81(2) min	β^+	^{208}Yb	(1/2) ⁻			
^{212}Yb	70	142	211.99640(21)	4.69(1) min	β^+	^{209}Yb	(1/2) ⁻			
^{213}Yb	70	143	212.99640(21)	4.69(1) min	β^+	^{210}Yb	(1/2) ⁻			
^{214}Yb	70	144	213.99640(21)	1.48(13) min	β^+	^{211}Yb	(1/2) ⁻			
^{215}Yb	70	145	214.99640(21)	1.81(2) min	β^+	^{212}Yb	(1/2) ⁻			
^{216}Yb	70	146	215.99640(21)	4.69(1) min	β^+	^{213}Yb	(1/2) ⁻			
^{217}Yb	70	147	216.99640(21)	4.69(1) min	β^+	^{214}Yb	(1/2) ⁻			
^{218}Yb	70	148	217.99640(21)	1.48(13) min	β^+	^{215}Yb	(1/2) ⁻			
^{219}Yb	70	149	218.99640(21)	1.81(2) min	β^+	^{216}Yb	(1/2) ⁻			
^{220}Yb	70	150	219.99640(21)	4.69(1) min	β^+	^{217}Yb	(1/2) ⁻			
^{221}Yb	70	151	220.99640(21)	4.69(1) min	β^+	^{218}Yb	(1/2) ⁻			
^{222}Yb	70	152	221.99640(21)	1.48(13) min	β^+	^{219}Yb	(1/2) ⁻			
^{223}Yb	70	153	222.99640(21)	1.81(2) min	β^+	^{220}Yb	(1/2) ⁻			
^{224}Yb	70	154	223.99640(21)	4.69(1) min	β^+	^{221}Yb	(1/2) ⁻			
^{225}Yb	70	155	224.99640(21)	4.69(1) min	β^+	^{222}Yb	(1/2) ⁻			
^{226}Yb	70	156	225.99640(21)	1.48(13) min	β^+	^{223}Yb	(1/2) ⁻			
^{227}Yb	70	157	226.99640(21)	1.81(2) min	β^+	^{224}Yb	(1/2) ⁻			
^{228}Yb	70	158	227.99640(21)	4.69(1) min	β^+	^{225}Yb	(1/2) ⁻			
^{229}Yb	70	159	228.99640(21)	4.69(1) min	β^+	^{226}Yb	(1/2) ⁻			
^{230}Yb	70	160	229.99640(21)	1.48(13) min	β^+	^{227}Yb	(1/2) ⁻			
^{231}Yb	70	161	230.99640(21)	1.81(2) min	β^+	^{228}Yb	(1/2) ⁻			
^{232}Yb	70	162	231.99640(21)	4.69(1) min	β^+	^{229}Yb	(1/2) ⁻			
^{233}Yb	70	163	232.99640(21)	4.69(1) min	β^+	^{230}Yb	(1/2) ⁻			
^{234}Yb	70	164	233.99640(21)	1.48(13) min	β^+	^{231}Yb	(1/2) ⁻			
^{235}Yb	70	165	234.99640(21)	1.81(2) min	β^+	^{232}Yb	(1/2) ⁻			
^{236}Yb	70	166	235.99640(21)	4.69(1) min	β^+	^{233}Yb	(1/2) ⁻			
^{237}Yb	70	167	236.99640(21)	4.69(1) min	β^+	^{234}Yb	(1/2) ⁻			
^{238}Yb	70	168	237.99640(21)	1.48(13) min	β^+	^{235}Yb	(1/2) ⁻			
^{239}Yb	70	169	238.99640(21)	1.81(2) min	β^+	^{236}Yb	(1/2) ⁻			
^{240}Yb	70	170	239.99640(21)	4.69(1) min	β^+	^{237}Yb	(1/2) ⁻			
^{241}Yb	70	171	240.99640(21)	4.69(1) min	β^+	^{238}Yb	(1/2) ⁻			
^{242}Yb	70	172	241.99640(21)	1.48(13) min	β^+	^{239}Yb	(1/2) ⁻			
^{243}Yb	70	173	242.99640(21)	1.81(2) min	β^+	^{240}Yb	(1/2) ⁻			
^{244}Yb	70	174	243.99640(21)	4.69(1) min	β^+	^{241}Yb	(1/2) ⁻			
^{245}Yb	70	175	244.99640(21)	4.69(1) min	β^+	^{242}Yb	(1/2) ⁻			
^{246}Yb	70	176	245.99640(21)	1.48(13) min	β^+	^{243}Yb	(1/2) ⁻			
^{247}Yb	70	177	246.99640(21)	1.81(2) min	β^+	^{244}Yb	(1/2) ⁻			
^{248}Yb	70	178	247.99640(21)	4.69(1) min	$\beta^+</$					

Precision mass measurements: 10^{-10}



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The most precise atomic mass measurements in Penning traps

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Table 10

Atomic masses of the most abundant isotopes of strontium and ytterbium measured at FSU [100].

Atom	FSU mass (u)	σ_m/m (ppt)
^{88}Sr	85.909 260 730 9(91)	105
^{87}Sr	86.908 877 497 0(91)	105
^{86}Sr	87.905 612 257 1(97)	110
^{140}Yb	169.934 767 241(18)	105
^{171}Yb	170.936 331 514(19)	110
^{172}Yb	171.936 386 655(18)	105
^{173}Yb	172.938 216 213(18)	105
^{174}Yb	173.938 867 539(18)	105
^{176}Yb	175.942 574 702(22)	125

Partial solution, comparing different isotope shift, searching of nonlinearity in “King plot”

King's factorisation formula (King, 1963):

$$\delta\nu_i^{AA'} \equiv \nu_i^A - \nu_i^{A'} = K_i \mu_{AA'} + F_i \delta\langle r^2 \rangle_{AA'},$$

($\mu_{AA'} \equiv 1/m_A - 1/m_{A'} = (A' - A)/(AA') \text{ amu}^{-1}$, where amu $\approx 0.931 \text{ GeV}$)

only depend on e-transition

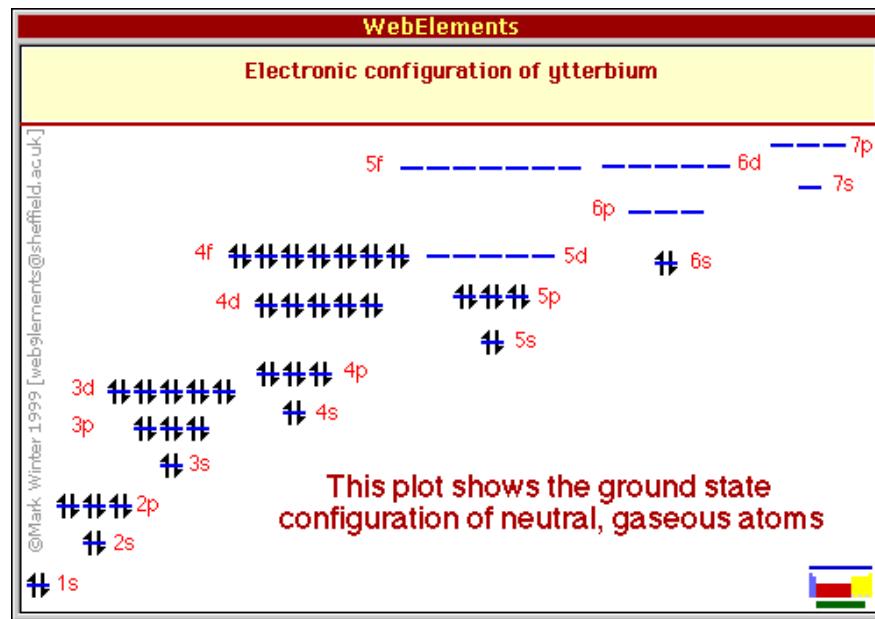
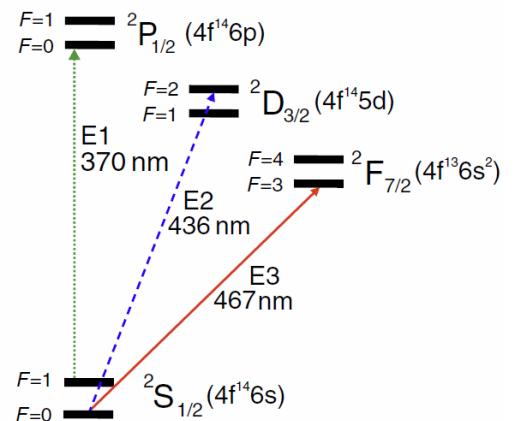
only depend on nucleus

We can solve for $\delta\langle r^2 \rangle_{AA'}$ to get a linear relation:

$$m\delta\nu_{AA'}^2 = F_{21}m\delta\nu_{AA'}^1 + K_{21},$$

(with $K_{21} \equiv (K_2 - F_{21}K_1)$ and $F_{21} \equiv F_2/F_1$ and $m\delta\nu_{AA'}^i \equiv \delta\nu_{AA'}^i/\mu_{AA'}$.)

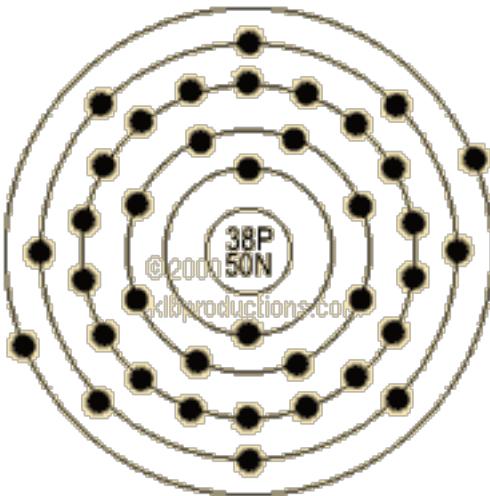
Ex.: Yb⁺ with Z=70, n=6 and A=168(4)-174(6).



isotope symbol	Z	N	isotopic mass (u)	excitation energy	half-life	decay mode(s) ⁽²⁾	daughter isotope(s) ⁽²⁾	nuclear spin	representative isotopic composition (atom fraction)	range or natural variation (atom fraction)
^{168}Yb	70	98	167.96742(54)	250 ms	β^+		^{167}Yb	0+		
^{169}Yb	70	99	168.96884(54)	9.9(3) s	β^+		^{168}Yb	(19.91-48.1)		
^{170}Yb	70	100	169.96842(43)	700 ms (20 ns)	β^+		^{169}Yb	0+		
^{171}Yb	70	101	170.96840(32)	1.68(1) s	β^+	β^+, γ (trans)	^{170}Yb	(10.24)		
^{172}Yb			170.96800(8) keV	1.68(1) s	β^+	β^+, γ (trans)	^{171}Yb	(11.12)		
^{173}Yb			170.96800(8) keV	2.6(7) μ s				19/2-		
^{174}Yb			170.96800(8) keV	20(1) μ s				27/2-		
^{175}Yb			170.96800(8) keV	9.6(6) s	β^+	β^+, γ (trans)	^{174}Yb	0+		
^{176}Yb			170.96800(8) keV	4.2(2) s	β^+ (50%)	β^+, γ (50%)	^{175}Yb	(11.12)		
^{177}Yb			170.96800(8) keV	1.0(2) μ m	β^+, β^- (50%)	β^+, β^- (50%)	^{176}Yb	(27/2+)		
^{178}Yb			170.96800(8) keV	0.49(2) s	β^+, γ (trans)	β^+, γ (trans)	^{177}Yb	0+		
^{179}Yb			170.96800(8) keV	0.31(2) s	β^+	β^+, γ (trans)	^{178}Yb	(11.12)		
^{180}Yb			170.96800(8) keV	2.1(1) s	β^+	β^+, γ (trans)	^{179}Yb	(11.12)		
^{181}Yb			170.96800(8) keV	38.6(10) s	β^+	β^+, γ (trans)	^{180}Yb	1/2-		
^{182}Yb			170.96800(8) keV	1.48(13) min	β^+	β^+, γ (trans)	^{181}Yb	0+		
^{183}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{182}Yb	3/2-		
^{184}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{183}Yb	0+		
^{185}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{184}Yb	1/2+		
^{186}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{185}Yb	0+		
^{187}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{186}Yb	1/2-		
^{188}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{187}Yb	(0.021)		
^{189}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{188}Yb	3/2-		
^{190}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{189}Yb	0+		
^{191}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{190}Yb	1/2+		
^{192}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{191}Yb	0+		
^{193}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{192}Yb	1/2-		
^{194}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{193}Yb	(0.021)		
^{195}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{194}Yb	3/2-		
^{196}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{195}Yb	0+		
^{197}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{196}Yb	1/2+		
^{198}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{197}Yb	0+		
^{199}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{198}Yb	1/2-		
^{200}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{199}Yb	(0.021)		
^{201}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{200}Yb	3/2-		
^{202}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{201}Yb	0+		
^{203}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{202}Yb	1/2+		
^{204}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{203}Yb	0+		
^{205}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{204}Yb	1/2-		
^{206}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{205}Yb	(0.021)		
^{207}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{206}Yb	3/2-		
^{208}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{207}Yb	0+		
^{209}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{208}Yb	1/2+		
^{210}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{209}Yb	0+		
^{211}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{210}Yb	1/2-		
^{212}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{211}Yb	(0.021)		
^{213}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{212}Yb	3/2-		
^{214}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{213}Yb	0+		
^{215}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{214}Yb	1/2+		
^{216}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{215}Yb	0+		
^{217}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{216}Yb	1/2-		
^{218}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{217}Yb	(0.021)		
^{219}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{218}Yb	3/2-		
^{220}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{219}Yb	0+		
^{221}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{220}Yb	1/2+		
^{222}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{221}Yb	0+		
^{223}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{222}Yb	1/2-		
^{224}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{223}Yb	(0.021)		
^{225}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{224}Yb	3/2-		
^{226}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{225}Yb	0+		
^{227}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{226}Yb	1/2+		
^{228}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{227}Yb	0+		
^{229}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{228}Yb	1/2-		
^{230}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{229}Yb	(0.021)		
^{231}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{230}Yb	3/2-		
^{232}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{231}Yb	0+		
^{233}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{232}Yb	1/2+		
^{234}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{233}Yb	0+		
^{235}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{234}Yb	1/2-		
^{236}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{235}Yb	(0.021)		
^{237}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{236}Yb	3/2-		
^{238}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{237}Yb	0+		
^{239}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{238}Yb	1/2+		
^{240}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{239}Yb	0+		
^{241}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{240}Yb	1/2-		
^{242}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{241}Yb	(0.021)		
^{243}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{242}Yb	3/2-		
^{244}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{243}Yb	0+		
^{245}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{244}Yb	1/2+		
^{246}Yb			170.96800(8) keV	10.4(7) min	β^+	β^+, γ (trans)	^{245}Yb	0+		
^{247}Yb			170.96800(8) keV	11.5(2) min	β^+	β^+, γ (trans)	^{246}Yb	1/2-		
^{248}Yb			170.96800(8) keV	32.1(3) min	β^+	β^+, γ (trans)	^{247}Yb	(0.021)		
^{249}Yb			170.96800(8) keV	1.8(1) min	β^+	β^+, γ (trans)	^{248}Yb	3/2-		
^{250}Yb			170.96800(8) keV	4.6(3) min	β^+	β^+, γ (trans)	^{249}Yb	0+		
^{251}Yb			170.96800(8) keV	4.0(3) min	β^+	β^+, γ (trans)	^{250}Yb	1/2+		
^{252}Yb			1							

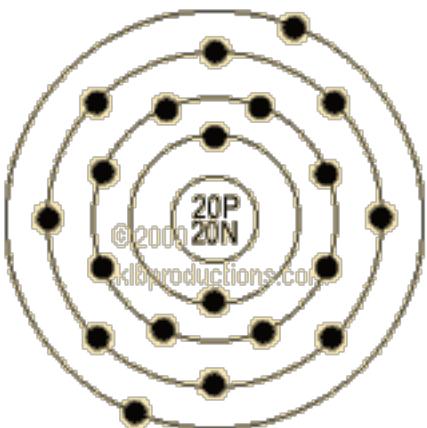
Ex.: Sr⁽⁺⁾ with Z=38, n=5 and A=84-88 (90).

- **Electron Configuration:** $1s^2 2s^2 p^6 3s^2 p^6 d^{10} 4s^2 p^6 5s^{2(1)}$
- **Electrons per Energy Level:** 2,8,18,8,2(1)

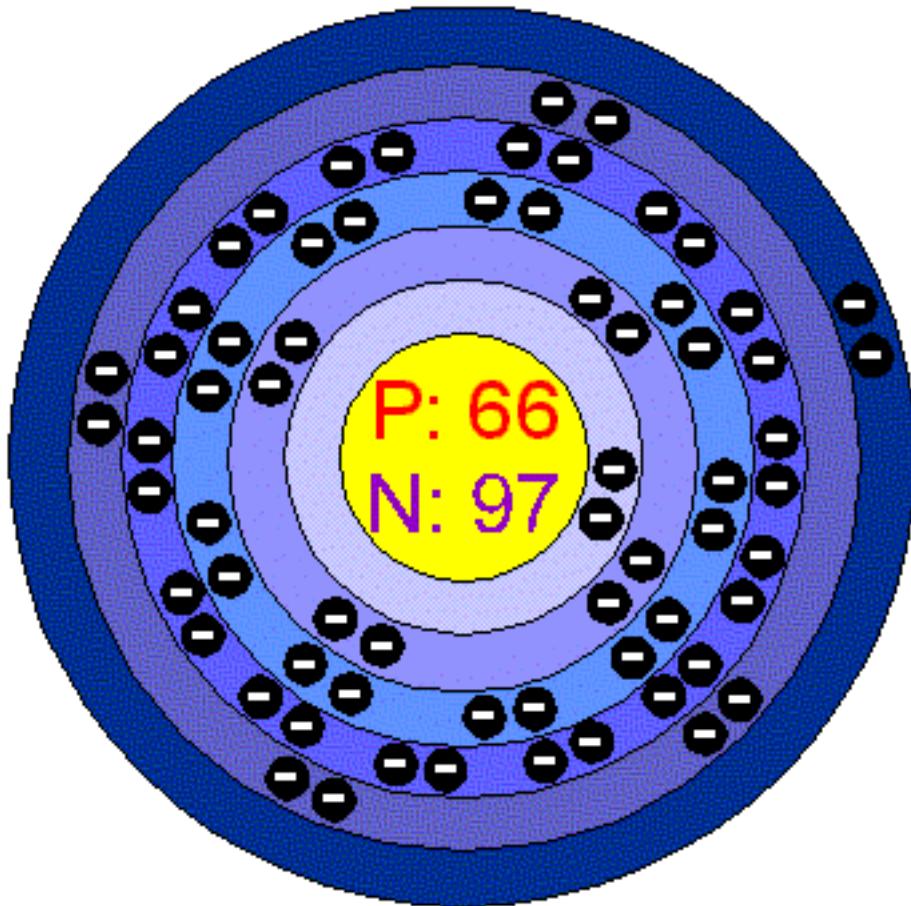


Ex.: Ca⁽⁺⁾ with Z=20, n=4 and A=40-48.

- **Electron Configuration:** $1s^2 \ 2s^2 p^6 \ 3s^2 p^6 \ 4s^1$
- **Electrons per Energy Level:** 2,8,8,2(1)
-



Ex.: Dy with Z=66, n=6 and A=158-164.



Number of Energy Levels: 6
First Energy Level: 2
Second Energy Level: 8
Third Energy Level: 18
Fourth Energy Level: 28
Fifth Energy Level: 8
Sixth Energy Level: 2