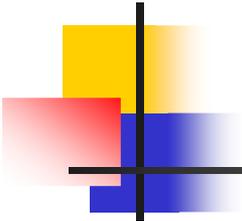


# The LHC: Physics without mathematics !

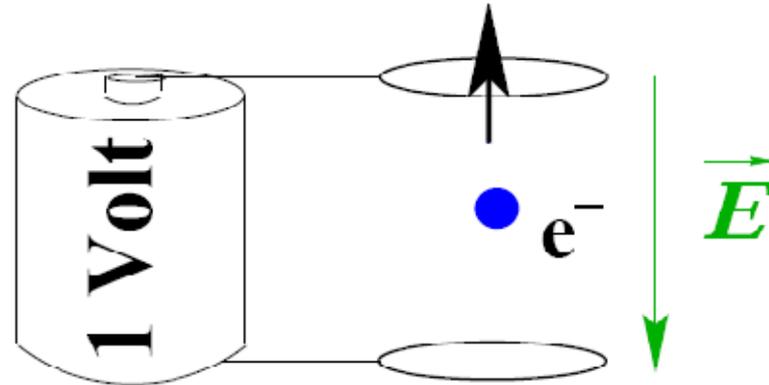
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D. Brandt, CERN



# Some generalities ...

# Units: the electronvolt (eV)



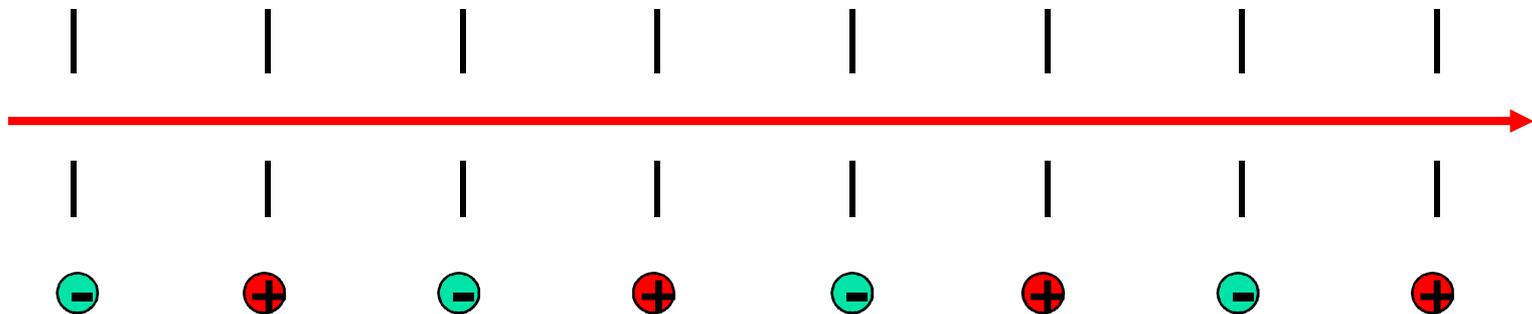
The **electronvolt (eV)** is the energy gained by an electron travelling, in vacuum, between two points with a voltage difference of 1 Volt.  $1 \text{ eV} = 1.602 \cdot 10^{-19} \text{ Joule}$

We also frequently use the electronvolt to express masses from  $E=mc^2$ :  $1 \text{ eV}/c^2 = 1.783 \cdot 10^{-36} \text{ kg}$

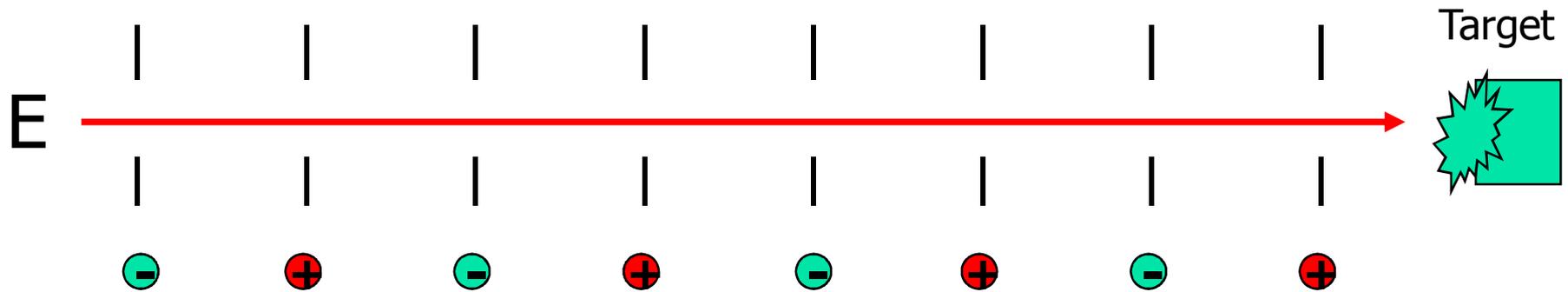
# What is a Particle Accelerator?

➤ a machine to accelerate some particles ! **How is it done ?**

➤ Many different possibilities, but rather easy from the general principle:



# Ideal linear machines (linacs)



$$\text{Available Energy : } E_{\text{c.m.}} = m \cdot (2+2\gamma)^{1/2} = (2m \cdot (m+E))^{1/2}$$

$$\text{with } \gamma = E/E_0$$

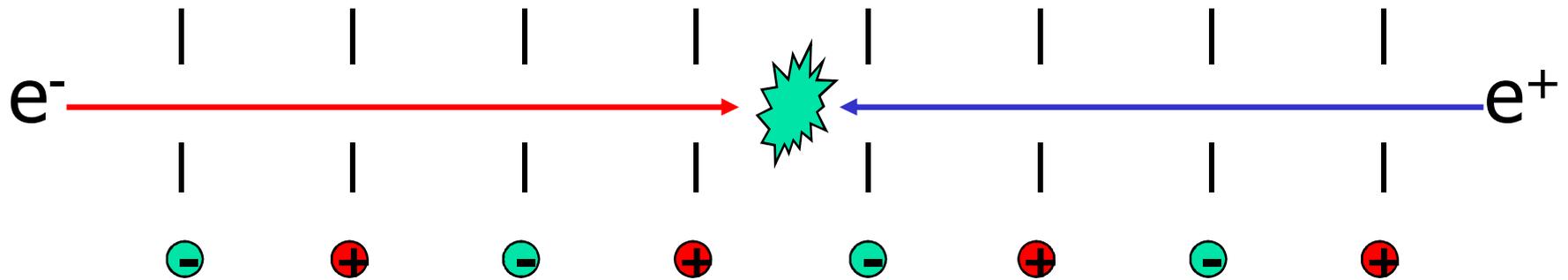
Advantages: Single pass

High intensity

Drawbacks: Single pass

Available Energy

# Improved solution for $E_{c.m.}$



Available Energy :  $E_{c.m.} = 2m\gamma = 2E$

with  $\gamma = E/E_0$

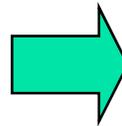
Advantages: High intensity

Drawbacks: Single pass  
Space required

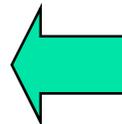
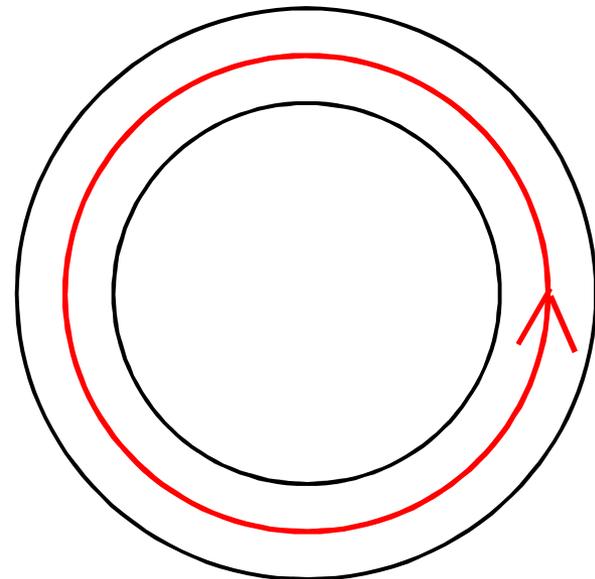
# Keep particles: circular machines

Basic idea is to keep the particles in the machine for many turns.

Move from the linear design



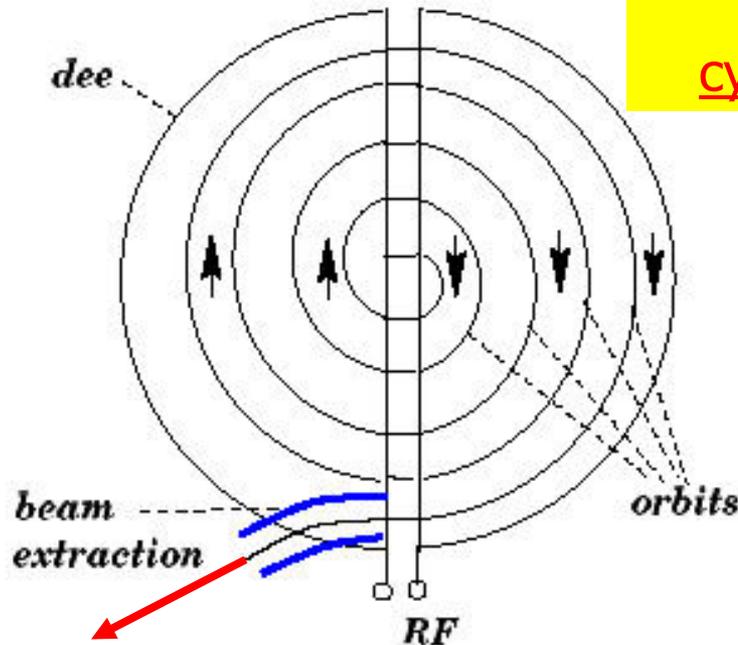
To a circular one:



➤ Need Bending

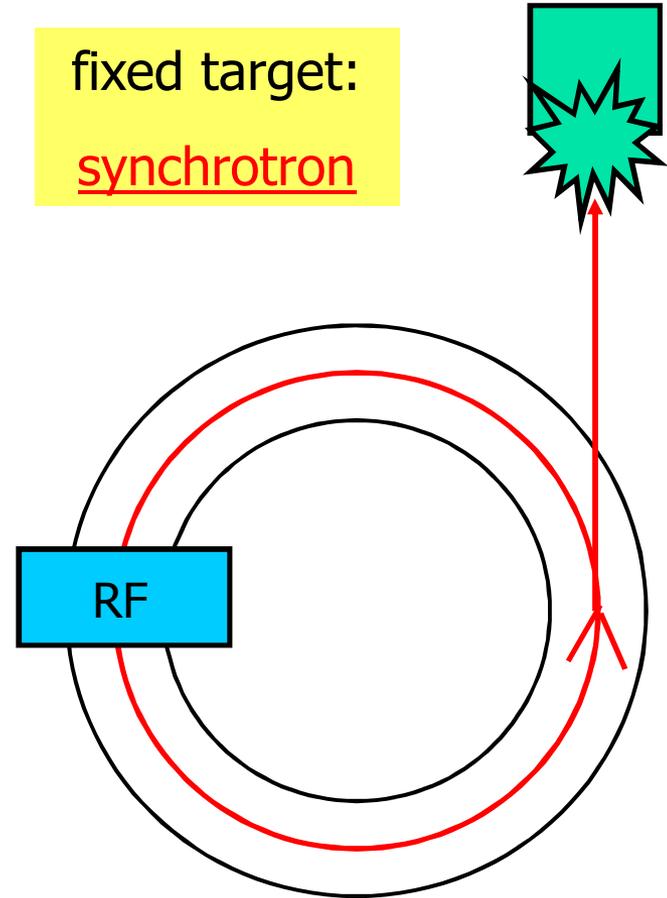
➤ Need **Dipoles!**

# Circular machines ( $E_{c.m.} \sim (mE)^{1/2}$ )



fixed target:  
cyclotron

huge dipole, compact design,  
**B = constant**  
low energy, single pass.



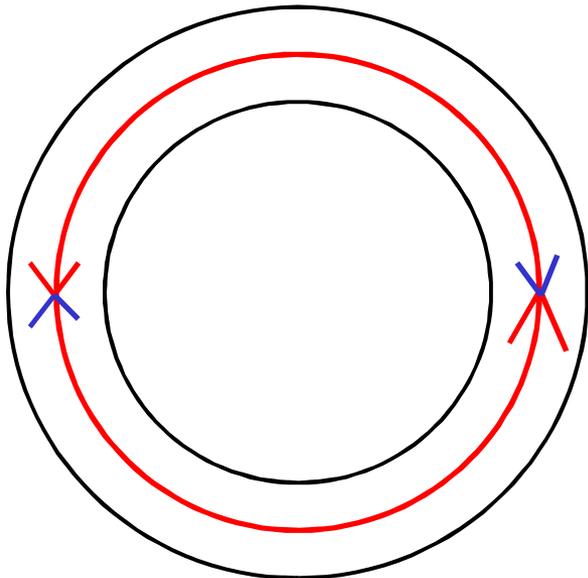
fixed target:  
synchrotron

**varying B**, small magnets, high energy

# Colliders ( $E_{\text{c.m.}}=2E$ )

## Colliders:

electron – positron  
proton - antiproton

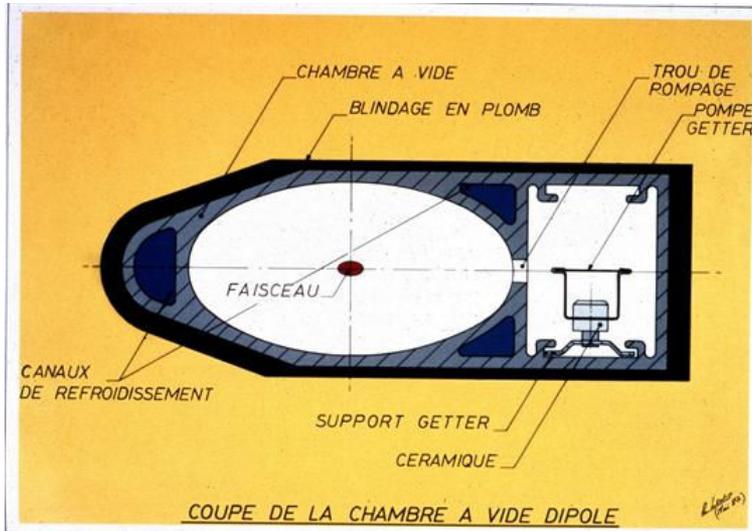


Colliders with the same type of particles (e.g. p-p) require two separate chambers. The beam are brought into a common chamber around the interaction regions

Ex: LHC

8 possible interaction regions  
4 experiments collecting data

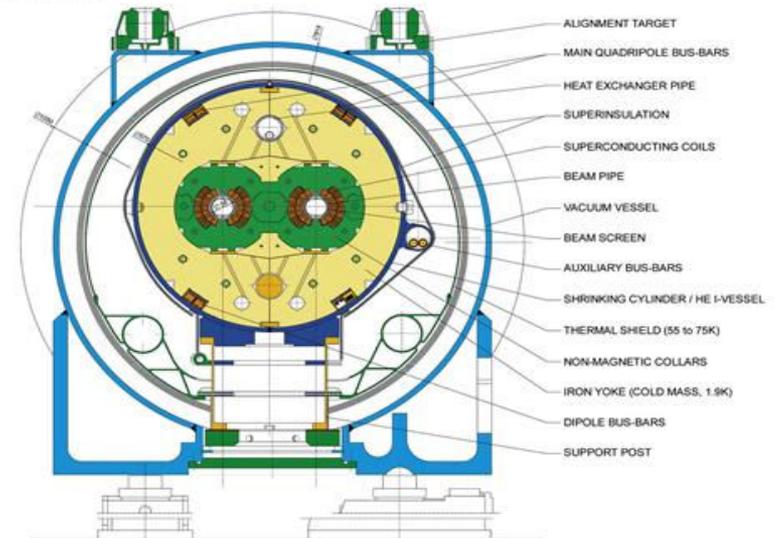
# Colliders ( $e^+ - e^-$ ) et ( $p - p$ )

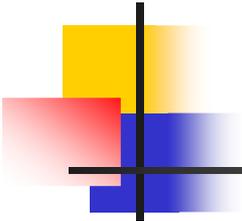


LEP

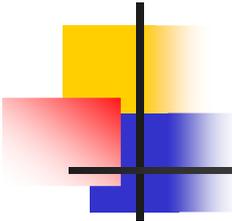
LHC

LHC DIPOLE : STANDARD CROSS-SECTION





# Transverse Dynamics



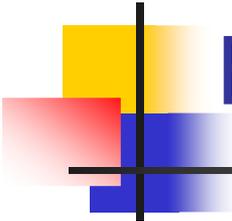
# Beam Dynamics (1)

In order to describe the motion of the particles, each particle is characterised by:

- Its azimuthal position along the machine:  $s$
- Its momentum:  $p$
- Its horizontal position:  $x$
- Its horizontal slope:  $x'$
- Its vertical position:  $y$
- Its vertical slope:  $y'$

i.e. a sixth dimensional phase space

$(s, p, x, x', y, y')$

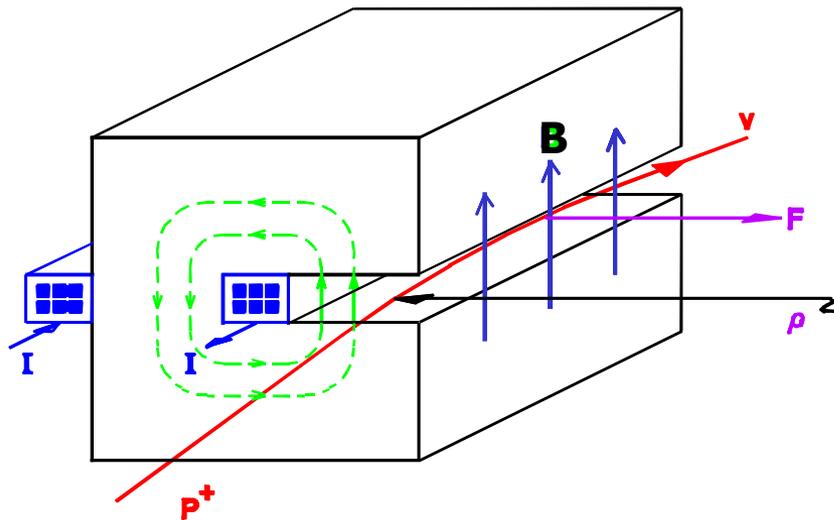


# Beam Dynamics (2)

---

- In an accelerator designed to operate at the energy  $E_{nom}$ , all particles having  $(s, E_{nom}, 0, 0, 0, 0)$  will happily fly through the center of the vacuum chamber without any problem. These are “ideal particles”.
- The difficulties start when:
  - one introduces **dipole magnets**
  - the energy  $E \neq E_{nom}$  or  $(p-p_{nom}/p_{nom}) = \Delta p/p_{nom} \neq 0$
  - either of  $x, x', y, y' \neq 0$

# Circular machines: Dipoles



$$p = m_0 \cdot c \cdot (\beta\gamma)$$



Classical mechanics:

Equilibrium between two forces

Lorentz force

Centrifugal force

$$F = e \cdot (\underline{v} \times \underline{B})$$

$$F = mv^2/\rho$$

$$evB = mv^2/\rho$$

Magnetic rigidity:

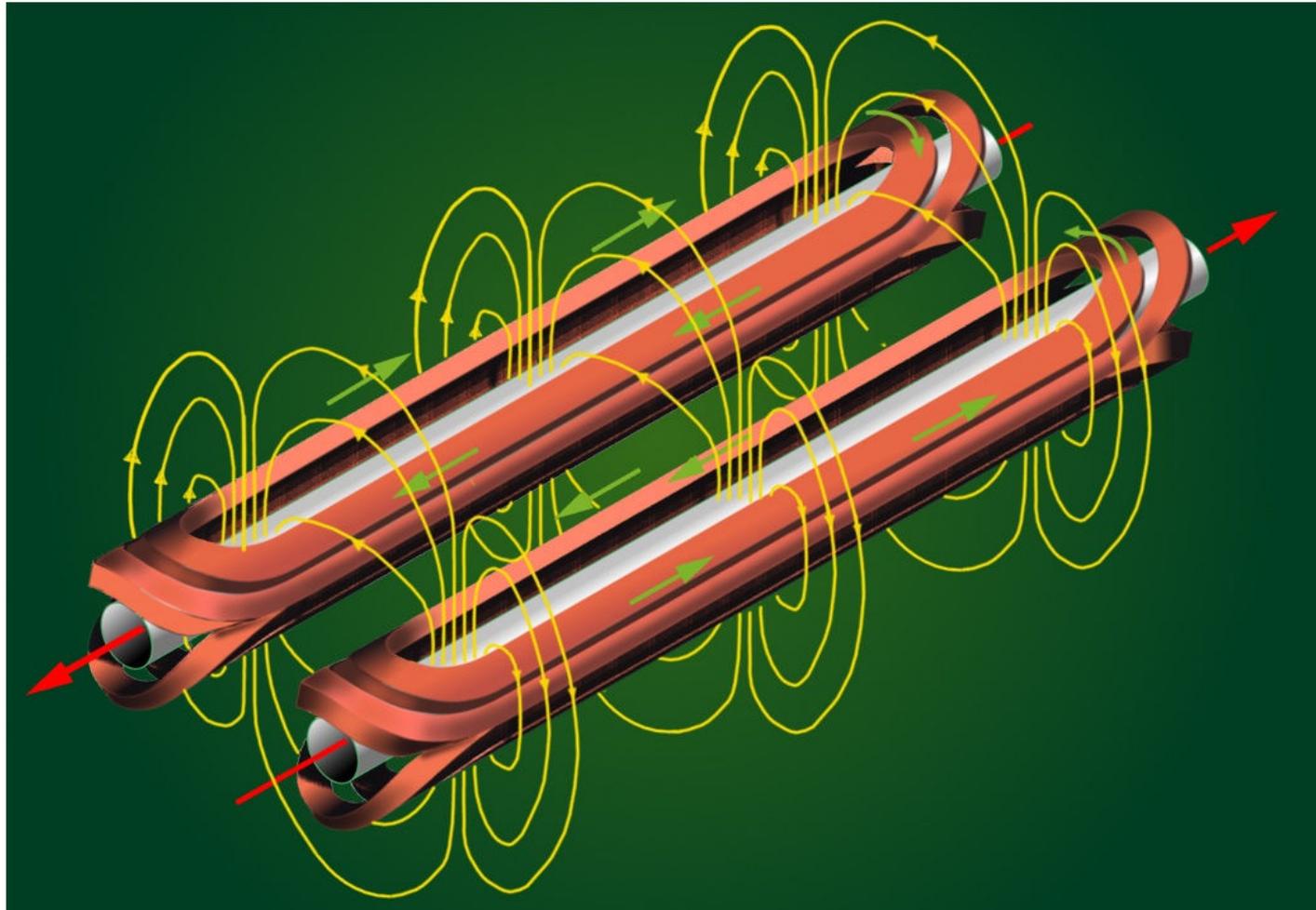
$$B\rho = mv/e = p/e$$

Relation also holds for relativistic case provided the classical momentum  $mv$  is replaced by the relativistic momentum  $p$

# Dipoles (1):



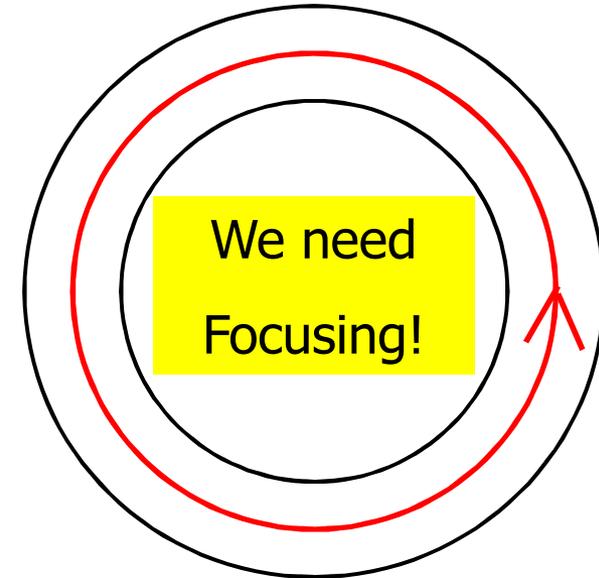
# Dipoles (2):



# Ideal circular machine:

- Neglecting radiation losses in the dipoles
- Neglecting gravitation

ideal particle would happily circulate on axis in the machine for ever!



Unfortunately: real life is different!

Gravitation:  $\Delta y = 20$  mm in 64 msec!

Alignment of the machine

Limited physical aperture

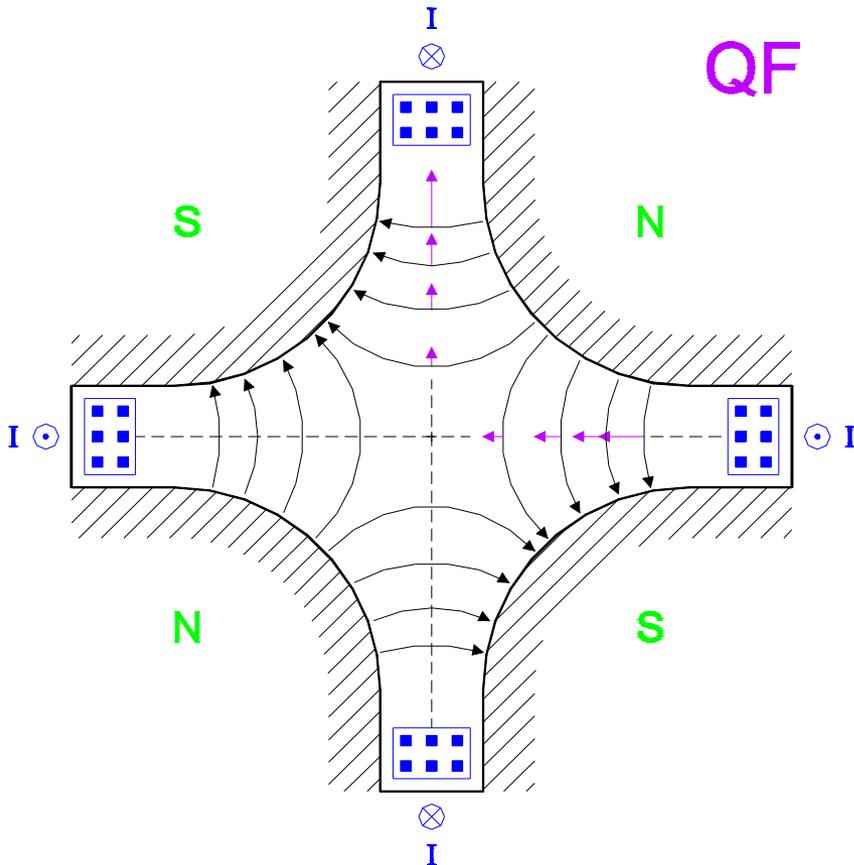
Ground motion

Field imperfections

Energy error of particles **and/or**  $(x, x')_{inj} \neq (x, x')_{nominal}$

Error in magnet strength (power supplies and calibration)

# Focusing with quadrupoles



$$F_x = -g \cdot x$$

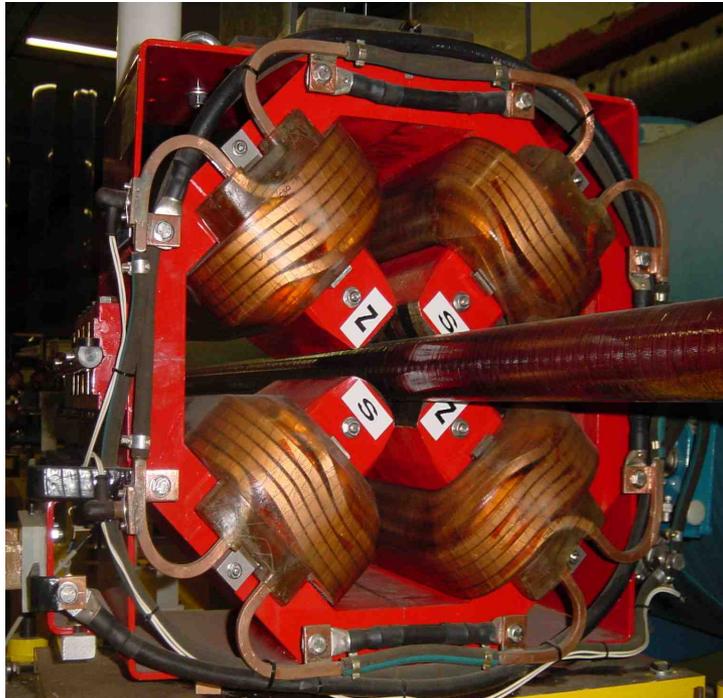
$$F_y = g \cdot y$$

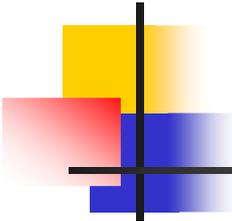
Force increases **linearly** with displacement.

Unfortunately, effect is **opposite** in the two planes (H and V).

Remember: **this** quadrupole is **focusing** in the **horizontal** plane but **defocusing** in the **vertical** plane!

# Quadrupoles:





# Focusing properties ...

---

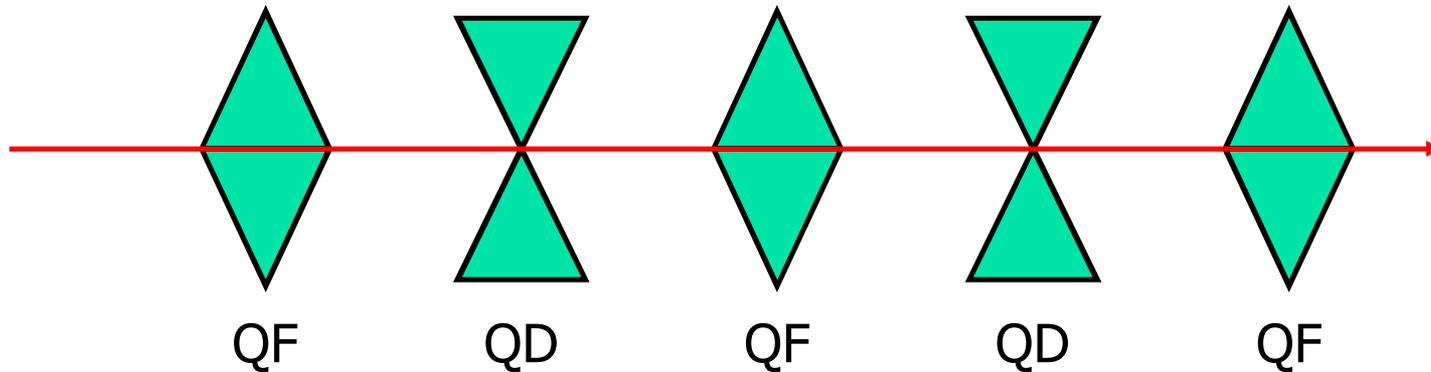
A quadrupole provides the required effect in one plane...

but the opposite effect in the other plane!

Is it really interesting ?

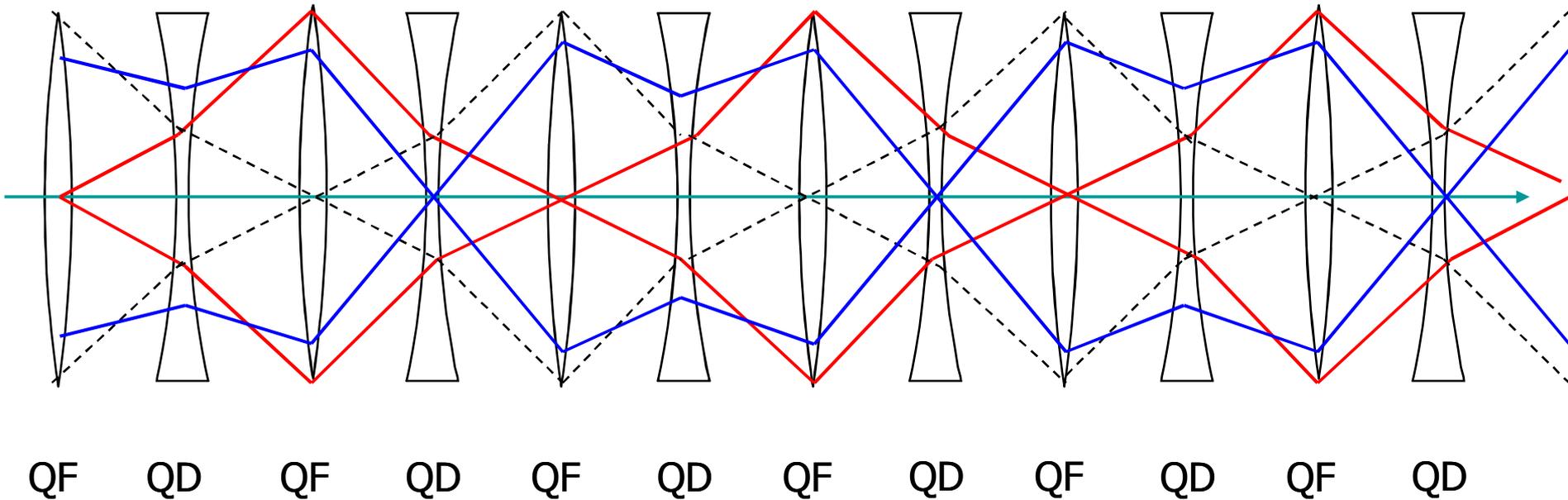
# Alternating gradient focusing

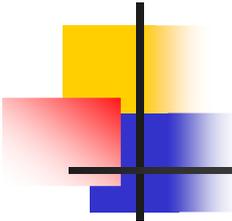
Basic new idea:  
Alternate QF and QD



valid for one plane only (H or V) !

# Alternating gradient focusing





# Alternating gradient focusing:

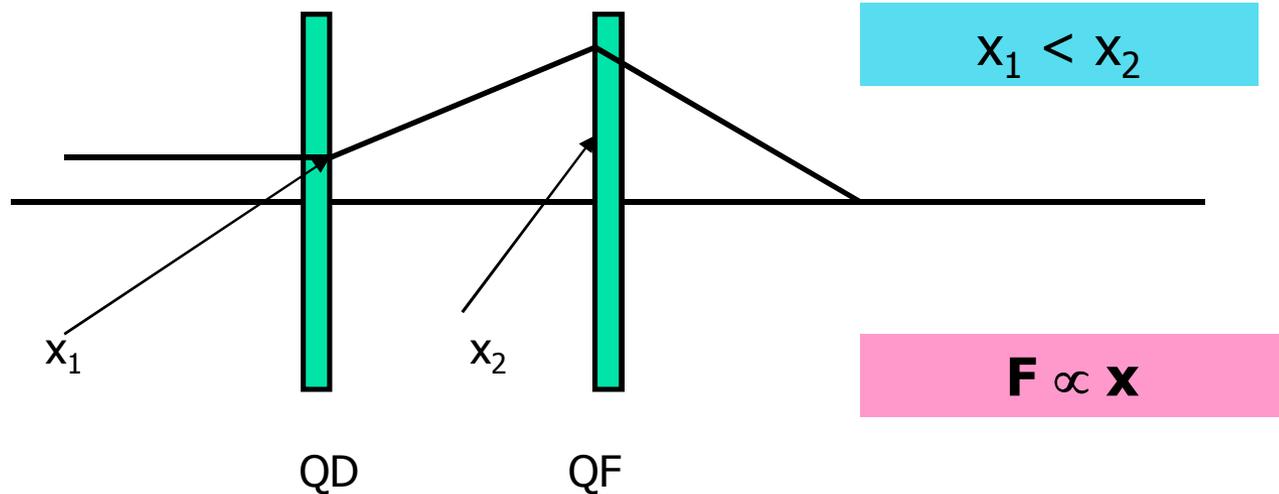
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Particles for which  $x, x', y, y' \neq 0$  thus oscillate around the ideal particle ...

but the trajectories remain inside the vacuum chamber !

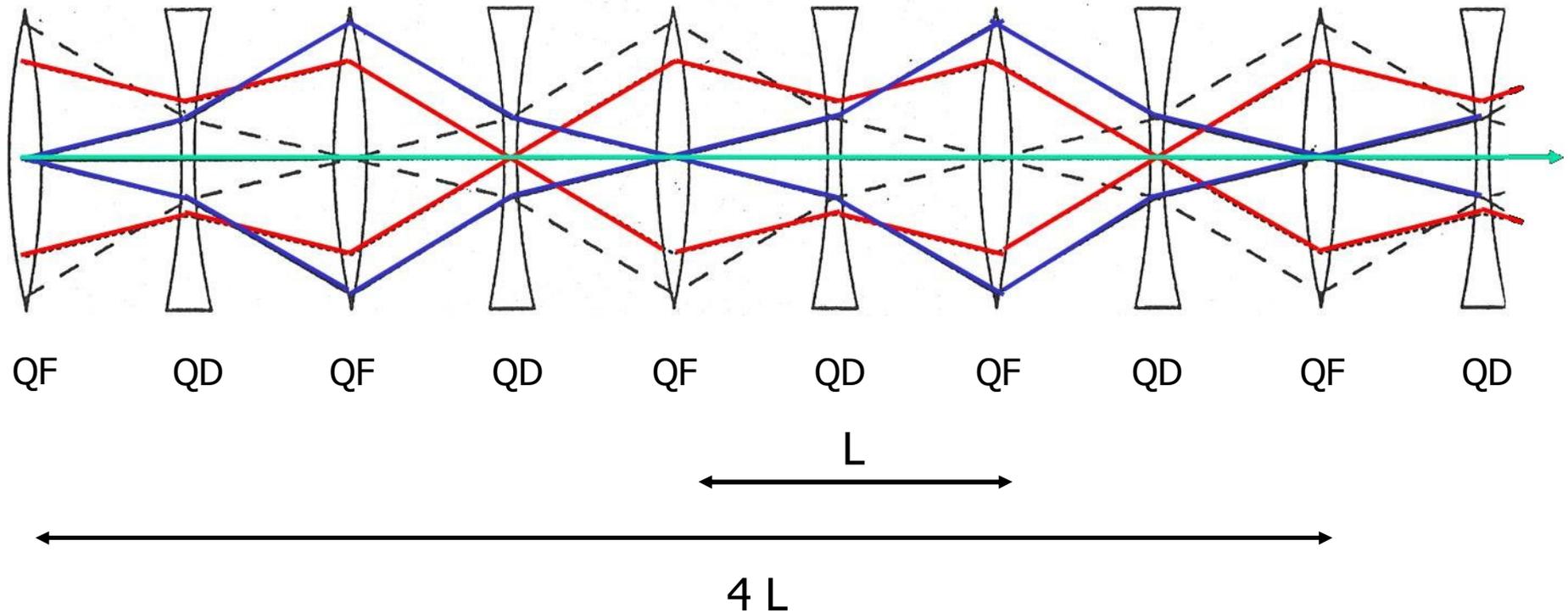
# Why net focusing effect?

Purely intuitively:



Rigorous treatment rather straightforward !

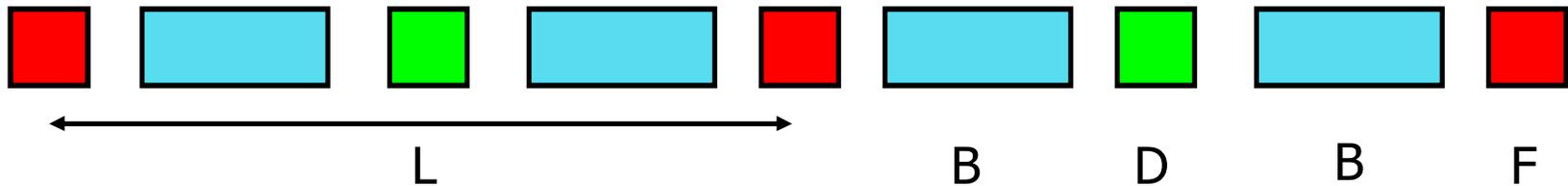
# The concept of the « FODO cell »



One complete oscillation in 4 cells  $\Rightarrow 90^\circ / \text{cell} \Rightarrow \mu = 90^\circ$

# Circular machines (no errors!)

The accelerator is composed of a **periodic** repetition of **cells**:



➤ The phase advance per cell  $\mu$  can be modified, in each plane, by varying the strength of the quadrupoles.

➤ The ideal particle will follow a **particular** trajectory, which **closes on itself** after one revolution: **the closed orbit**.

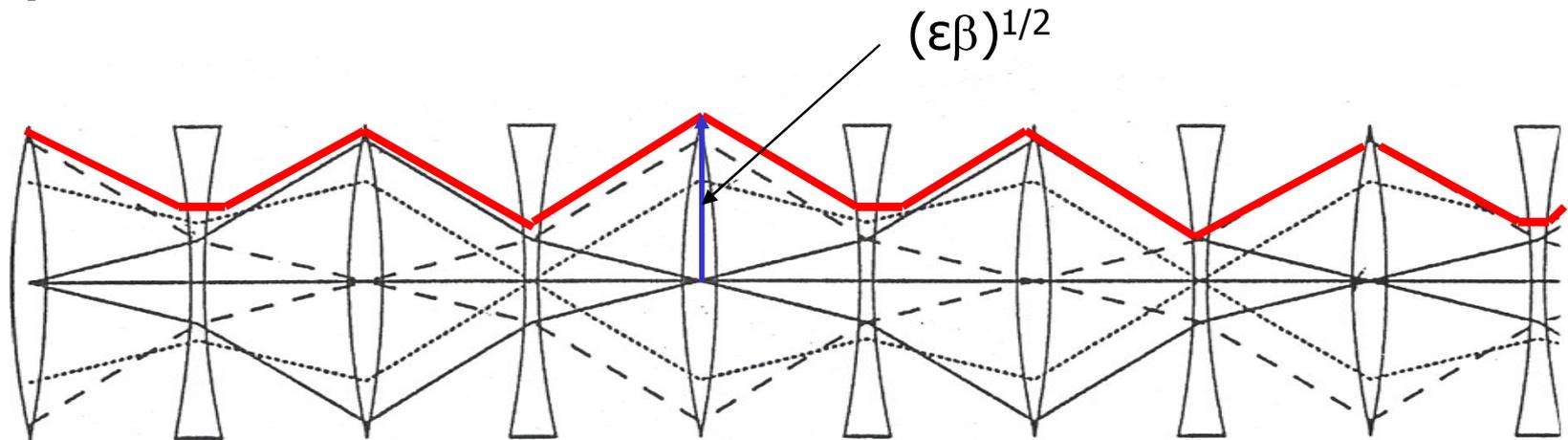
➤ The real particles will perform oscillations **around the closed orbit**.

➤ The number of **oscillations for a complete revolution** is called the **Tune Q** of the machine ( $Q_x$  and  $Q_y$ ).

# Regular periodic lattice: The Arc



# The beta function $\beta(s)$



The  $\beta$ -function is the **envelope** around all the trajectories of the particles circulating in the machine.

The  $\beta$ -function has a **minimum at the QD** and a **maximum at the QF**, ensuring the net focusing effect of the lattice.

It is a **periodic function** (repetition of cells). The oscillations of the particles are called **betatron motion** or **betatron oscillations**.

# Why introducing this function?

The  $\beta$  function is a fundamental parameter, because it is directly related to the beam size (**measurable quantity** !):

Beam size [m]

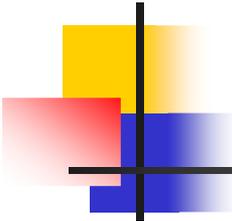
$$\sigma_{x,y}(s) = (\varepsilon \cdot \beta_{x,y}(s))^{1/2}$$

$\sigma$  (IP) = 17  $\mu\text{m}$   
at 7 TeV ( $\beta=0.55$  m)

The emittance  $\varepsilon$  characterises the quality of the injected beam (kind of measure how the particles depart from ideal ones). It is an **invariant** at a given energy.

$\varepsilon$  = beam property

$\beta$  = machine property (quads)



# Off momentum particles:

---

- These are “non-ideal” particles, in the sense that they do not have the right energy, i.e. all particles with  $\Delta p/p \neq 0$

What happens to these particles when traversing the magnets ?

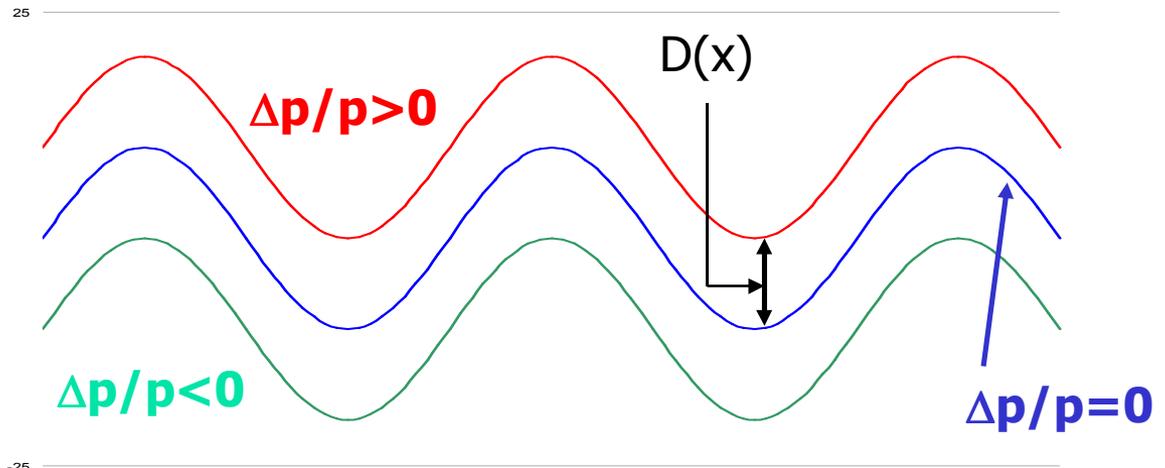
# Off momentum particles ( $\Delta p/p \neq 0$ )

## Effect from Dipoles

- If  $\Delta p/p > 0$ , particles are **less** bent in the dipoles → should spiral out !
- If  $\Delta p/p < 0$ , particles are **more** bent in the dipoles → should spiral in !

**No!**

There is an equilibrium with the restoring force of the quadrupoles

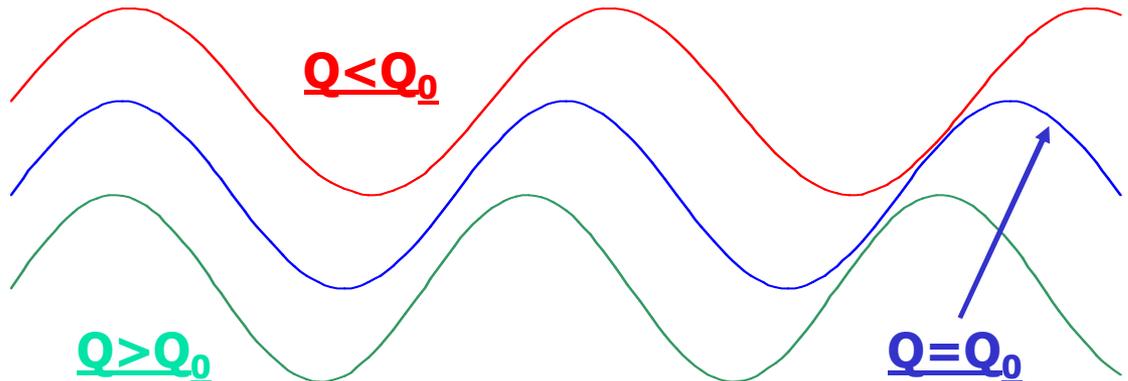


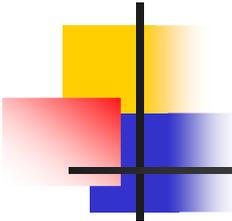
# Off momentum particles ( $\Delta p/p \neq 0$ )

## Effect from Quadrupoles

- If  $\Delta p/p > 0$ , particles are **less** focused in the quadrupoles → **lower Q !**
- If  $\Delta p/p < 0$ , particles are **more** focused in the quadrupoles → **higher Q !**

Particles with different momenta would have a different **betatron tune**  $Q=f(\Delta p/p)$ !





# The chromaticity $Q'$

Particles with different momenta ( $\Delta p/p$ ) would thus have different tunes  $Q$ .  
So what ?

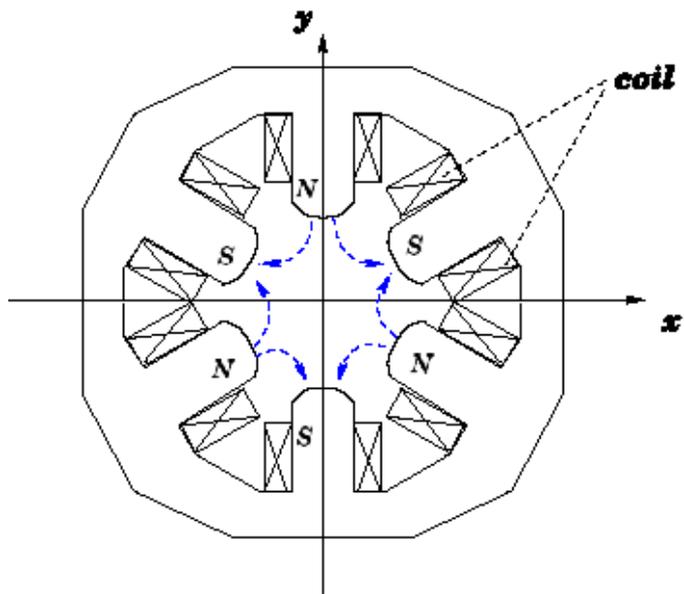
unfortunately

- The tune dependence on momentum is of **fundamental** importance for the **stability** of the machine. It is described by the **chromaticity** of the machine  $Q'$ :

$$Q' = \Delta Q / (\Delta p/p)$$

The chromaticity has to be carefully **controlled and corrected** for stability reasons.

# The sextupoles (SF and SD)

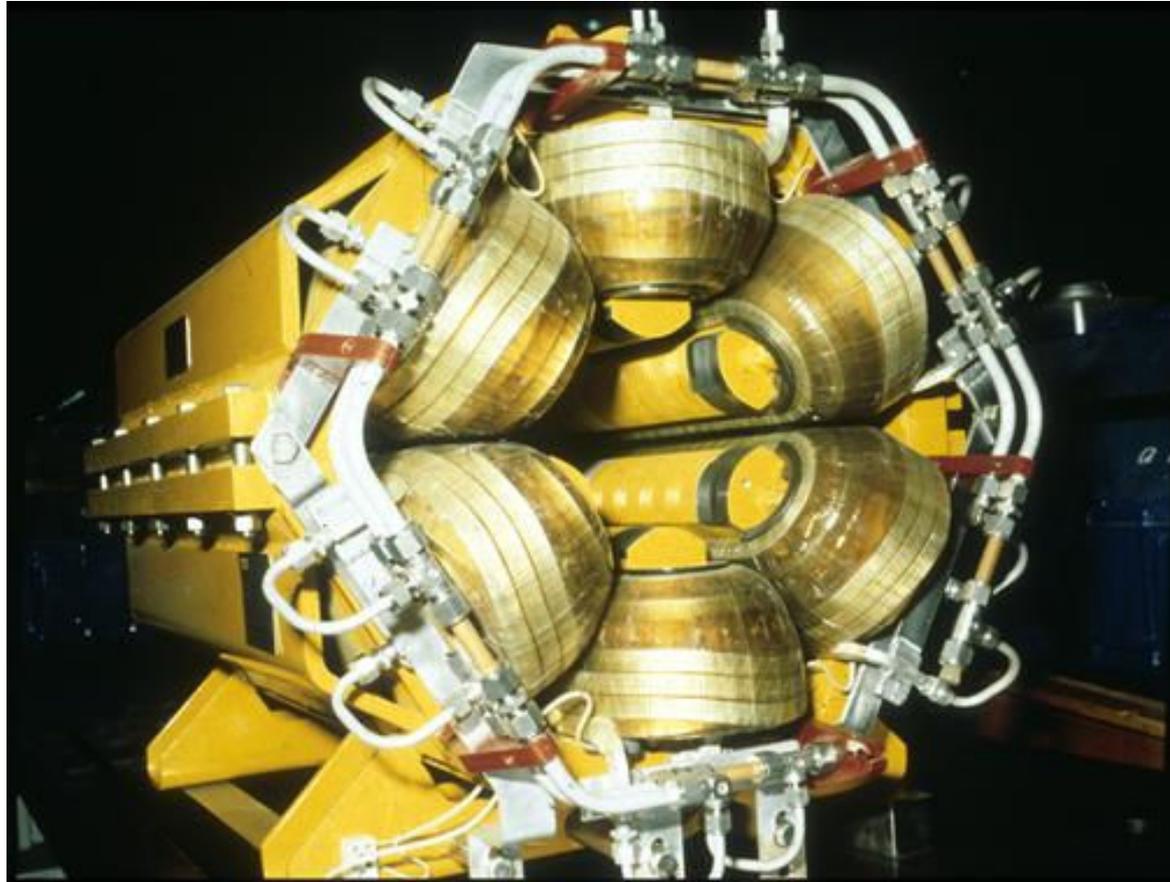


$$\triangleright \Delta X' \propto X^2$$

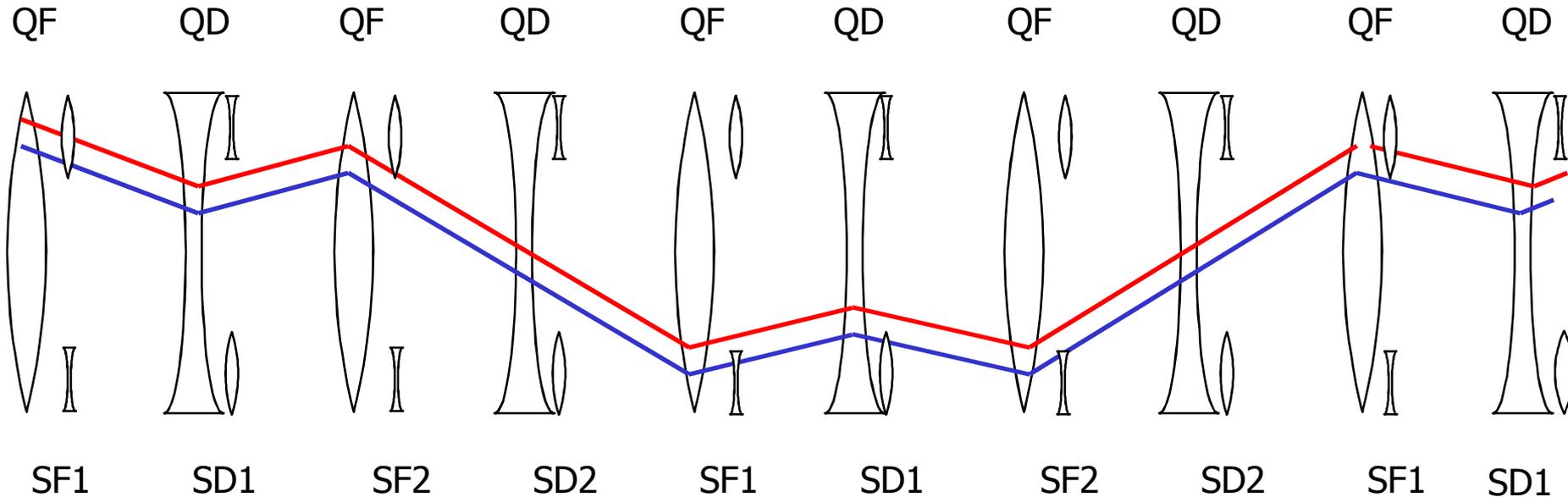
- A SF sextupole basically « adds » focusing for the particles with  $\Delta p/p > 0$ , and « reduces » it for  $\Delta p/p < 0$ .
- The chromaticity is corrected by adding a sextupole after each quadrupole of the FODO lattice.

# Sextupoles:

SPS



# Chromaticity correction

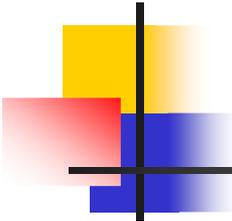


The undesired effect of sextupoles on particles with the **nominal energy** can be avoided by grouping the sextupoles into « families ».

Nr. of families:

$$N = (k * 180^\circ) / \mu = \text{Integer}$$

$$\text{e.g. } 180^\circ / 90^\circ = 2$$



# Transverse stability of the beam:

---

So, **apparently**, the tunes  $Q_x$  and  $Q_y$  have to be **selected** and **controlled** very accurately. Why this ?

LHC in collision:

$$Q_x = 64.31$$

$$Q_y = 59.32$$

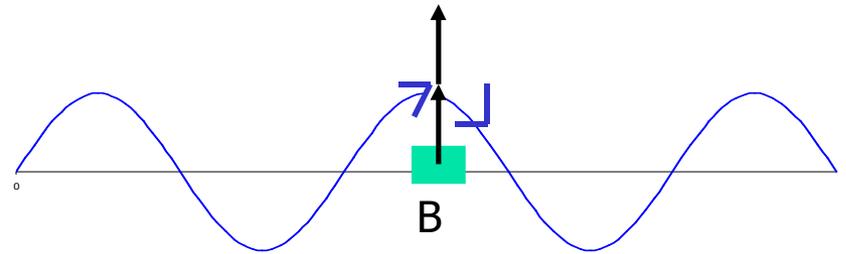
# Forbidden values for Q

- An error in a **dipole** gives a kick which has always the same sign!

Integer Tune  $Q = N$

**Forbidden!**

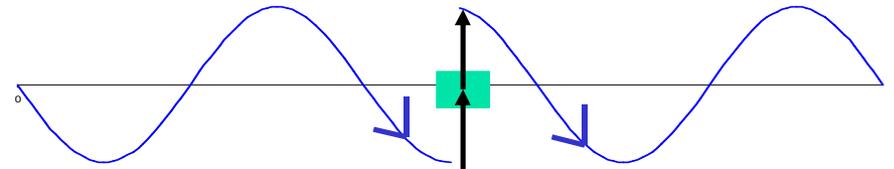
The perturbation adds up!



Half-integer Tune  $Q = N + 0.5$

O.K. for an error in a dipole!

The perturbation cancels after each turn!



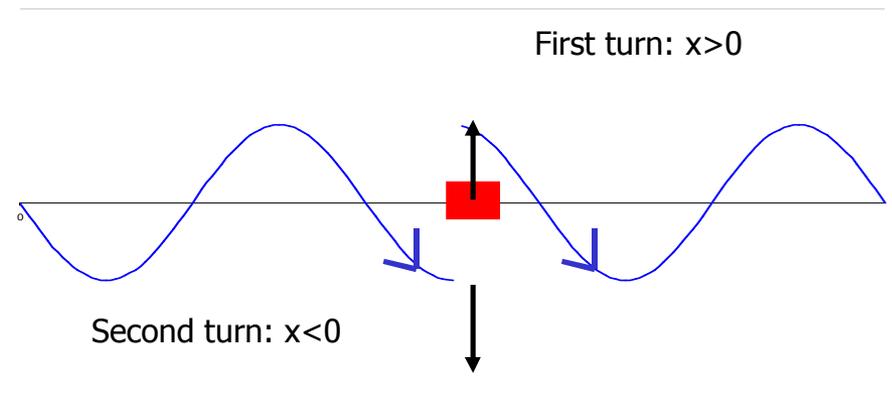
# Forbidden values for Q

- An error in a **quadrupole** gives a kick whose sign depends on  $x$  ( $F \propto x$ )

Half-integer Tune  $Q = N + 0.5$

**Forbidden !**

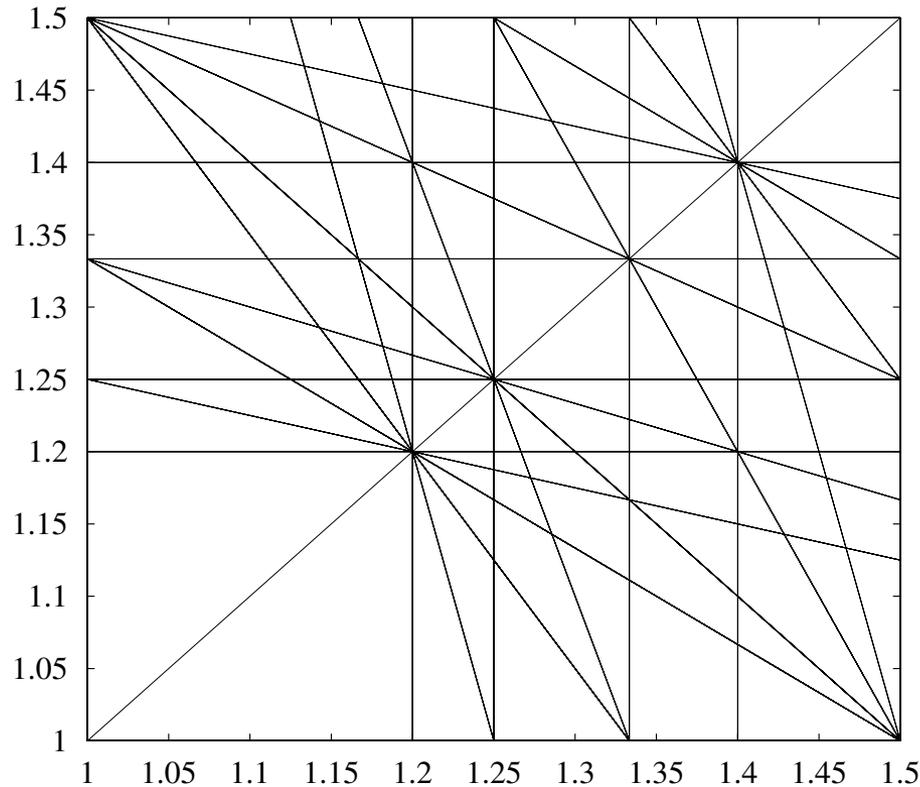
The amplitude of the oscillation is steadily increasing!



1/3 integer tune would be o.k. for a quadrupole, but **NOT** for a sextupole ...

# Tune diagram for leptons:

$Q_y$

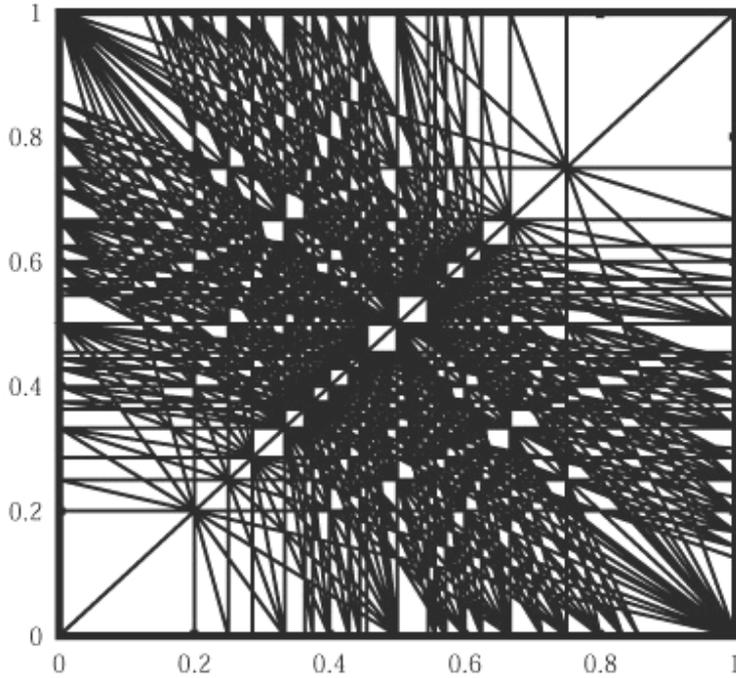


$Q_x$

Tune values  
( $Q_x$  and/or  $Q_y$ )  
which are **forbidden**  
in order to avoid  
**resonances**

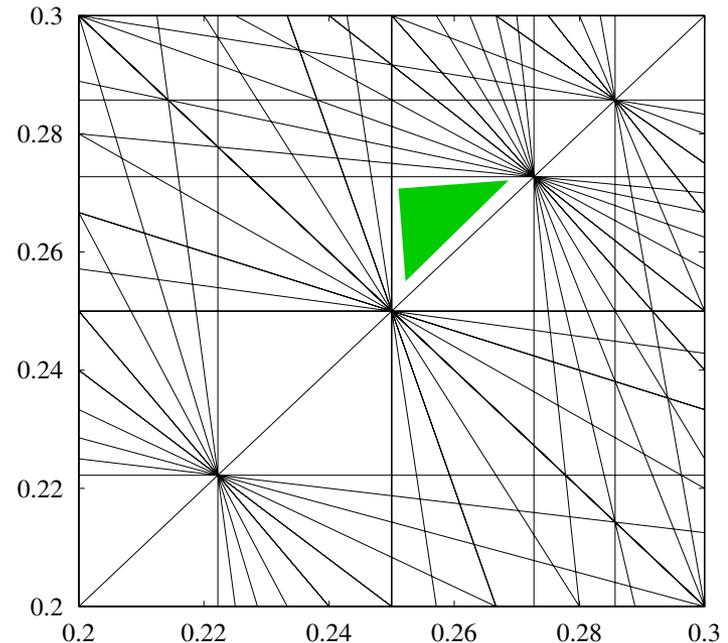
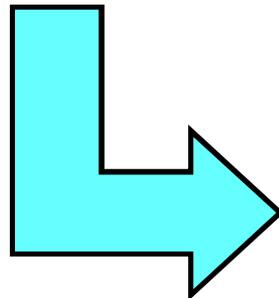
The lowest the order of  
the resonance, the most  
dangerous it is.

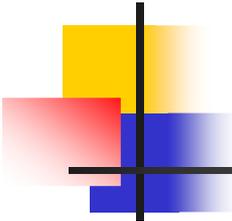
# Tune diagram for protons



Due to the energy spread in the beam, we have to accommodate an « area » rather than a point!

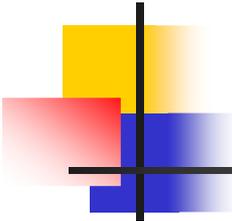
LHC





# Summary for the transverse planes:

- A particle is described by its **position** and its **slope** ( $x, x'$ ) and ( $y, y'$ )
- The circular trajectory is achieved with **dipoles**.
- The particles are kept together in the chamber with **quadrupoles**.
- The particles perform **betatron oscillations** around the **closed orbit**.
- The number of oscillations per turn (**the tune Q**) has to be carefully selected in order to avoid **resonances**.
- The phase advance per cell ( $\mu$ ) can be modified with **quadrupoles**.
- The **natural chromaticity** of the machine  $Q'$  ( $<0$ ) is compensated with **sextupoles**.



# Where are we ?

---

We have covered the **transverse beam dynamics** and we have learned that, essentially, the machine is composed of a periodic repetition of dipoles, quadrupoles and sextupoles.

**Is there still anything missing ?**

A system to accelerate the particles

A system to get efficient collisions