Flavour symmetries and FCNC processes

K. Olive & L. Velasco-Sevilla

Theoretical Physics Institute, U of Minnesota, U.S.A

Outline

- Fermion Masses and supersymmetry
- Minimal Flavour Violation
- Flavour violation with an underlying supergravity theory
- GUT boundary conditions and RGE evolution
- $B(B \to X_s \gamma)$ and $B(B \to l^+ l^-)$
- Summary

Fermion masses and flavour symmetries

Use experimental information + your favourite form of mass matrices

Experimental information

- **Fermion Masses**

$$\{\mathbf{m_u},\ \mathbf{m_c},\ \mathbf{m_t}\} = \{(\mathbf{0.0015}, \mathbf{0.004}), (\mathbf{1.15}, \mathbf{1.35}), \mathbf{174.3} \pm \mathbf{5.1}\}\mathrm{GeV}$$

$$\{\mathbf{m_d},\ \mathbf{m_s},\ \mathbf{m_b}\} = \{(\mathbf{0.004}, \mathbf{0.008}), (\mathbf{0.080}, \mathbf{0.130}), (\mathbf{4.1}, \mathbf{4.4})\}\ \mathrm{GeV}$$

- CKM matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} \mathbf{1} - \frac{\lambda^{\mathbf{2}}}{\mathbf{2}} & \lambda & A\lambda^{\mathbf{3}}(\rho - \mathbf{i}\eta) \\ -\lambda & \mathbf{1} - \frac{\lambda^{\mathbf{2}}}{\mathbf{2}} & \mathbf{A}\lambda^{\mathbf{2}} \\ \mathbf{A}\lambda^{\mathbf{3}}(\mathbf{1} - \rho - \mathbf{i}\eta) & -\mathbf{A}\lambda^{\mathbf{2}} & \mathbf{1} \end{pmatrix}$$

$$= \begin{pmatrix} (\mathbf{0.9739}, \mathbf{0.9751}) & (\mathbf{0.221}, \mathbf{0.227}) & (\mathbf{0.0029}, \mathbf{0.0045}) \\ (\mathbf{0.221}, \mathbf{0.227}) & (\mathbf{0.9730}, \mathbf{0.9744}) & (\mathbf{0.039}, \mathbf{0.044}) \\ (\mathbf{0.0048}, \mathbf{0.014}) & (\mathbf{0.037}, \mathbf{0.043}) & (\mathbf{0.9990}, \mathbf{0.9992}) \end{pmatrix}$$

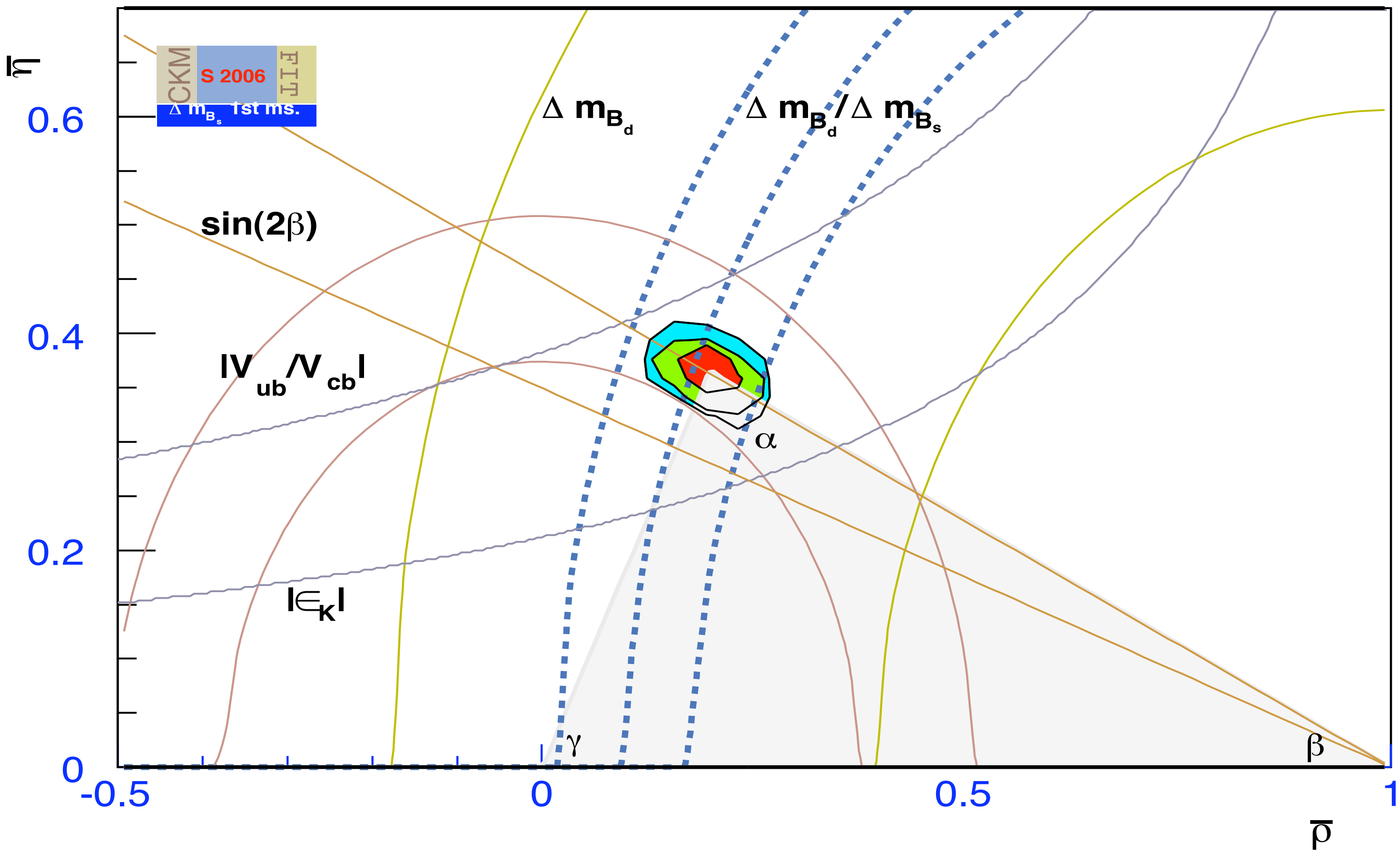


Figure 1: $(\bar{\rho}, \bar{\eta})$ including the measurement on Δm_{B_s}.

Favourite form of mass matrices

Determine your form of mass matrices

$$M^u_{\rm diag} = L^{u\dagger} M^u R^u, \qquad M^d_{\rm diag} = L^{d\dagger} M^d R^d$$

$$V_{\rm CKM} = L^{u\dagger} L^d \downarrow \qquad U_{\rm MNS} = L^{l\dagger} L^\nu$$

$\longrightarrow$ **we can determine the structure above the diagonal and the eigenvalues and constrain elements below the diagonal.**

We have many possibilities for the structure of mass matrix but a **natural description of masses in terms of** $\varepsilon = O(\lambda),\ \lambda = 0.224$ **it is a hierarchical description**

$$M^d = m_b \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^2 \\ & & 1 \end{pmatrix},\ M^u = m_t \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^6 & \varepsilon^{\geq 6} \\ & \varepsilon^{\geq 4} & \varepsilon^4 \\ & & 1 \end{pmatrix}, M^e = m_\tau \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^{\geq 2} \\ & & 1 \end{pmatrix}$$

Need to make extra assumptions

- Elements below diagonal: Symmetric matrix, anti-symmetric
- Which powers to keep in certain places?

$\longrightarrow$ Gatto-Sartori-Tonin Relation $V_{us} = |s^d_{12} - e^{i\phi_1} s^u_{12}| \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|$

Choose your flavour symmetries

Which GUT?, which horizontal symmetry?

Some possibilities

Just GUT's [Senjanovic et. al.]

$SU(5) + \nu_R$ + horizontal symmetries [Masina & Savoy, Z. Tavartkiladze, Z. Berezhiani, K. Babu et. al.]

$SO(10)$ + non-Abelian horizontal symmetries [Ross, V-S, Raby & Dermisek, M-C. Chen & K.T. Mahanthapa, Bando & et al.]

Just horizontal symmetries, e.g. $U(1)$ [Dreiner & Thormeier et. al.]

Emerging scenarios

Symmetric	Non-symmetric
Non-Abelian	Abelian or Non-Abelian
$m^f_{11} = 0$	$m^f_{11} \neq 0$
$\downarrow$	$\downarrow$
$SU(4)_C \times SU(2)_R \times SU(2)_L$	$SU(5)$

Flavour structure of Yukawa couplings

At EW scale we set up the off diagonl Y^d entries with the CKM mixings, and we also assume the same mixings for the charged lepton sector.

$$Y^u = \begin{pmatrix} 0 & O(\epsilon_u^3) & O(\epsilon_u^3) \\ O(\epsilon_u^3) & c_{22}^u \epsilon_u^2 & O(\epsilon_u^2) \\ O(\epsilon_u^3) & O(\epsilon_u^2) & c_{33}^u \end{pmatrix}, \quad Y^d = \begin{pmatrix} 0 & O(\epsilon_d^3) & O(\epsilon_d^3) \\ O(\epsilon_d^3) & c_{22}^d \epsilon_d^2 & c_{23}^d \epsilon_d^2 \\ O(\epsilon_d^3) & c_{23}^d \epsilon_d^2 & c_{33}^d \end{pmatrix},$$

$$Y^e = \begin{pmatrix} 0 & O(\epsilon_d^3) & O(\epsilon_d^3) \\ O(\epsilon_d^3) & c_{22}^e \epsilon_d^2 & c_{23}^e \epsilon_d^2 \\ O(\epsilon_d^3) & c_{23}^e \epsilon_d^2 & c_{33}^e \end{pmatrix}.$$

How to determine the Flavour Structure in the Supersymmetric Sector?

Minimal Flavour Violation hypothesis

The global symmetry in the gauge sector of the SM

$$U(3)^5 = SU(3)_Q \times SU(3)_U \times SU(3)_D \times \ldots$$

Broken only by the Yukawa couplings

$$Y_d \to \overline{3}_Q \times 3_d, \quad Y_u \to \overline{3}_Q \times 3_u, \quad Y_e \to \overline{3}_L \times 3_e,$$

Specific **symmetry+ symmetry-breaking pattern** $\rightarrow$ **responsible for the supression of FCNC, CPV effects,** etc..

However in general soft breaking terms of the MSSM allow a richer structure

$$\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{1}{2}\left(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}\tilde{W} + M_3 \tilde{g}\tilde{g}\right) + h.c. \\
& -\left(\bar{\tilde{Q}} a_u H_u \tilde{u} + \bar{\tilde{Q}} a_d H_d \tilde{d} + \bar{\tilde{L}} a_e H_d \tilde{e}\right) + h.c. \\
& -\tilde{Q} M^2_{\tilde{Q}} \tilde{Q}^\dagger - \tilde{L} M^2_{\tilde{L}} \tilde{L}^\dagger - \tilde{u} M^2_{\tilde{u}} \tilde{u}^\dagger - \tilde{d} M^2_{\tilde{d}} \tilde{d}^\dagger - \tilde{e} M^2_{\tilde{e}} \tilde{e}^\dagger \\
& -m^2_{H_u} H^*_u H_u - m^2_{H_d} H^*_d H_d - (B\mu H_u H_d + c.c.)
\end{aligned}$$

Within the MSSM, the MFV hypothesis implies a strong restriction on the soft terms

$$M^2_{\tilde{Q}} \tilde{Q} \tilde{Q}^\dagger \quad \propto \quad \sum x_n (Y_u Y_u^\dagger)^n \sim x_o I + x_1 Y_u Y_u^\dagger$$

$$(\delta^D_{ij})_{LL} \quad \propto \quad y_t^2 (V_{\mathrm{CKM}\,ti})^* (V_{\mathrm{CKM}\,ti})$$

As a consequence we have the same CKM factors as in the SM: only flavour-independent magnitude of FCNC amplitudes can be modified

$$A(b \to s\gamma) \propto \left[(V_{\mathrm{CKM}})^*_{ts} (V_{\mathrm{CKM}})_{tb}\right], \quad \Delta M_{B_d} \propto \left[(V_{\mathrm{CKM}})_{tb} (V_{\mathrm{CKM}})_{td}\right]^2$$

→

Excellent supression of BSM effects in flavour parameters (e.g. SM CKM fits)

Flavour violation with an underlying supergravity theory

Trilinear terms

$$Y^f_{ij} = b^f_{\phi_a} \frac{\langle \theta_{a,f} \rangle^{\alpha^f_{ij}}}{M}, \quad \epsilon_f = f(\frac{\langle \theta_f \rangle}{M_f}),$$

where the order of magnitud of ϵ_f is fixed through the minimization of the scalar potential involving them.

These fields also couple to the s-fermions through the trilinear terms

$$(a^f)_{ij} H_f Q_i q^c_j$$

The generic form of the trilinear couplings matrices in the main supersymmetric models of breaking, i.e. supergravity mediation, gauge mediation or anomaly mediation, is of the form

$$a^f_{ij} = Y^f_{ij}(A^f_0)ij, \quad \leftarrow \quad a^f_{ij} = Y^f_{ij}(A^f_0)$$

In minimal supergravity A_0 becomes a constant and hence the proportionally of MFV is achieved, if just the evolution of one family is considered.

Once a family symmetry is considered, there are additional terms to the trilinear couplings giver by derivatives of the Yukawa couplings with respect to the flavon fields:

$$
\begin{aligned}
a_{ijf} &= Y_{ijf} F^{\alpha} \partial_{\alpha} \left[\tilde{K}(X_p, \theta_a) + \ln(K^g_g(H_f) K^l_l(\psi) K^m_m(\psi)) \right] \\
&+ F^{\theta_a} \partial_{\theta_a} Y_{ijf} + (a_{D_A})_{ijf}, \\
&\alpha = X_p, \theta_a.
\end{aligned}
$$

F^{θ_a} is proportional to the vacuum expectation value of the corresponding flavon field, $\langle \theta_a \rangle$:

$$F^{\theta_a} = f_{\theta_a} m_{3/2} \langle \theta_a \rangle,$$

where f_{θ_a} is a consant determined by the flavour symmetry.

The Yukawa couplings in term of the flavon fields are generically written as $\frac{\langle \theta_{a,f} \rangle}{M}^{\alpha^f_{ij}}$

$$\mathbf{a^f_{ij}} = \mathbf{Y^f_{ij}}((\mathbf{A_0})_{\mathbf{ij}} + \mathbf{k^f_{ij}}),$$

k^f_{ij} are the coefficients (e.g.) produced when taking the derivatives with respect to the flavon fields times a mass term:

$$k^f_{ij} = f_{\phi_a} \alpha^f_{ij} m_{3/2}.$$

Thus a generic form of the trilinear couplings under these conditions is:

$$a^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A^u_{22} Y^u_{22} & 0 \\ 0 & 0 & A^u_{33} Y^u_{33} \end{pmatrix}, \quad a^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & A^d_{22} Y^d_{22} & A^d_{23} Y^d_{23} \\ 0 & A^d_{32} Y^d_{32} & A^u_{33} Y^u_{33} \end{pmatrix},$$

Soft squared mass masses

Once the flavour symmetry is specified, the Kähler potential can be trivially written as

$$\begin{aligned} K &= \sum_\psi \psi^i \psi^{\dagger \bar{j}} K_{i\bar{j}}(\psi), \quad \psi = u_R,\, d_R,\, e_R,\, \nu_R,\, Q_L,\, L_L, \\ K_{i\bar{j}}(\psi) &= \delta_{i\bar{j}} \left[c(\psi) + d(X_p, \psi) X^p X^\dagger_p \right] \\ &+ \frac{\theta^a_i \theta^\dagger_{a\bar{j}}}{M^2(\theta_a)} \left[c(\theta_a, \psi) + d(\theta_a, X_p, \psi) X^p X^\dagger_p \right]. \end{aligned}$$

Then the squared mass matrices are given by

$$\begin{aligned} m^2_{i\bar{j}} &= m^2_{3/2} K_{i\bar{j}} - F^{\bar{X}_p} \left[\partial_{\bar{X}_p} \partial_{X_q} K_{i\bar{j}} - (\partial_{\bar{X}_p} K_{i\bar{l}})(K^{-1})_{\bar{l}m} (\partial_{X_q} K_{m\bar{j}}) \right] F^{X_q} \\ &- F^{\overline{\theta}_a} \left[\partial_{\overline{\theta}_a} \partial_{\theta_b} K_{i\bar{j}} - (\partial_{\overline{\theta}_a} K_{i\bar{l}})(K^{-1})_{\bar{l}m} (\partial_{\theta_b} K_{m\bar{j}}) \right] F^{\theta_b} + (m^2_{D_A})_{i\bar{j}}, \end{aligned}$$

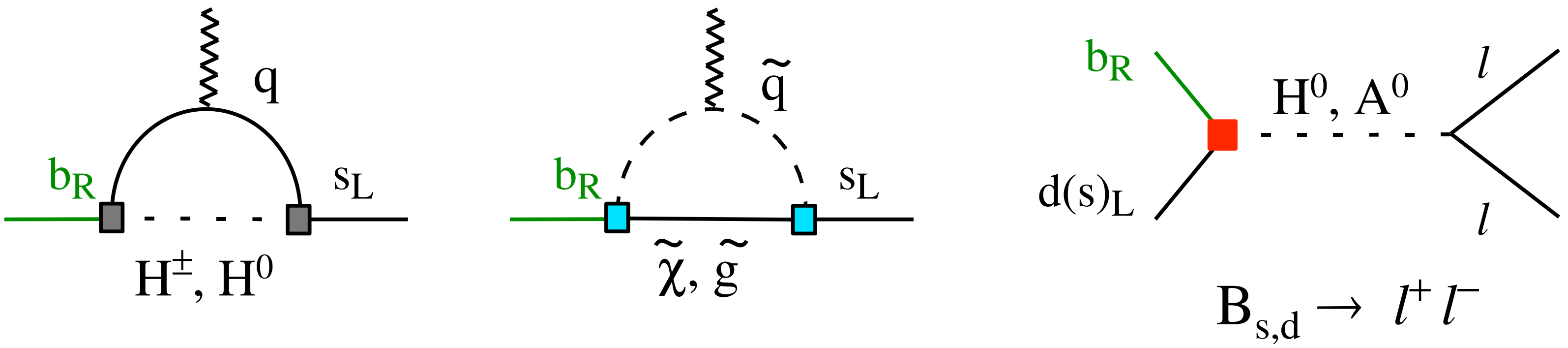
For example in $\mathbf{SU(3)}$ flavour models, we have an structure like

$$m^2_{i\bar{j}} = m^2_{3/2}\delta_{i\bar{j}} - m^2_o \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & O\left(\frac{\langle\theta\rangle^2}{M^2}\right) \\ 0 & O\left(\frac{\langle\theta\rangle^2}{M^2}\right) & r_3 \end{pmatrix}.$$

Hence we consider in the analysis the following form of soft-squared masses

$$M^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}1} & 0 & 0 \\ 0 & m^2_{\tilde{f}2} & m^2_{\tilde{f}23} \\ 0 & m^{2\dagger}_{\tilde{f}23} & m^2_{\tilde{f}3} \end{pmatrix}, \quad f = Q, u, d, L, e.$$

$\mathrm{B}(B \to X_s \gamma)$ and $\mathrm{B}(B \to X_s l^+ l^-)$



$$\mathbf{B(B\!\to} X_s\gamma\mathbf{)=(3.55}\pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$$

$$\mathbf{B(B\!\to} X_s l^+ l^-\mathbf{)=(1.6}\pm \pm 0.51) \times 10^{-6}$$

3x3 flavour-violating

$$\mathcal{M}^2_{\tilde{d}} = \begin{pmatrix} \hat{m}^2_{\tilde{Q}} + m_d^2 + D_{dLL} & v_1 \hat{T}_D - \mu^* m_d \tan\beta \\ v_1 \hat{T}_D^\dagger - \mu m_d \tan\beta & \hat{m}^2_{\tilde{d}} + m_d^2 + D_{dRR} \end{pmatrix} \equiv \begin{pmatrix} (\mathcal{M}^2_{\tilde{d}})^{LL} & (\mathcal{M}^2_{\tilde{d}})^{LR} \\ (\mathcal{M}^2_{\tilde{d}})^{RL} & (\mathcal{M}^2_{\tilde{d}})^{RR} \end{pmatrix}$$

The starting point in the calculation of inclusive B decay rates is the low-energy effective Hamiltonian

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}\, V_{ts}^* V_{tb} \sum_i C_i(\mu_b) O_i(\mu_b)\,.$$

In the massless strange quark limit, the operators relevant to our discussion are

$$\begin{aligned} O_2 &= \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L\,, \\ O_7 &= \frac{e\, m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R\,, \\ O_8 &= \frac{g_s m_b}{16\pi^2}\, \bar{s}_L \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_R\,. \end{aligned}$$

When the mass of the strange quark is taken into account we need to consider the operators

$$\tilde{O}_7 = \frac{e\, m_b}{16\pi^2}\, \bar{s}_R \sigma_{\mu\nu} F^{\mu\nu} b_L \,,$$
$$\tilde{O}_8 = \frac{g_s m_b}{16\pi^2}\, \bar{s}_R \sigma_{\mu\nu} G_a^{\mu\nu} t_a b_L \,.$$

The prediction for the $B \to X_s\gamma$ branching ratio is obtained by normalizing the result for the corresponding decay rate to that for the semileptonic decay rate, thereby eliminating a strong dependence on the b-quark mass:

$$\mathrm{B}(B \to X_s\gamma)\Big|_{E_\gamma > (1-\delta)E_\gamma^{\max}} = B(B \to X_c e\bar{\nu})_{\exp} \frac{\Gamma(B \to X_s\gamma)|_{E_\gamma > (1-\delta)E_\gamma^{\max}}}{\Gamma(B \to X_c e\bar{\nu})} .$$

$$\mathrm{B}(B \to X_s\gamma)\Big|_{E_\gamma > E_0} = B(B \to X_c e\bar{\nu})_{\exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\mathrm{em}}}{\pi} \left[P(E_0) + N(E_0)\right] \frac{1}{r(\Gamma_u/\Gamma_c}$$

Instead of $|C_7^{\mathrm{eff}}(M_W)|^2$ to $B(X_s \to b\gamma)$, we have

$$|C_7^{\mathrm{eff}}(\mu_b)|^2 \rightarrow P(E_0) + N(E_0)$$
$$P(E_0) = \left|X^(0)_{\tilde{g}} + X_c + X_t + \epsilon_w\right|^2 + B(E_0),$$

GUT boundary conditions and RGE evolution

Boundary conditions at GUT scale

$$Y^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^u_{22}\epsilon_u^2 & 0 \\ 0 & 0 & c^u_{33} \end{pmatrix}, \quad Y^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^d_{22}\epsilon_d^2 & c^d_{23}\epsilon_d^2 \\ 0 & c^d_{23}\epsilon_d^2 & c^d_{33} \end{pmatrix},$$

$$Y^e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^e_{22}\epsilon_d^2 & c^e_{23}\epsilon_d^2 \\ 0 & c^e_{23}\epsilon_d^2 & c^e_{33} \end{pmatrix}.$$

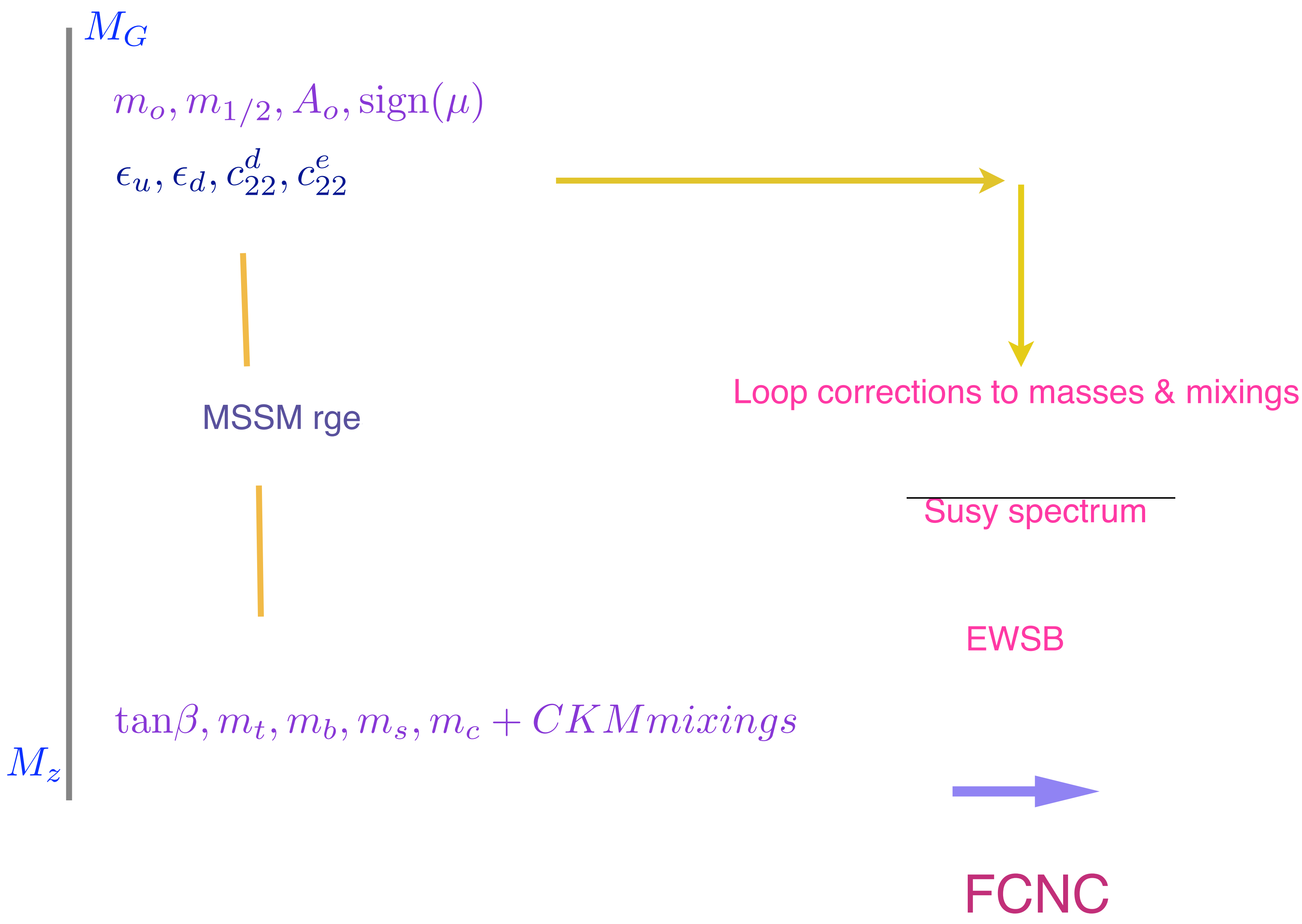
With the above parameterization of Yukawa matrices then soft masses are as follows:

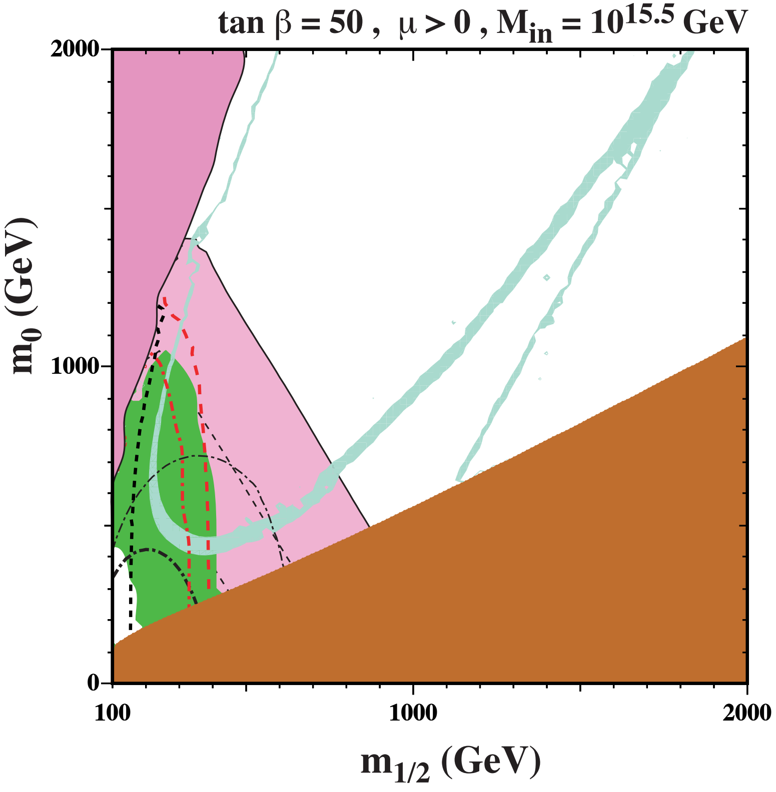
$$M^2_{\tilde{Q}} = M^2_{\tilde{u}_R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_u^2 \\ 0 & \epsilon_u^2 & 1 \end{pmatrix} m_0^2, \quad M^2_{\tilde{d}_R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_d^2 \\ 0 & \epsilon_d^2 & 1 \end{pmatrix} m_0^2$$

$$M^2_{\tilde{L}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_u^2 \\ 0 & \epsilon_u^2 & 1 \end{pmatrix} m_0^2, \quad M^2_{\tilde{e}_R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_d^2 \\ 0 & \epsilon_d^2 & 1 \end{pmatrix} m_0^2$$

$$a^u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^u_{22}\epsilon^u & 0 \\ 0 & 0 & c^u_{33} \end{pmatrix} m_a, \qquad a^d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^d_{22}\epsilon^2_d & c^d_{23}\epsilon^2_d \\ 0 & c^d_{23}\epsilon^2_d & c^d_{33} \end{pmatrix} m_a,$$

$$a^e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c^e_{22}\epsilon^2_d & c^e_{23}\epsilon^2_d \\ 0 & c^e_{23}\epsilon^2_d & c^e_{33} \end{pmatrix} m_a$$


M_G
$m_o, m_{1/2}, A_o, \mathrm{sign}(\mu)$
$\epsilon_u, \epsilon_d, c^d_{22}, c^e_{22}$
Loop corrections to masses & mixings
MSSM rge
Susy spectrum
EWSB
$\tan\beta, m_t, m_b, m_s, m_c + CKM mixings$
M_z
FCNC

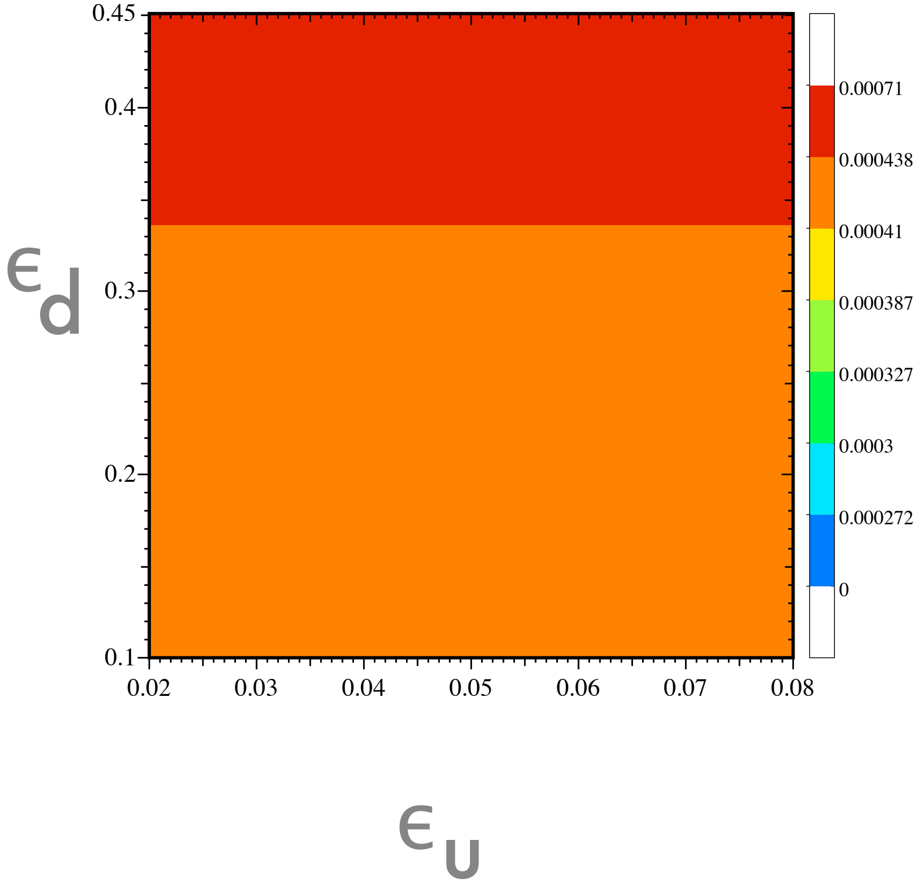


tan β = 50 , μ > 0 , M_in = 10^15.5 GeV
2000
1000
0
100
1000
2000
m_0 (GeV)
m_1/2 (GeV)

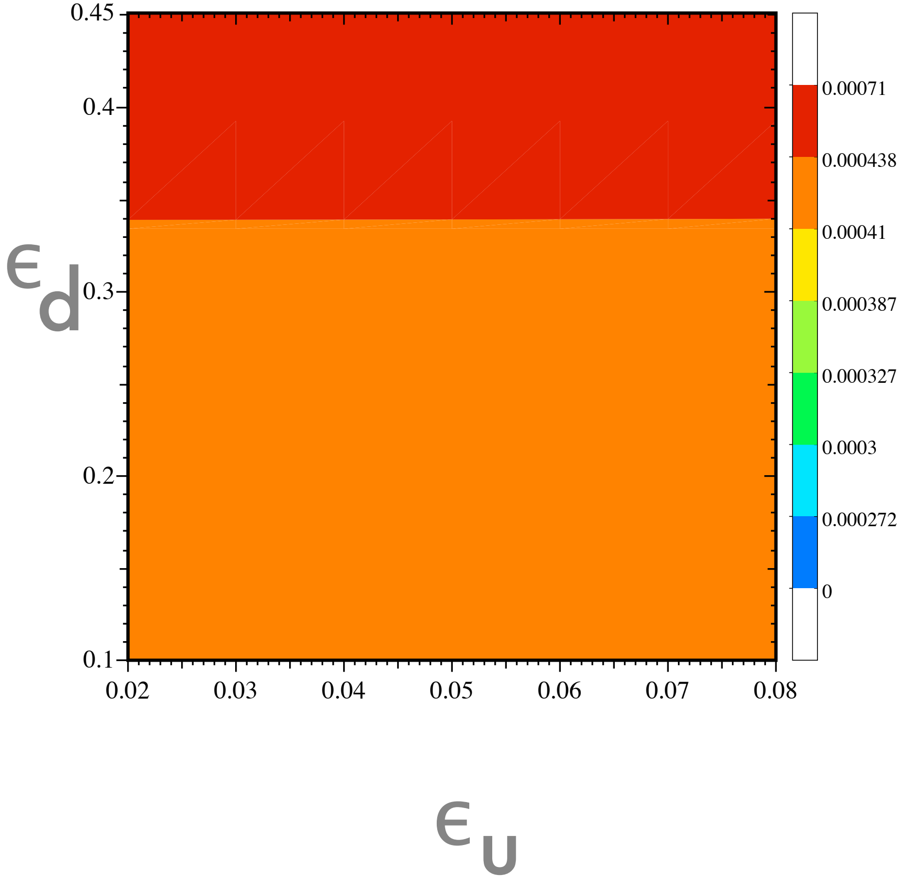
Example $m_o = 400$ GeV, $m_{1/2} = 670$ GeV, $A_o = 0, \text{sign}(\mu) > 0$
$\tan\beta = 50$,

$[m_t = 171.4,\ m_b = 4.25,\ m_s = 0.095\ , m_c = 1.3\, m_\mu = 0.106]$ [GeV]

b → s γ

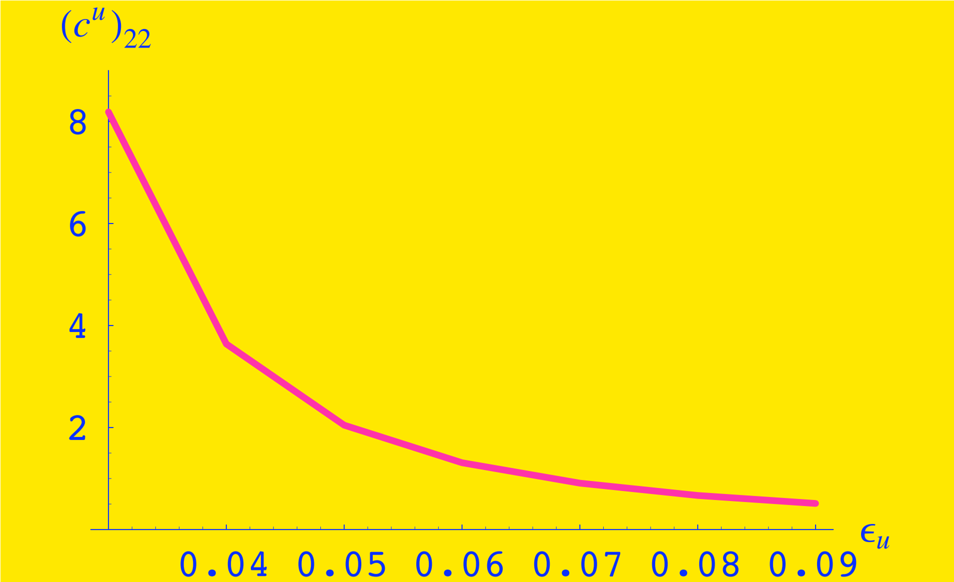


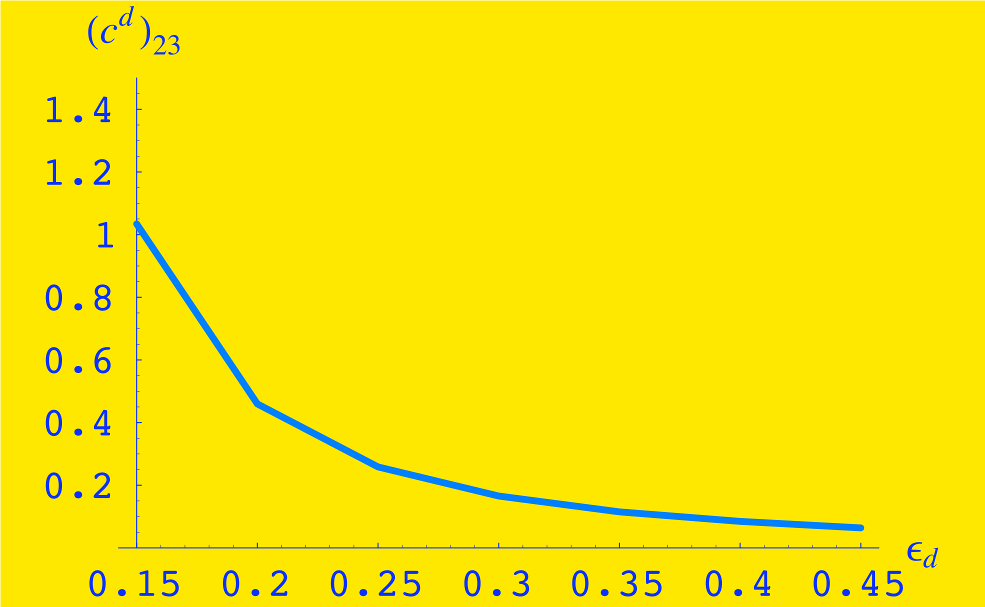
b → s γ

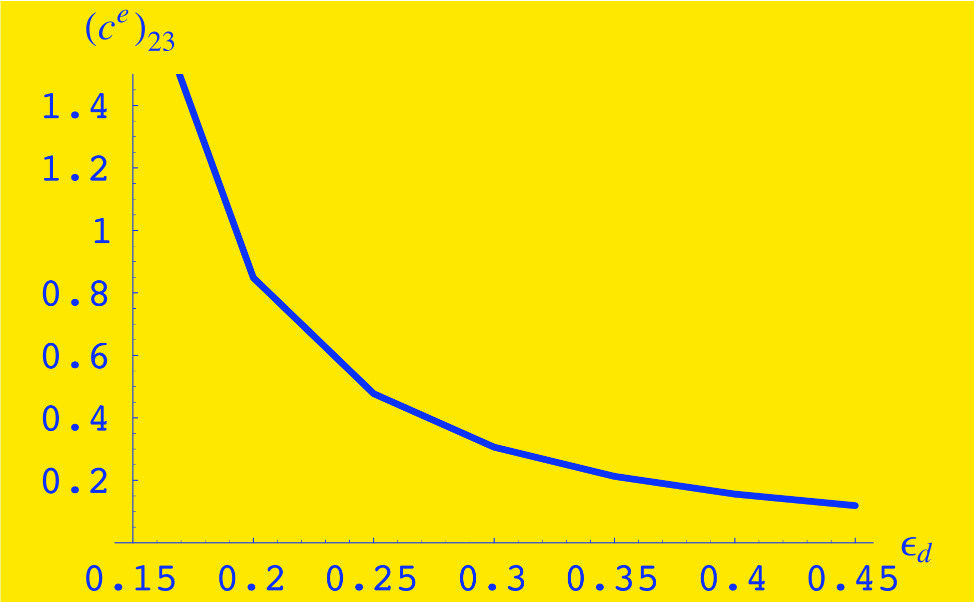


Within parametric errors, most part of the region lies within the 2σ region

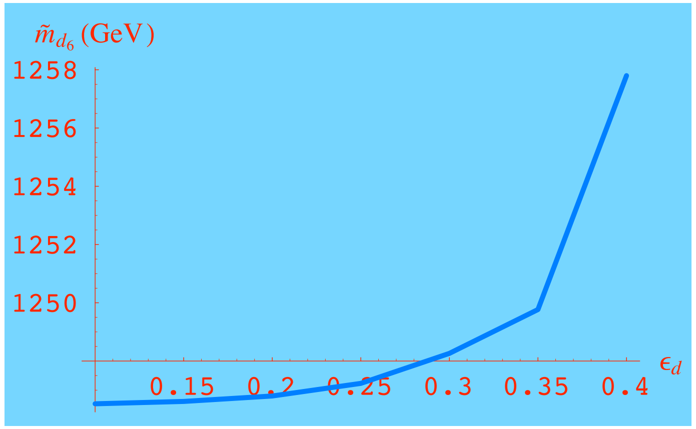
Determination of Yukawa structure

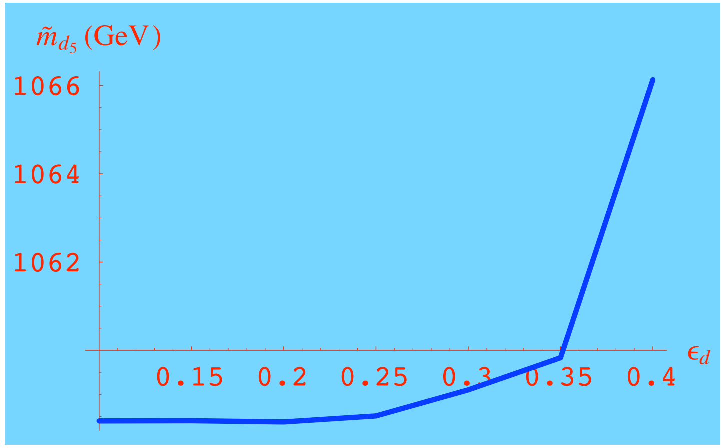


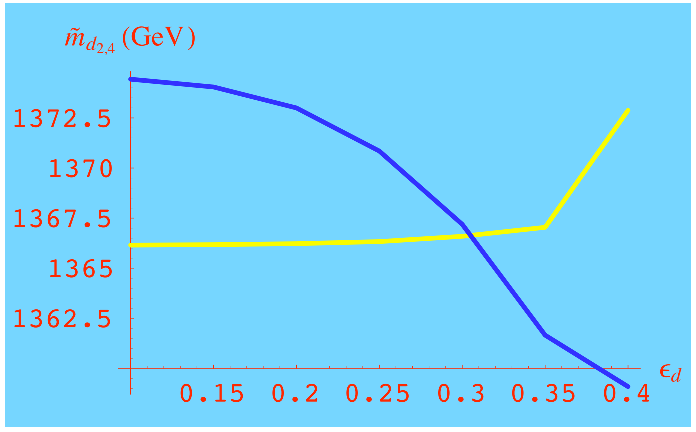


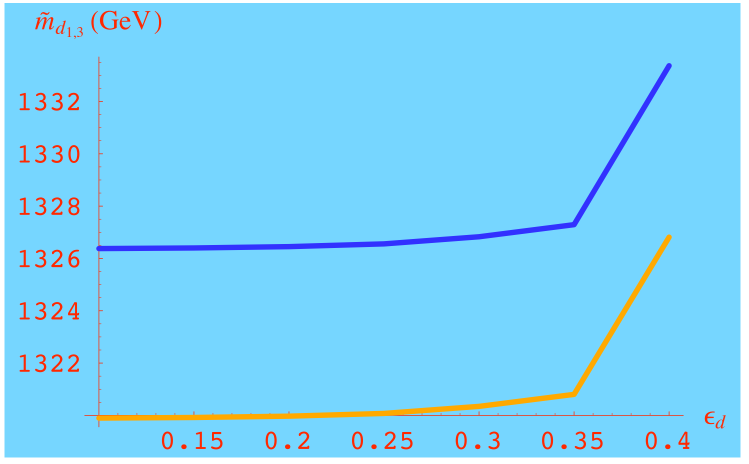


s-fermion mass spectrum









Summary