Gauge Coupling Unification and Light Exotica

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Outline of my Talk

- Motivation
- Standard Model and Grand Unification: A Recap
- A Special Class of Light Exotic Particles
- In Theory: Beyond the Standard Model Constructions
- In Experiment: How can we observe them?
- Conclusions
The Standard Model

Gauge group:

\[ \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \]

Particle content:

| \( Q \) | (3, 2)_{1/3} | \( L \) | (1, 2)_{-1} | \( H \) | (1, 2)_{1} |
| \( \bar{u} \) | (\overline{3}, 1)_{-4/3} | \( \bar{e} \) | (1, 1)_{2} | \( \bar{H} \) | (1, 2)_{-1} |
| \( \bar{d} \) | (\overline{3}, 1)_{2/3} | \( \bar{\nu} \) | (1, 1)_{0} | | |
Running gauge couplings seem to meet at $10^{15}$ GeV 😊
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Looking more closely, couplings do not unify
Hints at Physics Beyond the SM?

- Running gauge couplings seem to meet at $10^{15}$ GeV 🥳
- Looking more closely, couplings do not unify 😞
- Supersymmetry helps: Unification at $10^{16}$ GeV 😊
Assume SU(5) or SO(10) at fundamental scale $\sim 10^{16}$ GeV


\[ \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \]
Assume SU(5) or SO(10) at fundamental scale \( \sim 10^{16} \) GeV


\[
SO(10) \rightarrow SU(5) \times U(1)_X
\]
Assume SU(5) or SO(10) at fundamental scale $\sim 10^{16}$ GeV


\[ \begin{align*}
\text{SO}(10) & \rightarrow \text{SU}(5) \times U(1)_X \\
& \rightarrow \text{SU}(3) \times \text{SU}(2) \times U(1)_Y \times U(1)_X
\end{align*} \]
Assume SU(5) or SO(10) at fundamental scale $\sim 10^{16}$ GeV


\[
\begin{align*}
\alpha_1 & \quad \alpha_2 & \quad \alpha_3 & \quad \alpha_4 \\
\times & \quad \times & \quad \times & \\
\alpha_5 & \\
\end{align*}
\]

\[
\text{SO}(10) \quad \rightarrow \quad \text{SU}(5) \times U(1)_X \quad \rightarrow \quad \text{SU}(3) \times \text{SU}(2) \times U(1)_Y \times U(1)_X
\]
Assume $SU(5)$ or $SO(10)$ at fundamental scale $\sim 10^{16}$ GeV


\[ SO(10) \rightarrow SU(5) \times U(1)_X \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)_X \]

16 \rightarrow 10 + \overline{5} + 1
Assume SU(5) or SO(10) at fundamental scale $\sim 10^{16}$ GeV


\[
\begin{align*}
\text{SO(10)} & \rightarrow \text{SU(5)} \times U(1)_X \rightarrow \text{SU(3)} \times \text{SU(2)} \times U(1)_Y \times U(1)_X \\
16 & \rightarrow 10 + \overline{5} + 1 \\
& \rightarrow (3,2)_{1/3} + (\overline{3},1)_{-4/3} + (1,1)_{-2} + (\overline{3},1)_{2/3} + (1,2)_{-1} + (1,1)_0
\end{align*}
\]

$Q$, $\tilde{u}$, $\tilde{e}$, $\tilde{d}$, $L$, $\tilde{\nu}$
Predictions from Grand Unification

- Gauge coupling unification ★★★★
- One family of quarks and leptons in one irrep ★★★★
- Quantization of electric charge ★★★★
- $\sin^2 \theta_w$ in agreement w/experiment ★★
- $\frac{m_b}{m_{\tau}}$ ratio ★
- Right-handed neutrino ★
- Smallness of neutrino masses ★★
Our Rationale ...

- We want to keep these nice predictions
- Matter comes in complete GUT multiplets
  In SU(5) models:
  Quarks and leptons in $10 + \bar{5} + 1$, Higgs in $5 + \bar{5}$
  $24$ to break SU(5) to SM or $45$ for realistic masses
  In SO(10) models:
  Quarks and leptons in $16$, Higgs in $10$
- Incomplete multiplets spoil GUT relations
Heavy vs. Light Exotic Particles

Standard ansatz:
If exotics are present, make them heavy

E.g. an exotic right-handed quark \( \bar{q} = (3, 1)_{1/3} \)

Electric charge \( Q = T_3 + \frac{Y}{2} = 1/6 \)

Bound state “exotic baryon” \( [\bar{q}uu]_Q=3/2 \)

Presence of \( \bar{q} \) would spoil standard GUT relations

Introduce mass term \( M\bar{q}\bar{q} \) and let \( M \gg M_{\text{GUT}} \)

\( \bar{q} \) does not contribute to renormalization group running

\( \sim \) everything is fine
Non-standard ansatz:

Can there light exotic particles, i.e. $M \ll M_{GUT}$, such that all/most standard GUT predictions hold?


Consider following set of light exotics:

$$(3, 1)_{1/3} + (1, 1)_{-1} + (1, 2)_0 + (1, 1)_{\pm 1}$$

$\tilde{q} + \tilde{e}^- + \tilde{L} + \tilde{e}_{\pm}$

How do they affect the GUT predictions?
Running Gauge Couplings

\( \beta \)-function

\[
\beta(g) = -\frac{1}{16\pi^2} \left[ \frac{11}{3} \ell(\text{vector}) - \frac{2}{3} \ell(\text{Weyl fermion}) - \frac{1}{6} \ell(\text{spinless}) \right] g^3 + \ldots
\]

or in supersymmetric form

\[
\beta(g) = -\frac{1}{16\pi^2} \sum_{i} b_i \left[ 3\ell(\text{vector}) - \ell(\text{chiral}) \right] g^3 + \ldots
\]

Here,

\[
\ell(\Lambda) = \frac{1}{2} \frac{\dim(\Lambda)}{\dim(g)} \langle \Lambda, \Lambda + 2\delta \rangle
\]

is the index of the representation, \( g \) the gauge group, \( \Lambda \) the highest weight, \( \delta = (1, \ldots, 1) \).

For the Standard Model particle content:

\[
b_1 = -\frac{33}{5}, \quad b_2 = -1, \quad b_3 = 3
\]
Consider now the contribution of

\[(3, 1)_{1/3} + (1, 1)_{-1} + (1, 2)_0 + (1, 1)_{\pm 1}\]

\[\tilde{q} + \tilde{e}_- + \tilde{L} + \tilde{e}_+\]

to the renormalization group parameters:

\[\Delta b_3 = 3\ell(\text{vector}) - \ell(\text{chiral}) = 0 - 1/2 = -1/2\]

\[\Delta b_2 = 3\ell(\text{vector}) - \ell(\text{chiral}) = 0 - 1/2 = -1/2\]

\[\Delta b_1 = 3\ell(\text{vector}) - \ell(\text{chiral})\]

\[= -\frac{3}{5} \cdot \frac{1}{4} \cdot \left[ 3 \cdot \left(\frac{1}{3}\right)^2 + 1 \cdot (-1)^2 + 2 \cdot 0^2 + 1 \cdot (+1)^2 + 1 \cdot (-1)^2 \right]\]

\[= -1/2\]

\((\times 2 \text{ for vectorlike exotics})\)
Running gauge couplings . . .

\[
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left( \frac{M_{\text{GUT}}}{\mu} \right), \quad i = 1, 2, 3, \quad \text{e.g.} \quad \mu = M_Z
\]

3 equations, 1 unknown $M_{\text{GUT}} \sim$ Calculate $M_{\text{GUT}}$ and predict $\alpha_3$ or $\sin^2 \theta$
How the Predictions Change

Running gauge couplings . . .

\[
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left( \frac{M_{\text{GUT}}}{\mu} \right), \quad i = 1, 2, 3, \quad \text{e.g.} \quad \mu = M_Z
\]

3 equations, 1 unknown \( M_{\text{GUT}} \sim \) Calculate \( M_{\text{GUT}} \) and predict \( \alpha_3 \) or \( \sin^2 \theta \)

1. \textbf{GUT scale}

\[
M_{\text{GUT}} = \mu \exp \left[ \frac{2\pi}{b_2 - b_1} \left( \frac{3}{5\alpha_{\text{em}}(\mu)} - \frac{8}{5\alpha_2(\mu)} \right) \right]
\]
Running gauge couplings . . .

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left( \frac{M_{\text{GUT}}}{\mu} \right), \quad i = 1, 2, 3, \quad \text{e.g.} \quad \mu = M_Z$$

3 equations, 1 unknown $M_{\text{GUT}} \sim$ Calculate $M_{\text{GUT}}$ and predict $\alpha_3$ or $\sin^2 \theta$

② Strong coupling constant

$$\frac{8}{5\alpha_3(\mu)} = \frac{3}{5\alpha_{\text{em}}(\mu)} - \frac{b_3 - b_1}{2\pi} \log \left( \frac{M_{\text{GUT}}}{\mu} \right)$$
Running gauge couplings . . .

\[ \frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left( \frac{M_{\text{GUT}}}{\mu} \right), \quad i = 1, 2, 3, \quad \text{e.g.} \quad \mu = M_Z \]

3 equations, 1 unknown $M_{\text{GUT}} \sim$ Calculate $M_{\text{GUT}}$ and predict $\alpha_3$ or $\sin^2 \theta$

3 Weinberg angle

\[ \sin^2 \theta(\mu) = \frac{3}{8} \left[ 1 - \frac{1}{2\pi} (b_2 - b_1) \frac{5\alpha_{\text{em}}}{3} \log \left( \frac{M_{\text{GUT}}}{\mu} \right) \right] \]
How the Predictions Change

Running gauge couplings . . .

\[
\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_{\text{GUT}}} - \frac{1}{2\pi} b_i \log \left( \frac{M_{\text{GUT}}}{\mu} \right), \quad i = 1, 2, 3, \quad \text{e.g.} \quad \mu = M_Z
\]

3 equations, 1 unknown \( M_{\text{GUT}} \sim \) Calculate \( M_{\text{GUT}} \) and predict \( \alpha_3 \) or \( \sin^2 \theta \)

4 GUT coupling constant

\[
\frac{1}{\alpha_{\text{GUT}}} = \frac{1}{\alpha_2(\mu)} + \frac{b_2}{b_2 - b_1} \left( \frac{3}{5\alpha_{\text{em}}(\mu)} - \frac{8}{5\alpha_2(\mu)} \right)
\]
5d orbifold GUT with 2 end-of-the-world-branes

T. Kobayashi, S. Raby, R. Zhang “Searching for realistic 4d string models…”
Nucl.Phys.B704:3-55,2005

\[
\begin{align*}
\text{SO}_{10} \text{ brane} & \quad \text{SU}_6 \times \text{SU}_{2R} \text{ brane} \\
2 \times (16) & \quad \text{Gauge} - V, \Sigma (78) \\
& \quad 27 + \overline{27} \\
& \quad 3 \times (27 + \overline{27})
\end{align*}
\]

\[
\begin{align*}
\text{SU}(6) \times \text{SU}(2)_R & \quad \rightarrow \quad \text{SU}(4) \times \text{SU}(2)_L \times \text{SU}(2)_R \\
(6,1) + (1,2) & \quad \rightarrow \quad (4,1,1) + (1,2,1) + (1,1,2) \\
& \quad \rightarrow \quad (3,1)_{1/3} + (1,1)_{-1} + (1,2)_0 + (1,1)_{\pm 1}
\end{align*}
\]

\[
\begin{align*}
\tilde{q} & \quad + \quad \tilde{e}_- & \quad + \quad \tilde{L} & \quad + \quad \tilde{e}_{+}
\end{align*}
\]
Models w/Light Exotics

Limit of 10d heterotic string orbifold

5th twisted sector of $\mathbb{Z}_6$-II orbifold
Limit of 10d heterotic string orbifold

The $\mathbb{Z}_6$-II can be rewritten as a $\mathbb{Z}_2 \times \mathbb{Z}_3$ orbifold

$\sim$ First do the $\mathbb{Z}_3$ twist, then the $\mathbb{Z}_2$ twist
Limit of 10d heterotic string orbifold

The $\mathbb{Z}_6$-II can be rewritten as a $\mathbb{Z}_2 \times \mathbb{Z}_3$ orbifold

$\sim$ First do the $\mathbb{Z}_3$ twist, then the $\mathbb{Z}_2$ twist
Experimental Consequences

Exotic particles (henceforth “exotica”)

\[(3, 1)_{1/3} + (1, 1)_{-1} + (1, 2)_0 + (1, 1)_{\pm 1} \]

\[
\begin{align*}
\tilde{q} & + \tilde{e}_- & + \tilde{L} & + \tilde{e}_\pm \\
\end{align*}
\]

- **Leptons**
  \[
  \tilde{e}_{-1/2}, \quad \tilde{L}_{\pm 1/2}, \quad \tilde{e}_{\pm 1/2}
  \]

- **Baryons**
  \[
  [\tilde{q}(uu)]_{3/2}, \quad [\tilde{q}(ud)_s]_{1/2}, \quad [\tilde{q}(dd)]_{-1/2}, \quad [\tilde{q}(ud)_a]_{1/2}
  \]

- **Mesons**
  \[
  [\tilde{q}u]_{+1/2}, \quad [\tilde{q}d]_{-1/2}
  \]
Experimental Consequences

General, model independent features

- Fermionic exotica expected to be lighter than scalar partners ("s-exotica")
- Scalar exotica decay to fermionic partners
- Fermionic partners will be stable, unless flavor symmetry is broken
- Fractionally charged particles cannot be screened by surrounding matter \(\rightsquigarrow\) Clear signature
- Leptonic exotica will look like “heavy muons”

For more details on decay channels and generalization to models w/hidden sector see arXiv:0705.0294 [hep-ph]
Conclusions

- Matter representations need not be in complete SU(5) representations to preserve most successful predictions of Grand Unification
- Only $\alpha_{\text{GUT}}$ is affected
- As a consequence, exotic particles need not necessarily be of order GUT scale or higher
- New directions for string/GUT model building
- Novel signatures @ LHC and Tevatron
- “Generic” in string constructions ($\sim 5\%$), might thus be relevant for the real world