Deformation of Dijkgraaf-Vafa Relation via Spontaneously Broken $\mathcal{N}$=2 Supersymmetry

Kazunobu Maruyoshi

Osaka City University, Japan

Collaborated with Hiroshi Itoyama.

Ref.) Hiroshi Itoyama and K. M.,
Phys.Lett.B650 (2007) 298, arXiv:0704.1060[hep-th]

SUSY07 @Karlsruhe University, 31 July 2007

Partial breaking of $\mathcal{N}$=2 supersymmetry

[Antoniadis-Partouche-Taylor]

[Itoyama-Fujiwara-Sakaguchi]

U(N) gauge model in which $\mathcal{N}$=2 supersymmetry is broken to $\mathcal{N}$=1 spontaneously has some interesting properties.

The remarkable one is that the model includes $\mathcal{N}$=1, U(N) super Yang-Mills with tree level superpotential as a particular limit;

U(N) gauge model with spontaneously broken $\mathcal{N}=2$ supersymmetry $\xrightarrow[e, m, \xi \to \infty]{}$ $\mathcal{N}=1$, U(N) super Yang-Mills + $W_{\mathrm{tree}}(\Phi)$

(a large FI parameters limit)

Motivation

a large FI parameters limit

$e, m, \xi \to \infty$

U(N) gauge model with spontaneously broken $\mathcal{N}=2$ supersymmetry $\longrightarrow$ $\mathcal{N}=1$, U(N) super Yang-Mills + $W_{\text{tree}}(\Phi)$

low energy

$$W_{\text{eff}}(S) = N\frac{\partial}{\partial S}F_{\text{free}}(S) + \cdots$$

???

low energy

$$W_{\text{eff}}(S) = N\frac{\partial}{\partial S}F_{\text{free}}(S)$$

[Dijkgraaf-Vafa]

$$S = -\frac{1}{64\pi^2}\mathrm{Tr}_{\mathrm{U(N)}}\mathcal{W}^\alpha\mathcal{W}_\alpha$$

The (bare) Lagrangian [Fujiwara-Itoyama-Sakaguchi]

The Lagrangian of U(N) gauge model with spontaneously broken $\mathcal{N}$=2 supersymmetry is

$$\mathcal{L} = \int d^4\theta \left[\frac{-i}{2}\mathrm{Tr}\left(\bar{\Phi}e^{adV}\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi} - h.c.\right) + \xi V^0\right] + \int d^2\theta\left(-\frac{i}{4}\frac{\partial^2\mathcal{F}(\Phi)}{\partial\Phi^a\partial\Phi^b}\mathcal{W}^a\mathcal{W}^b + e\Phi^0 + m\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi^0}\right) + h.c.$$

We choose the prepotential as

$$\mathcal{F}(\Phi) = \sum_{k=0}^{n+1}\frac{g_k}{(k+1)!}\mathrm{Tr}\Phi^{k+1}$$

(degree n+2)

V^a: vector superfield

Φ^a: chiral superfield

$a = 0, 1, \ldots, N^2 - 1$

$\uparrow$

overall U(1)

Spontaneous breaking of $\mathcal{N}$=2 susy

- Vacuum condition: $\frac{\partial V}{\partial \phi^a} = 0 \longrightarrow \langle \frac{\partial^2 \mathcal{F}}{\partial \phi^0 \partial \phi^0} \rangle = -\left(\frac{e}{m} + i\frac{\xi}{m} \right)$
- The gauge symmetry breaking: $U(N) \to \prod_{i=1}^{n} U(N_i)$
- The Nambu-Goldstone fermion is in the overall U(1) vector part:

$$\langle \delta \left(\frac{\lambda^0 - \psi^0}{\sqrt{2}} \right) \rangle \neq 0 \qquad \langle \delta \left(\frac{\lambda^0 + \psi^0}{\sqrt{2}} \right) \rangle = 0$$

- The mass spectrum

  - $\mathcal{N}$=1 massless $\prod_{i=1}^{n} U(N_i)$ vector multiplet
  - $\mathcal{N}$=1 massive $\prod_{i=1}^{n} U(N_i)$ adjoint chiral multiplet
  - $\mathcal{N}$=1 massive vector multiplets corresponding to broken generators

Large FI parameters limit

Let us take the limit: $(e, m, \xi) = \Lambda(e', m', \xi'), \quad \Lambda \to \infty.$

(with $\tilde{g}_k \equiv m g_k$ fixed for $k \geq 2$)

$$\mathcal{F}(\Phi) = \sum_{k=0}^{n+1} \frac{g_k}{(k+1)!} \mathrm{Tr}\Phi^{k+1} = g_0 \mathrm{Tr}\Phi + \frac{g_1}{2} \mathrm{Tr}\Phi^2 + \mathcal{O}(\Lambda^{-1})$$

◆In this limit the model reduces to $\mathcal{N}$=1 U(N) SYM with $W_{\mathrm{tree}}(\Phi)$.

$$\mathcal{L} \longrightarrow \mathcal{L}_{DV} = \mathrm{Im}\left[\frac{-e' + i\xi'}{m'}\left(2\int d^4\theta \mathrm{Tr}\bar{\Phi} e^{adV}\Phi + \int d^2\theta \mathrm{Tr}\mathcal{W}\mathcal{W}\right)\right] + \int d^2\theta W(\Phi) + h.c.,$$

◆The overall U(1) part (the Nambu-Goldstone fermion) is decoupled.

Effective superpotential

- We consider the effective superpotential by integrating out massive modes Φ and $\bar{\Phi}$.

- We treat $\mathcal{W}$ (or V) as the background field.

  The result is represented by $S = -\frac{1}{64\pi^2}\mathrm{Tr}_{U(N)}\mathcal{W}^\alpha\mathcal{W}_\alpha$

- We assume large FI parameters and see the difference between our model and $\mathcal{N}$=1, U(N) SYM, in the leading 1/m order.

Our result

- We assume large FI parameters and see the difference between our model and $\mathcal{N}$=1, U(N) SYM, in the leading 1/m order.

Summary of our result: ℓ-loop contribution to $W_{\text{eff}}(S)$

$$W^{(\ell)}_{\text{eff}}(S) = \underbrace{N\frac{\partial \mathcal{F}^{(\ell)}(S)}{\partial S}}_{\text{The leading order terms which also exist in DV case}} \underbrace{- \frac{16\pi^2 i\tilde{g}_3}{\tilde{g}_2}\left(\frac{\partial \mathcal{F}^{(\ell)}(S)}{\partial S}\right)\frac{S}{m} + W^{(\ell)}_{\text{vertex}}}_{\text{order 1/m terms (leading difference)}} + \mathcal{O}((1/m)^2)$$

The leading order terms which also exist in DV case

order 1/m terms (leading difference)

cf.) Dijkgraaf-Vafa relation: $W_{\text{eff}}(S) = N\frac{\partial}{\partial S}F_{\text{free}}(S)$

Diagrammatical computation 1

We firstly integrate out $\bar{\Phi}$ and consider the perturbation theory with Φ.

- propagator

$$\boxed{\Delta(p,\pi) = \int_0^\infty ds e^{-s(p^2+m'+\frac{1}{2}ad\mathcal{W}^\alpha\pi_\alpha-ig_3'\mathcal{W}\mathcal{W})}}$$

- vertices

1st type….. $m\frac{g_k}{k!}\mathrm{Tr}\Phi^k$, for $k = 3\ldots n+1$

new terms !

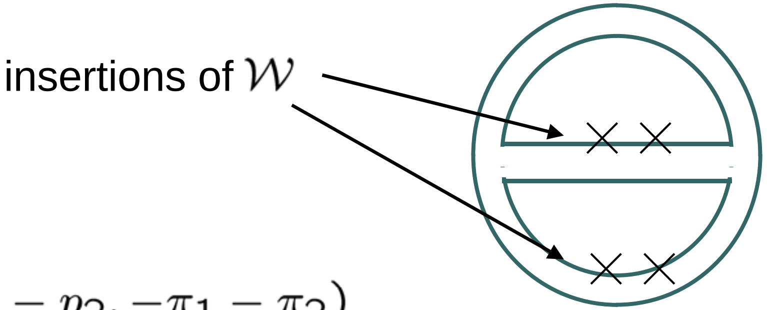
2nd type….. $-\frac{i}{4}\sum_{s=0}^{k-1}\frac{g_k}{k!}\mathrm{Tr}(\mathcal{W}\Phi^s\mathcal{W}\Phi^{k-1-s})$, for $k = 4\ldots n+1$

$$\left(\mathcal{L} = \int d^2\theta \frac{-i}{4}\frac{\partial^2\mathcal{F}(\Phi)}{\partial\Phi^a\partial\Phi^b}\mathcal{W}^a\mathcal{W}^b + \ldots\right)$$

The new terms do contribute to the effective superpotential !

Diagrammatical computation 2

- 2-loop example



$$\int \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}d^2\pi_1 d^2\pi_2$$
$$\Delta(p_1,\pi_1)\Delta(p_2,\pi_2)\Delta(-p_1-p_2,-\pi_1-\pi_2)$$

$$= \int ds_1 ds_2 ds_3 e^{-(\sum s_i)m'}\{3NS^2 - ig_3'(\sum s_i)S^3\}$$

$$(S = -\frac{1}{64\pi^2}\mathrm{Tr}_{U(N)}\mathcal{W}^\alpha\mathcal{W}_\alpha)$$

$$\Delta(p,\pi) = \int_0^\infty ds e^{-s(p^2+m'+\frac{1}{2}ad\mathcal{W}^\alpha\pi_\alpha)}\ e^{sig_3'\mathcal{W}\mathcal{W}}$$

$$\left(e^{-\frac{s}{2}ad\mathcal{W}^\alpha\pi_\alpha} = 1 - \frac{s}{2}ad\mathcal{W}^\alpha\pi_\alpha + \frac{s^2}{8}(ad\mathcal{W}^\alpha\pi_\alpha)^2\right)$$

Alternative method (generalized Konishi anomaly)

Let us define the generating functions of the one-point functions:

$$R(z) = -\frac{1}{64\pi^2}\mathrm{Tr}\left\langle \mathcal{W}^\alpha \mathcal{W}_\alpha \frac{1}{z-\Phi}\right\rangle, \quad T(z) = \mathrm{Tr}\left\langle \frac{1}{z-\Phi}\right\rangle .$$

In terms of these, the effective superpotential is

$$\frac{\partial W_{eff}}{\partial g_k} = \frac{m}{k!}\int dz z^k T(z) + \frac{16\pi^2 i}{(k-1)!}\int dz z^{k-1} R(z),$$

$R(z)$ and $T(z)$ satisfy the following equations:

$$\begin{aligned} R(z)^2 &= W'(z)R(z) + \frac{1}{4}g(z), \\ 2R(z)T(z) &= W'(z)T(z) + \frac{1}{4}c(z) + 16\pi^2 i\mathcal{F}'''(z)R(z) + \frac{1}{4}\tilde{c}(z), \end{aligned}$$

which follow from the generalized Konishi anomaly equations.

Conclusion

We have shown that Dijkgraaf-Vafa relation is deformed by spontaneously broken $\mathcal{N}$=2 supersymmetry by two method:

- the diagrammatical computation

$$W_{\text{eff}}^{(\ell)}(S) = N\frac{\partial \mathcal{F}^{(\ell)}(S)}{\partial S} - \frac{16\pi^2 i\tilde{g}_3}{\tilde{g}_2}\left(\frac{\partial \mathcal{F}^{(\ell)}(S)}{\partial S}\right)\frac{S}{m} + W_{\text{vertex}}^{(\ell)} + \mathcal{O}((1/m)^2)$$

- the argument based on the generalized Konishi anomaly.

$\mathcal{N}$=2 supersymmetry

R transformation act on L:

$$R\mathcal{L}(\xi)R^{-1} = \mathcal{L}(-\xi)$$

1st susy transformation: $\delta^{(1,\xi)}\mathcal{L}(\xi) = 0$

This is ordinary $\mathcal{N}$=1 susy transformation.

2nd susy transformation: $\delta^{(2,\xi)}\mathcal{L}(\xi) = 0$

The definition of $\delta^{(2,\xi)}$ is $\delta^{(2,\xi)} = R\delta^{(1,-\xi)}R^{-1}$

Thus,
$$\begin{aligned}
\delta^{(2,\xi)}\mathcal{L}(\xi) &= \left(R\delta^{(1,-\xi)}R^{-1}\right)\left(R\mathcal{L}(-\xi)R^{-1}\right) \\
&= R\left(\delta^{(1,-\xi)}\mathcal{L}(-\xi)\right)R^{-1} \\
&= 0
\end{aligned}$$