Cycling in the Throat

Ivonne Zavala
IPPP, Durham

Based on: JHEP04(2007)026 and Spinflation, to appear.
In collaboration with:

D.Easson, R.Gregory, G.Tasinato and D. Mota

D-branes and Flux Compactifications

- A D3-brane (or anti-D-brane) wandering in a type IIB warped flux compactification.
  - ➡ Internal fluxes: Reduced susy, warping, hierarchies, moduli stabilisation, etc.

- A large variety of SUGRA backgrounds with internal fluxes has been explored in the context of the AdS/CFT correspondence.
  In this talk: Klebanov-Strassler (KS) solution.

Applications to string cosmology

- A D-brane (or anti-D-brane) wandering in a flux compactification experiences a speed limit due to brane action. Thus, kinetic terms can become negligible, compared with potential terms, which then dominate and -> inflation. (DBI inflation) [Silverstein-Tong, etc.]

- Besides potential cosmological applications of this effect, it is interesting to explore the consequences of a non-standard DBI brane action for trajectories of branes in warped backgrounds in more generality.

- In particular, motion of the brane along the angular coordinates should have interesting effects. Angular momentum gives rise to centrifugal forces and thus, brane bounces generic along radial direction.

Outline

- Brane evolution along the radial and angular directions without including gravitational backreaction (mirage cosmologies [Kehagias-Kiritsis]).
[Easson-Gregory-Tasinato-IZ]

- Take into account gravitational backreaction by coupling the DBI action to gravity.
Analyse the resulting (cosmological) brane evolution: Spinflation
[Easson-Gregory-Mota-Tasinato-IZ]

D-Brane Dynamics

Brane motion described by DBI+WZ action $(q = \pm 1)$

$$\mathcal{L} = -m \left\{ h^{-1} \left[\sqrt{1 - h\, v^2} - q \right] \right\} \quad \Rightarrow hv^2 < 1$$

where $m = T_3\, g_s^{-1} \int d^3x$

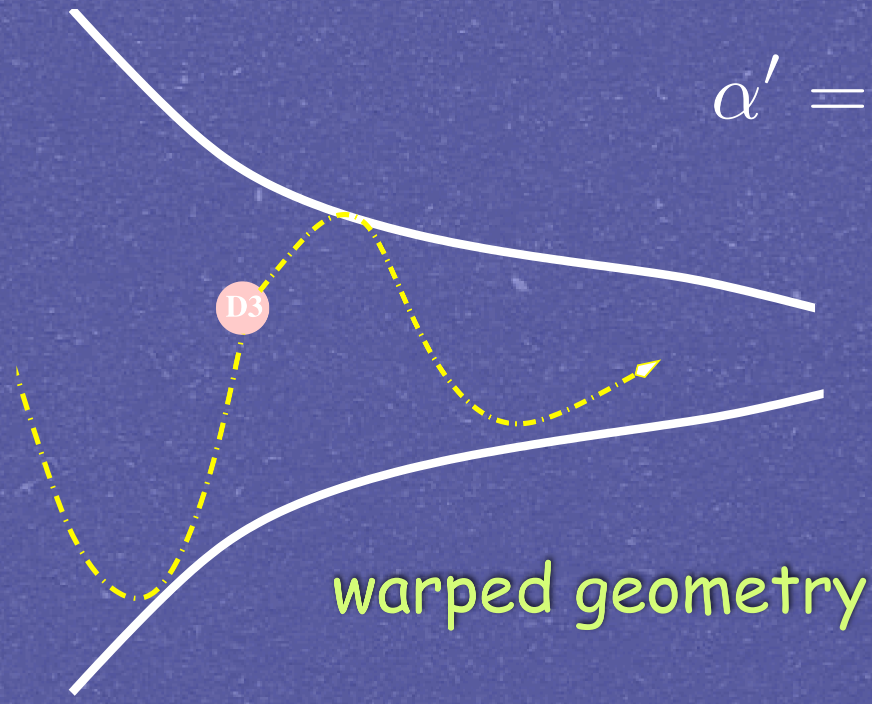
$T_3 = ((2\pi)^3 \alpha'^2)^{-1}$

$\alpha' = \ell_s^2$

$$ds_{10}^2 = h^{-1/2}(\eta) dx_\mu dx^\mu + h^{1/2}(\eta) ds_6^2$$

$$v^2 = g_{\eta\eta}\dot{\eta}^2 + g_{rs}\dot{y}^r \dot{y}^s$$

g_{MN} with $M = \mu\,, \nu\,, s$



$h(\eta)$ is the warp factor of background metric:

Klebanov-Strassler.

Brane trajectories (q=1, $y^r = \theta$)

- Conserved quantities

$$E = \frac{(\gamma - 1)}{h} \; ; \qquad l_\theta = g_{\theta\theta}\,\dot{\theta}\,\gamma$$

$$\gamma = \frac{1}{\sqrt{1 - hv^2}} = \sqrt{\frac{1 + h\,\ell^2(\eta)}{1 - h\,g_{\eta\eta}\dot{\eta}^2}} \; ; \qquad \ell^2(\eta) = g^{rs} l_r l_s$$

- Brane trajectories described by equation of motion:

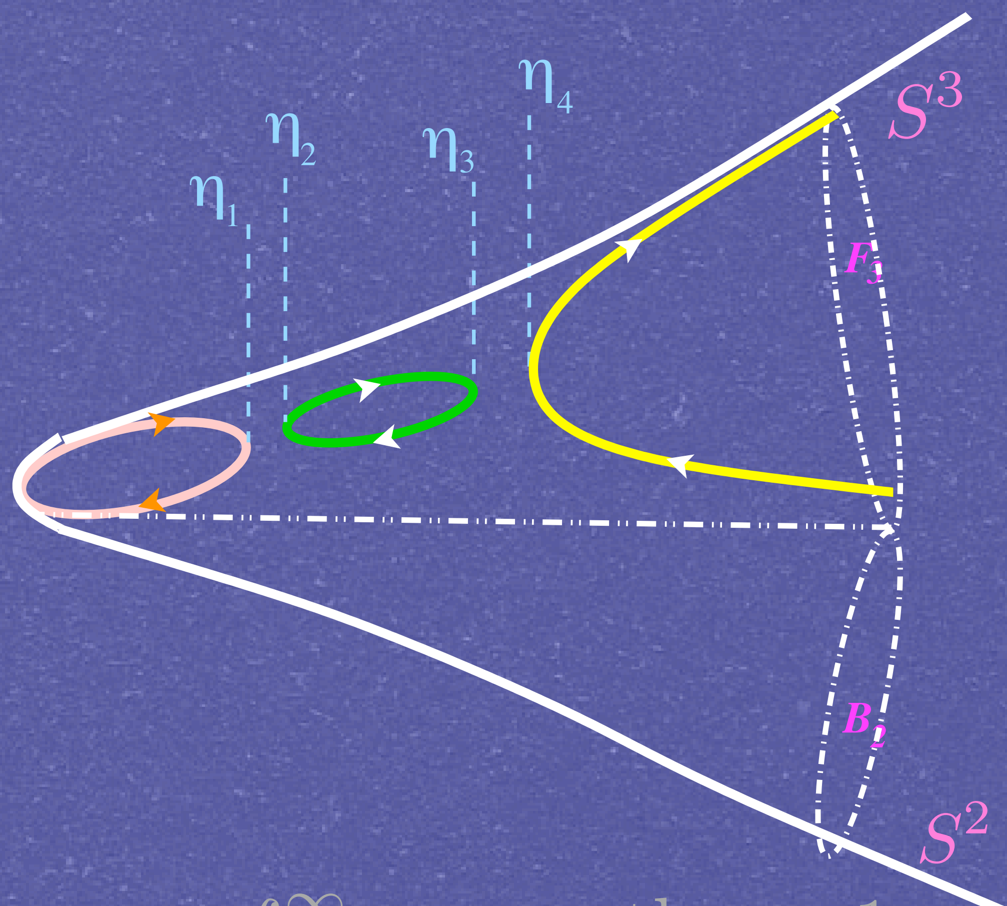
$$\boxed{\dot{\eta}^2 = \frac{g^{\eta\eta}\,\left[E(h\,E + 2) - \ell^2(\eta)\right]}{(h\,E + 1)^2}}$$

Klebanov-Strassler background

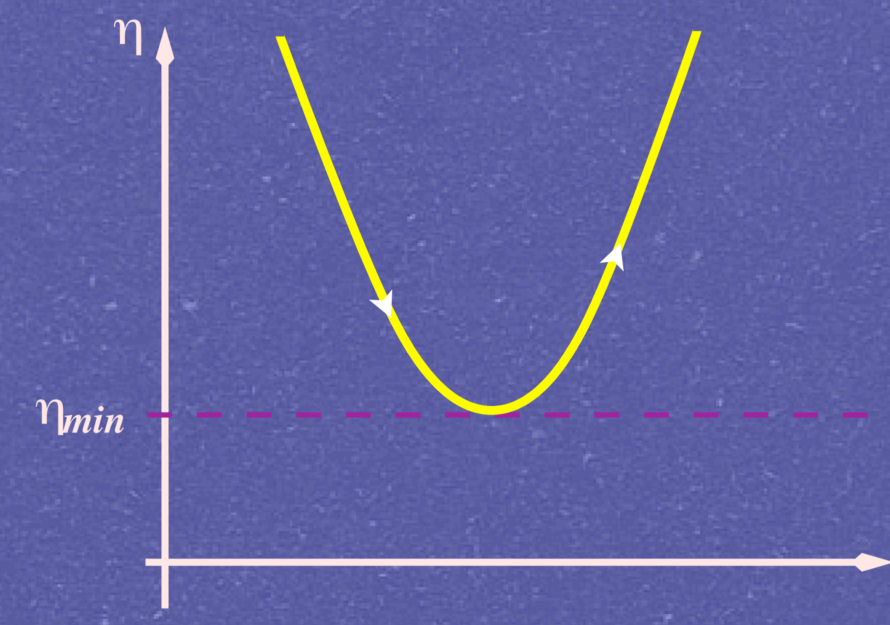
F_3, H_3, F_5 fluxes

$$ds_6^2 = ds^2_{\mathcal{T}^{1,1}} = \textit{Deformed conifold}$$

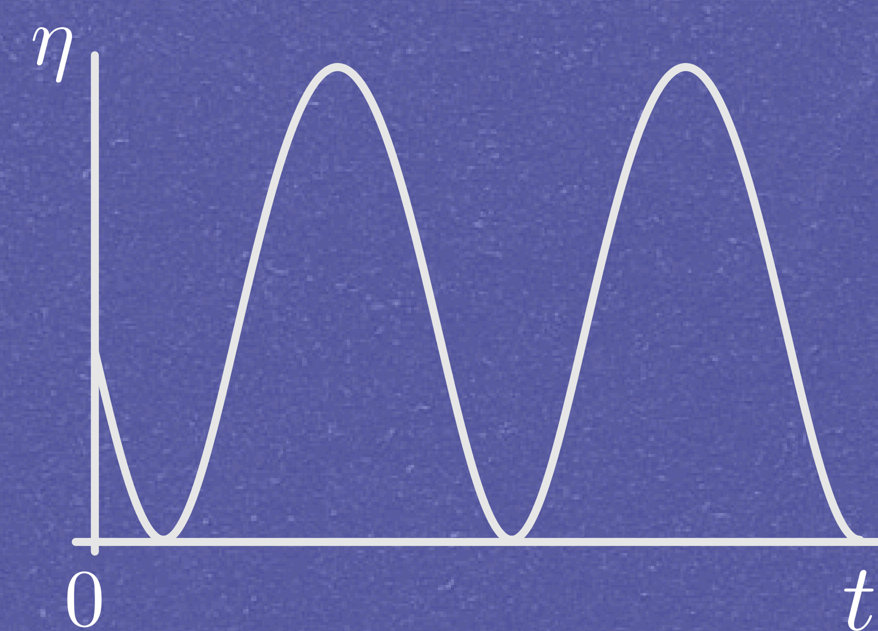
$$h(\eta) = (g_s M \ell_s^2)^2 2^{2/3}\, \epsilon^{-8/3}\, \mathcal{I}(\eta)$$



$$\mathcal{I}(\eta) = \int_\eta^\infty dx\, \frac{x\coth x - 1}{\sinh^2 x}\,(\sinh(2\,x) - 2\,x)^{1/3}\,.$$



bouncing branes



cyclic branes

Induced expansion: Mirage Cosmology

$$ds_4^2 = h^{-1/2}\left(-(1-h\,v^2)\,dt^2 + dx_i dx^i\right)$$
$$= -d\tau^2 + a^2(\tau) dx_i dx^i\,,$$

$$H_{ind}^2 = \left(\frac{h'}{4\,h^{3/4}}\right)^2 g^{\eta\eta}\left[E\,(h\,E+2q) - \ell^2(\eta)\right]$$

where $a(\tau) = h^{-1/4}(\tau)$ and $H_{ind} = \frac{1}{a}\frac{d\,a}{d\eta}\frac{d\,\eta}{d\tau}$

- Induced bouncing and cyclic universes

The effects of angular motion

- ★ Slows down the brane as it moves down the throat.
- ★ Provides a centrifugal barrier that allows bounded trajectories to exist, for a D3-brane.
- ★ Gives rise to bouncing as well as cyclic mirage cosmologies.

Coupling to gravity: Spinflation

- What happens to cyclic/bouncing trajectories/cosmologies when gravitational backreaction is taken into account?

- What is the effect of brane angular motion (spin) on 4D cosmological expansion, in particular on acceleration ?

Coupled system

$$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g}\, R$$

$$- \, g_s^{-1} \int d^4x \sqrt{-g} \left[h^{-1} \sqrt{1 + h\, g_{mn} g^{\mu\nu} \partial_\mu \phi^m \partial_\nu \phi^n} - q\, h^{-1} + V(\phi^m) \right]$$

where

$$\phi = \sqrt{T_3}\, \eta \qquad T_3 = ((2\pi)^3 \alpha'^2)^{-1}$$

$$M_{Pl}^2 = V_6 / \kappa_{10}^2 \qquad \kappa_{10}^2 = \frac{(2\pi)^7}{2} g_s^2 \alpha'^4$$

$g_{\mu\nu}$ Four dimensional FRW metric

$$\boxed{h g_{\phi\phi}\,\dot{\phi}^2 = 1 - \left(1 + \frac{h\ell^2(\phi)}{a^6}\right)\cdot\left(q + h\left(\frac{H^2}{\beta} - V\right)\right)^{-2}}$$

$$H^2 = \beta\, E\,; \qquad \beta = \frac{1}{3g_s(M_{Pl}/M_s)^2}\,; \qquad H = \frac{\dot{a}}{a}$$

$$E = \frac{(\gamma - 1)}{h} + V; \qquad P = \frac{(1 - \gamma^{-1})}{h} - V; \qquad V = m^2\phi^2$$

$$\gamma = \frac{1}{\sqrt{1 - hv^2}} = \sqrt{\frac{1 + h\,\ell^2(\phi)}{1 - h\,g_{\phi\phi}\dot{\phi}^2}}$$

$$l_\theta = a^3 g_{\theta\theta}\,\dot{\theta}\,\gamma$$

$$\ell^2(\phi) = g^{rs} l_r l_s$$

Remarks

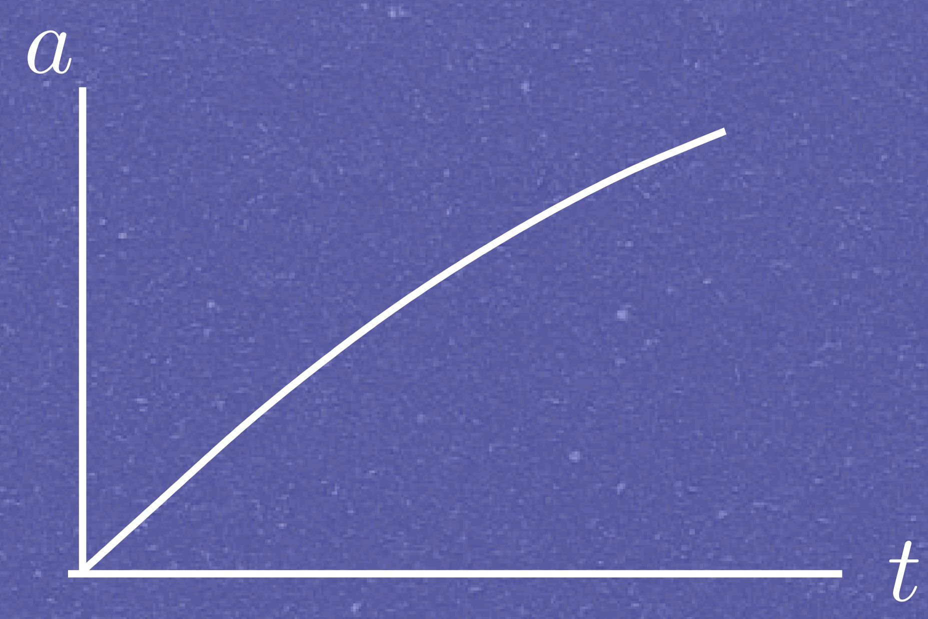
- $\beta \to 0 \ \ (M_{Pl} \to \infty)$ we recover previous results.
- Energy conditions are satisfied, thus bouncing & cyclic cosmologies cannot arise.
- Cyclic and bouncing brane trajectories survive for a while.
- Angular momentum term gets damped due to cosmological expansion. However, it can provide a source of acceleration.
- Near bouncing points, brane experiences kicks of acceleration.
- Nature of geometry allows cyclic trajectories, whether angular momentum is present or not.

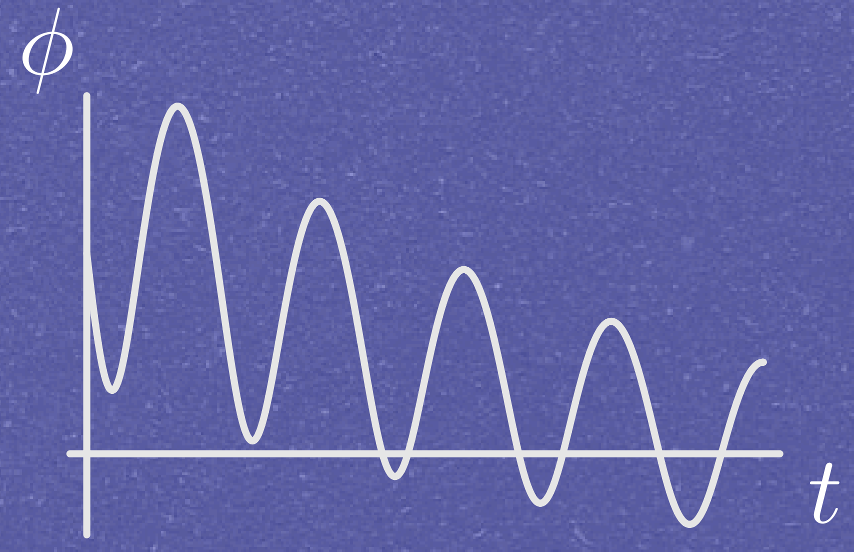
Constraints

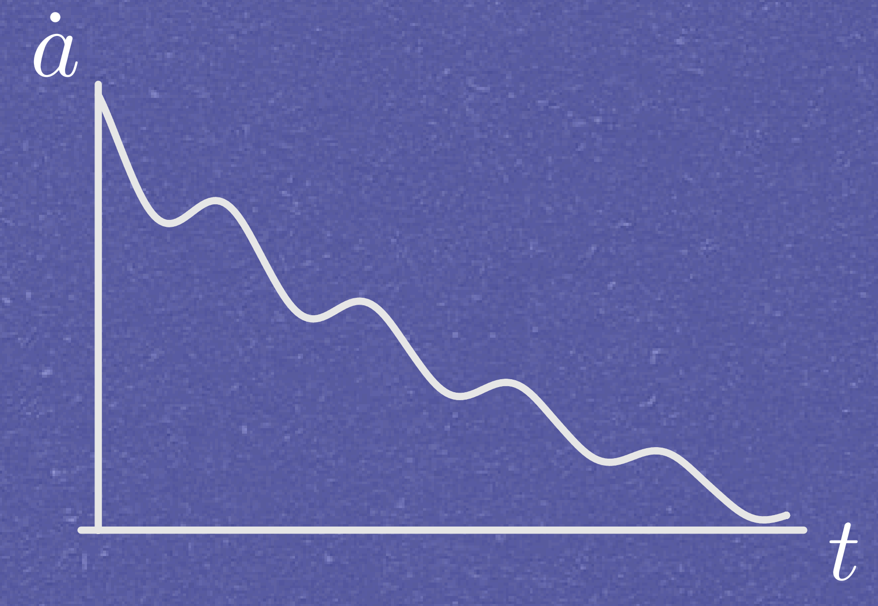
- Backreaction issues:
  - ✓ acceleration of the brane has to be small in string units.
  - ✓ curvature of the brane as it moves at speed close to that of light.
- Cut off (UV) scale: range of the scalar field
  - ✓ Depends of amount of flux M (AdS is restricted. KS fits better.)
- String scale cannot be too large nor too small.
  - ✓ It is related to the amount of flux, the internal overall volume and the mass of ϕ

Cycling in KS

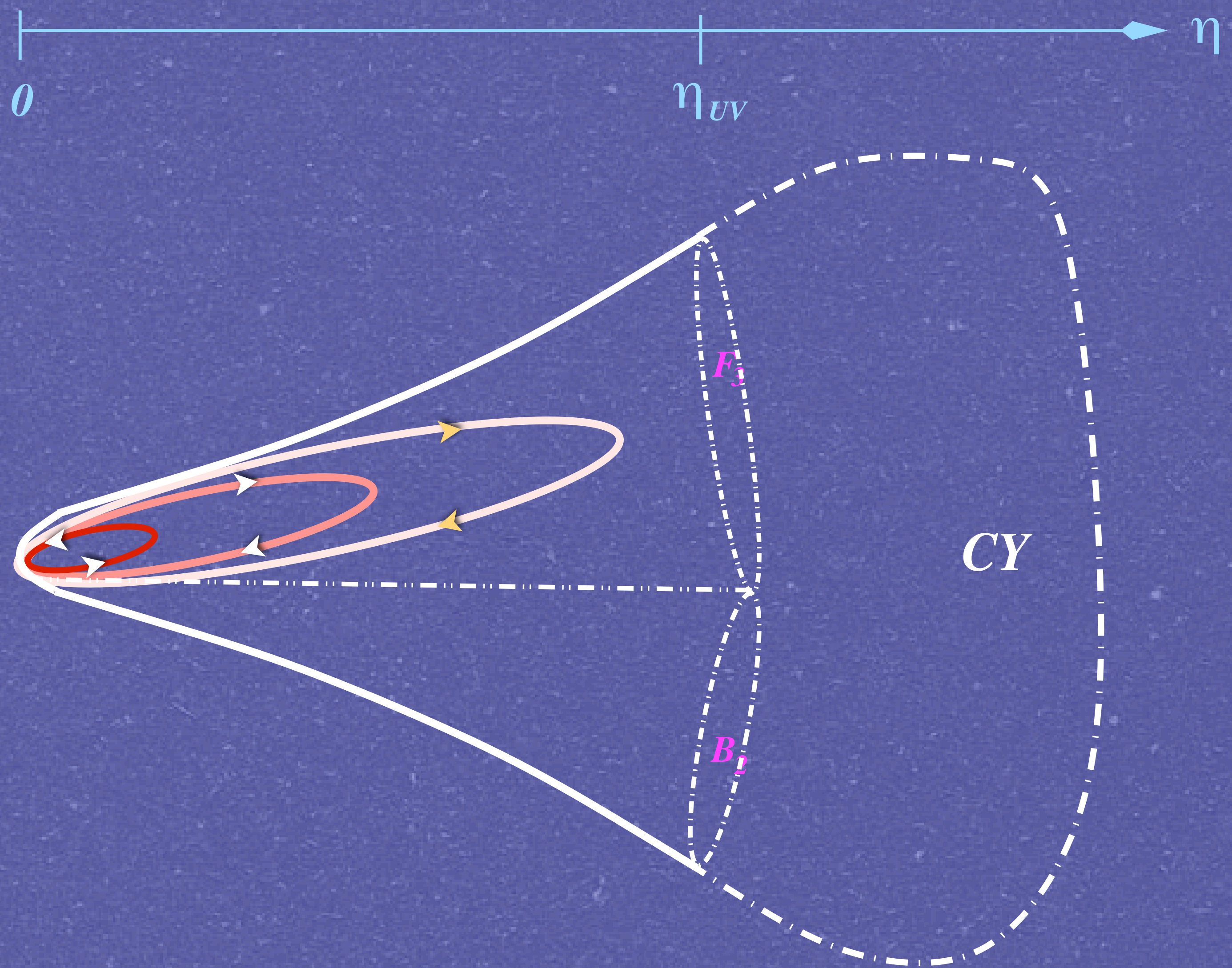
Non-accelerating solutions (flat potential): damped cyclic trajectories with kicks of acceleration around turning points

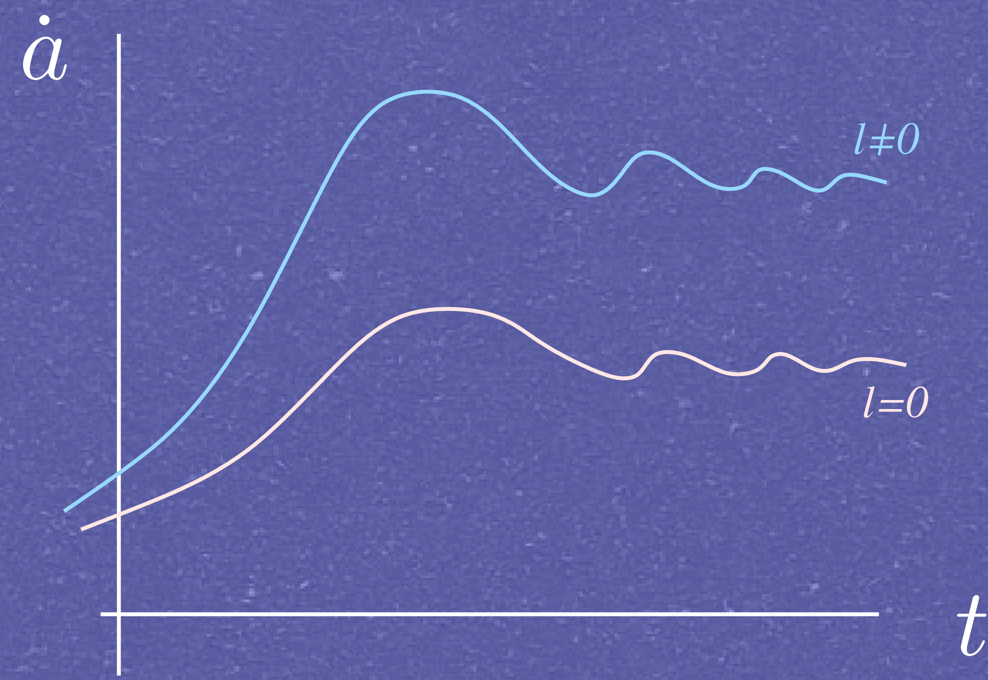


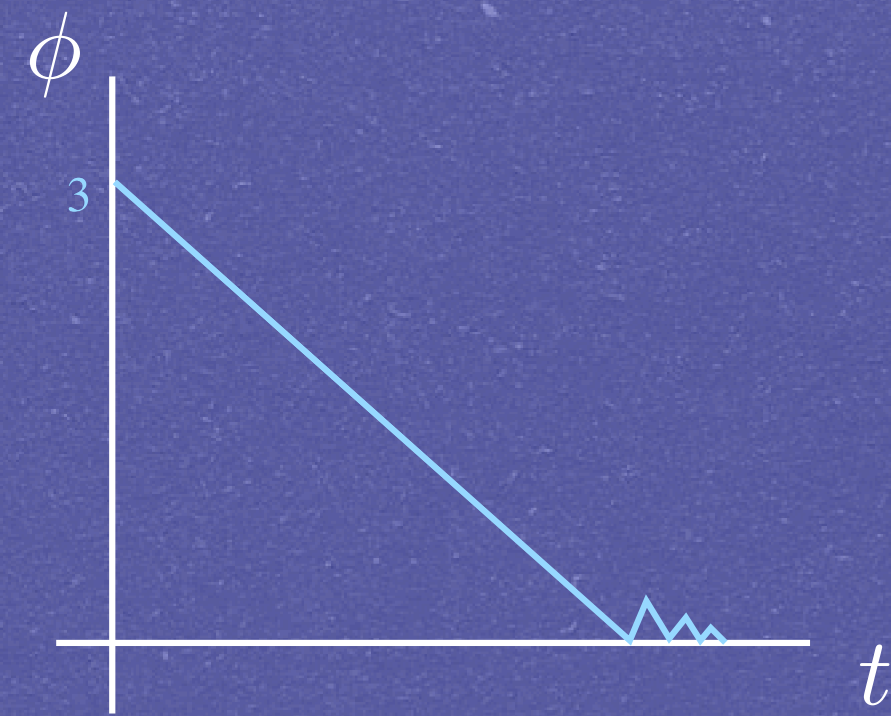




Cycling in KS







$$m^2(g_s M)^2 > \frac{1}{\beta} \Rightarrow$$ inflation

$$H^2 \to \begin{cases} 0 & \ell = 0 \\ 1/a^6 & \ell \neq 0 \end{cases}$$ late time evolution

Perturbations in Spinflation

★ Since the brane moves along different directions, various fields can contribute to the evolution of the perturbations

✓ we have a multifield inflation with non-standard DBI-kinetic terms => Non-Adiabatic modes appear

★ One can extract some general features, without explicitly solving the equations

✓ The entropy perturbation evolves independently of the curvature perturbation at large scales

✓ Curvature and entropy perturbations evolve at different speeds. (Curvature perturbations move with a speed $c_S^2 = \gamma^{-2} \ll 1$. Entropy perturbations move at speed of light)

Perturbations in Spinflation (cont.)

$$\delta\ddot{\sigma}_\Phi + \left(3H + 3\frac{\dot{\gamma}}{\gamma}\right)\delta\dot{\sigma}_\Phi + \left(U_{\sigma\Phi} + c_s^2\frac{k^2}{a^2}\right)\delta\sigma_\Phi = -\left(\frac{H\,\dot{\sigma}c_S^2}{a^3\,(E+P)}\right)\left[\frac{a^3\tan\alpha}{Hc_S^2}\left(\dot{P} - c_S^2\dot{E}\right)\frac{\delta s}{\dot{\sigma}}\right]^{\cdot}$$

where $$U_{\sigma\Phi} \equiv \frac{\dot{\sigma}H^2c_S^2}{a^3\,(E+P)}\left[\left(\frac{\dot{H}}{H} - \frac{\ddot{\sigma}}{\dot{\sigma}}\right)\frac{a^3(E+P)}{\dot{\sigma}H^2c_S^2}\right]^{\cdot}$$

$$\delta\ddot{s} + \left(3H + \frac{\dot{\gamma}}{\gamma}\right)\delta\dot{s} + \left(U_s + \frac{k^2}{a^2}\right)\delta s = -\frac{k^2}{a^2}\frac{\dot{\sigma}\ \tan\alpha\,H}{a\,(E+P)^2}\left(\dot{P} - c_S^2\dot{E}\right)\xi$$

where $$U_s = \tan\alpha\left[3H\tan\alpha\left(\frac{\ddot{\sigma}}{\dot{\sigma}} - \frac{\dot{\alpha}}{\tan\alpha} + 3Hc_S^2 - \cos\alpha\frac{f'\,\dot{\sigma}}{f}\right) + \dot{\sigma}\left(\frac{\dot{\alpha}}{\dot{\sigma}} - \frac{f'\,\cos\alpha}{f\ \tan\alpha}\right)\right]^{\cdot}$$

and $$d\,\sigma = \cos\alpha\,d\phi + f(\phi)\sin\alpha\,d\theta \qquad d\,s = f(\phi)\cos\alpha\,d\theta - \sin\alpha\,d\phi$$

$$\cos\alpha = \frac{\dot{\phi}}{\sqrt{2X}} \qquad \sin\alpha = \frac{f(\phi)\,\dot{\theta}}{\sqrt{2X}} \qquad 2X \equiv h\left(g_{\phi\phi}\,\dot{\phi}^2 + g_{\theta\theta}\,\dot{\theta}^2\right)$$

Comments

- ★ Explored consequences of putting a D3 probe to spin in a warped flux SUGRA background (KS).
  Both when gravity is not included and when it is.
- ★ Panoramic view of possibilities for brane trajectories and cosmologies once the angular momentum is turned on.
- ★ When gravity is taken into account cyclic and bouncing cosmologies disappear, but cyclic and bouncing trajectories persist.
- ★ Angular momentum is a source of accelerated expansion, thus helping with (a handful of e-folds) inflation: Spinflation.