Moduli stabilization in (string) model building: gauge fluxes and loops

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Introduction: the string-pheno paradigm

- Low energy string theory: $d=10$, $N=I/II$ SUGRA.
- Necessary a compatification on a 6d space $K$, such that SUSY is reduced to $N=1$ in 4d.

The choice of $K$:

I - Topological properties
   - “topological” properties of the 4d model;

II - Metric properties (Size & Shape)
   - “parameters” of the 4d model.

Point: I - Size & Shape are vev’s of dynamical fields;
   II - Flat potential at tree-level.

Which control on the phenomenology of the model?
- More in general we have to choose a \textit{background} for all the 4d scalars (internal components of metric, \textit{p}-forms ...)

Non-trivial background for the closed string \textit{p}-forms wrapped in the internal space (IIB Strings)

\textbf{\(\rightarrow\)} Stabilization of shape (complex structure) moduli. 
\hspace{10pt} Giddings, Kachru, Polchinski ‘01

\textbf{\(\rightarrow\)} In case there is a \textit{single} size (Kähler) modulus extra effects (gaugino condensation) can fix it.
\hspace{10pt} Kachru, Kallosh, Linde, Trivedi ‘03

The minimal option is very specific: an extension is necessary.

Include the effect of

- gauge (open string) fluxes \(\rightarrow\) D-term stabilization;
- loop corrections;
- \(\alpha'\) corrections.
Study of the effects due to gauge fluxes and loop corrections in a 6d toy model

I - Review of the KKLT proposal:
- basic ingredients (fluxes & gaugino condensation)
- the sequestered “uplifting” sector.

II - Realization and extension (two Kahler moduli)
from 6d perspective.
- 6d SUGRA + SYM compactified on $T^2/Z_2$;
- Scherk-Schwarz mechanism as a source of $W_0$;
- The presence of gauge fluxes: D-term potential;
- Loop corrections;
- The complete potential: complete stabilization.
The KKLT proposal: basic issues

- Take a compactification of Type IIB string on a CY with a single Kähler modulus $S$.

- Include closed string fluxes
  → stabilization of complex structure moduli, that can be integrated out. A constant superpotential term $W_0$.

- Include non-perturbative effects (gaugino condensation)
  \[ W = W_0 + e^{-S} \]
  → stabilization of $S$ at a SUSY AdS minimum, with $S > 1$, $V_{\text{Min}} \sim -|W_0|^2$.

- Include a SUSY breaking mechanism
  → SUSY breaking and “uplifting” of the minimum.
The uplifting sector: sequestering in the throat

- The flux modifies the geometric background: 
  "throats" develops: regions where $K^{10} \sim \text{AdS}_5 \times M^5$

- The AdS$_5$ can be seen as a realization of the Randall-Sundrum model: use the same language.

- The bottom of the throat (IR brane) is sequestered from the rest of the space, the top of the throat or UV brane, that is the visible brane.

- The details of the SUSY breaking sector are invisible in the visible sector: the SUSY breaking sector can be modelled in any way, the visible effects are just the same.

Choi, Falkowski, Nilles, Olechowski; Lebedev, Nilles, Ratz; Brümmer, Hebecker, MT., ...
6d SUGRA

- The bosonic 6d action is

\[ (-g_6)^{-\frac{1}{2}} \mathcal{L} = -\frac{1}{2} R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{1}{24} e^{2\phi} H_{MNP} H^{MNP} - \frac{1}{4} e^\phi F_{MNP} F^{MNP} \]

with

\[ H_{MNP} = \partial_M B_{NP} + F_{MNP} A_P + \text{cyclic perm.} = (dB + F \wedge A)_{MNP} \]

and is invariant under the gauge transformations

\[ \delta A = d\Lambda, \quad \delta B = -\Lambda F + dC \]

where \( \Lambda \) is a scalar and parametrizes the “\( F \)” gauge symmetry and \( C \) is a 1-form and parametrizes the “\( B \)” gauge symmetry.

This action can be seen as the outcome of a K3 compactification of string theory, in case the internal moduli fields are neglected.
Compactification to 4d: effective SUGRA

- We can consider a compactification on an internal $T^2/Z_2$.

$$(g_6)_{MN} = \begin{pmatrix} r^{-2}(g_4)_{\mu \nu} & 0 \\ 0 & r^2(g_2)_{mn} \end{pmatrix}, \quad (g_2)_{mn} = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}$$

the dimensional reduction produces the following fields
- 4d metric $g_4$ + internal metric components $r, \tau_1, \tau_2$;
- 4d $B$ field, i.e. one scalar $c$ + internal $B_{56} = b$;
- 4d gauge field $F$;
- dilaton.

- $g_4$ and $F$ fill the standard 4d SUGRA/SYM action;
- the scalars are organised in 3 chiral multiplets, $S, T, \tau$, with Kähler potential

$$K = -\log(S + \bar{S}) - \log(T + \bar{T}) - \log(\tau + \bar{\tau})$$

- the gauge kinetic function is $2S$. 


Scherk-Schwarz mechanism: a source for $W_0$

- **R-Symmetry in 6d SUGRA**
Let 6d SUGRA be defined as a compactification of 10d SUGRA
  - $T^4$ compactification: the 10d Lorentz group is broken as $SO(1,9) \rightarrow SO(1,6) \times SO(4)_R$.
  - $K3$ compactification:
    - consider $K3 \sim T^4/Z_n$ for simplicity
    - let $SO(4)_R = SU(2)_{R1} \times SU(2)_{R2}$
    - take $Z_n$ in $SU(2)_{R1}$ \( \rightarrow \) $SU(2)_{R1}$ is broken but $SU(2)_{R2}$ remains as an active R-symmetry!

- **SS compactification of 6d SUGRA**
Consider a generic bulk field $\Phi$ and define
  \[
  \Phi(x^5 + 2\pi, x^6) = T_5 \Phi(x^5, x^6), \quad \Phi(x^5, x^6 + 2\pi) = T_6 \Phi(x^5, x^6)
  \]
with $T_5$ and $T_6$ being $SU(2)_R$ operators.
In case one of the matrices is non-trivial
  \[\rightarrow\] SS compactification

Dudas, Grojean ‘97;
Barbieri, Hall, Nomura ...;
- Consistency conditions: $T^2$ compactification

$T_i$ is the embedding in $SU(2)_R$ of the translation $t_i$ along $x^i$. Since $t_5 t_6 = t_6 t_5$ we need $T_5 T_6 = T_6 T_5$.

- Consistency conditions: $T^2/Z_N$ compactification

In case of an orbifold, also the orbifold rotation $r$ is embedded into the R-symmetry group, via a matrix $R$. Such a matrix is fixed (up to discrete choice) by the requirement of having SUSY in the 4d model, and is non-trivial.

Again, the commutation relations of $t_5$, $t_6$, and $r$ define commutation relations for $T_5$, $T_6$, and $R$. These are non-trivial, since $R$ is non-trivial.

In case a solution exists with $T_5$ and/or $T_6$ non-trivial

→ SS compactification

If then the non-trivial $T$’s can be chosen in a “continuos” way, linked to the identity, then the breaking is described by a constant superpotential term $W_0$.

Such is the case in $T^2/Z_2$ compactifications ...

... and only in this case in the 2d case.

Lee ‘05
**Gauge background: D-term potential**

- We can consider a constant background $F_{56} = f$.
- The fields $A^5$, $A^6$ are not globally defined:
  $$ A(z+\pi) = A(z) + d\Lambda_0 $$
- Thus also $B_{56}$ is not globally defined:
  since $H = dB + F \wedge A$ and $H$ is gauge invariant, it follows $B(z+\pi) = B(z) - \Lambda_0 F$, thus both $A$ and $B$ have a non-trivial profile in the internal space.
- In order to single out the zero modes of $A$ and $B$ we
  a) define $A = \langle A \rangle + \mathcal{A}$, splitting the background field, not globally defined, from the “quantum fluctuations”, globally defined and with standard constant zero-mode (standard KK massless state);
  b) redefine the field $B$ as $B = \mathcal{B} + \langle A \rangle \wedge \mathcal{A}$ so that the new field $\mathcal{B}$ is also globally defined with ....

Kaloper, Myers ’99; Villadoro PhD Thesis ‘06
- Given the redefinition:

\[ \delta B_{56} = -2\Lambda f \]

\[ \rightarrow \] B transforms (as expected)

\[ \rightarrow \] the gauge transformation is the double of what one would naively expect from \( H = dB + F \wedge A \)

- The “new” SUGRA is exactly the old one, provided that one redefines the field \( \hat{b} = B_{56} \) as \( b = B_{56} \). In this way the field \( T \), whose imaginary part is \( b \), transforms under the gauge transformation.

- Given such a transformation we can infer the D-term potential \( D = i K_I X^I \), where \( X^I \) is the Killing vector, in the present case being \( X^T = -i f \).

- Thus we have \( D = f / t \), and \( V_D = \frac{f^2}{2st^2} \).

- We can compute the potential also directly from the \( F^2 \) term in the lagrangian, the two results coincide.
**D-term + $W_0 +$ gaugino condensation : a clash?**

- Take the KKLT model  
  - single modulus $S$  
  - superpotential $W = W_0 + e^{-S}$

- Can we use a D-term potential to break SUSY and uplift the AdS minimum? No, for two reasons:

**I -** The D-term is associated with a gauge transformation involving one modulus. If there is only $S$ then it must transform, but this is incompatible with $W = W_0 + e^{-S}$.

Choi et al.; Dudas, Vempati; Villadoro, Zwirner

- Present case: **no clash!** The field transforming is $T$, and the field entering the gaugino condensation term is $S$.

  see also Haack et al. ‘06 for a realization with D7-branes

  (other way out: $A(M) e^{-S}$ Achucarro et al; Dudas et al; Haack et al.... )

**II -** D-terms and F-terms are related, and it is impossible to uplift a SUSY minimum ($F = 0$) via a D-term.

- Present case: **no clash!** The minimum with non-zero D-term is non-SUSY: $F_T$ is not zero! (but no uplift ... )
Loop corrections

- We can introduce in the system bulk fields (hypers) charged under the U(1) gauge group.
- These fields have a standard KK reduction in absence of a gauge background.
- In the presence of a gauge background the KK reduction is deeply modified:

\[ m_n^2 = \frac{2|f|}{r^4} \left( n + \frac{1}{2} \right) \] for bosons, \[ m_n^2 = \frac{2|f|}{r^4} \left( n + \frac{1}{2} \pm \frac{1}{2} \right) \] for fermions,

and the degeneracy can be deduced via the Dirac index:

\[ d_n = \frac{f}{2\pi} = N \]

- From the 4d spectrum the 1-loop potential follows

\[ V_{loop} = \frac{\alpha |f|^3}{(2\pi)^3 (st)^2} \]

Bachas ‘95
The complete potential: stabilization

**Ingredients:**

I - $W = W_0 + e^{-S}$ (from SS twist and gaugino condensation)

II - D-term potential

$$V_D = \frac{f^2}{2st^2}$$

III - Loop corrections

$$V_{loop} = \frac{\alpha |f|^3}{(2\pi)^3(st)^2}$$

**Step 1:**

Neglect $t$ and include only I: $\Rightarrow$ KKLT potential in $S$, $\tilde{V}(s)$

$s$ fixed in a SUSY AdS minimum

**Step 2:**

Include $t$ $\Rightarrow V = \tilde{V}(s)/t$ runaway behaviour in $t$

**Step 3:**

Include the D-term (II) $\Rightarrow$ stabilization of $t$ in a non-SUSY AdS minimum

**Step 4:**

Include the loop effect (III) $\Rightarrow$ no destabilization (but also no uplift)
Conclusions

- We have shown the role of gauge fluxes/D-terms in the stabilization of a 6d SUGRA model, that can be seen as a non-trivial extension of the KKLT model.
  - No clash D-term vs $W = W_0 + e^{-S}$: extra modulus!
  - D-term crucial in the stabilization the extra modulus.
  - No uplifting via the D-term.

- Computed the 1-loop corrections to the potential, and re-cast them as corrections to the Khäler potential.
  - No de-stabilization of the minimum.
  - No uplifting.

- “By-product”: we considered SS compactification in 2d as a source for $W_0$
  - Possible for $T^2$ or $T^2/Z_2$ compactifications;
  - Not possible for $T^2/Z_N$ compactifications.