

# Cosmology of Gravitino LSP Scenario with Right-Handed Sneutrino NLSP

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**Abstract.** We consider supersymmetric model with right-handed (s)neutrinos where the neutrino masses are purely Dirac-type. We discuss cosmology based on such a scenario, paying particular attention to the case that the gravitino is the lightest superparticles (LSP) while the right-handed sneutrino is the next-LSP. It will be shown that the cosmological constraints on the gravitino-LSP scenario (in particular, those from the big-bang nucleosynthesis) are drastically relaxed in such a case. We will also consider the implication of such scenario to the structure formation.

**PACS.** 12.60.Jv Supersymmetric models – 95.35.+d Dark matter – 98.80.Cq Particle-theory and field-theory models of the early Universe – 14.60.Pq Neutrino mass and mixing

## 1 Introduction

Existence of dark matter (DM) in our universe, which is strongly supported by a lot of recent cosmological observations [1, 2, 3], requires physics beyond the standard model (SM). Many possibilities of dark matter have been discussed in various frameworks of particle physics models so far [4]. Importantly, properties of the dark matter particle depend strongly on the particle physics model we consider.

In the framework of supersymmetry (SUSY), probably most popular candidate of dark matter is thermally produced lightest neutralino which is usually assumed to be the lightest superparticle (LSP). However, if we try to build a supersymmetric model which accommodates with all theoretical and experimental requirements, we expect that there exist new exotic would-be-DM particles which are not superpartners of the SM particles, for example, gravitino  $\psi_\mu$  and right-handed sneutrino  $\tilde{\nu}_R$ , which are superpartners of gravitino and right-handed neutrino, respectively. If the neutrino masses are Dirac-type, in particular,  $\tilde{\nu}_R$ s are expected to be as light as superpartners of SM particles in the framework of the gravity-mediated SUSY breaking. Existence of these exotic superparticles may significantly change the phenomenology of dark matter in supersymmetric models.

In this paper, we consider the supersymmetric model in which the neutrino masses are Dirac-type and discuss cosmological implications of such a scenario. It has already been pointed out that the  $\tilde{\nu}_R$ -DM scenario can be realized in Ref.[5]. Here, we consider another case where the gravitino is the LSP and one of the right-handed sneutrinos is the next-to-the-lightest superparticle (NLSP)[6]. If the gravitino is the LSP,

it may be a viable candidate of dark matter also in the case without the right-handed sneutrinos [7, 8, 9]. In such a case, however, stringent constraints on the scenario are obtained from the study of the gravitino production at the time of the reheating after inflation and also from the study of the big-bang nucleosynthesis (BBN) reactions. With the  $\tilde{\nu}_R$ -NLSP, we reconsider cosmological constraints on the  $\psi_\mu$ -LSP scenario. We pay particular attention to the BBN constraints and also to the constraints from the structure formation of the universe. We will see that the BBN constraints are significantly relaxed if there exists the  $\tilde{\nu}_R$ -NLSP.

## 2 Model Framework

In this section, we discuss the model in which gravitino is the LSP while right-handed sneutrino is the NLSP. We assume that neutrino masses are purely Dirac-type, and the superpotential of the model is written as

$$W = W_{\text{MSSM}} + y_\nu \hat{L} \hat{H}_u \hat{\nu}_R^c, \quad (1)$$

where  $W_{\text{MSSM}}$  is the superpotential of the minimal supersymmetric standard model (MSSM),  $\hat{L} = (\hat{\nu}_L, \hat{e}_L)$  and  $\hat{H}_u = (\hat{H}_u^+, \hat{H}_u^0)$  are left-handed lepton doublet and up-type Higgs doublet, respectively. In this article, “hat” is used for superfields, while “tilde” is for superpartners. Generation indices are omitted for simplicity.

In this model, neutrinos acquire their masses only through Yukawa interactions as  $m_\nu = y_\nu \langle H_u^0 \rangle = y_\nu v \sin \beta$ , where  $v$  is the vacuum expectation value (VEV) of the

standard model Higgs field ( $v \simeq 174$  GeV) and  $\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$ . Thus, the neutrino Yukawa coupling is determined by the neutrino mass through the equation:  $y_\nu \sin \beta = 3.0 \times 10^{-13} \times (m_\nu^2 / 2.8 \times 10^{-3} \text{ eV}^2)^{1/2}$ . Mass squared differences among neutrinos have already been determined accurately at neutrino oscillation experiments [10, 11]. In this article, we assume that the spectrum of neutrino masses is hierarchical, hence the largest neutrino Yukawa coupling is of the order of  $10^{-13}$ . We use  $y_\nu = 3.0 \times 10^{-13}$  for the numerical analysis in this article. With right-handed (s)neutrinos, new soft SUSY breaking terms are introduced in addition to the usual terms of the MSSM. Those are right-handed sneutrino mass terms and tri-linear coupling terms called  $A_\nu$ -terms. Breaking terms relevant to our analysis are

$$\begin{aligned} \mathcal{L}_{\text{SOFT}} = & -M_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L} - M_{\tilde{\nu}_R}^2 \tilde{\nu}_R^* \tilde{\nu}_R \\ & + (A_\nu \tilde{L} H_u \tilde{\nu}_R^c + \text{h.c.}), \end{aligned} \quad (2)$$

where all breaking parameters,  $M_{\tilde{L}}$ ,  $M_{\tilde{\nu}_R}$  and  $A_\nu$ , are defined at the electroweak (EW) scale. We parameterize  $A_\nu$  by using the dimensionless constant  $a_\nu$  as  $A_\nu = a_\nu y_\nu M_{\tilde{L}}$ . We adopt gravity-mediated SUSY breaking scenario and, in such a case,  $a_\nu$  is expected to be  $O(1)$ . Though the  $A_\nu$ -term induces the left-right mixing in the sneutrino mass matrix, the mixing is safely neglected in the calculation of mass eigenvalues due to the smallness of neutrino Yukawa coupling constants. Thus, the masses of sneutrinos are simply given by  $m_{\tilde{\nu}_L}^2 = M_{\tilde{L}}^2 + \frac{1}{2} \cos(2\beta) m_Z^2$ ,  $m_{\tilde{\nu}_R}^2 = M_{\tilde{\nu}_R}^2$ , where  $m_Z \simeq 91$  GeV is the Z boson mass. In the following discussion, we assume that all the right-handed sneutrinos are degenerate in mass for simplicity.

In this article, we consider the  $\psi_\mu$ -LSP scenario with  $\tilde{\nu}_R$ -NLSP. In such a case, the next-to-next-LSP (NNLSP) plays an important role in the thermal history of the universe. However, there are many possibilities of the NNLSP, depending on the detail of SUSY breaking scenario. Thus, we concentrate on the case that the NNLSP is the lightest neutralino, whose composition is Bino  $\tilde{B}$ . This situation is easily obtained if we consider the so-called constrained-MSSM type scenario. It is not difficult to extend our discussion to the scenario with other NNLSP candidate.

### 3 Constraints from BBN

It is well known that models with the  $\psi_\mu$ -LSP usually receive stringent constraints from the BBN scenario. In these models, LSP in the MSSM sector (which we call MSSM-LSP) is long-lived but decays to gravitino with hadrons, which may spoil the success of BBN scenario. The BBN constraints give the upper bound on  $Y_X E_{\text{vis}}$  as a function of  $\tau_X$ , where  $X$  stands for a long-lived but unstable particle,  $Y_X \equiv [n_X/s]_{t \ll \tau_X}$  (with  $n_X$  and  $s$  being the number density of  $X$  and the entropy density of the universe, respectively),  $E_{\text{vis}}$  is the mean energy of visible particles emitted in the  $X$  decay, and  $\tau_X$  is the lifetime of the particle  $X$  [12]. We use the upper

bound on  $Y_X E_{\text{vis}}$  obtained in the studies. In order to evaluate BBN constraints quantitatively, we calculate  $B_{\text{had}} Y_X E_{\text{vis}}$  as a function of  $\tau_{\tilde{B}}$ , then search for allowed parameter region.

First, we will see the decay of the MSSM-LSP, which is  $\tilde{B}$ -like neutralino, in order to calculate  $B_{\text{had}} E_{\text{vis}}$  and  $\tau_{\tilde{B}}$ . Main decay modes are the following two-body decays:  $\tilde{B} \rightarrow \tilde{\nu}_R \bar{\nu}_L$ ,  $\tilde{B} \rightarrow \psi_\mu \gamma$ , and  $\tilde{B} \rightarrow \psi_\mu Z$ . By the use of decay widths of these processes in Ref.[13]. Importantly, the decay mode  $\tilde{B} \rightarrow \tilde{\nu}_R \bar{\nu}_L$  competes with the mode  $\tilde{B} \rightarrow \psi_\mu \gamma$  or it even dominates in total decay in wide parameter region, especially when the gravitino mass  $m_{3/2}$  is larger than 0.1 GeV when  $m_{\tilde{\nu}_R} = 100$  GeV and  $a_\nu = 1$ . We have also checked that the lifetime of  $\tilde{B}$  is  $10^{2-3}$  seconds in that parameter region.

Without the  $\tilde{\nu}_R$ -NLSP, the decay mode  $\tilde{B} \rightarrow \psi_\mu \gamma / Z$  dominates in total decay, and many visible particles are emitted through photon or Z boson, which may spoil the success of the BBN scenario. As a result, the gravitino mass is strictly constrained as  $m_{3/2} \lesssim 0.1$  GeV for  $\tau_{\tilde{B}} \lesssim 1$  second [13]. In our scenario with the  $\tilde{\nu}_R$ -NLSP, however, less hadrons are emitted, though the Bino-like neutralino is long-lived<sup>1</sup>. Therefore, constraints from BBN is expected to be relaxed in the  $m_{3/2} \gtrsim 0.1$  GeV region.

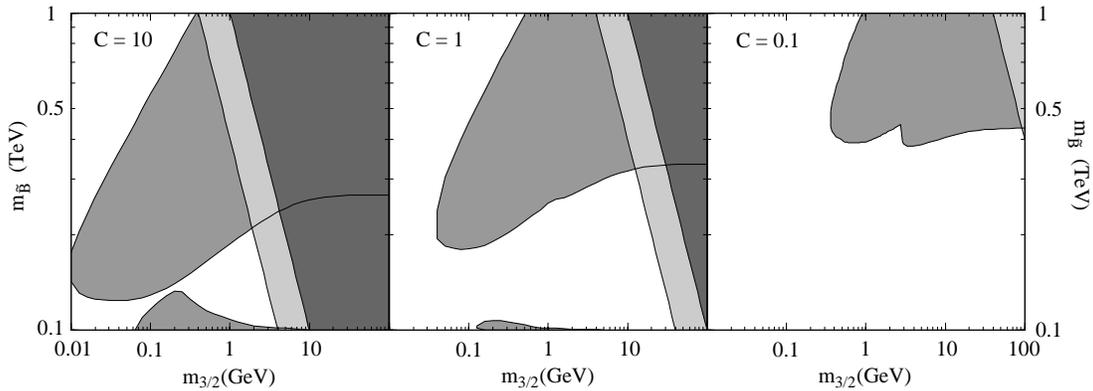
As we saw that no hadrons are emitted in two-body decays, in order to calculate  $B_{\text{had}} E_{\text{vis}}$ , we consider three- or four-body decays:  $\tilde{B} \rightarrow \psi_\mu f f$ ,  $\tilde{B} \rightarrow \tilde{\nu}_R e_L^+ f f'$ , and  $\tilde{B} \rightarrow \tilde{\nu}_R \bar{\nu}_L f f'$ , where  $f$  and  $f'$  denote fermion and anti-fermion, respectively. Although branching ratios of these processes are much smaller than 1, they have impacts on the BBN scenario.

Lastly, we determine  $Y_{\tilde{B}}$  by the use of following formula for (would-be) density parameter of  $\tilde{B}$  [13]:  $\Omega_{\tilde{B}} h^2 = C_{\text{model}} \times 0.1 [m_{\tilde{B}}/100 \text{ GeV}]^2$ , where  $m_{\tilde{B}}$  is Bino mass and the additional parameter  $C_{\text{model}}$  is introduced to take the model dependence into account in our analysis:  $C_{\text{model}} \sim 1$  for the neutralino in the bulk region,  $C_{\text{model}} \sim 0.1$  for that in the co-annihilation or funnel region, and  $C_{\text{model}} \sim 10$  for the pure Bino case without co-annihilation.

Our numerical results are shown in Fig.1, where the BBN constraints are depicted on the  $(m_{3/2}, m_{\tilde{B}})$  plane. We take  $C_{\text{model}} = 10, 1, 0.1$  in the left, middle, and right figures, respectively. Other parameters are set as  $m_{\tilde{\nu}_R} = 100$  GeV,  $a_\nu = 1$ , and  $m_{\tilde{\nu}_L} = 1.5 m_{\tilde{B}}$ . Shaded regions are ruled out by the BBN scenario. As shown in these figures, the constraints are drastically relaxed compared to those in models without the  $\tilde{\nu}_R$ -NLSP.

As shown in the figures, new allowed region appears; for example, for  $C_{\text{model}} = 1, 0.1$  GeV  $\lesssim m_{3/2} \lesssim 40$  GeV. In that region, the BBN constraints give the upper bound on  $m_{\tilde{B}}$ . Since the decay mode  $\tilde{B} \rightarrow \psi_\mu q \bar{q}$  is subdominant in this region, this bound comes mainly

<sup>1</sup> Left-handed neutrinos injected by the decay might possibly change the abundance of  ${}^4\text{He}$  [14]. However, we have checked that the BBN constraints on the neutrino injection are much less stringent than those on three- or four-body decays as shown in the following discussion.



**Fig. 1.** Constraints from the BBN on the  $(m_{3/2}, m_{\tilde{B}})$  plane: Parameters are chosen to be  $m_{\tilde{\nu}_R} = 100$  GeV,  $a_\nu = 1$ , and  $m_{\tilde{\nu}_L} = 1.5m_{\tilde{B}}$ . We set  $C_{\text{model}} = 10, 1$ , and  $0.1$  in the left, middle, and right figures, respectively. The middle shaded regions are ruled out by the BBN scenario, while dark and light shaded regions are excluded by the WMAP measurement and the structure formation of the universe, respectively.

from four-body decays,  $\tilde{B} \rightarrow \tilde{\nu}_R e_L^+ q \bar{q}'$  and  $\tilde{B} \rightarrow \tilde{\nu}_R \bar{\nu}_L q \bar{q}$ . This fact can be understood intuitively:  $B_{\text{had}}$  and  $E_{\text{vis}}$ , are enhanced when  $m_{\tilde{B}}$  is large. On the contrary, in the  $0.01 \text{ GeV} \lesssim m_{3/2} \lesssim 0.1 \text{ GeV}$  region, Bino-like neutralino decays mainly into the gravitino through the  $\tilde{B} \rightarrow \psi_\mu \gamma$  process with the lifetime  $\tau_{\tilde{B}} \lesssim 1$  second. Since the decay occurs before the BBN starts, it does not affect the BBN scenario. This situation also holds in the usual  $\psi_\mu$ -LSP scenario without the  $\tilde{\nu}_R$ -NLSP, and the same allowed region can be seen in Ref.[13]. In the case of  $C_{\text{model}} = 10(0.1)$  in the left (right) figure, the constraints from the BBN scenario is more (less) stringent than the  $C_{\text{model}} = 1$  case. As a result, the upper bound on  $m_{\tilde{B}}$  becomes smaller (larger). We also find that the region  $m_{\tilde{B}} \simeq 100 \text{ GeV}$  with  $0.1 \text{ GeV} \lesssim m_{3/2} \lesssim 1 \text{ GeV}$  is excluded in the left and middle figures: this is because the process  $\tilde{B} \rightarrow \tilde{\nu}_R \bar{\nu}_L$  is kinematically suppressed and branching ratio of the process  $\tilde{B} \rightarrow \psi_\mu Z$  is relatively enhanced.

In addition to the BBN constraints, we also depict other cosmological bounds in Fig.1: the gravitino abundance originating in  $\tilde{B}$  must not exceed the value observed in the WMAP,  $\Omega_{\text{DM}} h^2 \simeq 0.105$  [3], which is shown as a dark shaded region in Fig.1. This constraint gives the upper bound on  $m_{3/2}$ . Another constraint,  $\Omega_{3/2}^{\text{dec}} < 0.4\Omega_{\text{DM}}$ , is also depicted as a light shaded region, which comes from the structure formation of the universe as discussed in the next section.

## 4 Constraints from Structure Formation

As shown in the previous section, larger value of  $m_{3/2}$  is allowed compared to the case without  $\tilde{\nu}_R$ -NLSP. In the newly allowed parameter region, the MSSM-LSP decays mainly into  $\tilde{\nu}_R$ , and  $\tilde{\nu}_R$  decays into the gravitino. Since the gravitino is produced with large velocity dissipation at the late universe, it behaves as a warm dark matter, and as a result, may affect the structure formation of the universe. In this section, we consider the constraints from the structure formation.

$\tilde{\nu}_R$  decays to  $\psi_\mu$  mainly through two-body process,  $\tilde{\nu}_R \rightarrow \psi_\mu \nu_R$ . We find that the lifetime of  $\tilde{\nu}_R$   $\tau_{\tilde{\nu}_R}$  is  $10^2$ - $10^8$  seconds for  $m_{3/2} = 0.1$ - $100 \text{ GeV}$  with  $m_{\tilde{\nu}_R} = 100 \text{ GeV}$ . With the use of  $\tau_{\tilde{\nu}_R}$ , we calculate free-streaming length  $\lambda_{\text{FS}} \equiv \int_{\tau_{\tilde{\nu}_R}}^{t_{\text{EQ}}} dt v(t)/a(t)$ , where  $v(t)$  is the velocity of the  $\psi_\mu$ ,  $a(t)$  is the cosmic scale factor, and  $t_{\text{EQ}}$  is the time of the matter-radiation equality. As a result, we found  $\lambda_{\text{FS}} \simeq 6 \text{ Mpc}$  when  $m_{\tilde{\nu}_R} = 100 \text{ GeV}$  irrespective of the  $m_{3/2}$ . Thus, it indicates that the component of the dark matter (i.e., gravitino) from  $\tilde{\nu}_R$  decay acts as a warm dark matter (WDM). In addition to  $\tilde{\nu}_R$  decay, gravitinos are also produced by the thermal scattering at the reheating epoch after inflation. The abundance of the  $\psi_\mu$  from the scattering process is determined by the reheating temperature and the  $m_{3/2}$  [15]. Since the  $\psi_\mu$  from the scattering is non-relativistic at the time of the structure formation, it acts as a cold dark matter (CDM). Thus, we have to consider the constraints from the structure formation of the universe on the WDM+CDM scenario.

Constraints from the structure formation on the WDM+CDM scenario are studied in recent works [16, 17]. According to these studies, it turns out that the matter power spectrum has a step-like decrease around the free-streaming scale of the WDM component,  $k \sim 2\pi/\lambda_{\text{FS}}$ . This fact can be understood intuitively, because only the power spectrum of the WDM component dumps at the scale. On the other hand, the power spectrum is estimated from the observations of the cosmic microwave background [1, 3], the red shift surveys of galaxies [2], and so on. Even though the power spectrum has been experimentally determined accurately, an ambiguity still remains when  $k^{-1}$  is around  $1 \text{ Mpc}$ . Therefore, it is not clear whether the step-like decrease exists or not, if it is small enough. In this article, we put the conservative constraints on our model: the power spectrum, to be more precise, the magnitude of the step-like decrease, should be within the range of the 95% confidence level of the observational data [1]. This condition gives the upper bound on the portion of the WDM component.

In our model, the energy density of the dark matter is composed of two components,  $\rho_{\text{DM}} = \rho_{3/2}^{\text{dec}} + \rho_{3/2}^{\text{th}}$ , where  $\rho_{3/2}^{\text{dec}}$  and  $\rho_{3/2}^{\text{th}}$  are the energy densities of gravitino produced by the decay and by the thermal scattering processes, respectively. Introducing the fraction of WDM component  $f$ , we rewrite  $\rho_{\text{DM}}$  as

$$\begin{aligned}\rho_{\text{DM}} &= \rho_{3/2}^{\text{dec}} + \rho_{3/2}^{\text{th}} \\ &= f\rho_{\text{pureWDM}} + (1-f)\rho_{\text{pureCDM}},\end{aligned}\quad (3)$$

where  $\rho_{\text{pureWDM}}$  and  $\rho_{\text{pureCDM}}$  are the energy densities of pure WDM and CDM scenario, respectively. Considering the adiabatic density fluctuation, we calculate the power spectrum. As a result, we finally get  $f \lesssim 0.4$ . In terms of the density parameter, the constraints indicates  $\Omega_{3/2}^{\text{dec}} \lesssim 0.4\Omega_{\text{DM}}$ , which gives the upper bound on the gravitino mass as  $m_{3/2} \lesssim 40$  GeV and  $m_{3/2} \lesssim 4$  GeV for the cases of  $C_{\text{model}} = 1$  and 10, respectively (the light shaded regions in Fig.1).

The constraints do not depend highly on the detail of the observational data. In fact, in other recent observations, it is mentioned that the observational error on the power spectrum is around 15% [2, 16], leading to the constraint as  $f \lesssim 0.2$ , which is of the same order of magnitude as the result above. Therefore, our simple analysis is expected to give the viable constraint of the structure formation on the WDM+CDM scenario.

## 5 Conclusions

In this paper, we have studied the cosmological implications of the gravitino LSP scenario with the right-handed sneutrino NLSP in the framework where neutrino masses are purely Dirac-type. In the case that MSSM-LSP is Bino-like neutralino, it mainly decays into the right-handed sneutrino with the lifetime  $\tau_{\tilde{B}} \sim 10^2$ - $10^3$  seconds in the wide range of the parameter region. Though the MSSM-LSP is long-lived, no visible particles are produced in the leading process, thus constraints from the BBN scenario is drastically relaxed compared to the case without the right-handed sneutrino NLSP. With the quantitative analysis of the BBN constraints, we have found the new allowed region,  $0.1$  GeV  $\lesssim m_{3/2} \lesssim 40$  GeV, when  $m_{\tilde{\nu}_R} = 100$  GeV. In this region, the BBN constraints give the upper bound on the Bino mass as  $m_{\tilde{B}} \lesssim 200$ - $400$  GeV, which mainly comes from hadronic four-body decays. On the other hand, the upper bound on the gravitino mass is given by the constraints from the structure formation of the universe. In our scenario, some part of the gravitino is produced by the decay of right-handed sneutrino at the late universe. As a result, the gravitino freely streams in the universe and acts as a WDM. The gravitino is also produced from thermal scattering processes, which acts as a CDM. Taking the CDM contribution into account, we have considered the constraints on the WDM+CDM scenario from the observations of (small scale) structure formation, and finally found the upper bound on the gravitino mass.

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