Topological physics of little Higgs bosons

or

The Wess-Zumino-Witten term and the Higgs boson

Richard Hill

Fermilab

SUSY07, 27 July, 2007

Based on:

R.J. Hill and C.T. Hill,
hep-ph/0701044, PRD (Jan 2007),
hep-ph/0705.0697 (May 2007),

Very incomplete list of references

Composite models:

Kaplan, Georgi (84), Kaplan, Georgi, Dimopoulos (84), Georgi, Kaplan, Galison (84), Dugan, Georgi Kaplan (85)

Arkani-Hamed, Cohen, Georgi (01) Arkani-Hamed, Cohen, Katz, Kaplan, Schmaltz (03)

Nelson, Gregoire, Wacker (02)

Contino, Nomura, Pomaral (03) Agashe, Contino, Sundrum (2005)

Arkani-Hamed, Cohen, Katz, Nelson (02)

Low, Skiba, Smith (02)

Pierce, Perelstein, Peskin (04)

Han, Logan, Wang (06)

T parity:

Cheng, Low (03,04)

Birkedal-Hansen, Wacker (04) Birkedal-Hansen, Noble, Perelstein, Spray (06) Carena, Hubisz, Perelstein, Verdier (06)

KK parity:

Cheng, Feng, Matchev (02)

Servant, Tait (02) Bertone, Hooper, Silk (05)

What to take from this talk

- *Little Higgs:*
    - anomalies are fundamental in building a consistent model
    - T parity is generally violated (for UED models, KK parity is generally violated)

- *General interest*
    - new applications and playground for topological interactions
    - the simplest WZW term in four dimensions

As a most conservative motivation,

Should have alternatives to SUSY, and SM

A composite/little Higgs is such an alternative

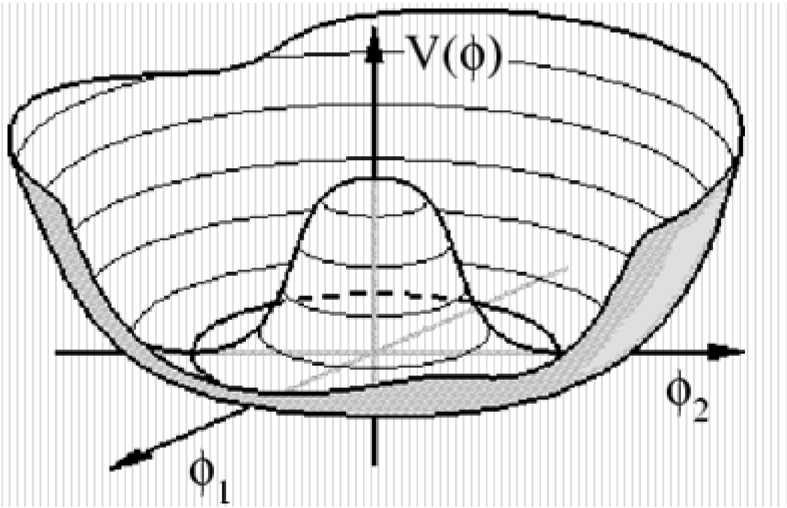
Can come from different perspectives:

Little Higgs
= high scale technicolor, with mechanism for Higgs to leak down to the weak scale
= technicolor without technicolor: a "chiral" Lagrangian leaving UV theory unspecified

Need to deal with anomalies and consistency conditions, even if fermions aren't mentioned by name

The simplest WZW term in 4 dimensions

Consider the NGB's of a spontaneously broken "flavor" symmetry, e.g. $SU(3) \to SU(2)$



Field space M = space of degenerate vacua, e.g. $SU(3)/SU(2) = S^5$

(just like SU(3)xSU(3)/SU(3)=SU(3))

What is the most general action that is:

- *globally SU(3) invariant*
- *four dimensional*
- *local*

Our field space for SU(3)/SU(2) is the five-sphere

$$\Phi = \begin{pmatrix} \phi^1 + i\phi^2 \\ \phi^3 + i\phi^4 \\ \phi^5 + i\phi^6 \end{pmatrix} \qquad \Phi^\dagger \Phi = \sum_{i=1}^{6} (\phi^i)^2 = 1$$

$$\Phi = \exp\left[i \begin{pmatrix} \eta & \cdot & \\ \cdot & \eta & H \\ & H^\dagger & -2\eta \end{pmatrix} \right] \begin{pmatrix} \cdot \\ \cdot \\ 1 \end{pmatrix}$$

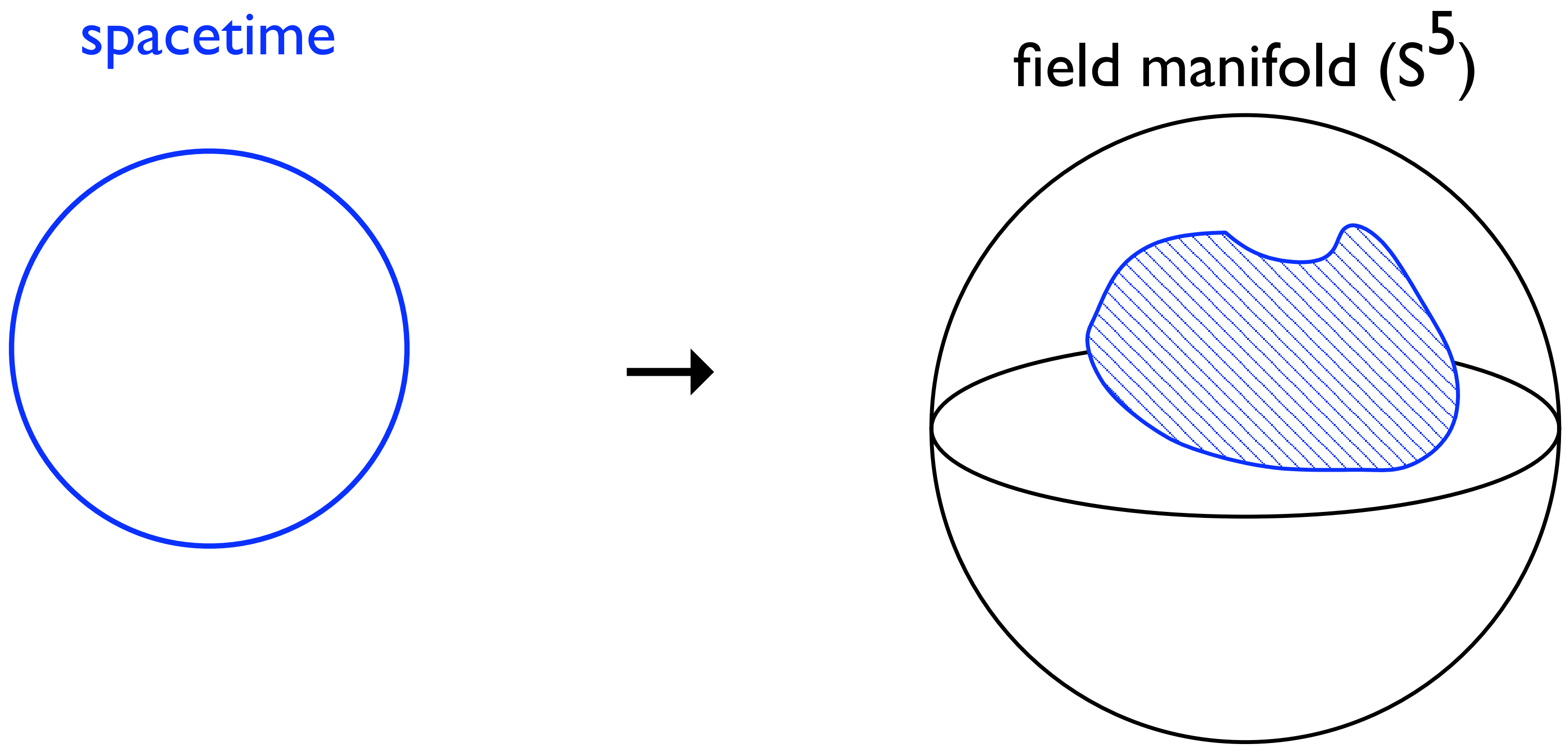
First pass:

$$\Gamma(\Phi) = \int d^4x \, |\partial_\mu \Phi|^2 + c_1 |\partial_\mu \Phi|^4 + c_2 \Phi^\dagger \partial^4 \Phi + \dots$$

Second pass:

$$\Gamma'(\Phi) = \text{number} \times \text{“area bounded by the image of spacetime on } S^5\text{”}$$

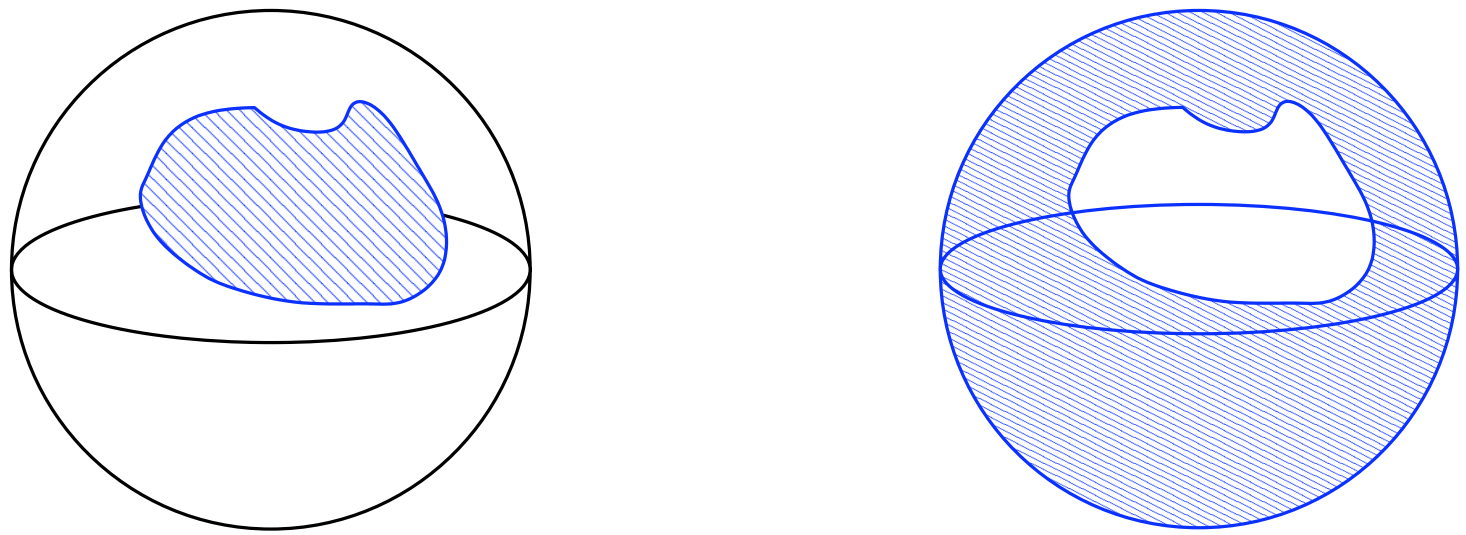
Together, Γ and Γ′ give the general effective action for Φ



Nothing subtle, just another way to build an action that is:

- *globally SU(3) invariant*
- *four dimensional*
- *local*

Quantization:



Can only be consistent if difference between choices of bounding surface is 2π x integer

Witten 1982

[Volume of S^5] = π^3 ⇒

$$\Gamma'(\Phi) = \text{integer} \times 2\pi \times \frac{1}{\pi^3} \int_{M^5} -\frac{i}{8} \Phi^\dagger d\Phi d\Phi^\dagger d\Phi d\Phi^\dagger d\Phi$$

volume element on sphere

Any candidate UV completion theory is labeled by an integer: 0, 1, 2, 3, ...

Less familiar example of WZW term: SU(n)/SU(n-1)

For example,
U(1)/1 = U(1) describes an axion

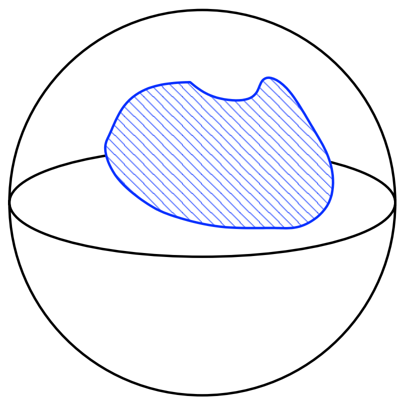
SU(2)xU(1)/U(1) describes the Goldstone bosons of the SM Higgs mechanism

SU(3)xU(1)/SU(2)xU(1) describes a simple Little Higgs model

All of these cases descend from SU(3)/SU(2) = S^5
In fact, this example deserves the label:

Simplest WZW Term in Four Dimensions

Abstracting the essential topology



For a given smooth $\Phi(x)$, there exists a bounding surface: $\pi_4(S^5)=0$

Most perverse we can be about choosing different bounding surfaces is to wrap around the sphere: $\pi_5(S^5)=Z$

These are obvious for the present case.

Also true for

$SU(n)xSU(n)/SU(n)=SU(n)$,

$SU(n)/SO(n)$

$SU(2n)/Sp(2n)$

Coupled to gauge fields, the WZW action has anomalies: **fermions without fermions**

Fermions from fermions:

In general, if we have a consistent anomaly:

$$\delta\Gamma = \int d^4x\, \epsilon^a(x) \mathcal{A}^a(A)$$

which vanishes for "*a*" in a anomaly-free subgroup H, then we can integrate to obtain an action for SU(n)/H:

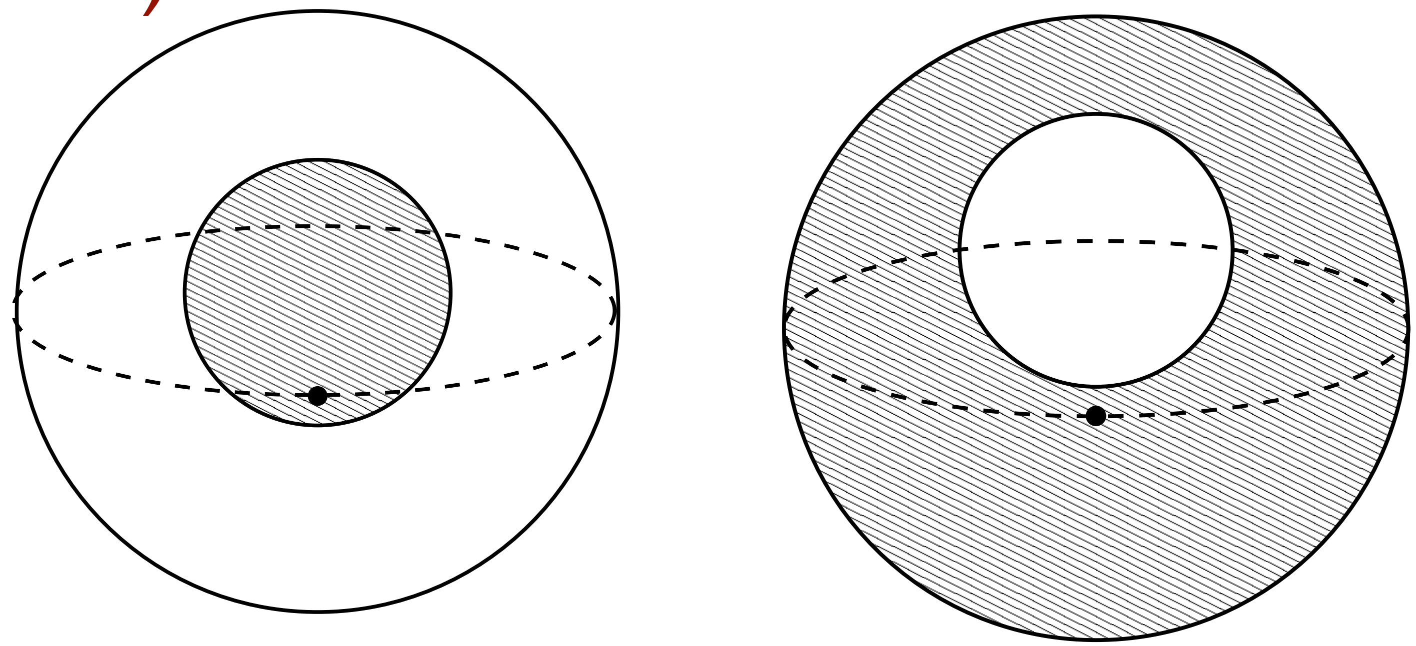
$$\Gamma = \int_0^1 dt \int d^4x\, \pi^a(x) \mathcal{A}^a\left(e^{-it\pi}(A + i\partial)e^{it\pi}\right)$$

Wess and Zumino 1971

These actions are in fact the same. In contrast to the geometrical construction:

- not explicitly four-dimensional
- not explicitly globally invariant
- seems explicitly local for *any* value of coefficient..

- appears that any coefficient of the action is possible (not quantized)



- *But*, in order that the action not be discontinuous under small changes in $\pi(x)$, quantization condition reappears

For "technicolor without technicolor" philosophy, can consider the "fermions without fermions" approach to WZW

In the end, it's just an operator that appears in the effective theory (with quantized coefficient)

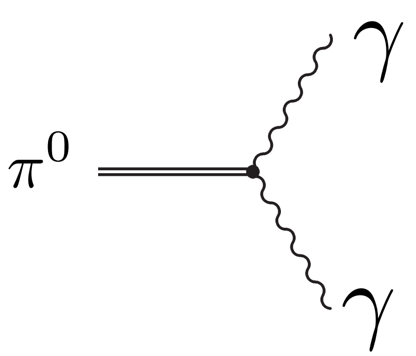
Chiral lagrangians and parities

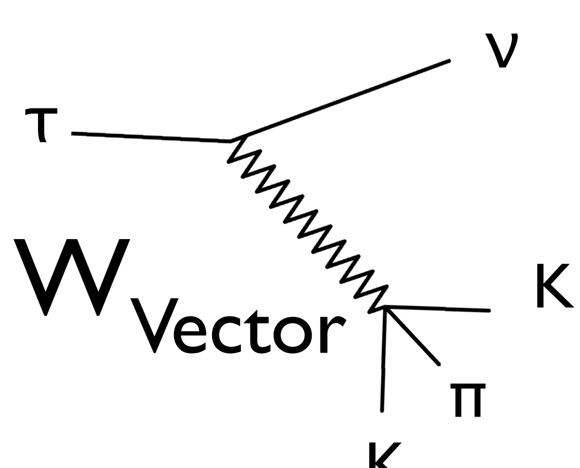
Consider the QCD chiral lagrangian for low-energy pion interactions.
Field space is SU(3)xSU(3)/SU(3) = SU(3): $U = e^{i\pi^a t^a}$

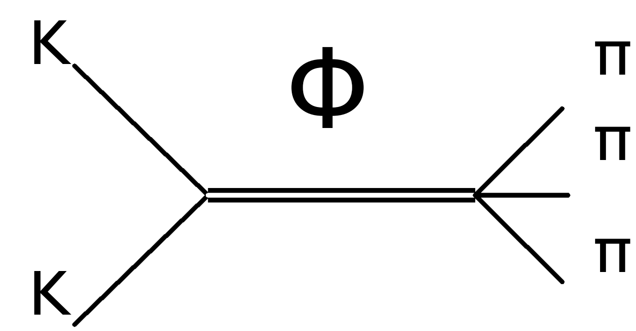
At first sight, it appears that the effective action conserves the internal parity $U \leftrightarrow U^\dagger$

$$\Gamma \sim \int d^4x \, \mathrm{Tr}\Big[|D_\mu U|^2 + c_1 |D_\mu U|^4 + c_2 D_\mu U D_\nu U^\dagger D_\mu U D_\nu U^\dagger + \dots \Big]$$

This would forbid interactions involving odd numbers of mesons, e.g. $\pi_0 \to \gamma\gamma$







NGB or "T" parity carries seeds of it's own destruction

For a general symmetry breaking pattern, the full symmetry group can be defined to act on NGB's as:

$$e^{i\pi} \longrightarrow e^{i\epsilon} e^{i\pi} e^{-i\epsilon'(\epsilon,\pi)}$$

general group element

element of unbroken group

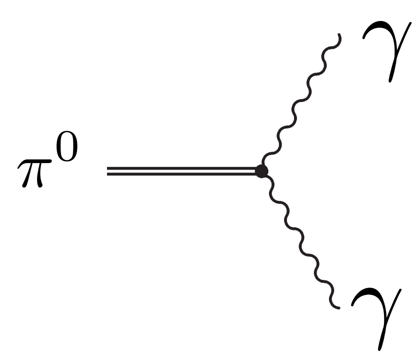
Callan, Coleman, Wess, Zumino 1969

Let R denote the parity: π is odd, ϵ' is even

$$e^{2i\pi'} = e^{i\pi'} R(e^{-i\pi'}) = e^{i\epsilon} e^{2i\pi} e^{-iR(\epsilon)}$$

Thus U=exp(2iπ) has the correct properties to build a topological action. This action is odd under both x→-x, and π→−π

IR probes of UV physics



theory: $\Gamma(\pi^0 \to \gamma\gamma) = \frac{N_c \alpha^2}{96\pi^2} \frac{m_\pi^3}{f_\pi^2} = N_c \times 2.4\,\mathrm{eV}$

experiment: $7.7 \pm 0.6\,\mathrm{eV}$

$\Rightarrow$ *number of colors = 3 !*

Recall QCD: important to know # colors (=3) to find out what's going on:

- baryons = 3 quarks
- structure of lepton sector highly constrained by anomaly cancellation: $3(-2(1/6)^3+(2/3)^3+(-1/3)^3)-2(-1/2)^3+(-1)^3 = 0$

What are the analogous probes for a composite Higgs?

- **Ingredients of a "little higgs" model**
  - 1) higgs is light (it's an NGB)
    - 2) EW-symmetric vacuum is destabilized (e.g. coupling to heavy top sector, or gauging broken generators)
      - 3) higgs potential is generated (e.g. after integrating out heavy scalars, or radiative corrections to chiral lagrangian)
        - 4) SM fermions get mass (coupling to Higgs = kaon induced by extended TC-like interactions)

Models generally have two identical sectors, or two identical gauge groups G1xG2
Natural to introduce the important concept of parity that exchanges sectors or gauge groups

Cheng and Low, 2003

An important minus sign

gauged generators: $\Lambda = \underset{\text{unbroken}}{\Lambda_V} + \underset{\text{broken}}{\Lambda_A}$

one-loop contribution to scalar masses:

$$\underset{\text{positive}}{m^2_{ab}} = M^2 \sum_\Lambda \underbrace{\mathrm{Tr}\left\{[\Lambda_V,[\Lambda_V,t^a_A]]t^b_A\right\}} - \underbrace{\mathrm{Tr}\left\{[\Lambda_A,[\Lambda_A,t^a_A]]t^b_A\right\}}$$

positive and nonzero
if NGB charged

e.g. Peskin 1980

e.g, π^+ heavier than π^0 due to EM corrections

- “LH cancellation”: *two terms cancel, $m^2 \approx 0$*
- “EWSB by vacuum misalignment”: *second term overwhelms first, EWSB*

In general, gauge broken symmetries → massive gauge bosons

What are the topological interactions of H,W,Z,γ,B’ ?

general structure of topological interaction of Higgs and gauge fields (this is for SU(3)xSU(3)/SU(3)):

heavy photon

anomalous terms cancelled by leptons, or other scalar sector

$$\Gamma_{WZW} = \frac{-N_c}{36\pi^2}\int d^4x\, \epsilon^{\mu\nu\rho\sigma}\hat{B}_\mu \Big\{ -\mathrm{Tr}(W_\nu \partial_\rho W_\sigma) + \frac{3i}{4}\mathrm{Tr}(W_\nu W_\rho W_\sigma) + \frac{1}{6}B_\nu \partial_\rho B_\sigma$$
$$\underbrace{+\frac{i}{F^2}\left[H^\dagger (F_W)_{\nu\rho} D_\sigma H - (D_\nu H^\dagger)(F_W)_{\rho\sigma} H\right] - \frac{i}{4F^2}\left[H^\dagger D_\nu H - (D_\nu H^\dagger) H\right](F_B)_{\rho\sigma}} \Big\} + \ldots .$$

gauge-invariant Gauge-Higgs interactions

A potential misperception

“The low-energy theory doesn’t have to be gauge invariant”

or

“The low-energy theory doesn’t have to have anomaly cancellation”

Hard to have a theory that is “just a little bit non-gauge-invariant”

Consider what happens in the SM...

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} c \\ s \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

The usual quarks and leptons, coupled to the usual SU(3)xSU(2)xU(1),

$$\sum_f \bar{f}(i\not{\partial} + g_1 Y \not{A}_1 + g_2 \not{A}_2^a t_2^a + g_3 \not{A}_3^a t_3^a) f$$

and Higgs:

$$\lambda_s(\bar{c}_L \ \bar{s}_L) H s_R + \lambda_c(\bar{c}_L \ \bar{s}_L) \tilde{H} c_R + h.c.$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \xcancel{\begin{pmatrix} c \\ s \end{pmatrix}}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

Suppose $\lambda_{s,c}$ are big, but the gauge couplings $g_{1,2}$ are small:

$$\lambda_s(\bar{c}_L \;\; \bar{s}_L) H s_R + \lambda_c(\bar{c}_L \;\; \bar{s}_L)\tilde{H} c_R + h.c.$$

$$\to m_s \bar{s} s + m_c \bar{c} c + \dots$$

Then s,c are heavy and low-energy theory contains:

$(u, d, \nu_{e,}, e, \nu_\mu, \mu, W, Z, \gamma)$

The low-energy theory is anomalous - or is it ?

We started with a gauge-invariant theory, so of course should end up with one

$$H \sim U \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad U = e^{ig/v}$$

After integrating out s,c, we end up with WZW term for SU(2)xU(1)/U(1):

$$\lambda_s (\bar{c}_L \ \bar{s}_L) H s_R + \lambda_c (\bar{c}_L \ \bar{s}_L) \tilde{H} c_R + h.c. \qquad \to \Gamma_{WZW}(U, A)$$

Low-energy theory actually contains:

$(u, d, \nu_{e,}, e, \nu_\mu, \mu, W, Z, \gamma)$ + (g^+, g^-, g^0)

Use this lesson in building little Higgs models. WZW term appears in two ways

1) anomaly cancellation between sectors

$$\Gamma_{\text{SM+other fermions}}$$

$$\Gamma_{WZW}(U_1) \longleftrightarrow \Gamma_{WZW}(U_2)$$

E.g. SU(3)/SU(2) little higgs model

Kaplan, Schmaltz 2003, Frampton, 1992, ...

- anomaly cancellation between generations
- WZW should (in general *must*) be used to represent the chiral anomalies of the UV completion theory

Use this lesson in building little Higgs models. WZW term appears in two ways

2) anomaly cancellation between fermions and scalars

$$\Gamma_{\text{SM+other fermions}}$$

$$\updownarrow$$

$$\Gamma_{WZW}(U)$$

Arkani-Hamed, Cohen, Katz, Nelson 2002, Katz, Lee, Nelson, Walker 2005, ...

E.g. SU(5)/SO(5) little higgs model

- anomaly cancellation involves SM or spectator fermions
- WZW should (in general *must*) be used for gauge anomaly cancellation

Suppose we build a theory that has a T-parity extending NGB parity to the SM + other fermion sector (using tools of nonlinear realizations, etc.)

WZW is generally present, and is odd under T-parity

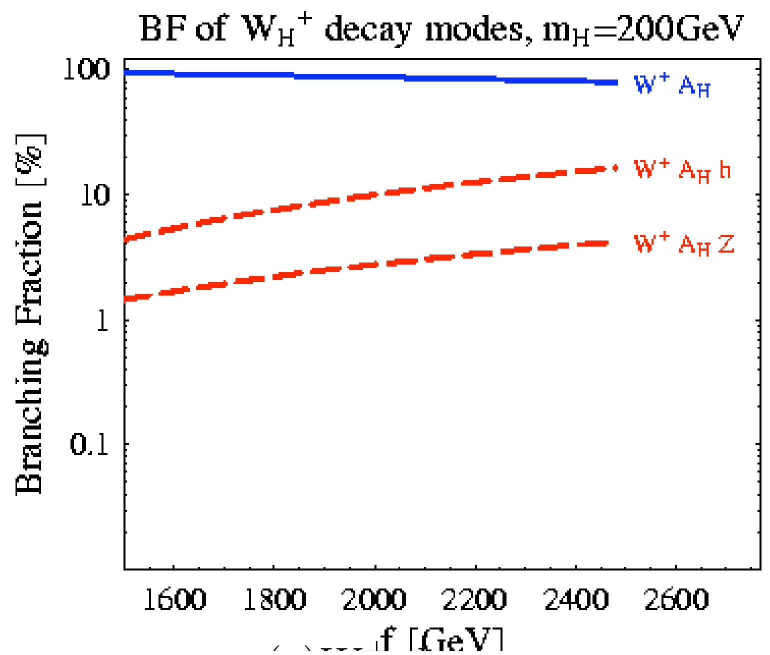
Constrained decay of T-odd particles

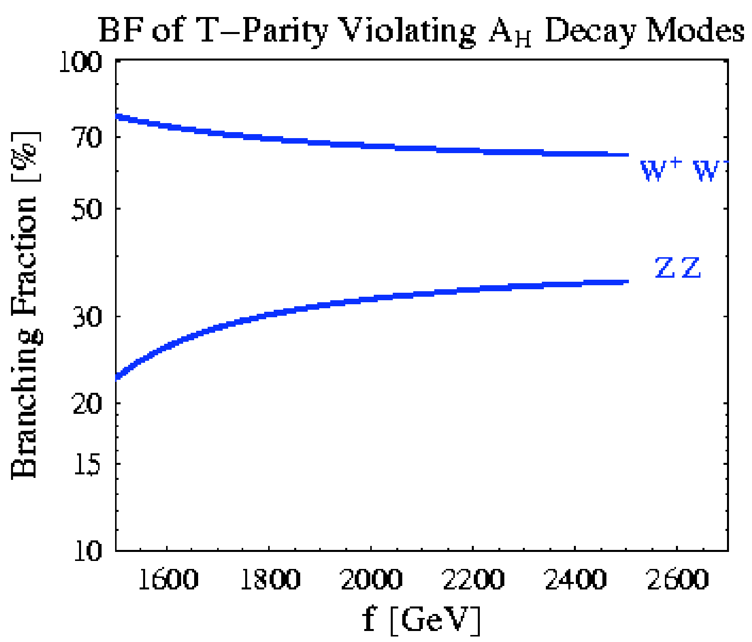
$$\Gamma(\tilde{B} \to ZZ) \approx N_c^2 \left[\frac{1}{2\pi} \left(\frac{\tilde{g}^3}{144\pi^2} \right)^2 \frac{m_Z^2}{m_{\tilde{B}}} \right]$$

measure number of “technicolors”

Collider phenomenology similar to R-parity violating SUSY - cascades down to LTP, then decay of LTP

Distinctive (VVVV+ leptons or + jets) final states





- Barger, Keung and Gao, 0707.3648[hep-ph]

- work in progress with K. Kong and E. Lunghi

Summary

- anomalies not just a nuisance - a low energy probe of UV completion physics
- can make a lot of progress by demanding consistency (anomaly cancellation) and writing the most general interactions
- T parity is generally violated by the WZW term
- Similarly, KK parity in extra dimensions is violated by the Chern Simons term
- can lead to highly constrained and interesting signatures