Supersymmetric three dimensional conformal sigma models

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Plan to talk

We consider three dimensional nonlinear sigma models using the Wilsonian renormalization group method.

In particularly, we investigate the renormalizability and the fixed point of the models.

1. Introduction (briefly review of WRG)
2. Two dimensional cases
3. Renormalizability of three dimensional sigma model
4. Conformal sigma models
5. Summary

1.Introduction

Non-Linear Sigma Model

Bosonic Non-linear sigma model

$$\mathcal{L} = \underline{g_{ij}}\partial_\mu \varphi^i \partial^\mu \varphi^j$$

The target space ・・・O(N) model

$$S^{N-1} \quad \mathcal{L} = \frac{1}{2}\underline{(\delta_{ij} + \frac{4\lambda^2 \varphi^i \varphi^j}{1-\lambda^2 \varphi^i \varphi^i})}\partial_\mu \varphi^i \partial^\mu \varphi^j$$

2-dim. Non-linear sigma model

(perturbatively renormalizable)

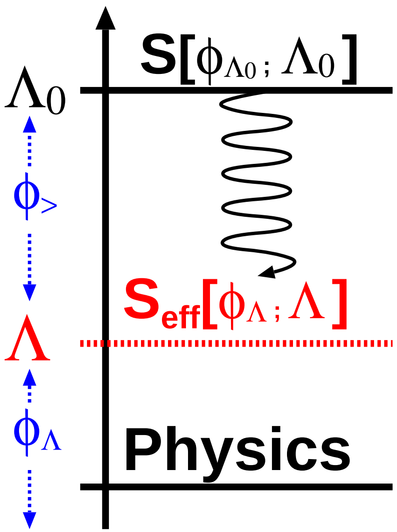
Toy model of 4-dim. Gauge theory

(Asymptotically free, instanton, mass gap etc.)

Polyakov action of string theory

3-dim. Non-linear sigma model

Wilsonian Renormalization Group Equation



We divide all fields ϕ into two groups,

high frequency modes and low frequency modes.

$$\phi_{\Lambda_0}(p) = \phi_\Lambda(p) + \phi_>(p)$$

The high frequency mode is integrated out.

$$e^{-S_{\text{eff}}[\phi_\Lambda,\Lambda]} = \int^{\Lambda_0}[d\phi_>]e^{-S[\phi_\Lambda+\phi_>,\Lambda_0]}$$

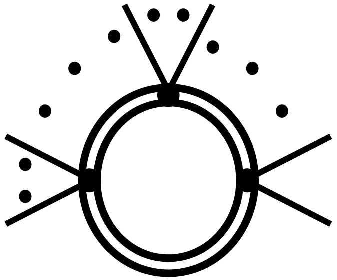
➢Infinitesimal change of cutoff $\Lambda \to e^{-\delta t}\Lambda = \Lambda - \delta\Lambda$

The partition function does not depend on Λ.

- Wegner-Houghton equation (sharp cutoff)
- Polchinski equation (smooth cutoff)
- Exact evolution equation (for 1PI effective action)

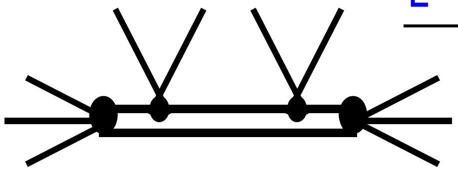
Wegner-Houghton eq

$$\frac{\partial S[\phi_\Lambda, \Lambda]}{\partial t} = \lim_{\delta t \to 0} \frac{1}{2\delta t} \int_{\Lambda - \delta\Lambda}^{\Lambda} \left[tr\ln \left(\frac{\delta^2 S}{\delta\phi\delta\phi} \right) - \frac{\delta S}{\delta\phi} \left(\frac{\delta^2 S}{\delta\phi\delta\phi} \right)^{-1} \frac{\delta S}{\delta\phi} \right]$$



Quantum correction

$$+ \left[d - \int_p \phi(p) (p \cdot \frac{\partial}{\partial p} + d_\phi + \gamma_\phi) \frac{\delta}{\delta\phi_p} \right] S[\phi_\Lambda, \Lambda]$$



Canonical scaling : Normalize kinetic terms

In this equation, all internal lines are the shell modes which have nonzero values in small regions.

More than two loop diagrams vanish in the $\delta t \to 0$ limit.

This is exact equation. We can consider (perturbatively) nonrenormalizable theories.

2. Two dimensional cases

Non-linear sigma models with N=2 SUSY in 3D (2D) is defined by Kaehler potential.

$$\mathcal{L} = \int d\theta^2 d\bar{\theta}^2 K(\phi, \bar{\phi})$$

$$= g_{ab^*}(\partial_\mu \varphi^a)(\partial^\mu \varphi^{*b}) + i g_{ab^*} \bar{\psi}^b \not{D} \psi^a + \frac{1}{4} R_{ab^* cd^*} \psi^a \psi^c \bar{\psi}^b \bar{\psi}^d$$

$$g_{ab^*} \equiv \frac{\partial^2 K}{\partial \varphi^a \partial \varphi^{b*}}$$

★The scalar field has zero canonical dimension.

$$dim[\varphi] = 0$$

Perturbatively renormalizable

$$\mathcal{L} = g_{ij}[\varphi, \varphi^*] \partial_\mu \varphi^i \partial^\mu \varphi^j$$

★In perturbative analysis, the 1-loop β function is proportional to the Ricci tensor of target spaces.

The perturbative results $$\beta(g_{i\bar{j}}) = \frac{1}{2\pi} R_{i\bar{j}}$$

Alvarez-Gaume, Freedman and Mukhi Ann. of Phys. 134 (1982) 392

K.Higashijima and E.I. Prog. Theor. Phys. 110 (2003) 107

Beta function from WRG

$$-\frac{d}{dt} g_{ab^*} = \frac{1}{2\pi} R_{ab^*} + \nabla_a \xi_{b^*} + \nabla_{b^*} \xi_a$$

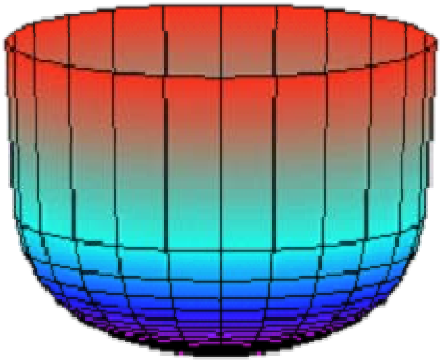
$$\xi^a = \gamma \varphi^a$$

Fixed Point Theories

★ $\gamma = 0$ ⇒ Ricci Flat solution

★ $\gamma \neq 0$ ⤵

Here we introduce a parameter which corresponds to the anomalous dimension of the scalar fields as follows: $a = -4\pi\gamma$



When *N*=1, the target manifold takes the form of a semi-infinite cigar with radius $\sqrt{\frac{1}{a}}$.

It is embedded in 3-dimensional flat Euclidean spaces.

Witten Phys.Rev.D44 (1991) 314

3.Three dimensional cases (renormalizability)

★The scalar field has nonzero canonical dimension.

$$dim[\varphi] = 1/2$$

$$\mathcal{L} = g_{ij}[\varphi, \varphi^*]\partial_\mu \varphi^i \partial^\mu \varphi^j$$

★ We need some nonperturbative renormalization methods.

WRG approach ⇐ Our works

Large-N expansion

CP^{N-1} model

Inami, Saito and Yamamoto Prog. Theor. Phys. 103 (2000) 1283

Beta fn. from WRG
(Ricci soliton equation)

$$-\frac{d}{dt}g_{ab^*} = \frac{1}{2\pi^2}R_{ab^*} - g_{ab^*} + \nabla_a \xi_{b^*} + \nabla_{b^*}\xi_a$$

$$\xi^a = \left(\tfrac{1}{2} + \gamma\right)\varphi^a = \Delta_\varphi \varphi^a$$

Renormalization condition

$$g_{ab^*}(\varphi = 0) = \delta_{ab^*}$$

The CP^{N-1} model :SU(N)/[SU(N-1) ×U(1)]

$$K[\Phi, \Phi^\dagger] = \frac{1}{\lambda^2}\ln(1 + \vec{\Phi}\vec{\Phi}^\dagger),$$

From this Kaehler potential, we derive the metric and Ricci tensor as follow:

$$g_{i\bar{j}} = \frac{\delta_{i\bar{j}}}{1+\lambda^2\varphi\varphi^*} - \frac{\lambda^2\varphi_i^*\varphi_{\bar{j}}}{(1+\lambda^2\varphi\varphi^*)}$$

$$R_{i\bar{j}} = N\lambda^2 g_{i\bar{j}}$$

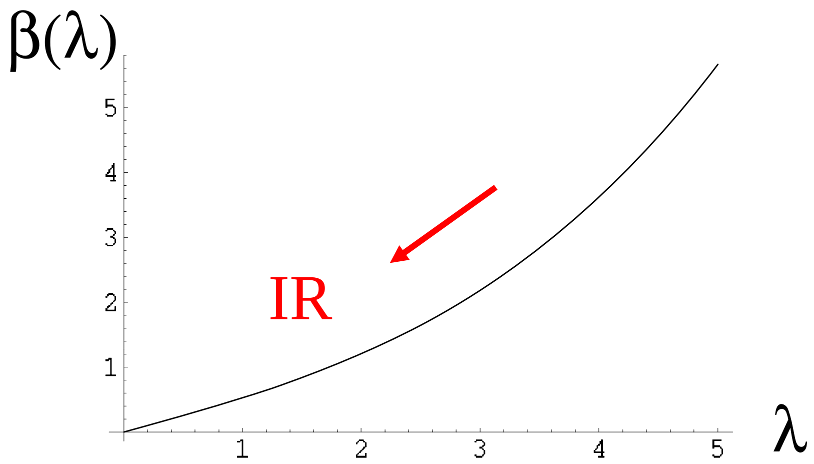
When the target space is an Einstein-Kaehler manifold, the βfunction of the coupling constant is obtained.

Einstein-Kaehler condition:

$$R_{i\bar{j}} = h\lambda^2 g_{i\bar{j}}.$$

$$\beta(\lambda) = -\frac{h\lambda^3}{4\pi^2} + \frac{1}{2}\lambda,$$
$$\gamma = -\frac{h\lambda^2}{4\pi^2}.$$

★ The constant *h* is negative (example Disc with Poincare metric)



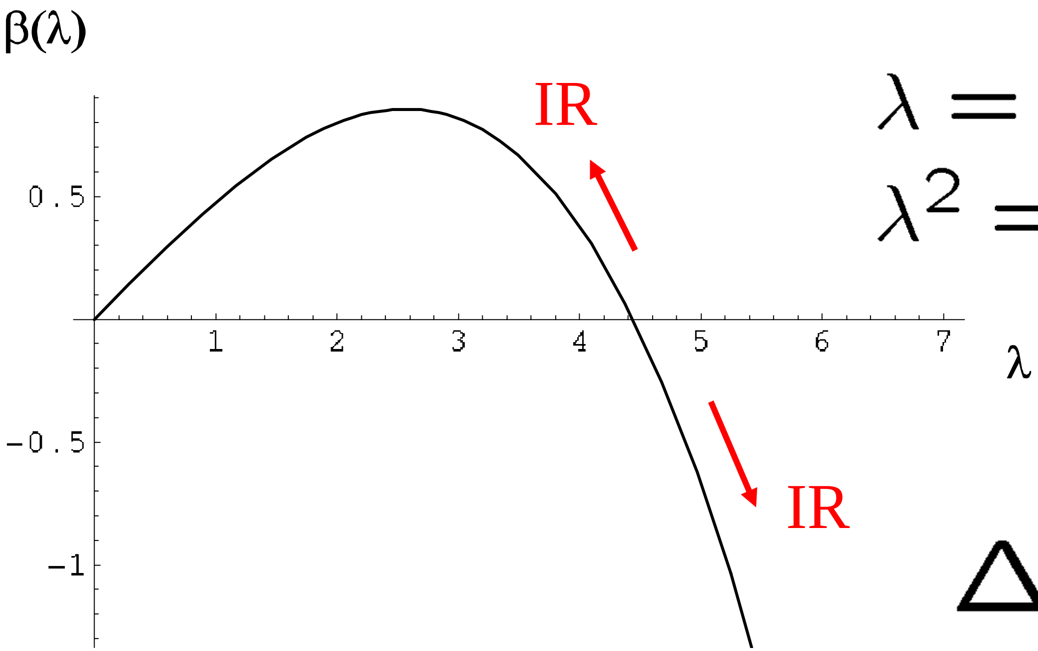
$$g_{i\bar{j}} = \frac{\delta_{i\bar{j}}}{(1-\lambda^2\varphi\varphi^*)^2}$$

$i, j=1$

We have only IR fixed point at λ=0.

★ If the constant h is positive, there are two fixed points:

Renormalizable



$$\lambda = 0 \text{ :IR fixed point}$$

$$\lambda^2 = \frac{2\pi^2}{h} \text{ :UV fixed point}$$

At UV fixed point

$$\gamma_c = -\frac{1}{2}$$

$$\Delta_\varphi \equiv d_\varphi + \gamma_\varphi = 0$$

It is possible to take the continuum limit by choosing the cutoff dependence of the "bare" coupling constant as

$$\lambda(\Lambda) \to \lambda_c - \frac{M}{\Lambda}.$$

M is a finite mass scale.

4.Conformal Non-linear sigma models

Fixed point theory obtained by solving an equation

$$\frac{1}{2\pi^2}R_{ab^*} - g_{ab^*} + \nabla_a \xi_{b^*} + \nabla_{b^*}\xi_a = 0$$

$$\xi^a = \left(\tfrac{1}{2} + \gamma\right)\varphi^a = \Delta_\varphi \varphi^a$$

At $\gamma = -\frac{1}{2}$ ⇒ $\Delta_\varphi = \gamma + \frac{1}{2} = 0$

Fixed point theories have Kaehler-Einstein mfd. with the special value of the radius.

$$R_{ab^*} - c\lambda^2 g_{ab^*} = 0$$

c is a constant which depends on models.

Hermitian symmetric space (HSS)

・・・・A special class of Kaehler- Einstein manifold with higher symmetry

New fixed points (γ≠-1/2)

◆Two dimensional fixed point target space for $\gamma \neq -\frac{1}{2}$

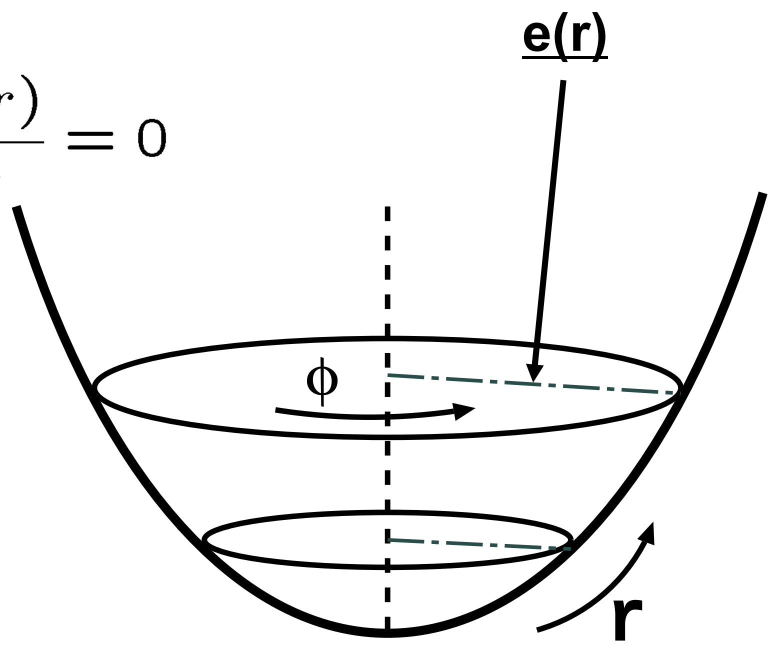
- The line element of target space

$$ds^2 = dr^2 + e(r)^2 d\phi^2$$

- RG equation for fixed point

$$\frac{1}{2\pi^2}\frac{\partial^2 e(r)}{\partial r^2} + e(r) + 2\Delta_\varphi e(r)\frac{\partial e(r)}{\partial r} = 0$$

$$(\Delta_\varphi = \frac{1}{2} + \gamma)$$

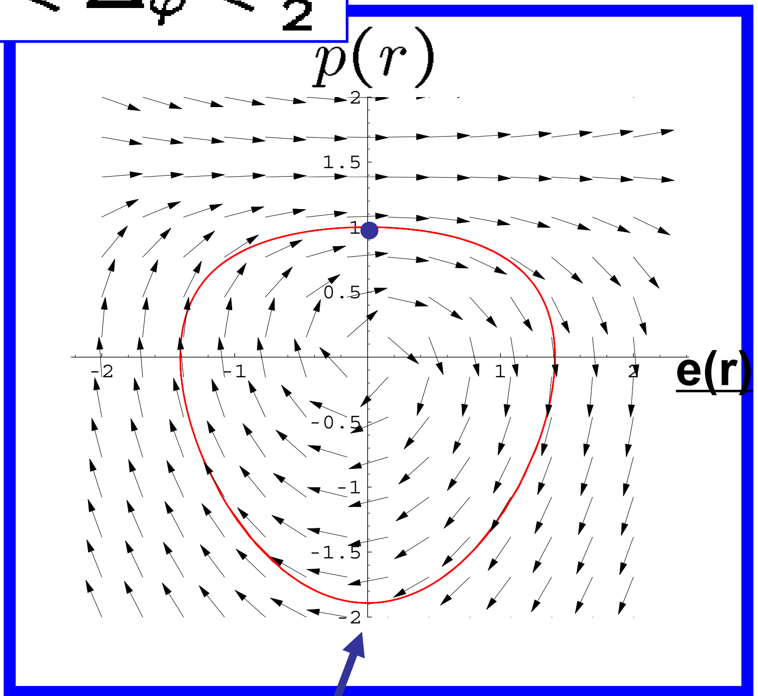


It is convenient to rewrite the 2nd order diff.eq. to a set of 1st order diff.eq.

$$\begin{cases} e'(r) = p(r) \\ p'(r) = -2\pi^2 e(r)\underline{(1 - 2\Delta_\varphi p(r))} \end{cases}$$

Deformed sphere

$0 < \Delta_\varphi < \frac{1}{2}$



$\Delta_\varphi = 0$ (CP^1) **: Sphere** S^2

$0 < \Delta_\varphi < \frac{1}{2}$ **: Deformed sphere**

$\Delta_\varphi = \frac{1}{2}$ **: Flat** R^2

At the point, the target mfd. is not locally flat.

It has deficit angle. Euler number is equal to S^2

Summary

- We study a perturbatively nonrenormalizable theory (3-dim. NLSM) using the WRG method.
- Some three dimensional nonlinear sigma models are renormalizable within a nonperturbative sense.
- We construct a class of 3-dim. conformal sigma models.