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Finite Unified Theories confronted with low-energy phenomenology

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Abstract. Finite Unified Theories (FUTs) are N=1 supersymmetric Grand Unified Theories that can be made all-loop finite. The requirement of all-loop finiteness leads to a severe reduction of the free parameters of the theory and, in turn, to a large number of predictions. Here SU(5) FUTs are investigated in the context of low-energy phenomenology observables. We present a detailed scanning of these FUTs, including theoretical uncertainties at the unification scale and applying all phenomenological constraints. Taking into account the restrictions from the top and bottom quark masses, we can discriminate between different models. Including further low-energy constraints such as B physics observables, the bound on the lightest Higgs boson mass and the cold dark matter density, we determine the predictions of the allowed parameter space for the Higgs boson sector and the supersymmetric particle spectrum of the model.

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1 Introduction

Finite Unified Theories are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, including the soft supersymmetry breaking sector. To construct GUTs with reduced independent parameters [1,2] one has to search for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. Of particular interest is the possibility to find RGI relations among couplings that guarantee finitenes to all-orders in perturbation theory [3,4]. In order to achieve the latter it is enough to study the uniqueness of the solutions to the one-loop finiteness conditions [3,4]. The constructed *finite unified* $N = 1$ supersymmetric SU(5) GUTs using the above tools, predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass [5]. The search for RGI relations and finiteness has been extended to the soft supersymmetry breaking sector (SSB) of these theories [6,7], which involves parameters of dimension one and two. Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the s-spectrum.

Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g. The superpotential of the theory is given by

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ , \qquad (1)$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G. All the one-loop β-functions of the theory vanish if the β-function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G)\,,\ \frac{1}{2}C_{ipq}C^{jpq} = 2\delta_i^j g^2 C_2(R_i)\ , \qquad (2)$$

where $\ell(R_i)$ is the Dynkin index of R_i, and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G. A theorem [3] guarantees the vanishing of the β-functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling. Alternatively, similar results can be obtained [4,8] using an analysis of the all-loop NSVZ gauge beta-function [9].

In the soft breaking sector, it was found that RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule at one-loop [10]. This result was generalized to two-loops for finite theories [11], and then to all-loops for general Gauge-Yukawa and finite unified theories [12]. Then the following soft

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scalar-mass sum rule is found [11]

$$\frac{(\ m_i^2 + m_j^2 + m_k^2\ )}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(1)} + O(g^4) \qquad (3)$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction

$$\Delta^{(1)} = -2\sum_l [(m_l^2/MM^\dagger) - (1/3)]\ \ell(R_l), \qquad (4)$$

$\Delta^{(1)}$ vanishes for the universal choice, i.e. when all the soft scalar masses are the same at the unification point.

2 FINITE UNIFIED THEORIES

The first one- and two-loop finite model was presented in [13]. A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
2. Three fermion generations, in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i = 1, 2, 3)$, which obviously should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of ref. [5], which will be labeled **A**, and a slight variation of this model (labeled **B**), which can also be obtained from the class of the models suggested in ref. [14] with a modification to suppress non-diagonal anomalous dimensions.

The superpotential which describes the two models takes the form [5,11]

$$\begin{aligned} W = &\sum_{i=1}^{3}\left[\ \frac{1}{2} g_i^u\, \mathbf{10}_i \mathbf{10}_i H_i + g_i^d\, \mathbf{10}_i \overline{\mathbf{5}}_i\, \overline{H}_i\ \right] \\ &+ g_{23}^u\, \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d\, \mathbf{10}_2 \overline{\mathbf{5}}_3\, \overline{H}_4 + g_{32}^d\, \mathbf{10}_3 \overline{\mathbf{5}}_2\, \overline{H}_4 \\ &+ \sum_{a=1}^{4} g_a^f\, H_a\, \mathbf{24}\, \overline{H}_a + \frac{g^\lambda}{3}\,(\mathbf{24})^3\ , \end{aligned} \qquad (5)$$

where H_a and $\overline{H}_a$ $(a = 1, \ldots, 4)$ stand for the Higgs quintets and anti-quintets.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for the models $\{\mathbf{A}\ ,\ \mathbf{B}\}$ are:

$$\begin{aligned} &(g_1^u)^2 = \{\frac{8}{5}, \frac{8}{5}\} g^2\ ,\ (g_1^d)^2 = \{\frac{6}{5}, \frac{6}{5}\} g^2\ , \\ &(g_2^u)^2 = (g_3^u)^2 = \{\frac{8}{5}, \frac{4}{5}\} g^2\ , \\ &(g_2^d)^2 = (g_3^d)^2 = \{\frac{6}{5}, \frac{3}{5}\} g^2\ , \\ &(g_{23}^u)^2 = \{0, \frac{4}{5}\} g^2\ ,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \{0, \frac{3}{5}\} g^2\ , \\ &(g^\lambda)^2 = \frac{15}{7} g^2\ ,\ (g_2^f)^2 = (g_3^f)^2 = \{0, \frac{1}{2}\} g^2\ , \\ &(g_1^f)^2 = 0\ ,\ (g_4^f)^2 = \{1, 0\} g^2\ . \end{aligned} \qquad (6)$$

According to the theorem of ref. [3] these models are finite to all orders. After the reduction of couplings the symmetry of W is enhanced [5,11].

The main difference of the models **A** and **B** is that three pairs of Higgs quintets and anti-quintets couple to the $\mathbf{24}$ for **B** so that it is not necessary to mix them with H_4 and $\overline{H}_4$ in order to achieve the triplet-doublet splitting after the symmetry breaking of $SU(5)$.

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale [11]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\overline{\mathbf{5}}}^2 + m_{\mathbf{10}}^2 = M^2 \quad \text{for} \quad \mathbf{A}\ , \qquad (7)$$

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2\ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}\ ,$$

$$m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} \quad \text{for} \quad \mathbf{B}, \qquad (8)$$

where we use as free parameters $m_{\overline{\mathbf{5}}} \equiv m_{\overline{\mathbf{5}}_3}$ and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for the model **A**, and $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ for **B**, in addition to M.

3 PREDICTIONS OF LOW ENERGY PARAMETERS

Since the gauge symmetry is spontaneously broken below $M_{\rm GUT}$, the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (6), the $h = -MC$ relation, and the soft scalar-mass sum rule (3) at $M_{\rm GUT}$, as applied in the two models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below $M_{\rm GUT}$ their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s (which we define as the geometric mean of the stop masses) and therefore below that scale the effective theory is just the SM.

We now present the comparison of the predictions of the four models with the experimental data, see ref. [15] for more details, starting with the heavy quark masses. In fig.1 we show the **FUTA** and **FUTB** predictions for $M_{\rm top}$ and $m_{\rm bot}(M_Z)$ as a function of the unified gaugino mass M, for the two cases $\mu < 0$ and $\mu > 0$. In the value of the bottom mass $m_{\rm bot}$, we have included the corrections coming from bottom squark-gluino loops and top squark-chargino loops [16]. We give the predictions for the running bottom quark mass evaluated at M_Z, $m_{\rm bot}(M_Z) = 2.825 \pm 0.1$ [17], to avoid the large QCD uncertainties inherent for the pole mass. The value of $m_{\rm bot}$ depends strongly on the sign of μ due to the above mentioned radiative corrections. For both models **A** and **B** the values for $\mu > 0$ are above the central experimental value, with $m_{\rm bot}(M_Z) \sim 4.0 - 5.0$ GeV. For $\mu < 0$, on the other hand, model **B** has overlap with the experimental allowed values, $m_{\rm bot}(M_Z) \sim 2.5 - 2.8$ GeV, whereas for model **A**, $m_{\rm bot}(M_Z) \sim 1.5 - 2.6$ GeV, there is only a

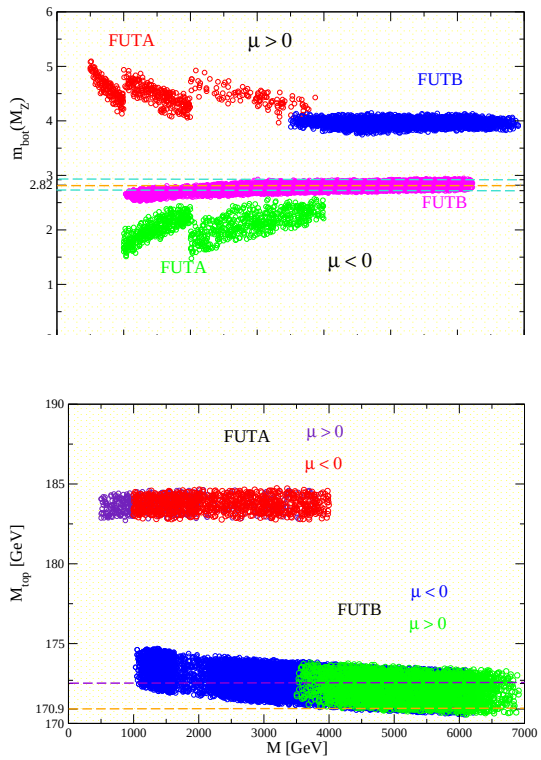

Fig. 1. The bottom quark mass at the Z boson scale (upper) and top quark pole mass (lower plot) are shown as function of M for both models.

small region of allowed parameter space at two sigma level, and only for large values of M. This clearly selects the negative sign of μ.

The predictions for the top quark mass $M_{\rm top}$ are ~ 183 and ~ 172 GeV in the models **A** and **B** respectively, as shown in the lower plot of fig. 1. Comparing these predictions with the most recent experimental value $M_{\rm top}^{exp} = (170.9 \pm 1.8)$ GeV [18], and recalling that the theoretical values for $M_{\rm top}$ may suffer from a correction of $\sim 4\%$ [19], we see that clearly model **B** is singled out. In addition the value of $\tan\beta$ is found to be $\tan\beta \sim 54$ and ~ 48 for models **A** and **B**, respectively. Thus the comparison of the model predictions with the experimental data is survived only by **FUTB** with $\mu < 0$.

We now analyze the impact of further low-energy observables on the model **FUTB** with $\mu < 0$. In the case where all the soft scalar masses are universal at the unfication scale, there is no region of M below $\mathcal{O}$(few TeV) in which $m_{\tilde\tau} > m_{\chi^0}$ is satisfied (where $m_{\tilde\tau}$ is the lightest $\tilde\tau$ mass, and m_{χ^0} the lightest neutralino mass). But once the universality condition is relaxed this problem can be solved naturally, thanks to the sum rule (3). Using this equation and imposing the conditions of (a) successful radiative electroweak symmetry breaking, (b) $m_{\tilde\tau}^2 > 0$ and (c) $m_{\tilde\tau} > m_{\chi^0}$, a comfortable parameter space is found for **FUTB** with $\mu < 0$ (and also for **FUTA** and both signs of μ).

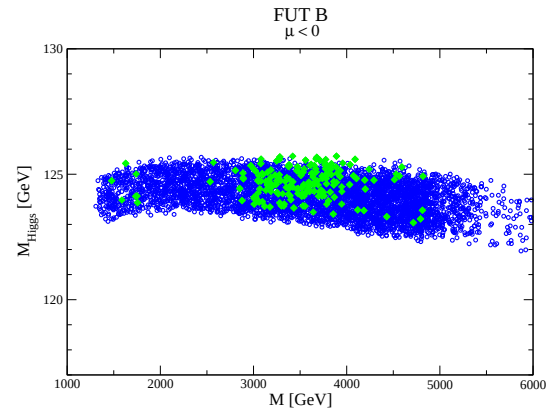

Fig. 2. The lightest Higgs mass, M_h, as function of M for the model **FUTB** with $\mu < 0$, see text.

As additional constraints we consider the following observables: the rare b decays BR($b \to s\gamma$) and BR($B_s \to \mu^+\mu^-$), the lightest Higgs boson mass as well as the density of cold dark matter in the Universe, assuming it consists mainly of neutralinos. More details and a complete set of references can be found in ref. [15].

For the branching ratio BR($b \to s\gamma$), we take the present experimental value estimated by the Heavy Flavour Averaging Group (HFAG) is [20]

$$\mathrm{BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}, \quad (9)$$

where the first error is the combined statistical and uncorrelated systematic uncertainty, the latter two errors are correlated systematic theoretical uncertainties and corrections respectively.

For the branching ratio BR($B_s \to \mu^+\mu^-$), the SM prediction is at the level of 10^{-9}, while the present experimental upper limit from the Tevatron is 5.8×10^{-8} at the 95% C.L. [21], providing the possibility for the MSSM to dominate the SM contribution.

Concerning the lightest Higgs boson mass, M_h, the SM bound of 114.4 GeV [22] can be used. For the prediction we use the code `FeynHiggs` [23,24,25,26].

The lightest supersymmetric particle (LSP) is an excellent candidate for cold dark matter (CDM) [27], with a density that falls naturally within the range

$$0.094 < \Omega_{\rm CDM} h^2 < 0.129 \quad (10)$$

favoured by a joint analysis of WMAP and other astrophysical and cosmological data [28].

The prediction for M_h of **FUTB** with $\mu < 0$ is shown in fig. 2. The constraints from the two B physics observables are taken into account. In addition the CDM constraint (evaluated with `Micromegas` [29]) is fulfilled for the lighter (green) points in the plot, see ref. [15] for details. The lightest Higgs mass ranges in

$$M_h \sim 121 - 126 \text{ GeV}, \quad (11)$$

where the uncertainty comes from variations of the soft scalar masses, and from finite (i.e. not logarithmically divergent) corrections in changing renormalization scheme. To this value one has to add ± 3 GeV

coming from unkonwn higher order corrections [25]. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Thus, taking into account the B physics constraints (and possibly the CDM constraints) results naturally in a light Higgs boson that fulfills the LEP bounds [22].

In the same way the whole SUSY particle spectrum can be derived. The resulting SUSY masses for **FUTB** with $\mu < 0$ are rather large. The lightest SUSY particle starts around 500 GeV, with the rest of the spectrum being very heavy. The observation of SUSY particles at the LHC or the ILC will only be possible in very favorable parts of the parameter space. For most parameter combination only a SM-like light Higgs boson in the range of eq. (11) can be observed.

We note that with such a heavy SUSY spectrum the anomalous magnetic moment of the muon, $(g-2)_\mu$ (with $a_\mu \equiv (g-2)_\mu/2$), gives only a negligible correction to the SM prediction. The comparison of the experimental result and the SM value [30]

$$a_\mu^{\rm exp} - a_\mu^{\rm theo} = (27.5 \pm 8.4) \times 10^{-10}. \qquad (12)$$

would disfavor **FUTB** with $\mu < 0$ by about $3\,\sigma$. However, since the SM is not regarded as excluded by $(g-2)_\mu$, we still see **FUTB** with $\mu < 0$ as the only surviving model. A more detailed numerical analysis, also using `Suspect` [31] for the RGE running, and including all theory uncertainties for the models will be presented in ref. [15].

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