Local SUSY-breaking minima in $N_f = N_c$ SQCD?

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Abstract. We study non-supersymmetric minima in $N_f = N_c$ SQCD conjectured by Intriligator, Seiberg and Shih (ISS). We show that the existence of such minima depends on the signs of three non-calculable parameters and that no evidence can be inferred by deforming the theory. We illustrate this by demonstrating that the conjectured minimum is destabilized in a different deformation of $N_f = N_c$ SQCD. We also comment briefly on the phenomenological consequences of this instability.

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1 Introduction

It was shown by Intriligator, Seiberg and Shih (ISS) [1] that supersymmetry breaking in meta-stable vacua is a common phenomenon which occurs in a wide range of models. In particular they showed that such vacua are present in massive supersymmetric QCD (SQCD) with the number of colors N_c and number of flavors N_f in the range $N_c < N_f < \frac{2}{3}N_c$. It was demonstrated that the supersymmetry breaking minimum is located near the origin of field space, far from the supersymmetric minimum, such that the lifetime of the supersymmetry breaking minimum is sufficiently long. The analysis was done using the Seiberg dual theory [2], which is weakly coupled.

ISS also considered the case $N_f = N_c$. This theory confines at low energies and the quantum moduli space is deformed. Since the deformation of the moduli space does not allow all the fields to be near the origin of the field space calculability is lost. In order to make further progress ISS deformed this theory adding one more massive flavor (which restores calculability) and then decoupling it, taking the mass of the additional flavor to be infinitely large. By using this deformation ISS conjectured that the local minimum also exists in $N_f = N_c$ SQCD.

In our current work [3] we revisit the case $N_f = N_c$. In the first part of my talk I will briefly review the main results of ISS. It will be also shown that the existence of the meta-stable vacuum in the massive $N_f = N_c$ SQCD is solely dependent on the signs of three non-calculable terms in the Kähler potential and hence no information about this minimum can be gained by deforming the theory. To illustrate this point I will introduce another deformation of $N_f = N_c$ SQCD. The potential of the proposed deformation possesses an extremum similar to ISS, but it is a saddle point rather than a minimum. I will also comment on the phenomenological consequences of this instability, concentrating on the "Pentagon" model [4].

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2 Review of the ISS model, $N_f = N_c$ conjecture

Consider SQCD with $N_c + 1 \leq N_f < \frac{3}{2}N_c$ and the tree level superpotential

$$W = (m_Q)_{ij} Q_i \bar{Q}_j \tag{1}$$

where Q and $\bar{Q}$ denote quarks and antiquarks respectively. Assume also that the rank of the matrix m_Q is larger than the number of colors in the magnetic dual theory $N_f - N_c$. Clearly, this model possesses a supersymmetric global vacuum. Nonetheless, as was shown in [1], the theory has a local non-supersymmetric vacuum near the origin of the moduli space. Supersymmetry is broken by a rank condition in the magnetic dual theory. One of the most crucial points for this analysis was that the supersymmetric quark masses $(m_Q)_{ij}$ are much lower than the scale of the magnetic theory Λ. The smallness of the parameter m_Q/Λ is important for two reasons. It enables us to increase the distance between the supersymmetric and non-supersymmetric vacua in field space, rendering the lifetime of the metastable solution sufficiently long. But it is also important in order to ensure the calculability of the model, ensuring that the contributions of non-calculable terms in Kähler potential are neglegible.

ISS also considered the case $N_f = N_c$. One can think about this theory as the limit of $N_f = N_c + 1$ SQCD when the mass of one of the flavors is taken to be infinitely large. Although the theory with $N_f = N_c + 1$ confines, it has the same calculable supersymmetry breaking meta-stable vacuum as the theories in the free magnetic phase. Naively, one could trace the behavior of this minimum when the limit $N_f = N_c$ is taken, and come to the conclusion that the theory with $N_f = N_c$ has the same minimum. Nevertheless, $N_f = N_c$ SQCD is very different from the cases with with larger number of flavors: even if we demand $m_Q \ll \Lambda$, the contribution of the noncalculable Kähler potential is not negligible, moreover it dominates the calculable contributions.

When $N_f = N_c$ SQCD confines the quadratic approximation for the Kähler potential fails and one should take into account higher order terms. Consider the Kähler metric at next to leading order: using the global symmetries of the theory it can be parametrized as follow[1] :

$$g^{-1}_{M^\dagger M} \sim \alpha \frac{\mathrm{Tr}(M^\dagger M)}{\Lambda^2} + \beta \frac{\mathrm{Tr}M \mathrm{Tr}M^\dagger}{\Lambda^2} + \gamma \frac{(B_+ + B_+^\dagger)^2}{\Lambda^2} + \cdots \tag{2}$$

Here we used the fact that the baryon symmetry is spontaneously broken. The solution, conjectured by ISS, is located along the baryonic branch and $\mathrm{Im}(B_+)$ is supposed to be a Goldstone boson. The odd powers of $\mathrm{Re}(B_+)$ are forbidden by an unbroken subgroup $\mathbb{Z}_{2N}$ of the anomalous axial symmetry $U(1)_A$. The coefficients $\alpha,\ \beta$ and γ are non-calculable and probably of order one; neither their precise numerical values nor their signs are known.

Assume for simplicity that the quark mass matrix is proportional to the identity. Then the non-vanishing F-term in the conjectured local minimum is $F_{\mathrm{Tr}M} = m_Q \Lambda$. We get the following contribution to the scalar potential from the non-calculable terms in Kähler metric:

$$V \sim m_Q^2 \left(\alpha\ \mathrm{Tr}(M^\dagger M) + \beta\ \mathrm{Tr}M \mathrm{Tr}M^\dagger + \gamma\ (B_+ + B_+)^2\right) \tag{3}$$

Namely we get non-calculable contributions to the squared masses of baryons and mesons of order m_Q^2. Note that these contributions are of the same order of magnitude both in $N_f = N_c$ and $N_f = N_c + 1$ cases. Now let us compare these contributions to the calculable contributions to the masses in the meta-stable vacuum.

Consider the theory with $N_f = N_c + 1$. Denote its confining scale by $\hat{\Lambda}$ to distinguish it from Λ, the confining scale of $N_f = N_c$. It was shown in [1] that most of the fields get squared masses of order $\mathcal{O}(m_Q \hat{\Lambda})$. There are also pseudo-moduli, which are stabilized at the one-loop level and get squared masses of order $\mathcal{O}\left(\frac{m_Q \hat{\Lambda}}{16\pi^2}\right)$. Clearly these contributions are larger than those of (3).

Now assume that we take the mass of one of the flavors (denote this mass by $(m_Q)_{N_c+1}$) to be large, leaving all other masses of the same order of magnitude m_Q and small. The tree level masses are now of order

$$m^2 \sim \mathcal{O}\left(\frac{m_Q^2 \hat{\Lambda}}{(m_Q)_{N_c+1}}\right) , \tag{4}$$

and the masses of pseudo-moduli are suppressed by an extra factor of $16\pi^2$. Taking the limit $(m_Q)_{N_c+1} \to \infty$ we get the theory with $N_f = N_c$ and the confining scale

$$\Lambda^{2N_c} = (m_Q)_{N_c+1} \hat{\Lambda}^{2N_c-1} \tag{5}$$

Clearly, taking this limit we should also take the scale $\hat{\Lambda}$ to zero, preserving the confining scale of $N_f = N_c$ finite. In this limit the expression (4) tends to zero, but the noncalculable contributions are supposed to stay finite. Namely when we get to the limit $N_f = N_c$ the existence of the minimum depends only on the signs of $\alpha,\ \beta,\ \gamma$.

3 Another deformation

In this section we introduce another deformation of massive $N_f = N_c$ SQCD. We will try to find in this deformation the same extremum, as was found by ISS, and to check whether it is a minimum.

Consider SQCD with $N_f = N_c$ and singlets S_{ij}, T and $\bar{T}$. Let S be in the $(\mathbf{N}, \bar{\mathbf{N}})$ representation of the global flavor symmetry $SU(N_f)_L \times SU(N_f)_R$[2] while T and $\bar{T}$ are singlets of the flavor symmetry. We couple these singlets to the quarks and add the singlet masses. The tree level superpotential is

$$\Delta W = \lambda S_{ij} Q_i \bar{Q}_j + (m_Q)_{ij} Q_i \bar{Q}_j + \kappa(T \det Q + \bar{T} \det \bar{Q}) + \frac{m_S}{2} S^2 + \frac{m_T}{2}(T^2 + \bar{T}^2) . \tag{6}$$

This model is nothing but the Intriligator-Thomas-Izawa-Yanagida model [5],[6] with massive singlets[3]. The singlet masses are necessary if we expect to observe any effect, proportional to the quark mass; without these masses m_Q can be absorbed into redefinitions of the singlets S_{ij}.

Hereafter we assume for simplicity that the quark mass matrix is proportional to the identity. The full low energy superpotential is

$$W = \mathcal{A}(\det M - B\bar{B} - \Lambda^{2N}) + \lambda \mathrm{Tr}(MS) + \kappa(BT + \bar{B}\bar{T}) \\ m_Q \mathrm{Tr}M + \frac{m_S}{2} S^2 + \frac{m_T}{2}(T^2 + \bar{T}^2) . \tag{7}$$

Since we are interested in vacua of the SQCD with $N_f = N_c$ without any singlets we will study our theory in the limit that the gauge singlets are almost decoupled from the other fields. Obviously we can not completely decouple the singlets from the theory since then calculability is lost. Hence we will carry out our calculation in the limit where the couplings are finite and small. At the end of this analysis we will revisit the issue of calculability and determine the precise range

[1] Hereafter B_+ and B_- denote the heavy and the light combination of baryons respectively

[2] Clearly with the massive quarks this symmetry is broken, but since the breaking is small we will still use these notations.

[3] Originally this model with massless singlets was constructed as an example of the model which breaks supersymmetry dynamically, but this is irrelevant for our further discussion.

of validity for our calculations. There are two equivalent ways to decouple each singlet from the theory: one can either take its mass to be infinitely large or take its coupling to the mesons and baryons to be small. Appropriately rescaling the fields one can see, that these two ways are equivalent: one should take $\lambda/m_S^2 \to 0$ to decouple the singlets S_{ij} and $\kappa^2/m_T^2 \to 0$ to decouple T and $\bar{T}$.

Now we look for the extrema of the potential at the decoupling limit. We expect to find both supersymmetric and non-supersymmetric solutions. The supersymmetric solution should stay at a finite distance from the origin once the decoupling limit is taken. In order to match the non-supersymmetric ISS solution the meson should be stabilized at the origin of fields space. Moreover, we expect that the vacuum energy of the non-supersymmetric solution scales as $V \sim m_Q^2$.

Assuming the superpotential (7) one can find three different supersymmetric solutions. In two of these solutions the meson scales as $M \sim \left(\frac{m_T}{\kappa^2}\right)^{\frac{1}{N-1}}$. Since the distance between these solutions and the origin is infinite at the decoupling limit we will not consider these solutions further. Only the third solution is relevant for our analysis : $M \sim \Lambda$ with all other fields being stabilized at zero in the decoupling limit. This solutions lies far enough from the origin, so if one succeeds to find a SUSY breaking minimum near the origin, it could be made meta-stable.

One also finds a non-supersymmetric extremum. Treating both B_+ and B_- as dynamical fields[4] and dropping all numerical factors of order one we find

$$M \sim \left(\frac{\lambda^2}{m_s}\frac{m_T}{\kappa^2}\right)^{\frac{1}{N-2}} \Lambda, \quad B_- \sim \Lambda \tag{8}$$

All other fields are stabilized near the origin in the decoupling limit. To recover the ISS solution we should decouple the singlets S_{ij} faster than T and $\bar{T}$:

$$\frac{\lambda^2/m_S}{\kappa^2/m_T} \to 0 \ . \tag{9}$$

In order to understand whether this solution is a stable minimum, we study the mass spectrum of this solution. All the fields B and T get positive squared masses, their spectrum is supersymmetric and decoupled from the rest of the fields. The fermionic spectrum of the fields TrM and TrS contains exactly two massless states (Goldstino) and two massive state with mass of order $m_S^2 + (\lambda\Lambda)^2$. In the ISS model the mesino was the Goldstino. Here the Goldstino is a superposition of the mesino and the singlet-fermion and as expected, it reduces to the mesino in the decoupling limit. The spectrum of this sector is not supersymmetric and there is only one SUSY-breaking entry to the mass matrix. Hereafter we call this entry ξ. The full

[4] Strictly speaking B_- should not be treated as dynamical. It is heavy and should be integrated out of the theory, but as we show in [3] such proper treatment does not change qualitatively any of our results.

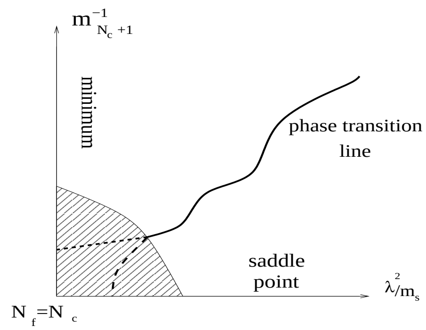

Fig. 1. Two deformations of $N_f = N_c$ SQCD. In the shaded region calculability is lost.

expression for ξ is rather cumbersome, but for large N it approximately behaves as

$$\xi \sim \frac{\lambda^2\Lambda^2}{m_S} m_Q^* \ . \tag{10}$$

As expected, it is proportional to m_Q, since the quark mass was supposed to be responsible for triggering of SUSY-breaking. The degeneracy between the scalar states is removed: the two states which are degenerate in the fermionic case are now split around the fermion mass and the splitting is proportional to ξ. But two of the four fermionic states were precisely massless! Therefore we get one tachyonic mode. Thus the point, which tends to the ISS-conjectured minimum is a saddle point in the ITIY deformation, with precisely one unstable direction. One could still hope that this tachyonic direction may be lifted by Coleman-Weinberg potential. Unfortunately it does not happen since $V_{\rm CW} \propto \xi$ and loop-suppressed, thus this contribution to the effective potential is smaller than a tree-level.

We can now estimate the range of validity of our calculations. In order to trust our calculations we should demand that the noncalculable contributions are much smaller than the calculable ones. The mass squared of the lowest (tachyonic) mode is roughly $|\xi|$ and as we have already mentioned the non-calculable contribution is always m_Q^2. Namely our calculations are valid in the range where (for large N)

$$\frac{(\lambda\Lambda)^2}{m_S} \gg m_Q \tag{11}$$

To maintain this condition one can choose for example $m_Q \ll \lambda\Lambda \ll m_S \lesssim \Lambda$. Note that m_Q should be the smallest mass parameter of the theory. Namely we can not take a "true" decoupling limit with $\lambda\Lambda$ lower than m_Q without loosing calculability.

This situation is summarized in figure 1. Consider the theory with $N_f = N_c + 1$ coupled to the massive singlets of ITIY theory. Clearly, when the couplings of quarks to the singlets are weak and the masses of the quarks are small the theory has a meta-stable minimum near the origin. On the other hand if the couplings to the singlets are significant and the mass of one of the flavors is very high we find a saddle point near the origin; the ISS extremum undergoes a phase transition. But note that if both $m^{-1}_{N_c+1}$ and $\frac{\lambda^2}{m_S}$ are small calculability is lost and we do not know how the phase transition line behaves. Hence it is impossible to determine if the extremum is a minimum or a saddle point in the $N_f = N_c$ SQCD.

4 Phenomenological consequences.

The instability described in the previous section, has direct consequences on the "Pentagon" model, which I will briefly discuss in this part of the talk.

First I introduce the "Pentagon", as it appears in the "remodelled" version [4]. Consider massive SQCD with $N_f = N_c = 5$, where the mass matrix is proportional to the identity in the flavor basis. This mass term breaks the flavor symmetry of the model down to $SU(5)_{diag}$. The Standard Model (SM) is embedded into this weakly gauged flavor $SU(5)_{diag}$. The model assumes that the local SUSY-breaking minimum, conjectured by ISS, exists near the origin of field space. SUSY is broken in this local minimum and the breaking is mediated to the SM fields by the off-diagonal components of the meson. In order to avoid fine-tuning and get viable phenomenology one should relate the μ-term to the superpartner scale. To get a μ-term of the desired magnitude the singlet S is introduced, which couples both to the Higgses of the SM and the quarks of the hidden sector. When this singlet develops a VEV of the right size, it produces a required μ-term. S couples to the quarks through the hypercharge in order not to break the SM subgroup of the $SU(5)$ and the explicit μ-term is forbidden by some discrete symmetry. The relevant part of the superpotential is:

$$W = m_Q \mathrm{Tr} M + S H_u H_d + \lambda S \mathrm{Tr}(YM) \ . \qquad (12)$$

Here Y denotes the hypercharge generator
$Y = \mathrm{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$.

It is clear from the previous discussion that one can not be sure that any meta-stable vacuum exists in the given setup. But at this stage let us believe the ISS conjecture and assume that the required minimum exists. First assume $m_Q \ll \Lambda_5$ (which is necessary for the conjectured vacuum to be meta-stable)[5] and $\lambda \ll 1$.

Our statement is that with these assumptions one can not get a viable spectrum for the superpartner masses from the "Pentagon" model. Recall that all the bosons will get noncalculable masses squared of order $m_Q \Lambda_5$. The masses of the fermions are much smaller since they involve some power of λ, and we have assumed that it is small. Namely we generate a large positive messenger supertrace. But as it was shown in [7] such a supertrace will give large *negative* contributions to the squark masses squared.

To avoid this phenomenological disaster one should take λ to be sufficiently large. But taking it to be large S acts precisely as $\mathrm{Tr} S$ in our previous discussion. Calculability is restored but the conjectured minimum is destabilized. Thus the "Pentagon" is excluded when λ is either too large or too small. In the intermediate range the model is completely non-calculable and one can make no statement about the model. Although it is quite possible that at this range of energies one gets very complicated potential with more local minima, it is also worth to notice, that this conjectured minimum can not be directly related to the ISS minimum. Since λ and m_Q should be both relatively large, one can not rely on the expansion (2) and we should know the entire Kähler potential in order to understand the nature of the conjectured minimum near the origin. Consequently, if the conjectured local minimum of the Pentagon exists, one can not rely on [1], making any statement about its metastability or the properties of the related sparticle spectrum.

[5] Hereafter Λ_5 denotes the "Pentagon" confining scale

5 Conclusions

We conclude that there is no clear indication whether the meta-stable SUSY breaking minimum exists in $N_f = N_c$ SQCD. We showed that no additional evidence can be gained by deforming the theory. A point which was a minimum of one deformation is a saddle point of another deformation. We have also demonstrated that the instability which we found significantly restricts the range, for which the "Pentagon" model is not excluded. In particular we emphasize that in this range the "Pentagon" is necessarily non-calculable and its phenomenologically viable minimum, even if it exists, can not be directly related to the ISS-conjectured minimum.

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