Studies of B decays with missing energy at Belle

The 15th International Conference on Supersymmetry and the Unification of Fundamental Interactions
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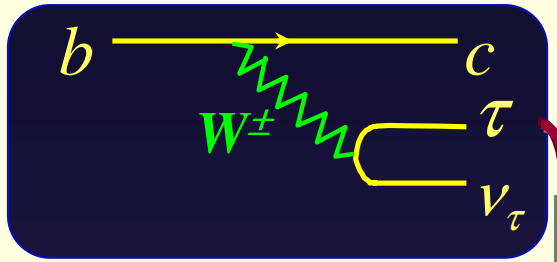
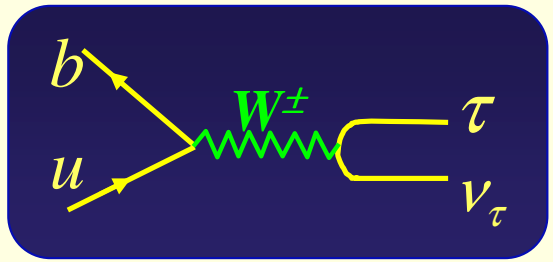
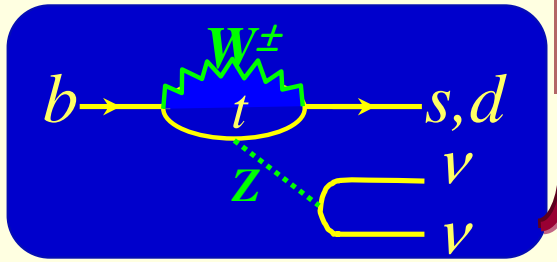
Maria Różańska
for the Belle Collaboration,
INP PAN, Kraków

OUTLINE

- motivation
- experimental techniques
- results
- summary

- $B \to h\ \bar{\nu}\nu$
- $B^+ \to \tau^+ \nu_\tau$
- $B^0 \to D^{*-} \tau^+ \nu_\tau$

Motivation

	expected decay rates	examples of SM amplitudes
$B^0 \to D^{*-}\tau^+\nu_\tau$	$\mathcal{O}(10^{-2})$	
$B^+ \to \tau^+\nu_\tau$	$\mathcal{O}(10^{-4})$	
$B \to h\ \bar{\nu}\nu$	$\leq\mathcal{O}(10^{-5})$	

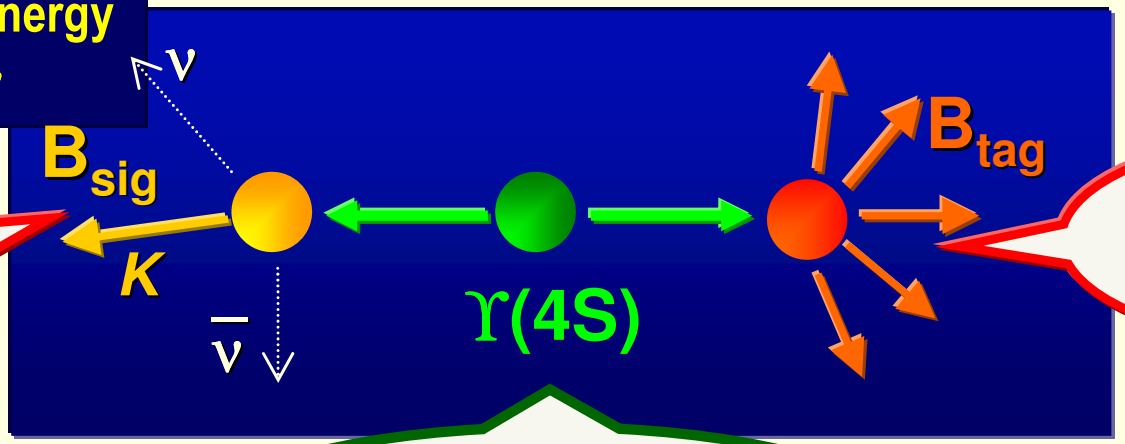
Small hadronic effects; theoretically clean.

Sensistive to New Pysics

poorly known: multiple ν's in final states ⇒ experimentaly difficult !

Experimental Techniques

B decay with missing energy
e.g. $B^+ \to K^+ \nu\nu$



signature: K + invisible

reconstruct B_{tag}

at B-factories: $e^+e^- \to \Upsilon(4S) \to B\bar{B}$

B_{tag} reconstruction:

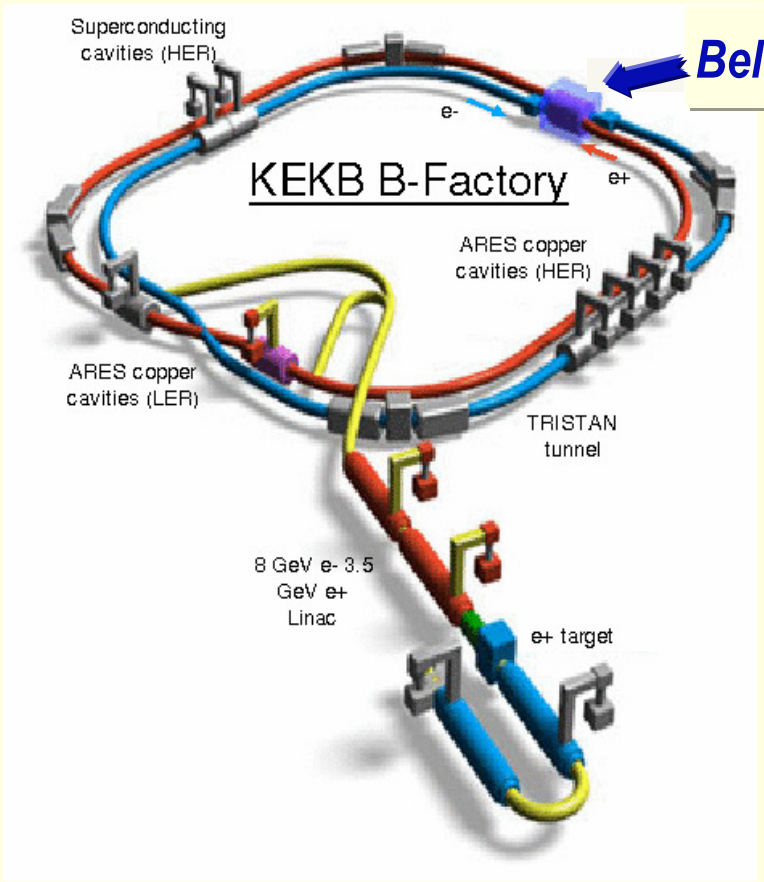
- $B\bar{B}$ event
- which particles belong to B_{sig}
- kinematical constraints on B_{sig}

$$\vec{p}_{sig} = -\vec{p}_{tag}$$

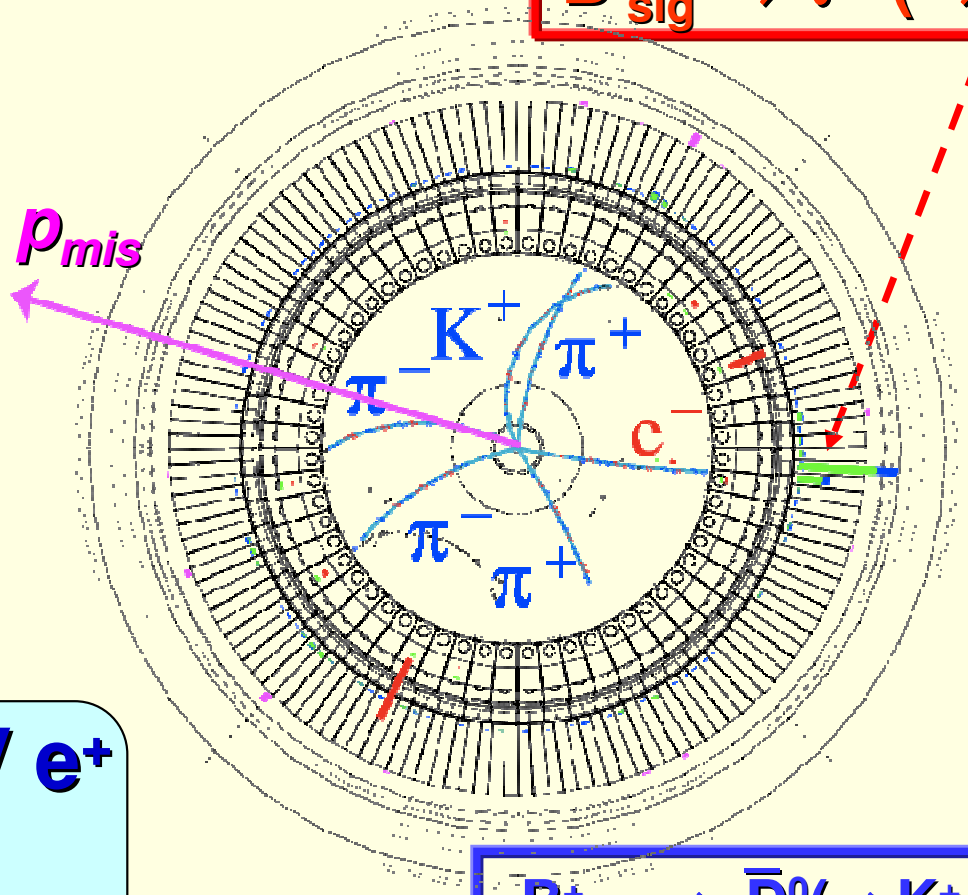
Two ways of B_{tag} reconstruction:

- Select B_{sig} candiate and check whether remaining particles consistent with B decay („inclusive" B_{tag} reconstruction)
- Reconstruct B_{tag} (in exclusive mode) and check whether remaining particles consistent with B_{sig} („exclusive" B_{tag} reconstruction)

KEKB / Belle



***Belle* detector:** multi-purpose, large-solid-angle magnetic spectrometer



8 GeV e^- × 3.5 GeV e^+

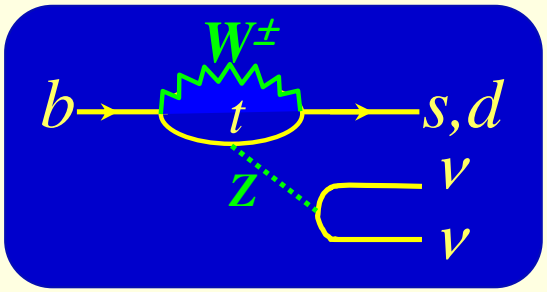
$L_{peak} = 1.71 \times 10^{34}$

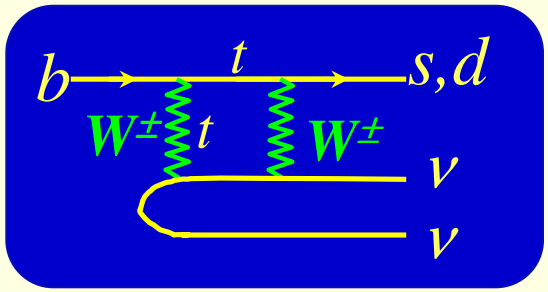
Integ. Lum. ~700 fb^{-1}

$B \to h^{(*)} \nu\nu$

Flavor Changing Neutral Current process:

Z-mediated electroweak penguin + box diagrams





Expected BF's in the SM:

$$BF(B \to K^* \nu\bar{\nu}) \cong 1.1 \times 10^{-5}$$

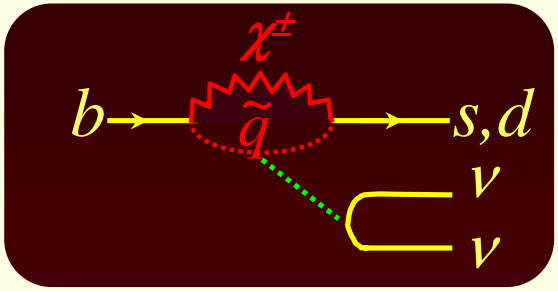
$$BF(B \to K \nu\bar{\nu}) \cong 5.3 \times 10^{-6}$$

$$BF(B \to \rho\nu\bar{\nu}) \cong 4.9 \times 10^{-7}$$

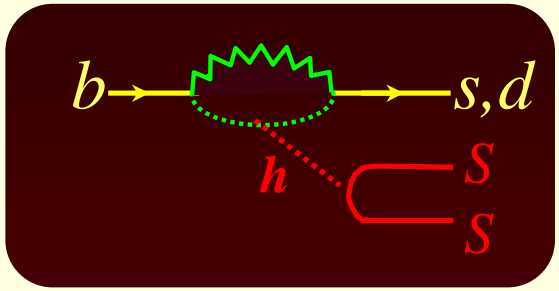
$$BF(B \to \pi\nu\bar{\nu}) \cong 2.2 \times 10^{-7}$$

J. H. Jeon *et al.*, PL B **636**, 270 (2006)

Sensistive to New Physics in loops, e.g.:

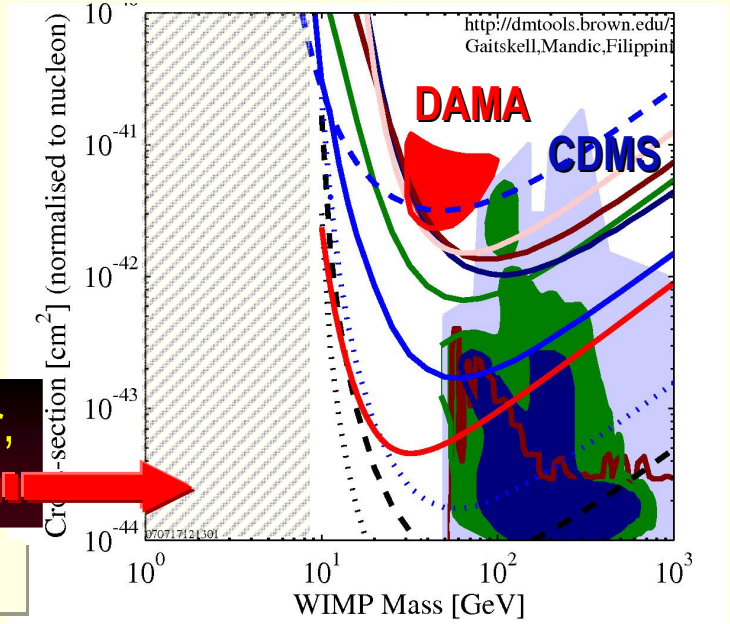


other weakly coupled particles:



possible window to light dark matter, not accessible in direct searches

e.g. C. Bird *et al.*, PRL **93**, 201803 (2004)

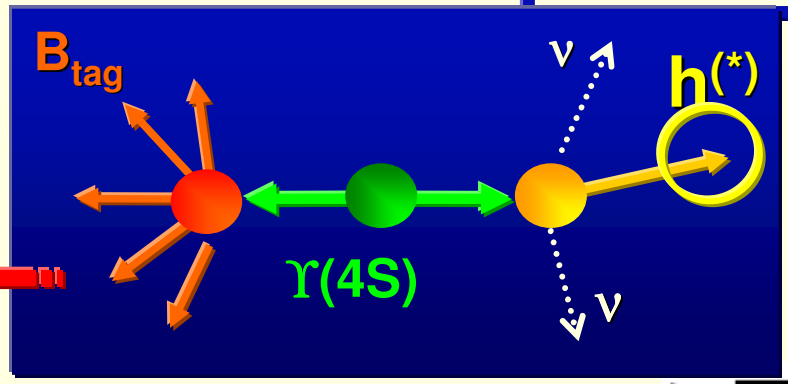


$B \to h^{(*)} \nu\nu$ - method

Reconstruct B_{tag} in hadronic mode:

$$B^0 \to \bar{D}^{(*)-}\pi^+/\rho^+/a_1^+/D_s^{(*)+}$$
$$B^+ \to \bar{D}^{(*)0}\pi^+/\rho^+/a_1^+/D_s^{(*)+}$$

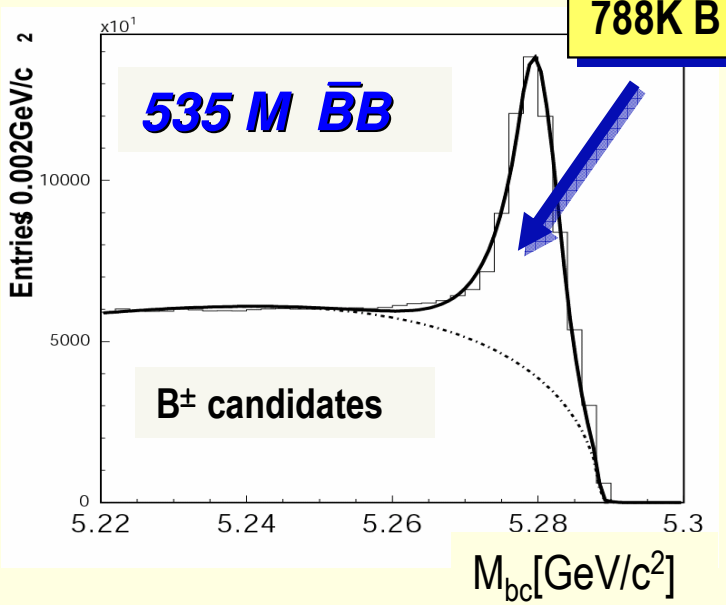
$$\Delta E = \Sigma E_i - E_{beam}$$
$$M_{bc} = \sqrt{E^2_{beam} - (\Sigma \mathbf{p}_i)^2}$$

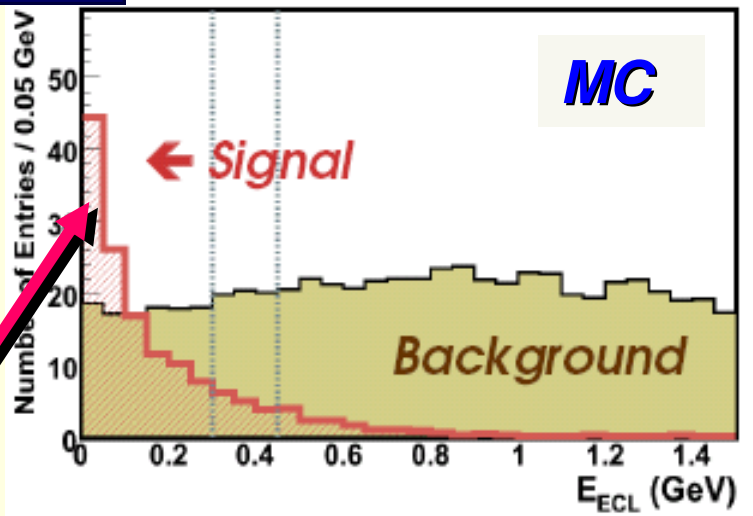


$h^{(*)} = K^{*+}, K^{*0}, K^+, K^0, \rho^+, \rho^0, \pi^+, \pi^0, \varphi$

signal signature:

$h^{(*)}$ + *nothing*





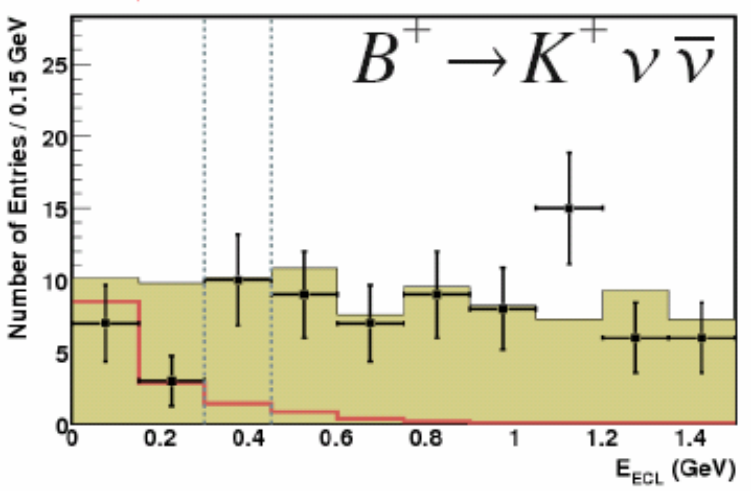
E_{ECL}: residual energy in calorimeter
for signal: $E_{ECL} \approx 0$

background suppression: $1.6 < p_{h^{(*)}} < 2.5$ GeV/c

suppress b→c

reject 2-body (eg. $B \to K^*\gamma$)

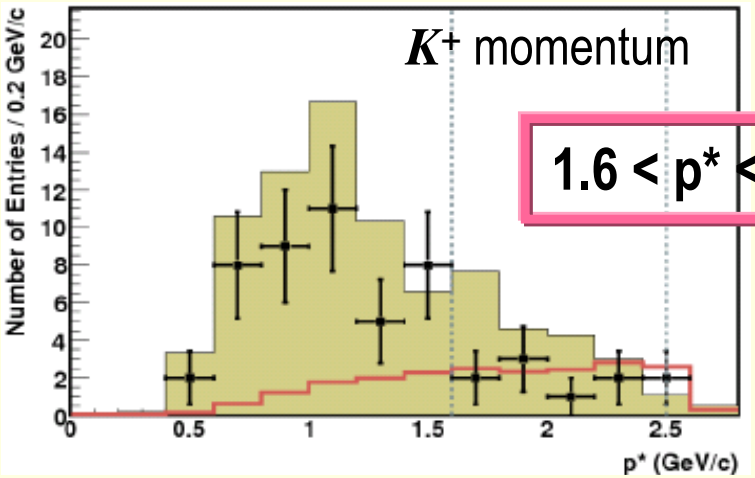
$B \to h^{(*)} \nu\nu$ - results



signal
SM $BF \times 20$

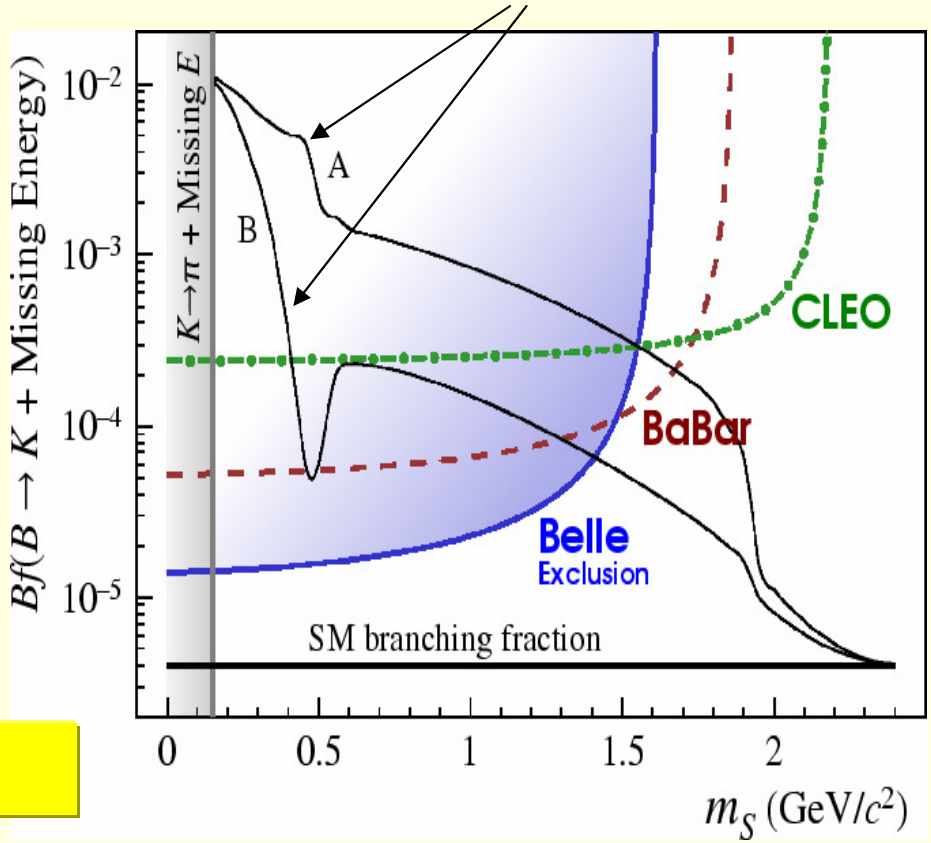
535 M $B\bar{B}$

hep-ex/0707.0138
submitted to PRL



$1.6 < p^* < 2.5$ GeV/c

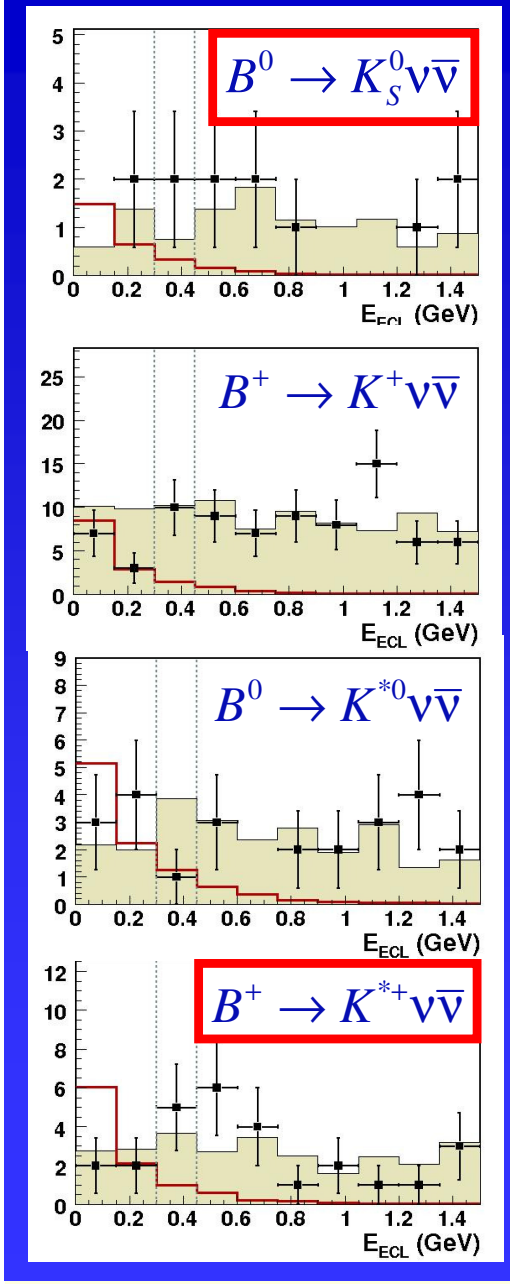
Theoretical predictions:
C. Bird *et al.*, PRL **93**, 201803 (2004)

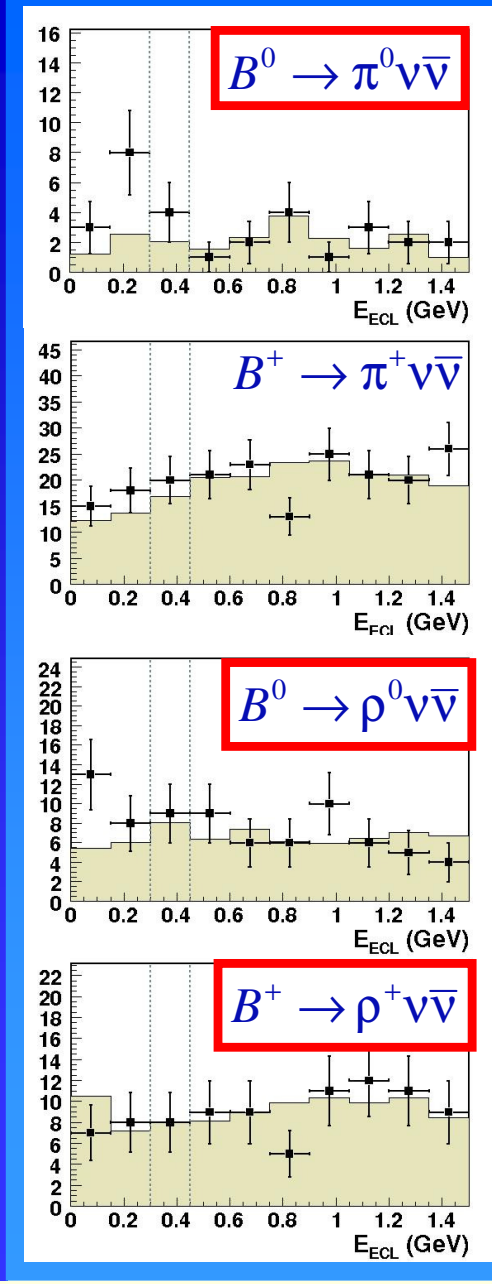


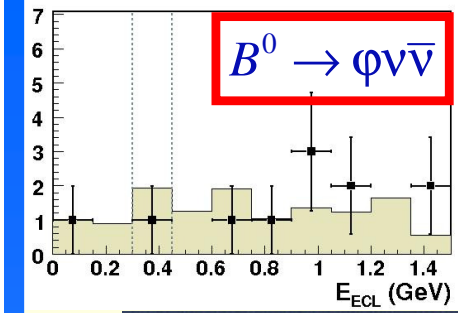
$N_b = 20.0 \pm 4.0$ $N_{obs} = 10$

$BF(B^+ \to K^+\nu\nu) < 1.4 \times 10^{-5}$ @90% CL

$B \to h^{(*)} \nu\nu$ - results



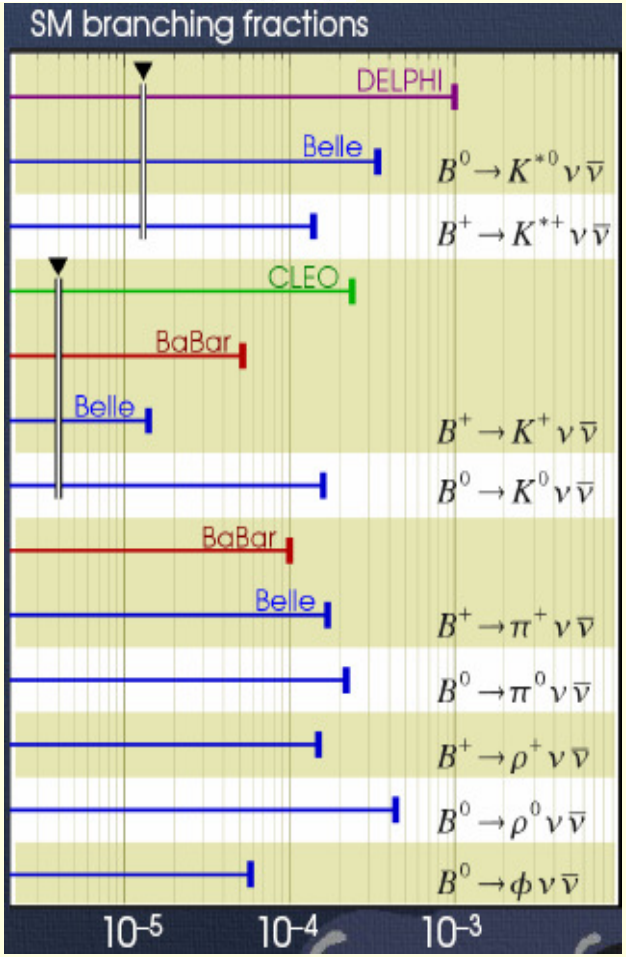




first measurements

535 M $B\bar{B}$

hep-ex/0707.0138
submitted to PRL

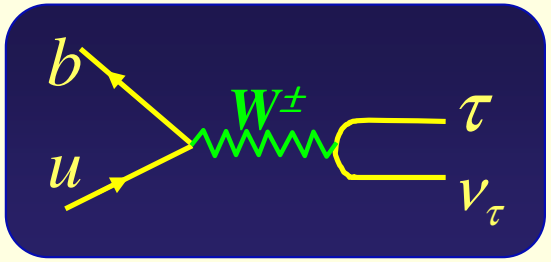


UL @ 90%CL

$B^0 \to K^{*0} \nu\bar{\nu}$	$< 3.4 \times 10^{-4}$
$B^+ \to K^{*+} \nu\bar{\nu}$	$< 1.4 \times 10^{-4}$
$B^+ \to K^+ \nu\bar{\nu}$	$< 1.4 \times 10^{-5}$
$B^0 \to K^0 \nu\bar{\nu}$	$< 1.6 \times 10^{-4}$
$B^+ \to \pi^+ \nu\bar{\nu}$	$< 1.7 \times 10^{-4}$
$B^0 \to \pi^0 \nu\bar{\nu}$	$< 2.2 \times 10^{-4}$
$B^+ \to \rho^+ \nu\bar{\nu}$	$< 4.4 \times 10^{-4}$
$B^0 \to \rho^0 \nu\bar{\nu}$	$< 1.5 \times 10^{-4}$
$B^0 \to \phi \nu\bar{\nu}$	$< 5.8 \times 10^{-5}$

$B \to \tau \nu_\tau$

purely leptonic B decay:
W-mediated annihilation



theoretically very clean, SM BF:

$$BF(B \to l\nu) = \frac{G_F^2 m_B}{8\pi} m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

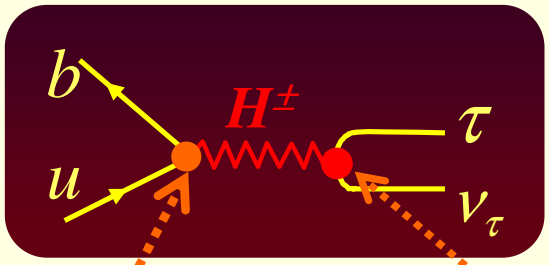
$$BF(B^+ \to \tau^+ \nu_\tau) = (1.59 \pm 0.40) \times 10^{-4}$$

B decay constant

f_B=0.216±0.022 GeV from LQCD HPQCD Collab., PRL **95**, 212001 (2005)

Sensistive to
Charged Higgs

providing f_B is known



$m_b \tan\beta + m_c \cot\beta$ $m_\tau \tan\beta$

Decay amplitude $\propto m_b m_\tau \tan^2\beta$

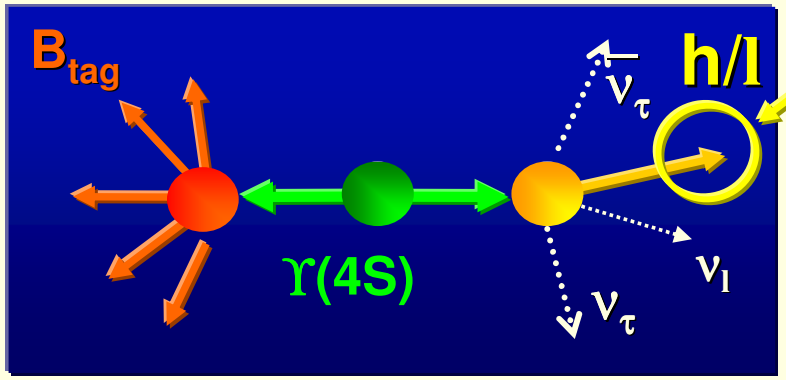
$H^\pm$ effects to branching fraction:

$$BF(B^+ \to \tau^+ \nu_\tau) = BF(B^+ \to \tau^+ \nu_\tau)_{SM} \times r_H$$

$$r_H = \left(1 - \frac{m_B^2}{m_H^2} \tan^2\beta\right)^2$$

W. S. Hou, PR D **48**, 2342 (1993)

$B \to \tau \nu_\tau$ - analysis



visible products of τ decay

$h = \rho^\pm, \pi^\pm, (3\pi)^\pm$, $l = e^\pm, \mu^\pm$

81% of all modes

449 M $B\bar{B}$

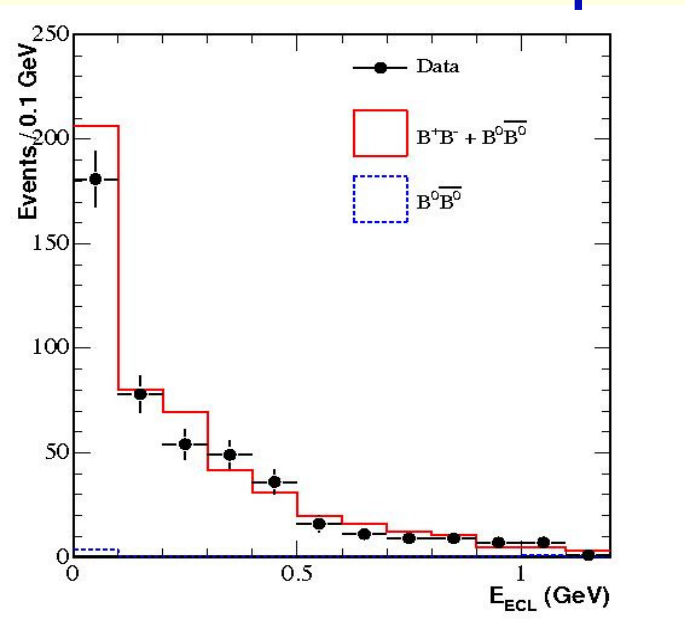
PRL **97**, 251802 (2006)

Find $17.2^{+5.3}_{-4.7}$ signal events from a fit to a sample of 54 events.

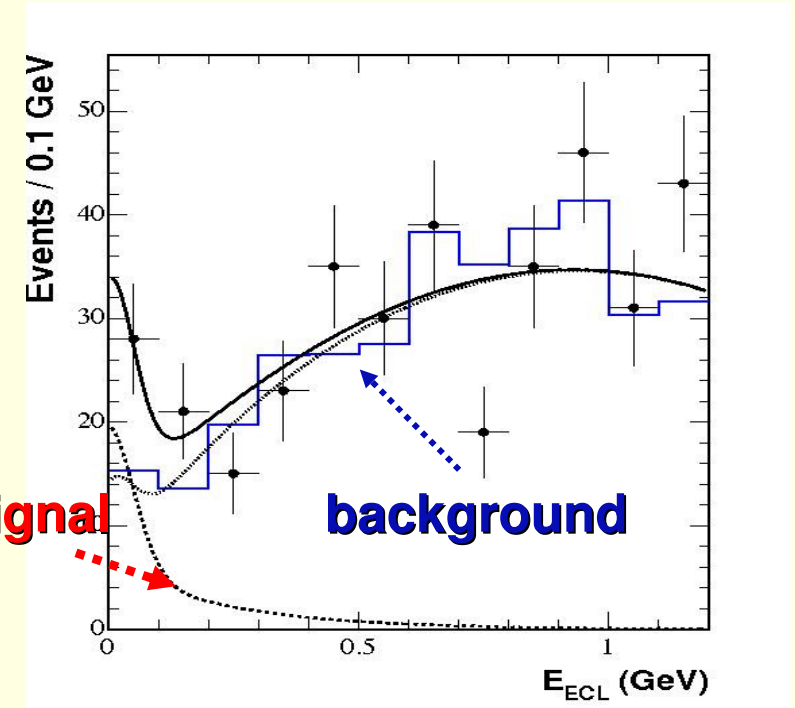
4.6σ stat. significance $\Rightarrow 3.5\sigma$ (syst. included)

validate E_{ECL} simulation using

$B \to D^{*0} l \nu$ control sample



FIRST EVIDENCE



$B \to \tau \nu_\tau$ - results

$$BF(B \to \tau\nu_\tau) = (1.79^{+0.56}_{-0.49}(stat)^{+0.46}_{-0.51}(syst)) \times 10^{-4}$$

449 M $B\bar{B}$

PRL **97**, 251802 (2006)

$$f_B = 229^{+36}_{-31}(stat)^{+34}_{-37}(syst)\ \text{MeV}$$

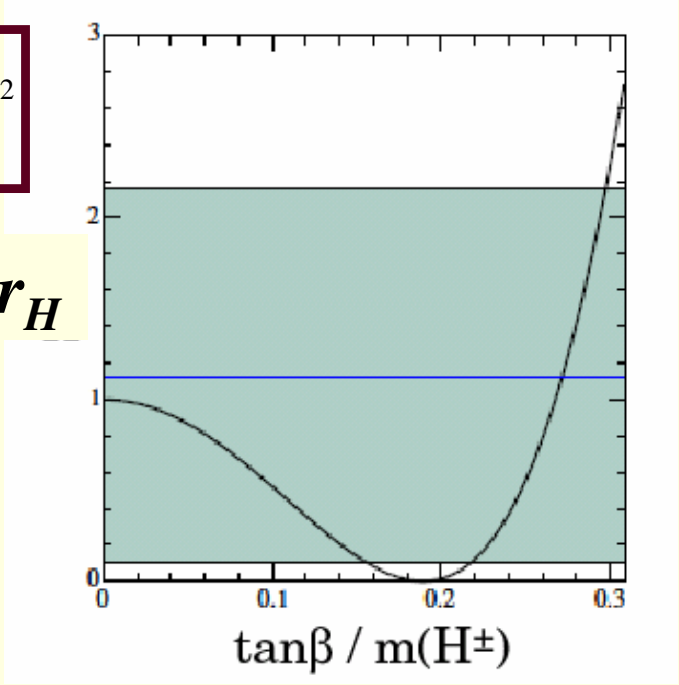
taking $|V_{ub}| = (4.39 \pm 0.33) \times 10^{-3}$ from HFAG

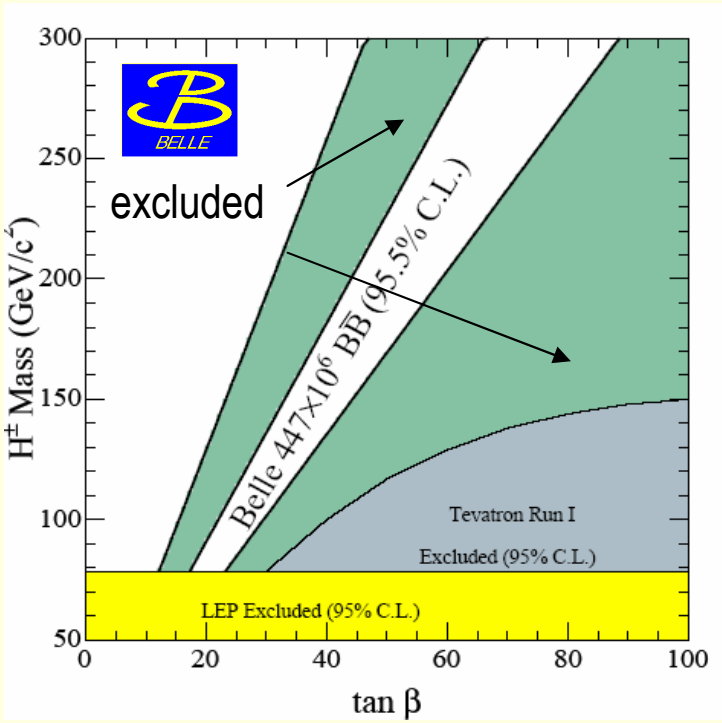
$$r_H = 1.13 \pm 0.51$$

$$(BF(B^+ \to \tau^+\nu_\tau)_{SM} = (1.59 \pm 0.40) \times 10^{-4})$$

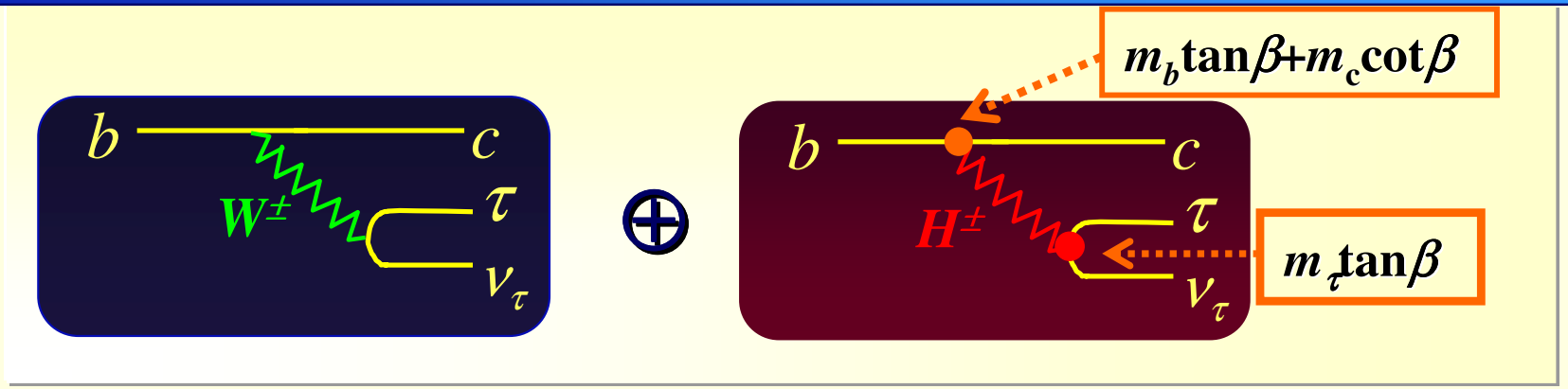
Constraint on Charged Higgs (2HDM II)

$$r_H = (1 - \frac{m_B^2}{m_H^2}\tan^2\beta)^2$$





$B \to D^{(*)} \tau \nu_\tau$



Theoretical tool:

Heavy Quark Effective Theory (HQET)

Expected SM BF's~$\mathcal{O}(10^{-2})$

- ✓ Sensitive to extended Higgs sector
- ✓ New Physics at tree level
- ✓ Sensitive observables
  e.g. τ polarization; possible $\mathcal{O}(1)$ effects

inclusive BF($b \to c\tau\nu_\tau$) = (2.48 ±0.26)% from LEP — PDG 2007

Y.Okada: *CP violation & CKM; plenary talk at ICHEP06*

H-b-u vertex measured in $B \to \tau\nu_\tau$

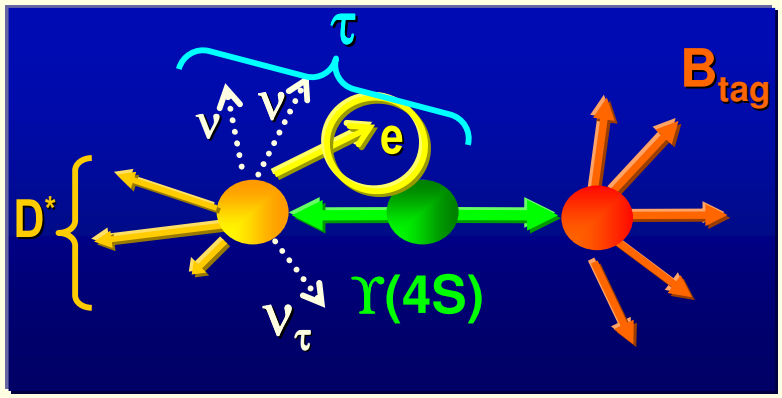
H-b-c vertex measured in $B \to D\tau\nu_\tau$

H-b-t vertex measured in direct production by LHC.

$B^0 \to D^{*-}\tau^+\nu_\tau$ - method

clean signature

$D^{*-}\, e^+ + p_{mis}$



reconstruct B_{tag} inclusively

$$\Delta E_{tag} = \Sigma E_i - E_{beam}$$

$$M_{tag} = \sqrt{E^2_{beam} - (\Sigma \mathbf{p}_i)^2}$$

$\Sigma Q = 0$,
no extra leptons,
$\Sigma N_{\bar{p}p} = 0$
$-0.25\ \text{GeV} < \Delta E_{tag} < 0.05\ \text{GeV}$

verify B_{tag} reconstruction

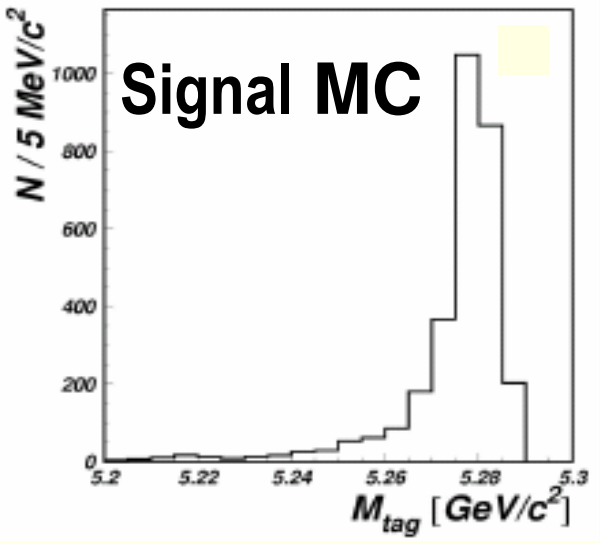
Control sample :
$B^0_{sig} \to D^{*-}\pi^+$

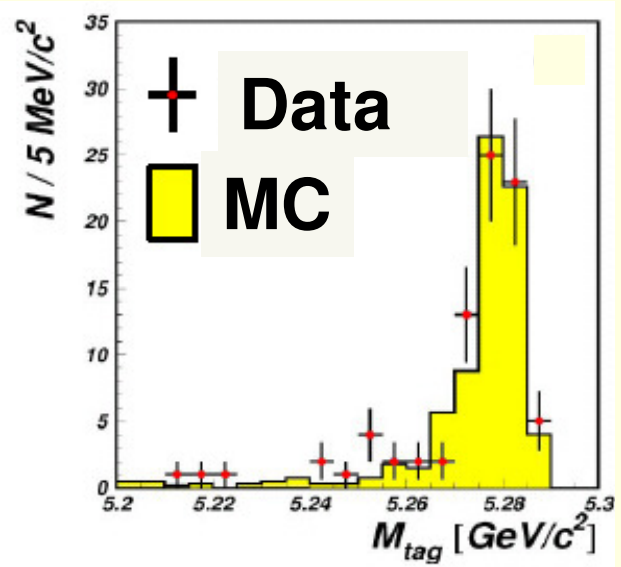
apply all the tag-side selection criteria

Signal sub-decay modes:

$D^{*-} \to \bar{D}^0\pi^-$
$\tau \to e\nu\nu$, $\bar{D}^0 \to K^+\pi^-$
$\tau \to e\nu\nu$, $\bar{D}^0 \to K^+\pi^-\pi^0$
$\tau \to \pi\nu$, $\bar{D}^0 \to K^+\pi^-$

$\tau \to \mu\nu\nu$ not used;
(μ-ID inefficient for soft leptons)



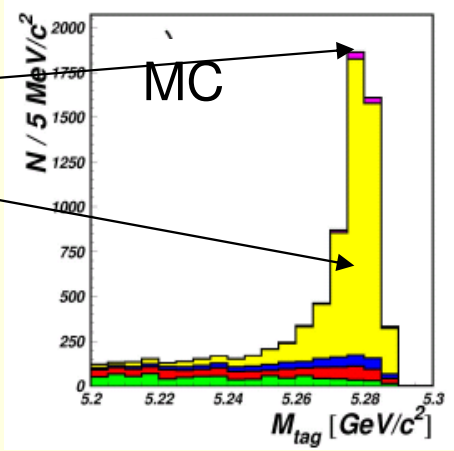


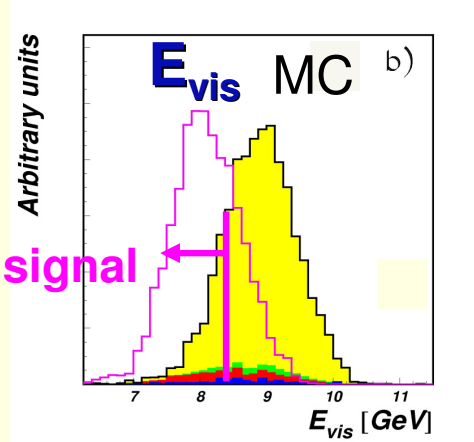
$B^0 \to D^{*-}\tau^+\nu_\tau$ - analysis

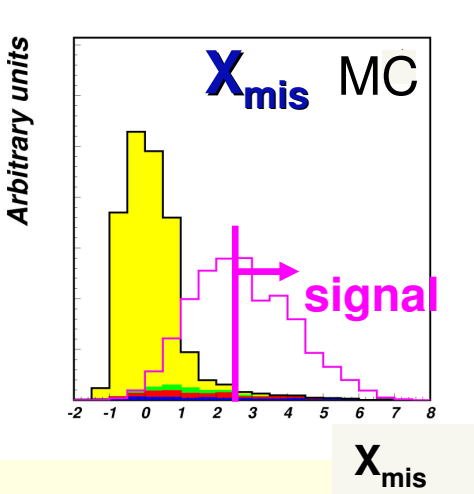
$\tau \to e\bar{\nu}\nu$ (3ν)

background suppression

- signal
- D*e ν
- D**e ν
- other B dec.
- continuum







exploit signal-side variables:

visible energy: E_{vis}<8.3 GeV

X_{mis}>2.75

$X_{mis} \equiv (E_{mis} - |\mathbf{p}_{D^*} + \mathbf{p}_{e/\pi}|)/|\mathbf{p}_B|$

$E_{mis} \equiv E_{beam} - E_{D^*} - E_e$: 1.9<$E_{mis}$<2.6 GeV

missing mass: $M_M^2 \equiv (E_{mis})^2 - (\mathbf{p}_{sig} - \mathbf{p}_{D^*} - \mathbf{p}_{e/\pi})^2$

virtual W mass: $M_W^2 \equiv (E_b - E_{D^*})^2 - (\mathbf{p}_{sig} - \mathbf{p}_{D^*})^2$

$\tau \to \pi\nu$ (2ν)

combinatorial background from hadronic B decays dominates

$E_{vis} < 8.3$ GeV, $X_{mis} > 1.5$

$M_W^2 - M_M^2 - M_\tau^2 + M_\pi^2 > 0$ (exact kinematical constraint)

energy of π $E_\pi > 0.6$ GeV

No K_L in the event (suppress $B \to D^{(*)}K_L + X$)

Number of tracks with bad impact parameter $N_{bad} < 4$ (suppress $B \to D^* n\bar{n} + X$)

$B^0 \to D^{*-} \tau^+ \nu_\tau$ - analysis

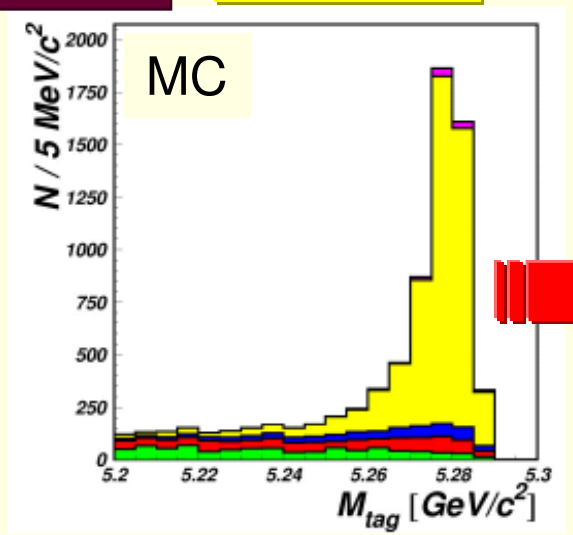
$\tau \to e\nu\nu$

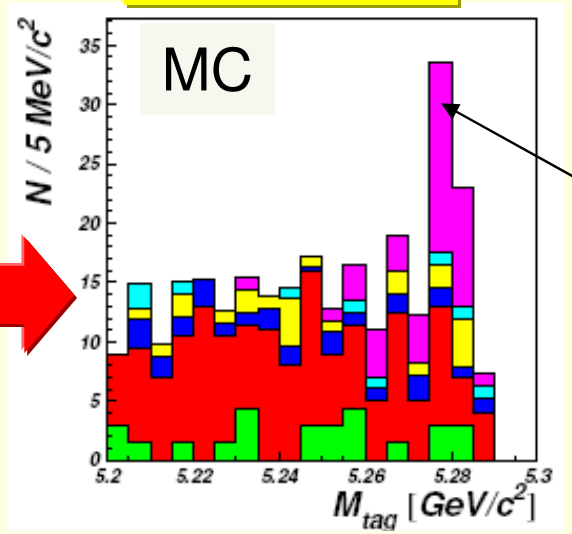
before

after cuts

535 M $B\bar{B}$

hep-ex/0707.0138
submitted to PRL





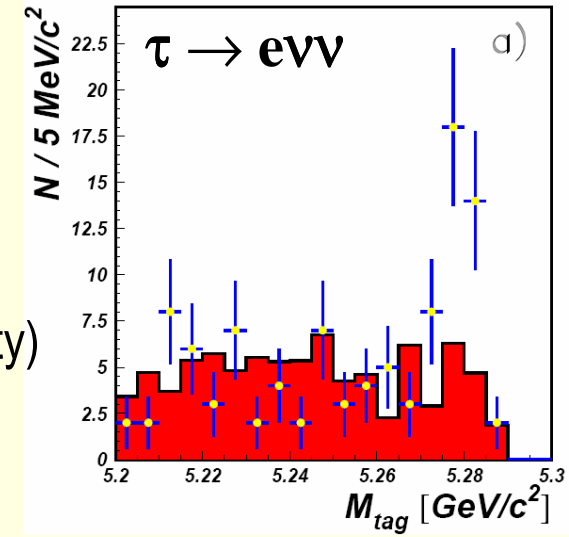
signal
D*e ν
D**e ν
other B decays
continuum

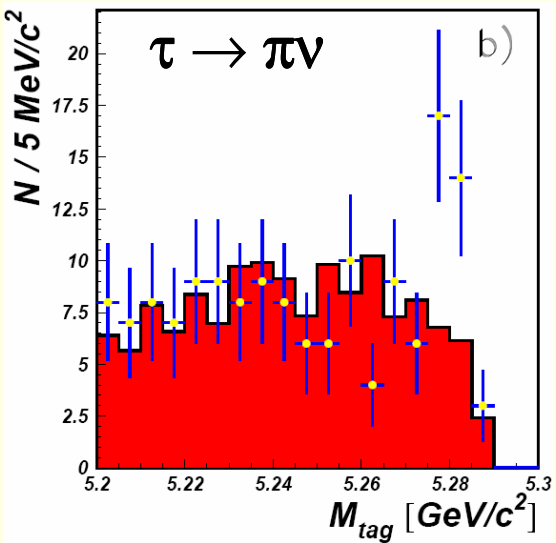
clear signal peak expected

AND OBSERVED IN DATA

DATA

Expected bckground
(MC scaled to data luminosity)





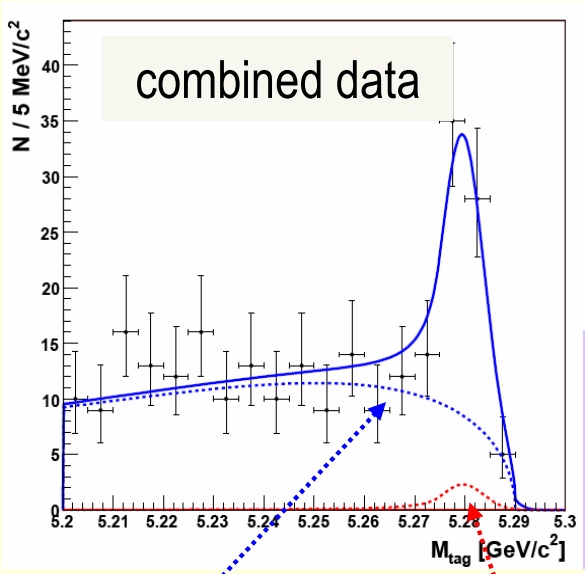
$B^0 \to D^{*-}\tau^+\nu_\tau$ - results

SIGNAL YIELD $N_s = 60^{+12}_{-11}$ **6.7σ (5.2σ with syst.)**

535 M $B\bar{B}$

from a combined maximum likelihood fit (with a single *BF*) to 3 M_{tag} distributions

hep-ex/0707.0138
submitted to PRL



combinatorial background

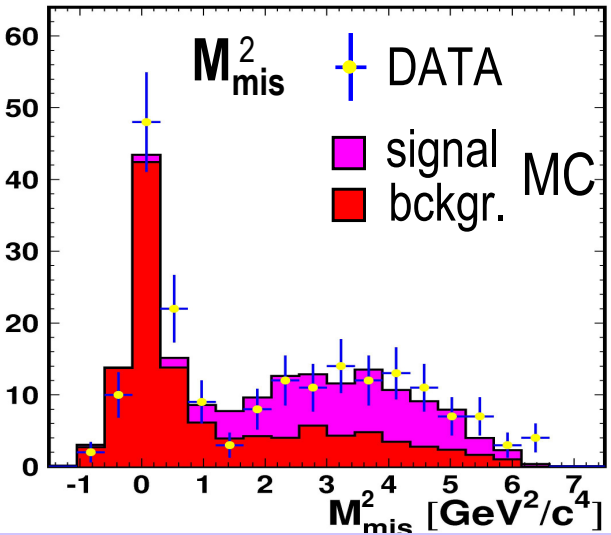
peaking background ($D^*e\nu$)

FIRST OBSERVATION

$$BF(B^0 \to D^{*-}\tau^+\nu_\tau) = (2.02^{+0.40}_{-0.37}(stat) \pm 0.37(syst)) \times 10^{-2}$$

CROSS-CHECKS

- seaparate fits to sub-channels
- check look-back plots
- signal yield from signal-side varibales



SM	channel	*BF*	fit variable
	$\tau\to e\nu\nu$, $D\to K\pi$	$2.44^{+0.74}_{-0.65}$%	M_{tag}
	$\tau\to e\nu\nu$, $D\to K\pi\pi^0$	$1.69^{+0.84}_{-0.74}$%	M_{tag}
	$\tau\to \pi\nu$, $D\to K\pi$	$2.02^{+0.68}_{-0.61}$%	M_{tag}
	combined	$2.02^{+0.40}_{-0.37} \pm 0.37$%	M_{tag}
	$\tau\to e\nu\nu$,	1.83 ±0.43%	M^2_{mis}
	$\tau\to \pi\nu$,	1.96 ±0.41%	M^2_{mis}
	combined	1.90 ±0.35%	M^2_{mis}
	$\tau\to \pi\nu$,	2.05 ±0.56%	$\cos\theta_{\nu_1\nu_2}$

SUMMARY

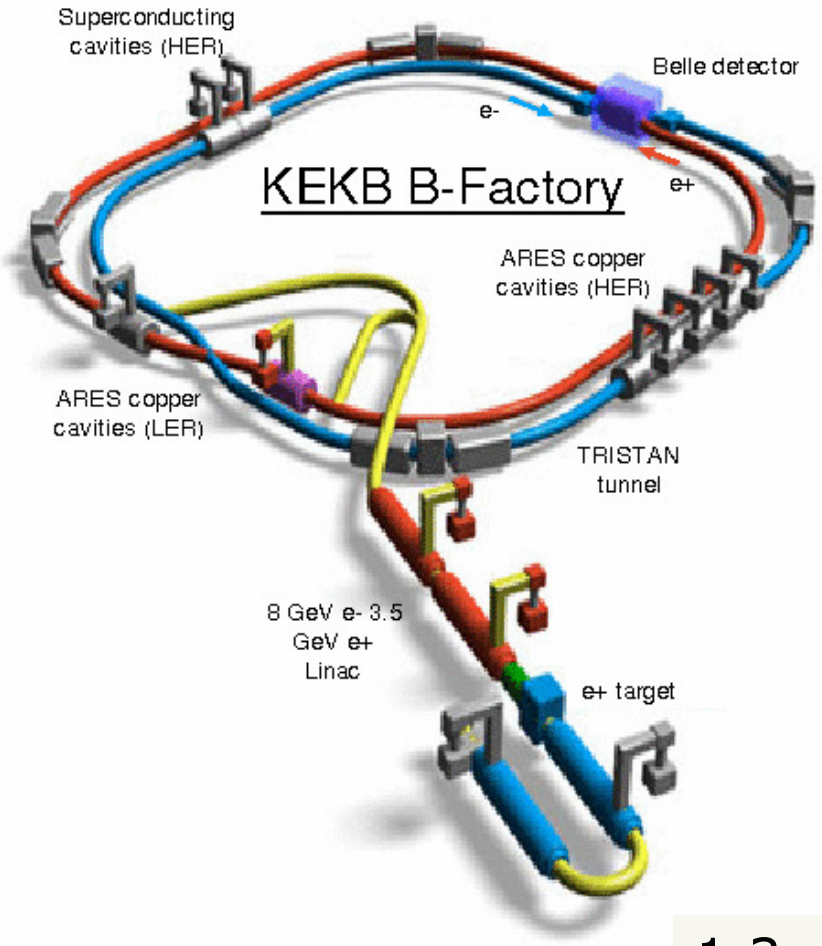
Reach program of $B \to E_{mis}$ studies is being pursued in Belle

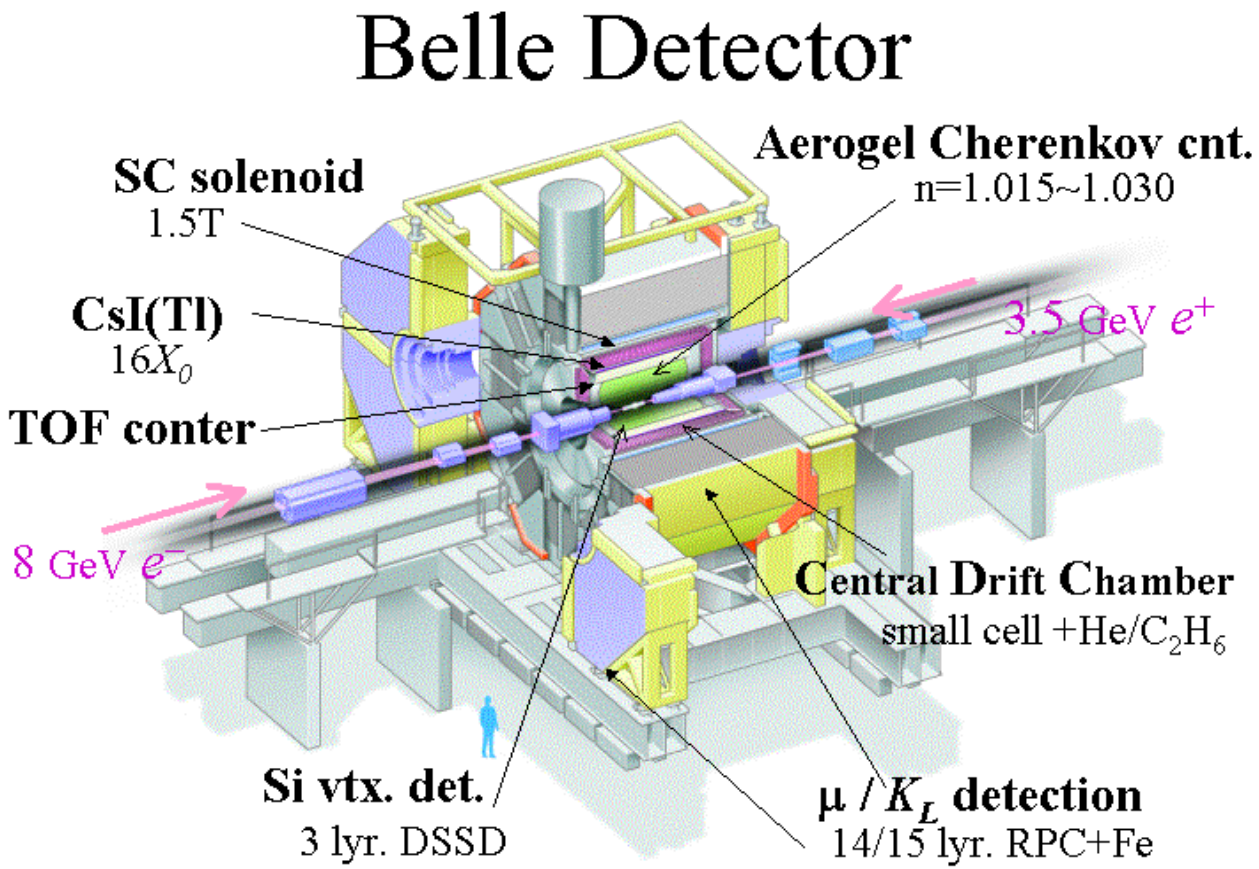
- **$B \to h^{(*)} \nu\nu$**
  - new measurements for 6 modes ($K^{*+}\nu\nu$, $K_S\nu\nu$,$\pi^0\nu\nu$, $\rho^0\nu\nu$, $\rho^+\nu\nu$, $\varphi\nu\nu$)
  - improved upper limits for $K^+\nu\nu$ and $K^{*0}\nu\nu$ modes
  - upper limit at 90% CL for $K^+\nu\nu$ mode $\approx 3\times BF_{SM}$
- **$B \to \tau\nu$**
  - first evidence (3.5σ) of purely leptonic B decay
  - constraints on $H^{\pm}$ competitive with direct searches
- **$B \to D^*\tau\nu$**
  - first observation (5.2σ) of exclusive B decay with $b \to c\tau\nu_\tau$ transition in the $B^0 \to D^{*-}\tau\nu_\tau$ mode

Looking forward to Super B-factory

BACKUP

Belle Experiment





1.3 million B $\bar{B}$ pairs / day

Total ~770 $\times$ 10^6 B $\bar{B}$ pairs

$B \to h^{(*)} \overline{\nu\nu}$: Systematic Uncertainties

Associated with the normalization (siganl efficiency and N(BB)) in %

Sources	K^{*0}	$K^{*+}_{(K_S\pi^+)}$	$K^{*+}_{(K^+\pi^0)}$	$K^{*+}_{\text{(combined)}}$	K^+	K_S	π^+	ρ^0	ϕ	π^0	ρ^+
$N(B\overline{B})$	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
Tracking	2.1	1.1	1.0	1.1	1.0	-	1.0	2.2	2.0	-	1.1
K_S/π^0 Rec.	-	4.9	4.0	4.4	-	4.9	-	-	-	4.0	4.0
Sub. BR	-	-	-	-	-	-	-	-	1.2	-	-
PID	1.3	0.5	0.8	0.7	0.7	-	0.5	1.0	2.0	-	0.5
MC stat.	3.5	3.4	3.3	2.4	1.9	3.2	2.0	3.3	2.3	2.8	3.4
Mass resol.	0.8	1.2	3.3	2.3	-	-	-	1.1	2.0	-	2.6
full-rec. tags	2.0	9.9	9.9	9.9	9.9	2.0	9.9	2.0	2.0	2.0	9.9
Veto eff.	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
Form factors	1.6	0.9	1.6	1.3	2.6	0.4	3.0	1.7	13.0	1.0	3.7
Sum	±5.9	±12.1	±12.2	±11.8	±10.9	±6.9	±11.0	±5.8	±14.2	±6.1	±12.5

Associated to BG signal-to-sideband ratio

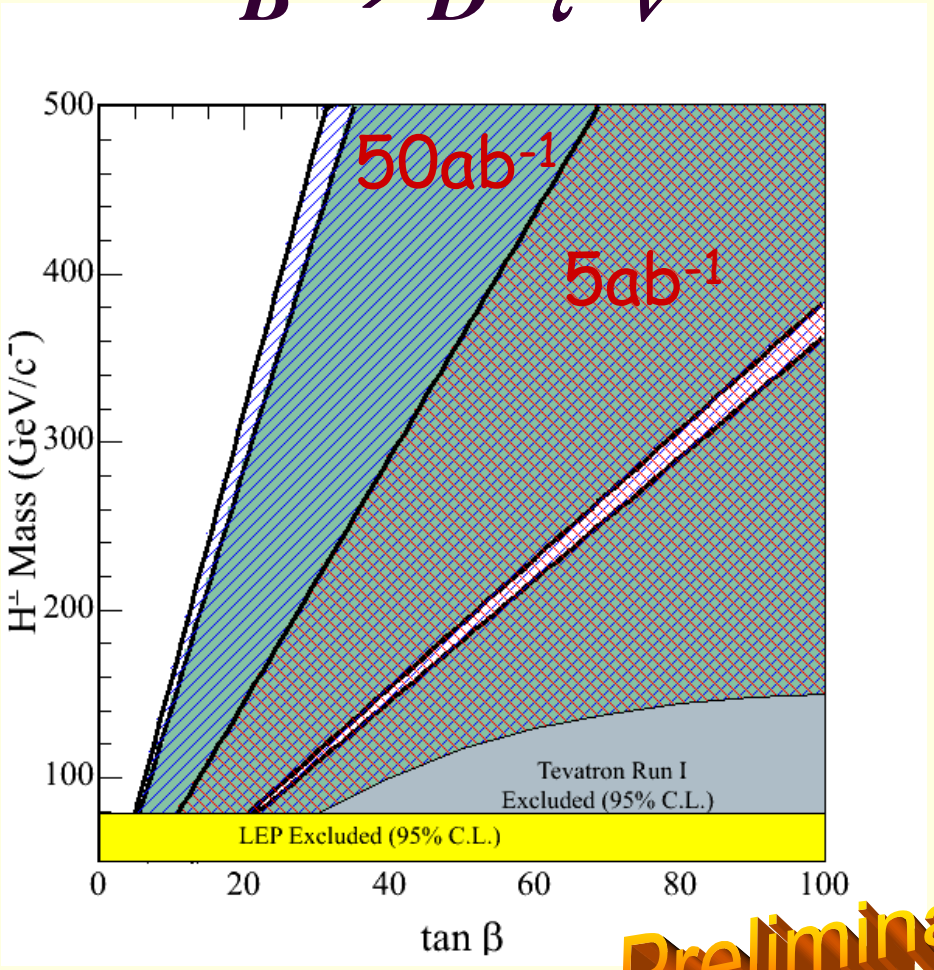
	K^{*0}	$K^{*+}_{(K_S\pi^+)}$	$K^{*+}_{(K^+\pi^0)}$	K^+	K_S	π^+	ρ^0	ϕ	π^0	ρ^+
Central value	0.260	0.328	0.303	0.333	0.249	0.174	0.250	0.212	0.252	0.269
MC stat.	0.057	0.115	0.078	0.043	0.073	0.013	0.028	0.069	0.053	0.025
MC/data diff.	0.020	0.025	0.023	0.025	0.019	0.013	0.019	0.016	0.019	0.021
Rare B decay	0.008	0.004	0.024	0.004	0.019	0.012	0.003	0.028	0.003	0.013
Sum	±0.061	±0.117	±0.084	±0.050	±0.077	±0.022	±0.034	±0.076	±0.056	±0.035

Prospects for $H^{\pm}$ sensitivity

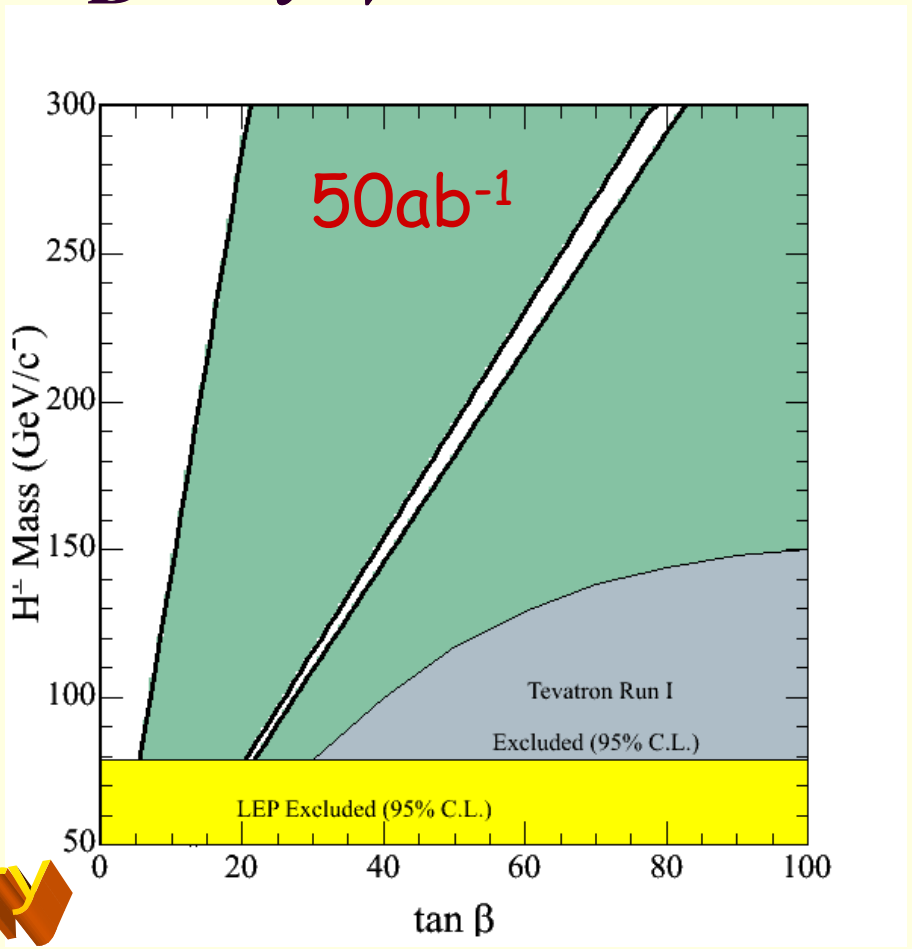
Super KEKB
uest for BSM

Super B Factory

$B \to D^{-} \tau^{+} \nu$



$B \to \tau^{+} \nu$



Preliminary

$B \to \tau \nu_\tau$ - signal selection

- τ lepton is identified in the 5 decay modes

$$\tau^- \to \mu^- \nu\bar{\nu},\ e^- \nu\bar{\nu},\ \pi^- \nu,\ \pi^- \pi^0 \nu,\ \pi^- \pi^+ \pi^- \nu$$

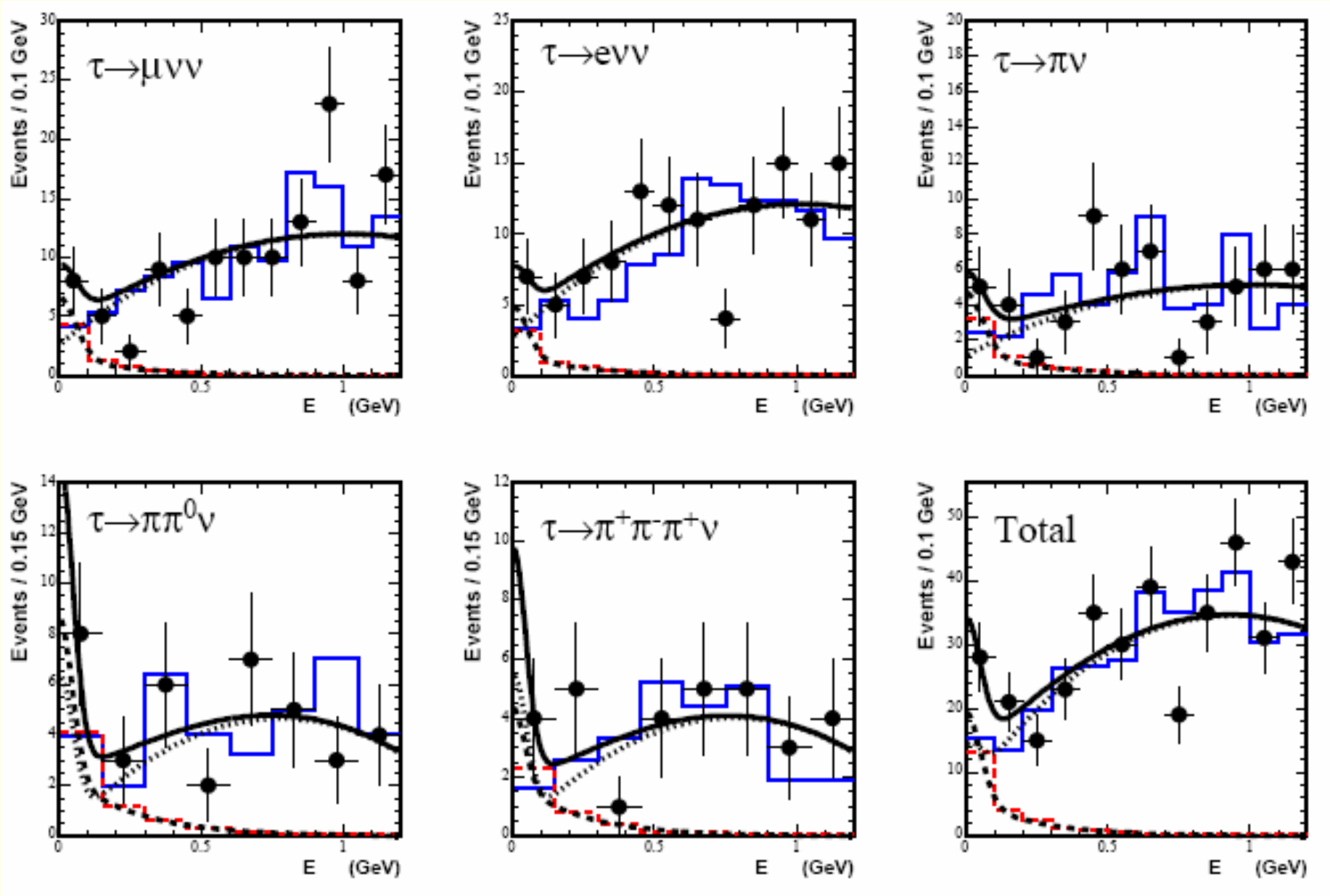
81% of all τ decay modes

$\tau^- \to \mu^- \nu\bar{\nu}$	$\tau^- \to e^- \nu\bar{\nu}$	$\tau^- \to \pi^- \nu$	$\tau^- \to \pi^- \pi^0 \nu$	$\tau^- \to \pi^- \pi^+ \pi^- \nu$
1 signal-side track				3 signal-side tracks
No signal-side π^0			1 signal-side π^0	No signal-side π^0
$E_{ECL} < 0.2$ GeV			$E_{ECL} < 0.3$ GeV	
$P^*_{\ell^-} > 0.3$ GeV		$P^*_{\pi^-} > 0.8$ GeV	$P^*_{\pi\pi^0} > 1.2$ GeV	$P^*_{3\pi} > 1.8$ GeV
$P^*_{miss} > 0.2$ GeV		$P^*_{miss} > 1.0$ GeV	$P^*_{miss} > 1.2$ GeV	$P^*_{miss} > 1.8$ GeV
			$\lvert M_{\rho^-} - M_{\pi\pi^0} \rvert < 0.15$ GeV	$\lvert M_{\rho^-} - M_{\pi^-\pi^-} \rvert < 0.15$ GeV
				$\lvert M_{a_1^-} - M_{3\pi} \rvert < 0.3$ GeV
$-0.86 < \cos\theta^*_{miss} < 0.95$				

Total efficiency with τ decay branching fraction : 15.81 $\pm$0.05%

All the selection criteria have been optimized to achieve the highest sensitivity

$B^+ \to \tau^+ \nu_\tau$: Fits to individual modes



$B \to \tau \nu_\tau$ - systematic uncertainty

- Signal selection efficiencies

Source	$\mu^- \nu \bar{\nu}$(%)	$e^- \nu \bar{\nu}$(%)	$\pi^- \nu$(%)	$\pi^- \pi^0 \nu$(%)	$\pi^+ \pi^- \pi^+ \nu$(%)
Tracking	1.0	1.0	1.0	1.0	3.0
τ decay BR	0.3	0.3	1.0	0.6	1.1
MC statistics	0.6	0.6	0.7	1.0	2.0
Lepton ID	2.1	2.1	-	-	-
π^0 reconstruction	-	-	-	3	-
$\pi^\pm$ ID	-	-	2.0	2.0	6.0

- Tag reconstruction efficiency : 10.5%

  Difference of yields between data and MC in the $B^- \to D^{*0} l^- \nu$ control sample
- Number of BB : 1%
- Signal yield : $^{+22.5\%}_{-25.7\%}$

  signal shape ambiguity estimated by varying the signal PDF parameters

  BG shape : changing PDF
- Total systematic uncertainty $^{+25.5\%}_{-28.4\%}$

$B^0 \to D^{*-} \tau^+ \nu_\tau$ Systematic errors

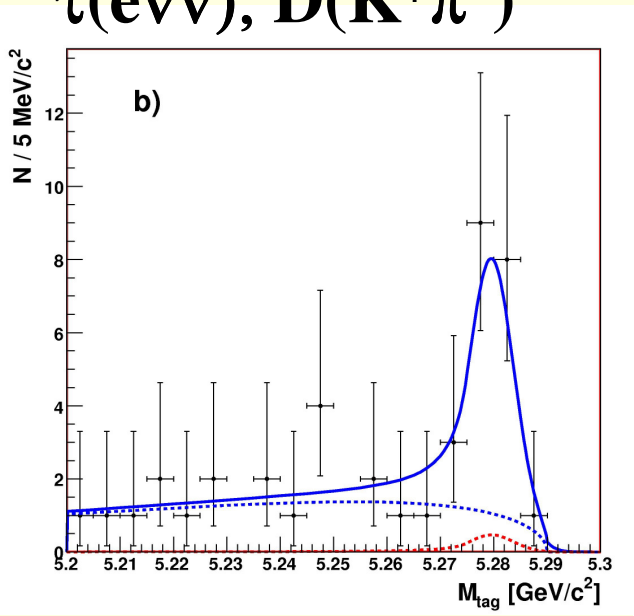
Source	$\tau \to e\nu\nu$, $D \to K\pi$	$\tau \to e\nu\nu$, $D \to K\pi\pi^0$	$\tau \to \pi\nu$, $D \to K\pi$
CB param.	± 2.8 %		
Argus param.	(+3.1, -4.4) %	(+2.7, - 2.6) %	(+4.0, -8.7) %
Peaking BG	(+ 8.2, - 4.4) %		
N_{BB}	± 1.3 %		
$\varepsilon_{D^*e/\pi}$	± 7.9 %	± 10.7 %	± 8.1 %
ε_{Btag}	± 10.9 %		
ε_{sel}	± 5.0 %		± 15.4 %
$Br(D^* \to D\pi)$	± 0.74 %		
$Br(D \to K\pi)$	± 1.84 %	-	± 1.84 %
$Br(D \to K\pi\pi^0)$	-	± 3.55 %	-
$Br(\tau \to e\nu\nu)$	± 0.28 %	± 0.28 %	-
$Br(\tau \to \pi\nu)$	-	-	± 0.64 %

Total uncertainty of branching fraction: 18.5 %

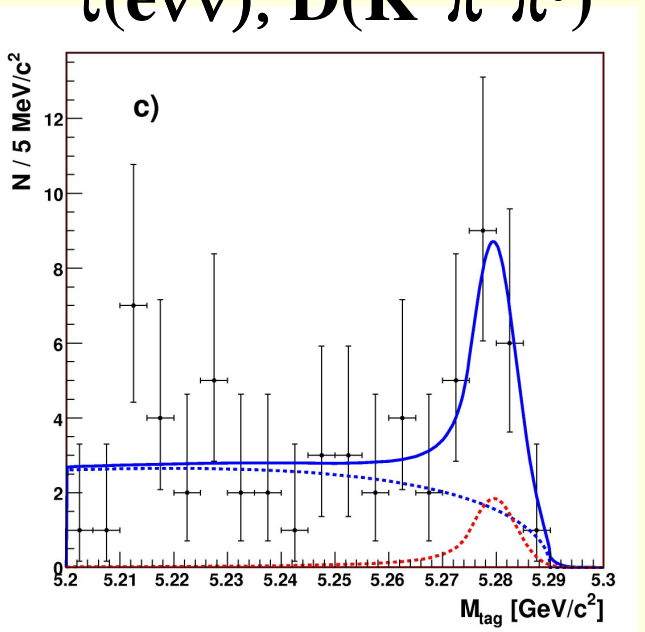
(*) peaking bg $1.2\ ^{+1.6}_{-1.5}$ $5.0\ ^{+2.6}_{-2.2}$ $-1.0\ ^{+3.6}_{-3.2}$

Checks: M_{tag} fit projections

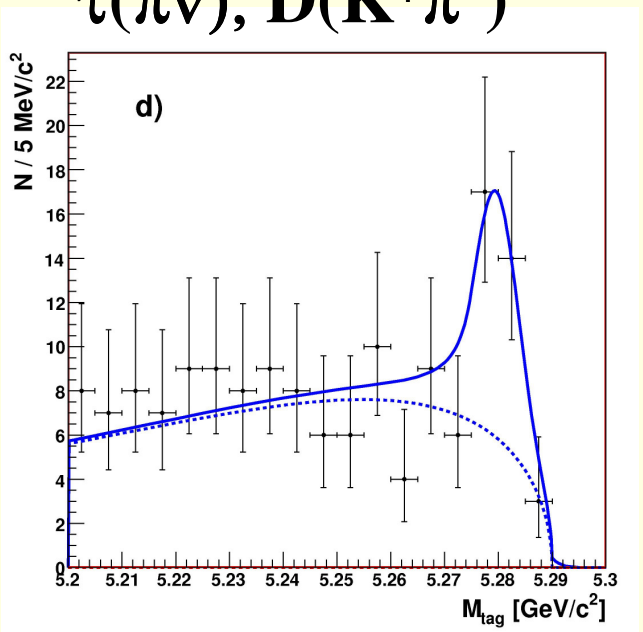
$\tau(e\nu\nu)$, $D(K^+\pi^-)$



$\tau(e\nu\nu)$, $D(K^+\pi^-\pi^0)$

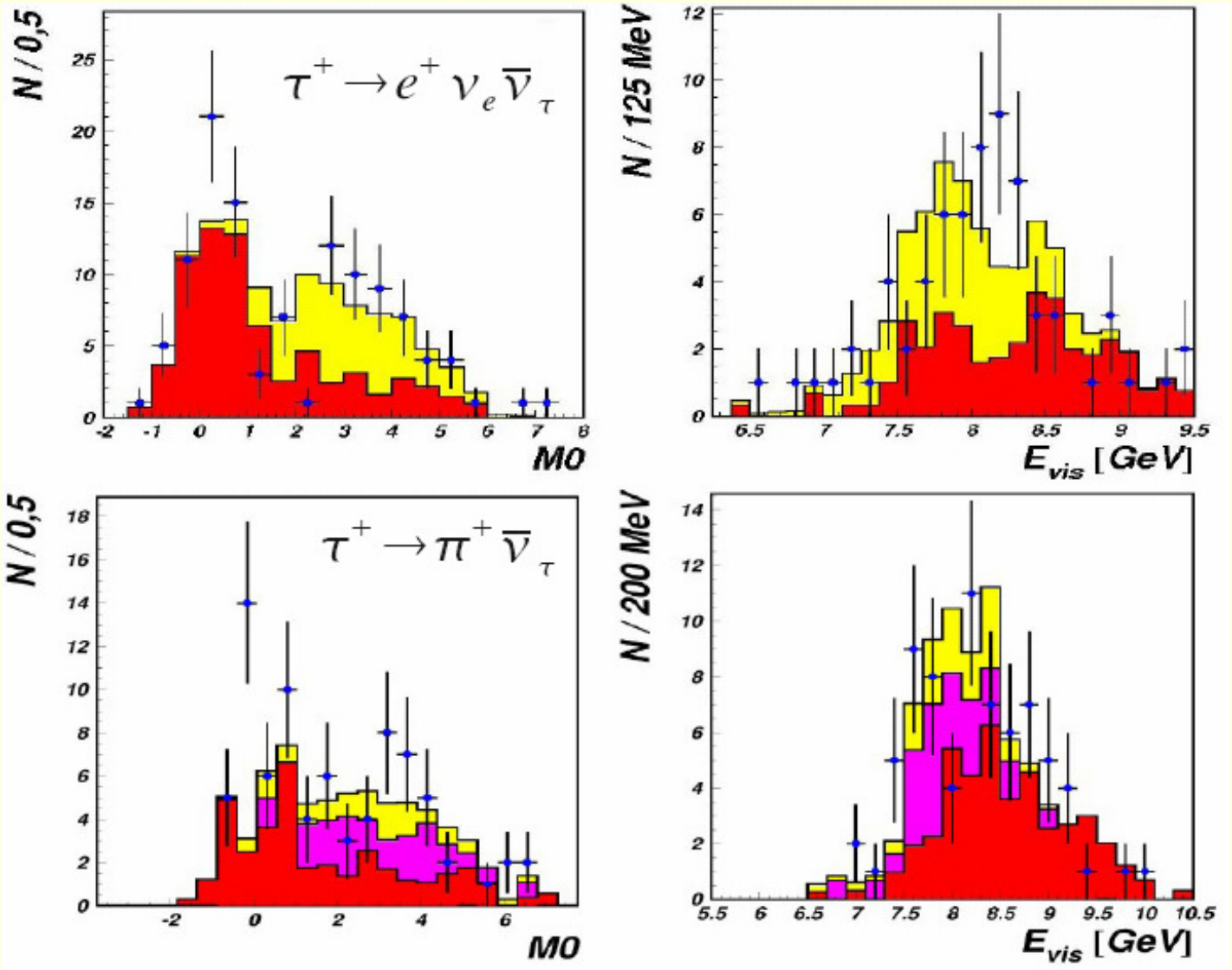


$\tau(\pi\nu)$, $D(K^+\pi^-)$



Checks: Look-back plots

Distribution of a variable in signal-box ("N-1" cuts)



data

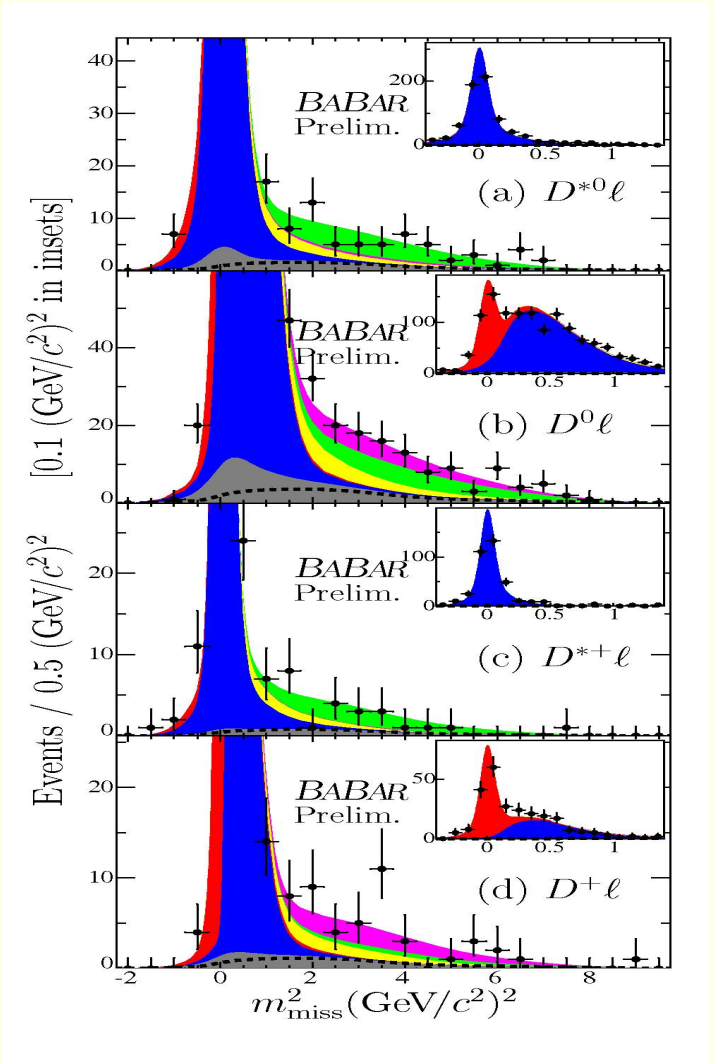
signal

other D*τν modes

background

$B \to D^{(*)} \tau \nu_\tau$ - BaBar preliminary

hep-ex/0707.2758



$BF(B^- \to D^0 \tau\nu) = (0.63 \pm 0.38 \pm 0.10 \pm 0.06)\%$,

$BF(B^- \to D^{*0} \tau\nu) = (2.35 \pm 0.49 \pm 0.22 \pm 0.18)\%$,

$BF(B^0 \to D^- \tau\nu) = (1.03 \pm 0.35 \pm 0.14 \pm 0.10)\%$,

$BF(B^- \to D^{*-} \tau\nu) = (1.15 \pm 0.53 \pm 0.04 \pm 0.04)\%$

Combined B^- and B^0 :

$BF(B \to D \tau\nu) = (0.90 \pm 0.26 \pm 0.11 \pm 0.06)\%$ (3.5σ),

$BF(B^- \to D^* \tau\nu) = (1.81 \pm 0.33 \pm 0.11 \pm 0.06)\%$ (6.2σ)