Lepton Flavour Violation
in SUSY-seesaw: an update

María J. Herrero
Dpt. Física Teórica/IFT, Universidad Autónoma, Madrid

From works
E. Arganda and M. H. PRD73, 055003 (2006)
E. Arganda, M. H. and A. Teixeira hep-ph/0707.2955
E. Arganda, M. H. and J. Portolés IFT-UAM/CSIC-07-27

SUSY07, Karlsruhe, 27-July-2007
Motivation

★ Lepton Flavour Violation (LFV) occurs in Nature, $\nu_i - \nu_j$ oscill.
★ In SM: no LFV if $m_\nu = 0$; very suppressed if $m_\nu \neq 0$
★ Many exp. bounds (present/future sensitivities): MEGA, SINDRUM, BABAR, Belle / MEG,...PRISM/PRIME

\[
\begin{align*}
\text{BR}(\mu \rightarrow e\gamma) &< 1.2 \times 10^{-11}/10^{-13} & \text{BR}(\tau \rightarrow \mu\eta) &< 6.5 \times 10^{-8} \\
\text{BR}(\tau \rightarrow \mu\gamma) &< 4.5 \times 10^{-8}/10^{-8} & \text{BR}(\tau \rightarrow \mu\eta') &< 1.3 \times 10^{-7} \\
\text{BR}(\tau \rightarrow e\gamma) &< 1.2 \times 10^{-7}/10^{-8} & \text{BR}(\tau \rightarrow \mu\pi) &< 1.2 \times 10^{-7} \\
\text{BR}(\mu \rightarrow 3e) &< 1.0 \times 10^{-12}/10^{-13} & \text{BR}(\tau \rightarrow \mu\rho) &< 2 \times 10^{-7} \\
\text{BR}(\tau \rightarrow 3\mu) &< 1.9 \times 10^{-7}/10^{-8} & \text{CR}(\mu - e, Au) &< 7 \times 10^{-13} \\
\text{BR}(\tau \rightarrow 3e) &< 2.0 \times 10^{-7}/10^{-8} & \text{CR}(\mu - e, Ti) &< 4.3 \times 10^{-12}/10^{-18}
\end{align*}
\]

★ Measurement of LFV ⇒ Window for new physics
★ Very sensitive to SUSY: if Majorana $\nu$, $Y_\nu$ can be $\mathcal{O}(1)$
  Large $Y_\nu$ induce, via SUSY loops, large LFV rates
★ If no LFV found ⇒
  Restrictions on SUSY and/or seesaw parameters
Our Work

- Prediction of LFV rates within SUSY-seesaw:
  - all $l_j \rightarrow l_i \gamma$
  - all $l_j \rightarrow 3l_i$
  - some semileptonic tau decays: $\tau \rightarrow \mu\eta$, $\tau \rightarrow \mu\pi$, $\tau \rightarrow \mu\rho$...
  - $\mu - e$ conversion in nuclei: Ti, Au,...

- Full one-loop computation of LFV BRs
- Require compatibility with $\nu$ data
- Compare with present/future LFV bounds/sensitivities
- Explore sensitivity to SUSY and seesaw parameters
- Study impact of $\theta_{13}$ on LFV, specially $\mu - e$
- Study of correlated/un-correlated processes: competing future sensitivities?
1-loop diagrams in $l_j \rightarrow 3l_i$, $\tau \rightarrow \mu \text{ hadrons, } \mu - e \text{ conversion}$

**Generic**

**Photon-mediated**

**Z boson-mediated**

**H boson-mediated**
Framework

- **Use seesaw (Type I)** for $\nu$ mass generation
- **Work within CMSSM + $3\nu_R$ (Majorana) + $3\tilde{\nu}_R$**
  
  Two scenarios for soft parameters at $M_X = 2 \times 10^{16}$ GeV:
  
  - Universal soft Higgs masses: **CMSSM-seesaw**
    
    \[(M_0, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu))\]
  
  - Non-universal soft Higgs masses: **NUHM-seesaw**
    
    \[(M_0, M_{1/2}, M_{H1}, M_{H2}, A_0, \tan \beta, \text{sign}(\mu))\]

- **LFV generated by 1-loop running from $M_X$ to $M_Z$**
  
  Full RGEs including $\nu$ and $\tilde{\nu}$ sectors (**No Llog approx**)

- **Mass eigenstates for all SUSY and Higgs particles** (**No MIA approx**)

- **Numerical estimates:**
  
  - SPheno 2.2.2 (W. Porod) for int. of RGEs and SUSY spectrum
  
  - Additional subroutines for all LFV processes (by us)
    
    Also subroutines for checks of BAU, EDM and $(g - 2)\mu$
Seesaw parameters versus neutrino data

SeeSaw equation: \( m_\nu = -m_D^T m_N^{-1} m_D \)

Solution: 
\[
    m_D = i \sqrt{m_N^{\text{diag}}} \, R \sqrt{m_\nu^{\text{diag}}} \, U^\dagger_{MNS}
\]

[Casas, Ibarra ('01)]

\( R \) is a 3 × 3 complex matrix and orthogonal

\[
    R = \begin{pmatrix}
        c_2 c_3 & -c_1 s_3 - s_1 s_2 c_3 & s_1 s_3 - c_1 s_2 c_3 \\
        c_2 s_3 & c_1 c_3 - s_1 s_2 s_3 & -s_1 c_3 - c_1 s_2 s_3 \\
        s_2 & s_1 c_2 & c_1 c_2
    \end{pmatrix},
\]

\( c_i = \cos \theta_i, \ s_i = \sin \theta_i, \ \theta_{1,2,3} \) complex

Parameters: \( \theta_{ij}, \delta, \alpha, \beta, m_\nu, m_{N_i}, \theta_i \) \( (18) \); \( m_{N_i}, \ \theta_i \) drive the size of \( Y_\nu \)

Hierarchical \( \nu \)'s: \( m_{\nu_1}^2 << m_{\nu_2}^2 = \Delta m_{\text{sol}}^2 + m_{\nu_1}^2 << m_{\nu_3}^2 = \Delta m_{\text{atm}}^2 + m_{\nu_1}^2 \)

2 Scenarios

- Degenerate \( N \)'s
  \( m_{N_1} = m_{N_2} = m_{N_3} = m_N \)
- Hierarchical \( N \)'s
  \( m_{N_1} << m_{N_2} << m_{N_3} \)
Our choice of input parameters
Constrained MSSM +3ν_R + 3\tilde{\nu}_R + seesaw

- **CMSSM:**
  \[
  \begin{align*}
  M_0, M_{1/2}, A_0 & \text{ (at } M_X \sim 2 \times 10^{16} \text{ GeV)} \\
  \tan \beta & = <H_2> / <H_1> \text{ (at EW scale)} \\
  \text{sign}(\mu) & \text{ (}\mu\text{ derived from EW breaking)}
  \end{align*}
  \]
  Choose SPS points

- **NUHM:** \((M_0, M_{1/2}, M_{H_1}, M_{H_2}, A_0, \tan \beta, \text{sign}(\mu))\)
  Choose \(M_0 = M_{1/2}, M_{H_1}^2 = M_0^2(1 + \delta_1), M_{H_2}^2 = M_0^2(1 + \delta_2)\)

- **Seesaw parameters**
  \[
  \begin{align*}
  m_{\nu_{1,2,3}} & \text{ (set by data)} \\
  m_{N_{1,2,3}} & \text{ (input)} \\
  U_{MNS} & \text{ (set by data)} \\
  R(\theta_1, \theta_2, \theta_3) & \text{ (input)}
  \end{align*}
  \]

- **For numerical estimates:**
  \[
  \begin{align*}
  (\Delta m^2)_{12} & = \Delta m^2_{\text{sol}} = 8 \times 10^{-5} \text{ eV}^2 \\
  (\Delta m^2)_{23} & = \Delta m^2_{\text{atm}} = 2.5 \times 10^{-3} \text{eV}^2 \\
  \theta_{12} & = 30^0; \ \theta_{23} = 45^0; \ \delta = \alpha = \beta = 0; \ 0 \leq \theta_{13} \leq 10^0 \\
  250 \text{ GeV} & < M_0, M_{1/2} < 1000 \text{ GeV}, -500 \text{ GeV} < A_0 < 500 \text{ GeV} \\
  5 & < \tan \beta < 50, -2 < \delta_{1,2} < 2
  \end{align*}
  \]
Results for CMSSM-seesaw
The most relevant parameter:
Hierarchical: $m_{N_3}$ / Degenerate: $m_N$

- Most obs. reach exp. lim. at $m_{N_3} \in [10^{13}, 10^{15}]$ GeV, $(Y_\nu)_{33,32} \sim 0.1 - 1$
- $\text{BR}_4 (\tan\beta = 50) > \text{BR}_{1b} \sim \text{BR}_{1a} > \text{BR}_3 \sim \text{BR}_2 > \text{BR}_5$
- $\text{BR} \sim |m_{N_3} \log m_{N_3}|^2$ except for SPS5: Llog fails in $\sim 10^4$!!
- Present: the most restrictive one is $\mu \rightarrow e\gamma$ (if $\theta_{13} \neq 0$)
  Bounds for SPS1a $m_{N_3} < 10^{13} - 10^{14}$ GeV
  Next are $\mu - e, \mu \rightarrow 3e; \tau \rightarrow \mu\gamma$ competitive if $\theta_{13} \simeq 0$ and N’s hier.
- Future: $\mu - e$ conversion the best: sensitive to $m_{N_3} > 10^{12}$ GeV
Large Yukawa couplings: role of $\theta_i$

Hierarchical $m_{N_i}$ and complex $\theta_i$

$(m_{N_1}, m_{N_2}, m_{N_3}) = (10^8, 2 \times 10^8, 10^{14})$ GeV, $\arg(\theta_1) = 0, \pi/10, \pi/8, \pi/6, \pi/4 (\theta_2 = \theta_3 = 0)$

**SPS 4**

Parameter $m_{N_1}$ ranges from $10^8$ to $2 \times 10^8$ GeV, $m_{N_2}$ from $2 \times 10^8$ to $10^{14}$ GeV, $m_{N_3}$ from $2 \times 10^8$ to $10^{14}$ GeV.

- BRs for $0 < |\theta_i| < \pi$, $0 < \arg\theta_i < \pi/2$ can increase up to $10^2 - 10^4$ respect to $\theta_i = 0$.
- BRs above present experimental bounds: mainly $\mu \to e\gamma$, $\mu \to 3e$ and $\tau \to \mu\gamma$.
- Similar results for $\theta_2$. BRs nearly constant with $\theta_3$ in the case of hier. N's.
Role of $\theta_i$ (cont.)

N's Deg. Eq. dep. all $\theta_i$

$\star$ CR($\mu - e$) reach exp. bound even for $m_N = 10^{13}$ GeV if complex $\theta_i \neq 0$

$\star$ BR($\tau \rightarrow \mu \pi^+ \pi^-$) reach exp. bound at high $m_{N_3} \sim 10^{14}$ GeV and large complex $\theta_{1,2}$
High sensitivity to $\theta_{13}$: the case $\theta_i = 0$

- $\mu \to e\gamma$, $\mu \to 3e$, $\mu - e$ extremely sensitive to $\theta_{13}$ if Hier.N’s: BRs $\times 10^5$ in $0^\circ < \theta_{13} < 10^\circ$
- $\tau \to e\gamma$, $\tau \to 3e$ also, but not within exp.reach ($\tau \to \mu\gamma$, $\tau \to 3\mu$ are not!!)
- Sensitivity of $\mu \to e\gamma$ clearly within exp. reach: $\text{BR}_{\text{all SPS}} > \text{BR}_{\text{exp}}^{\text{present}}$ for $\theta_{13} \gtrsim 5^\circ$ !!
Impact of $\theta_{13}$ on LFV processes

(All plotted points lead to 'viable BAU', respect EDM bounds, OK with $(g - 2)_\mu$)

\[-\pi/4 \lesssim \arg\theta_1 \lesssim \pi/4, 0 \lesssim \arg\theta_2 \lesssim \pi/4\]

Correlations in LFV BRs help: similar dependence on $m_{N_3}$ and $\tan \beta$

MEGA bound already disfavours $\theta_{13} \gtrsim 10^0 (2^0)$ for $m_{N_3} \gtrsim 10^{13} (10^{14})$ GeV

A measurement of BRs and $\theta_{13}$ will provide some insight into $m_{N_3}$!!
Results for NUHM-seesaw

- Looking for solutions with light Higgs sector as functions of non-universal soft parameters: $M_{H_{1,2}}^2 = M_0^2(1 + \delta_{1,2})$, $-2 < \delta_{1,2} < 2$

- Study of correlation loss between related LFV observables due to Higgs-mediated contributions that can dominate the photon-mediated ones. Relevant for:
  - $\mu - e$ conversion in nuclei versus $\mu \rightarrow e\gamma$
  - $\tau \rightarrow \mu\eta$ versus $\tau \rightarrow \mu\gamma$
  - $\tau \rightarrow \mu\eta$ versus $\tau \rightarrow \mu\rho$
  due to large Higgs couplings to strange quarks (not relevant for $l_j \rightarrow 3l_i$)

- Explore ratios of observables looking for enhancements respect to the universal case
We find solutions with light Higgs particles even for large $M_0 = M_{1/2} = M_{\text{SUSY}}$

- Ex.: for $M_{\text{SUSY}} = 850$ GeV, $\tan \beta = 50$, $A_0 = 0$, $\delta_1 = -1.8$, $\delta_2 = 0$, we find:
  - light Higgs: $m_{H^0} = 127$ GeV, $m_{h^0} = 123$ GeV, $m_{A^0} = 127$ GeV, $m_{H^+} = 155$ GeV
  - heavy SUSY: $m_{\tilde{t}_1} = 734$ GeV, $m_{\tilde{\nu}_1} = 971$ GeV, $m_{\tilde{\chi}^-_1} = 687$ GeV, $m_{\tilde{\chi}^0_1} = 362$ GeV
**μ − e conversion in nuclei: universality versus non-universality**

universality: $\gamma$ dominance  
non-universality: $H^0$ dominance

$H^0$ dominance $\Rightarrow$ heavy SUSY spectra do not decouple in $\mu − e$ conversion  
$CR(\mu − e, Au)$ above present experimental bound even for heavy SUSY
Correlation loss between $\mu - e$ conv. and $\mu \to e\gamma$

- Deviation from linearity due to Higgs-dominated rates. Visible at future?

- Ratio of $\mu - e$ to $\mu \to e\gamma$ can be a factor 10 larger than in photon-dominated scenarios like CMSSM (where this ratio is $\mathcal{O}(\alpha)$)
Higgs-dominance in semileptonic tau decays

- Heavy SUSY do not decouple in $\tau \rightarrow \mu \eta'$ for scenarios with Higgs-dominance $\tau \rightarrow \mu \eta'$ decay rates close to exp. bound if large $\theta_i$

- Ratio of $\tau \rightarrow \mu \eta$ to $\tau \rightarrow \mu \rho$ can be a factor 10 larger than in CMSSM
If SUSY exists:
LFV observables constitute an interesting lab that together with low-energy $\nu$-data can provide some insight into the heavy neutrino sector and SeeSaw parameters. Also into Higgs sector if NUHM like scenarios
Additional transparencies
Constraints from 'viable' BAU

BAU requires complex $R \neq 1 \Rightarrow$ complex $\theta_i \neq 0$. Most relevantly $\theta_2$

$n_B/n_\gamma \in \text{interval} \Rightarrow \text{(Re}(\theta_2),\text{Im}(\theta_2)) \in \text{area ('ring')} \quad \text{(WMAP in darkest ring)}$

Implications for LFV

- 'viable' BAU $\iff n_b/n_\gamma \in [10^{-10}, 10^{-9}]$ (WMAP $\sim 6.1 \times 10^{-10}, '06$)
  
  BAU [disfav]-[fav]-[disfav]-[fav]-[disfav] pattern in $0<|\theta_2|<3$

  The BAU [fav] windows occur at small ($\neq 0$) $|\theta_2| \lesssim 1.5$

- **smaller** $|\theta_2|$ $\Rightarrow$ **smaller** LFV rates

- The existence, location and size of the windows depend on $m_{N_1}$

  $m_{N_1} \sim O(10^{10})$ GeV BAU [fav] windows at $|\theta_2| \sim O(1)$ and $|\theta_2| \sim O(10^{-2})$

  $m_{N_1} \sim O(10^9)$ GeV only one window at $|\theta_2| \sim O(5 \times 10^{-1})$
Contributions to $\Delta a_{\mu}^{\text{SUSY}}$

$\Delta a_{\mu}^{\text{SUSY}} \in [10^{-8}, 10^{-9}]$: compatible with $a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 3.32 \times 10^{-9} (3.8\sigma)$
Similar for SPS1a,1b. Slightly worse prospects for SPS2,3. SPS5 the worst.

SPS4 the most restrictive one (due to $\tan \beta = 50$):

Present bounds from $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ already exclude $m_{N_3} \gtrsim 10^{14} \text{ GeV}$!!
SUSY SPS points (I)

SPS1a

\[ M_0 = 100 \text{ GeV} \]
\[ M_{1/2} = 250 \text{ GeV} \]
\[ A_0 = -100 \text{ GeV} \]
\[ \tan \beta = 10 \]
\[ \mu > 0 \]

SPS1b

\[ M_0 = 200 \text{ GeV} \]
\[ M_{1/2} = 400 \text{ GeV} \]
\[ A_0 = 0 \text{ GeV} \]
\[ \tan \beta = 30 \]
\[ \mu > 0 \]

SPS2

\[ M_0 = 1450 \text{ GeV} \]
\[ M_{1/2} = 300 \text{ GeV} \]
\[ A_0 = 0 \text{ GeV} \]
\[ \tan \beta = 10 \]
\[ \mu > 0 \]
SUSY SPS points (II)

SPS3

$M_0 = 90\text{ GeV}$

$M_{1/2} = 300\text{ GeV}$

$A_0 = 0\text{ GeV}$

$tan\beta = 10$

$\mu > 0$

SPS4

$M_0 = 400\text{ GeV}$

$M_{1/2} = 300\text{ GeV}$

$A_0 = 0\text{ GeV}$

$tan\beta = 50$

$\mu > 0$

SPS5

$M_0 = 150\text{ GeV}$

$M_{1/2} = 300\text{ GeV}$

$A_0 = -1000\text{ GeV}$

$tan\beta = 5$

$\mu > 0$