Running and Decoupling in the MSSM

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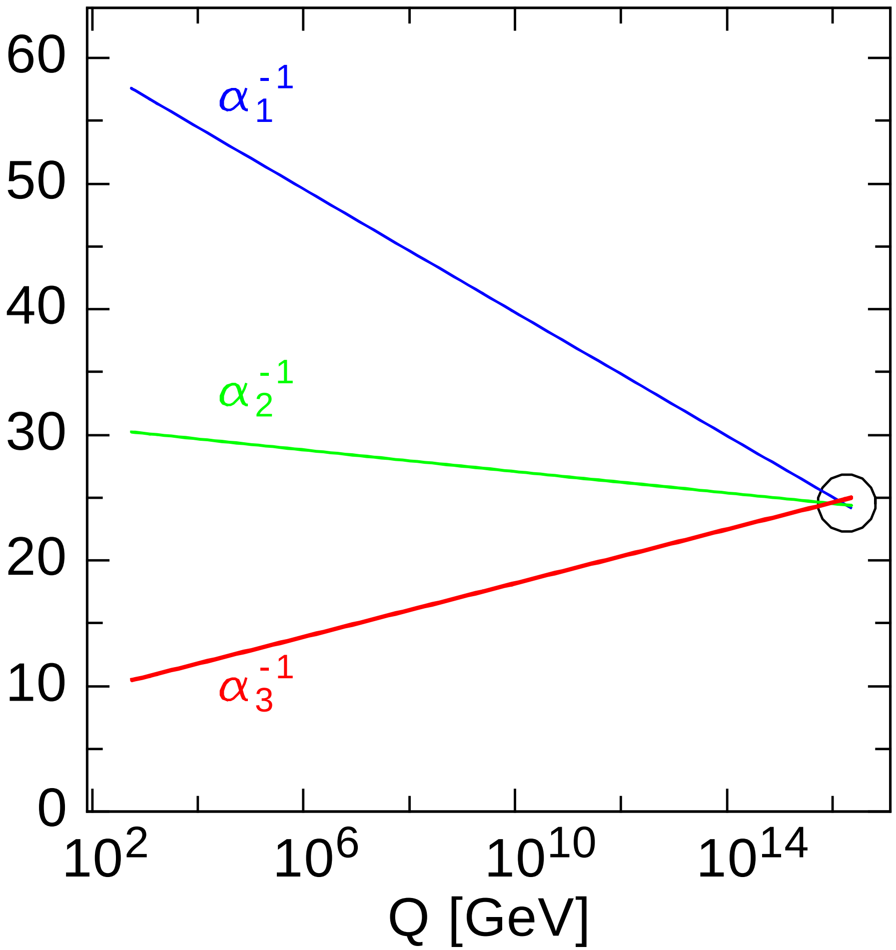
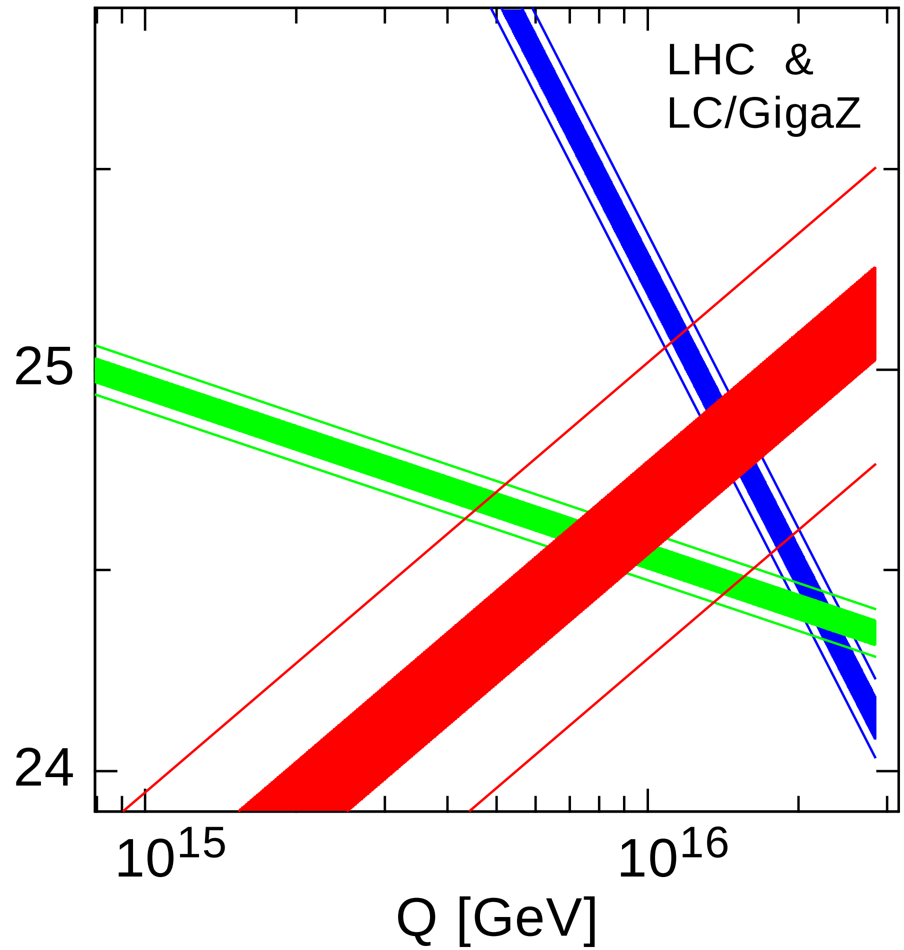
Motivation

- MSSM: gauge couplings tend to unify at $M_{\mathrm{GUT}} \simeq 10^{16}$GeV.
  Uncertainty on M_{GUT}, $\alpha(M_{\mathrm{GUT}})$, M_{SUSY} dominated by $\Delta\alpha_s(M_{\mathrm{GUT}})$.

Allanach et al '04





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- One can set constrains on the SUSY-breaking mechanism relating observables at the low-energy and GUT-scales
  - 2-loop MSSM RGEs *Sp. Martin and M. T. Vaughn '93*
    1-loop threshold corrections *D. Pierce et al '96* $\Rightarrow$
    Public Codes: ISAJET *H. Baer et al '03*, SuSpect *A. Djouadi et al '03*
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  - Our aim: 3-loop RGEs for SUSY-QCD sector
    2-loop threshold corrections *R. Harlander, L. M., M. Steinhauser '05*
    $\Rightarrow \alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ with 3-loop accuracy
    $\Rightarrow m_{\rm b}^{\overline{\rm DR}}(M_{\rm SUSY})$ with 3- and 4-loop accuracy

Running of α_s

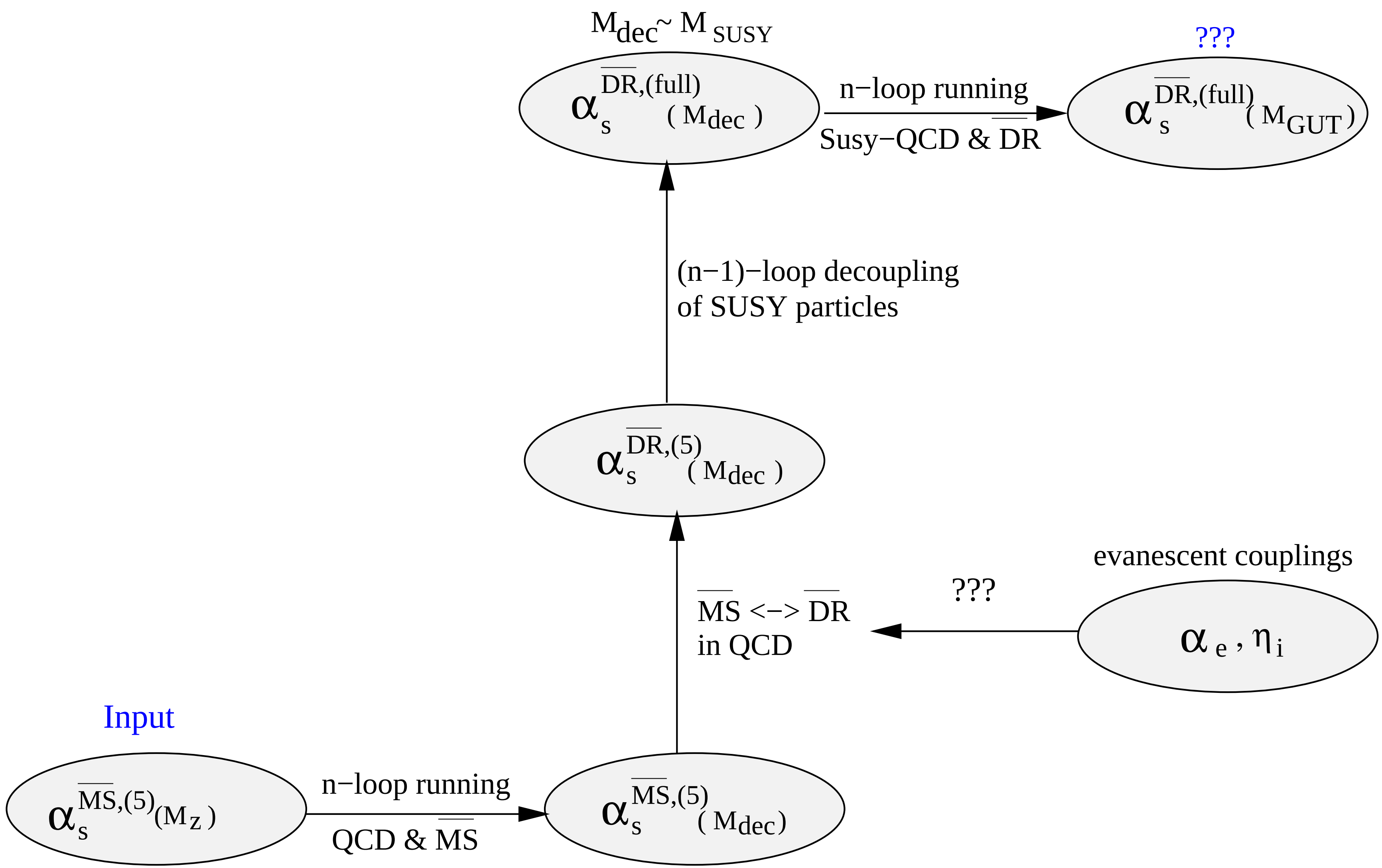
- Precision calculations in MSSM require a manifestly SUSY and gauge invariant Regularization scheme $\Rightarrow$ DRED
- Mass-independent Renormalization scheme
  Decoupling Theorem does not hold $\Rightarrow$ threshold effects should be added *by hand*
  - SUSY models with severely split mass spectrum
    Multi-Scale Approach: each particle decoupled at its own threshold
  - SUSY models with roughly degenerate mass spectrum
    Common Scale Approach: all SUSY particles decoupled at

$$\mu \simeq M_{\mathrm{SUSY}}$$

! implemented in almost all currently available codes

Running of α_s

- Input parameter: $\alpha_s^{\overline{\rm MS},(5)}(M_Z) \Rightarrow$ Output parameter: $\alpha_s^{\overline{\rm DR},(\rm full)}(M_{\rm GUT})$



DRED Framework

Quasi-4-dim. space (Q4S): $4 = d \oplus 4 - d$

- *Quasi*-4-dim metric tensor: $G_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$
- Dirac matrices in Q4S: $\Gamma_\mu = \gamma_\mu + \tilde{\gamma}_\mu$
- space-time coordinates continued from 4 to $d \leq 4$ dim.
- the number of field components unchanged
  - 4-dim gluon field: $A^a_\mu = V^a_\mu + S^a_\mu \,,$

$$
\begin{aligned}
V^a_\mu &= g_{\mu\nu}\, A^a_\nu = \quad d\text{- dim. vector} \\
S^a_\mu &= \tilde{g}_{\mu\nu}\, A^a_\nu = \quad \varepsilon \text{ scalar}
\end{aligned}
$$

under gauge transformations

Renormalization

$$\mathcal{L}_{\rm B} = \mathcal{L}^{d}_{\rm B} + \mathcal{L}^{\varepsilon}_{\rm B}$$

- $\mathcal{L}^{d}_{\rm B}$ same as in DREG
- $\mathcal{L}^{\varepsilon}_{\rm B}$ new contribution due to ε-scalars

$$\mathcal{L}^{d}_{\rm B} = -\frac{1}{4}G^{a,ij}G^{a}_{ij} - \frac{(\partial^i V^a_i)^2}{2(1-\xi)} + \mathcal{L}^{d}_{\rm ghost,B} + i\bar{\psi}^{\alpha}\gamma^{i}D^{\alpha\beta}_{i}\psi^{\beta}$$

$$\mathcal{L}^{\varepsilon}_{\rm B} = \frac{1}{2}(D^{ab}_{i}S^{b}_{\sigma})^2 - g\bar{\psi}\tilde{\gamma}_{\sigma}T^{a}\psi S^{a}_{\sigma} - \frac{1}{4}g^2 f^{abc}f^{ade}S^{b}_{\sigma}S^{c}_{\sigma'}S^{d}_{\sigma}S^{e}_{\sigma'}$$

- each term in $\mathcal{L}^{\varepsilon}_{\rm B}$ invariant under gauge transformations
  - no reason that Yukawa-type $\bar{\psi}\psi S$ and $\bar{\psi}\psi V$ vertices renormalize the same way [except for SUSY theories !]
  - $f - f$ structure not preserved under renormalization

Renormalization(2)

$$\begin{aligned}\mathcal{L}^\varepsilon &= \frac{1}{2} Z_3^\varepsilon (\partial_i S_\sigma)^2 + Z^{\varepsilon\varepsilon V} g f^{abc} \partial_i S_\sigma^a V^{b,i} S_\sigma^c \\ &+ Z^{\varepsilon\varepsilon VV} g^2 f^{abc} f^{ade} V_i^b S_\sigma^c V^{d,i} S_\sigma^e - Z_1^\varepsilon g_e \bar{\psi} T^a \tilde{\gamma}^\sigma \psi S_\sigma^a \\ &- \frac{1}{4} \sum_{r=1}^{p} Z_{1,r}^{4\varepsilon} \eta_r H_r^{abcd} S_\sigma^a S_{\sigma'}^c S_\sigma^b S_{\sigma'}^d\end{aligned}$$

- Evanescent Yukawa-type g_e and p quartic couplings η_r
- a possible choice of H^{abcd} for $SU(3)$ case

$$H_1 = \frac{1}{2}\left(f^{ace} f^{bde} + f^{ade} f^{bce}\right)$$

$$H_2 = \frac{1}{2} \delta^{ab} \delta^{cd} \qquad H_3 = \frac{1}{2} (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc})$$

QCD: 3-loop $\beta_s^{\overline{\mathrm{DR}}}$-function

- up to 2-loop order $\beta_s^{\overline{\mathrm{DR}}} = \beta_s^{\overline{\mathrm{MS}}}$
- explicit 3-loop computation *R. Harlander, P. Kant, L. M., M. Steinhauser '06*
  comprises Yukawa like evanescent coupling $\boldsymbol{\alpha_e}$

$$\beta_{\mathbf{s}}^{\overline{\mathrm{DR}},\mathbf{3l}}(\alpha_{\mathbf{s}}^{\overline{\mathrm{DR}}},\alpha_{\mathbf{e}}) = \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right]^3 \frac{\alpha_e}{\pi}\frac{3}{16}C_F^2 T n_f + \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\frac{\alpha_e}{\pi}\right]^2 C_F T n_f \left[\frac{C_A}{16} - \frac{C_F}{8} - \frac{T n_f}{16}\right]$$
$$- \left[\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right]^4 \left[\frac{3115}{3456}C_A^3 - \frac{1439}{1728}C_A^2 T n_f + \frac{1}{32}C_F^2 T n_f - \frac{193}{576}C_A C_F T n_f + \frac{79}{864}C_A T^2 n_f^2 + \frac{11}{144}C_F T^2 n_f^2\right]$$

- 4-loop order $\beta_s^{\overline{\mathrm{DR}}}$ *R. Harlander, T. Jones, P. Kant, L. M., M. Steinhauser '06*
  contains also quartic ε-scalar couplings $\boldsymbol{\eta_i}\,,(i=1,2,3)$

Conversion from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$

- Evanescent couplings should decouple from physical observables
- known through 3-loop *R. Harlander, T. Jones, P. Kant, L. M., M. Steinhauser '06*
- n-loop conversion relation needed for (n+1)-loop running analysis
- 2-loop conversion relation $\alpha_s^{\overline{\mathrm{MS}}} \leftrightarrows \alpha_s^{\overline{\mathrm{DR}}}, \alpha_s^{\overline{\mathrm{DR}}} = f(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e)$

$$\frac{\alpha_s^{\overline{\mathrm{DR}},(n_f)}}{\alpha_s^{\overline{\mathrm{MS}},(n_f)}} = 1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{C_A}{12} + \left[\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\right]^2 \frac{11}{72}C_A^2 - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi}\frac{\alpha_e}{\pi}\frac{1}{8}C_F T n_f$$

- $\alpha_e \neq \alpha_s^{\overline{\mathrm{DR}}}$ proves equivalence of **DRED** and **DREG** at 3-loops

Decoupling of SUSY particles

- SUSY-QCD: if SUSY preserved $\Rightarrow$ **one** coupling $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu)$

$$\begin{aligned}
\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu) &= \alpha_e^{(\mathrm{full})}(\mu) = \eta_1^{(\mathrm{full})}(\mu) \\
\eta_2^{(\mathrm{full})}(\mu) &= \eta_3^{(\mathrm{full})}(\mu) = 0 \\
\beta_s = \beta_e &= \beta_{\eta_1} \quad \text{and} \quad \beta_{\eta_2} = \beta_{\eta_3} = 0
\end{aligned}$$

- QCD($n_f = 5$) as the low-energy effective theory of SUSY-QCD

$\Rightarrow$ integrate out all SUSY-particles and top-quark at $\mu = \mu_{\mathrm{dec}}$

$$\begin{aligned}
\alpha_s^{\overline{\mathrm{DR}},(n_f)}(\mu_{\mathrm{dec}}) &= \zeta_s^{(n_f)}\, \alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mu_{\mathrm{dec}}) \\
\alpha_e^{q,(5)}(\mu_{\mathrm{dec}}) &= \zeta_e^{q}\, \alpha_e^{(\mathrm{full})}(\mu_{\mathrm{dec}})
\end{aligned}$$

$\zeta_s^{(n_f)}$ and ζ_e^q decoupling coefficients for α_s and α_e

Evaluation of $\alpha_s(\mu_{\rm GUT})$ from $\alpha_s(M_Z)$

- Iterative Method :
  1. Start with a trial value for $\alpha_e^{(5)}(\mu_{\rm dec})$
  2. Get $\alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm dec})$ and $\alpha_s^{\overline{\rm DR},(5)}(\mu_{\rm dec})$ through decoupling Eqs.
  3. Evaluate $\alpha_s^{\overline{\rm MS},(5)}(\mu_{\rm dec})$ and from that $\alpha_s^{\overline{\rm MS},(5)}(M_z)$
  4. Vary $\alpha_e^{(5)}(\mu_{\rm dec})$ until $\alpha_s^{\overline{\rm MS},(5)}(M_z)$ fits the experimental value
- Practical phenomenological analyses: approximate formula

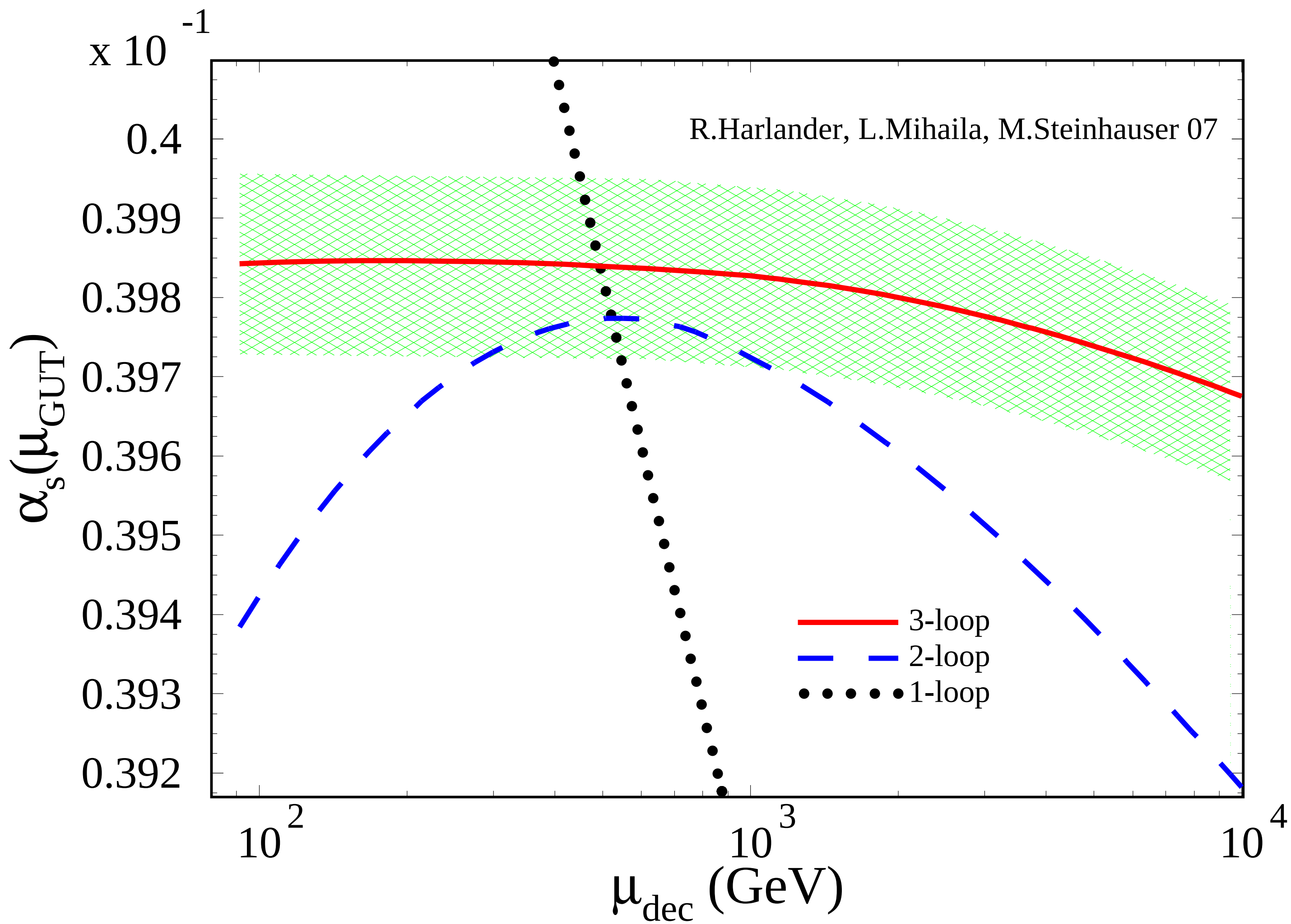
$$\alpha_s^{\overline{\rm DR},({\rm full})} = \alpha_s^{\overline{\rm MS},(n_f)}\left\{1+\frac{\alpha_s^{\overline{\rm MS},(n_f)}}{\pi}\left(\frac{1}{4}-\zeta_{s1}^{(n_f)}\right)\right. \\ \left. +\left(\frac{\alpha_s^{\overline{\rm MS},(n_f)}}{\pi}\right)^2\left[\frac{11}{8}-\frac{n_f}{12}-\frac{1}{2}\zeta_{s1}^{(n_f)}+2\,(\zeta_{s1}^{(n_f)})^2-\zeta_{s2}^{(n_f)}\right]\right\}$$

numerical deviation from the *Two-Step Approach* $\leq 0.1\%$

Numerical results

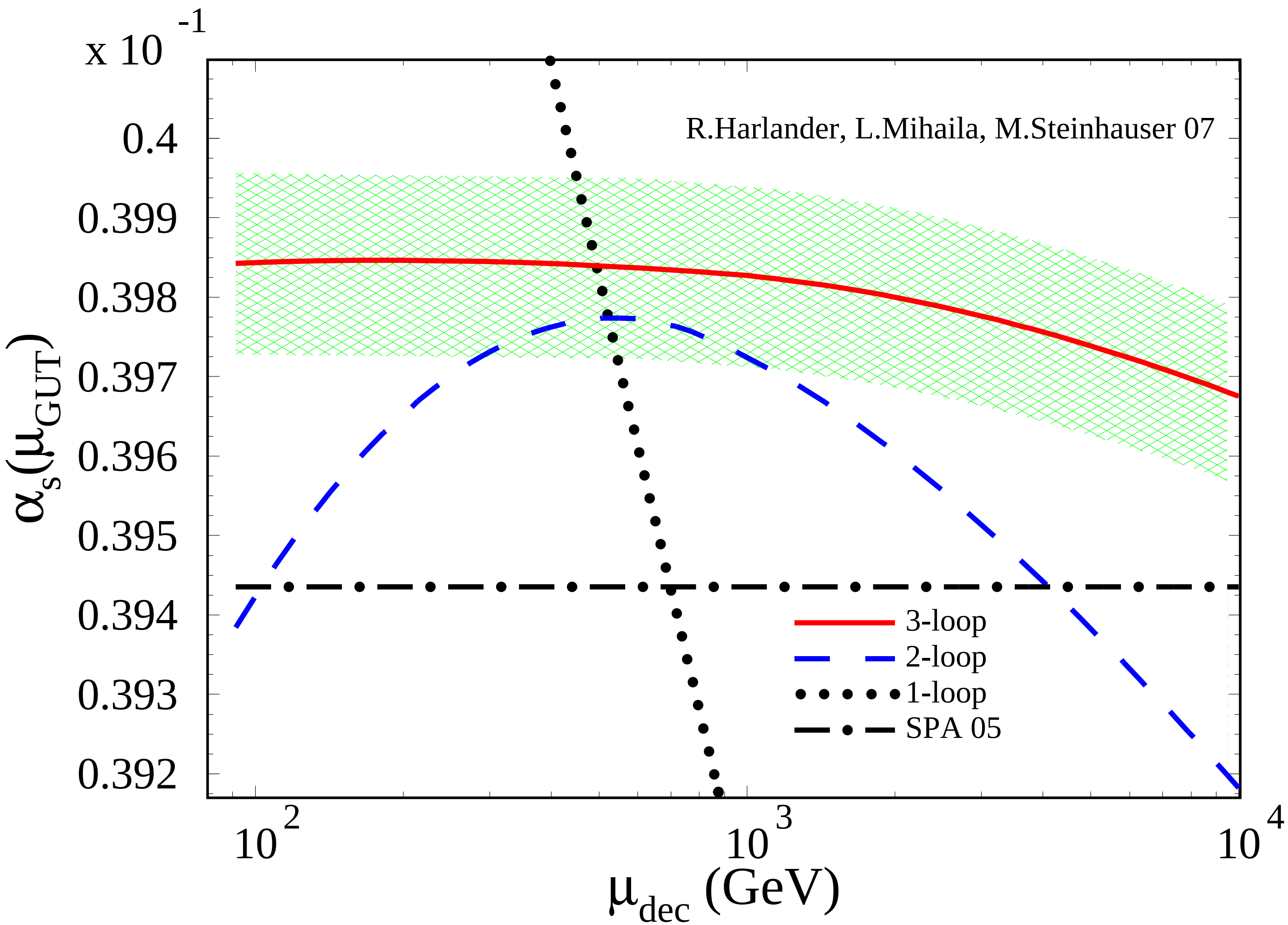
$\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z) = 0.1189 \pm 0.001$ *Bethke' 06*, $\quad M_Z = 91.1876$ GeV

and $\quad \tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV *SPS1a' '05*



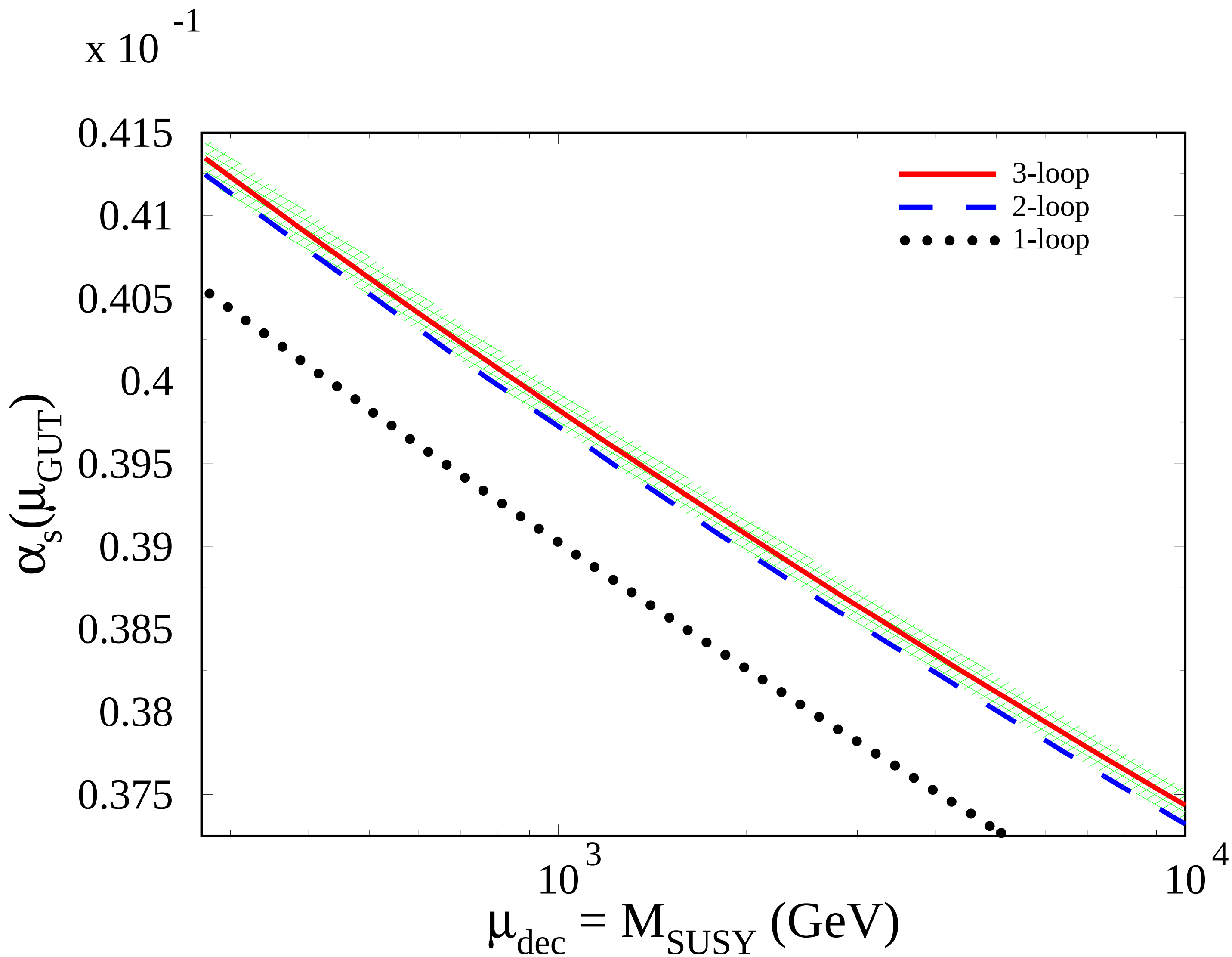
Numerical results

- Comparison with the Leading-Log Approximation *SPA-Convention'05* shows a big numerical deviation.



Numerical results

- Sensitivity of $\alpha_s(M_{\mathrm{GUT}})$ to SUSY-mass scale:



Evaluation of $m_b(\mu)$ in $\overline{\mathrm{DR}}$ scheme

- Yukawa sector of SUSY-GUT models $\Rightarrow m_{\mathrm{top}}, m_{\mathrm{bottom}}/m_{\mathrm{tau}}$
- SUSY models with large $\tan\beta$
  - SUSY mass spectrum and Higgs mass sensitive to bottom Yukawa coupling
  - relation between $Y_b(\mu)$ and $m^{\overline{\mathrm{DR}}}(\mu)$ affected by large SUSY radiative corrections
  - $m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ input parameter $\Rightarrow$ need to be known with the highest possible accuracy
- Relate $m_b^{\overline{\mathrm{DR}}}(M_{\mathrm{SUSY}})$ directly with $m_b^{\overline{\mathrm{MS}}}(m_b)$
- $m_b^{\overline{\mathrm{MS}}}(m_b)$ known with 4-loop accuracy *J. H. Kühn, M. Steinhauser, C. Sturm '07*

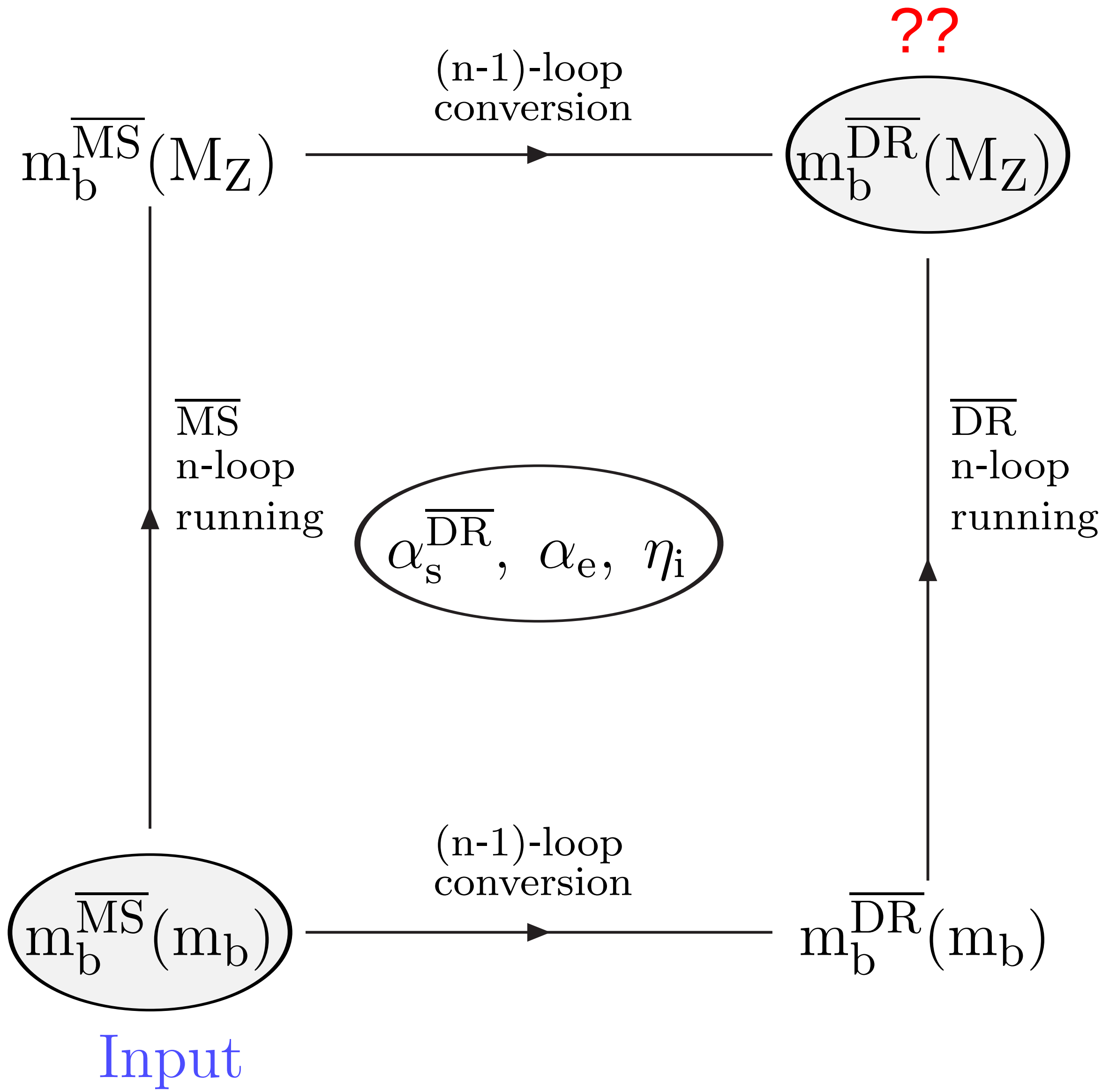
Relation $m^{\overline{\rm DR}} \leftrightarrow m^{\overline{\rm MS}}$

- Extract $m_{\rm b}^{\overline{\rm DR}}(M_{\rm SUSY})$ from accurately determined $m_{\rm b}^{\overline{\rm MS}}(m_b)$

$$m_b^{\overline{\rm DR}}(\mu) = m_b^{\overline{\rm MS}}(\mu)\Big[1 + \delta_m^{(1l)}(\alpha_e) + \delta_m^{(2l)}(\alpha_s^{\overline{\rm MS}}, \alpha_e) + \delta_m^{(3l)}(\alpha_s^{\overline{\rm MS}}, \alpha_e, \eta_i)\Big]\Big|_{\mu=\mu_S},$$

$\{\alpha_s^{\overline{\rm DR}}, \alpha_e, \eta_i\}\Big|_{\mu=\mu_S}$ have to be known.

  - Log contributions absent (mass-independent schemes)
- 2-step approach for computing $\boldsymbol{m}_{\rm b}^{\overline{\rm DR}}(M_Z)$ *H. Baer et al '02*
  - Running of $m_b(\mu)$ and conversion between $\overline{\rm MS} \leftrightarrow \overline{\rm DR}$
- Check if using QCD& $\overline{\rm DR}$ or QCD& $\overline{\rm MS}$ $\Rightarrow$ same result

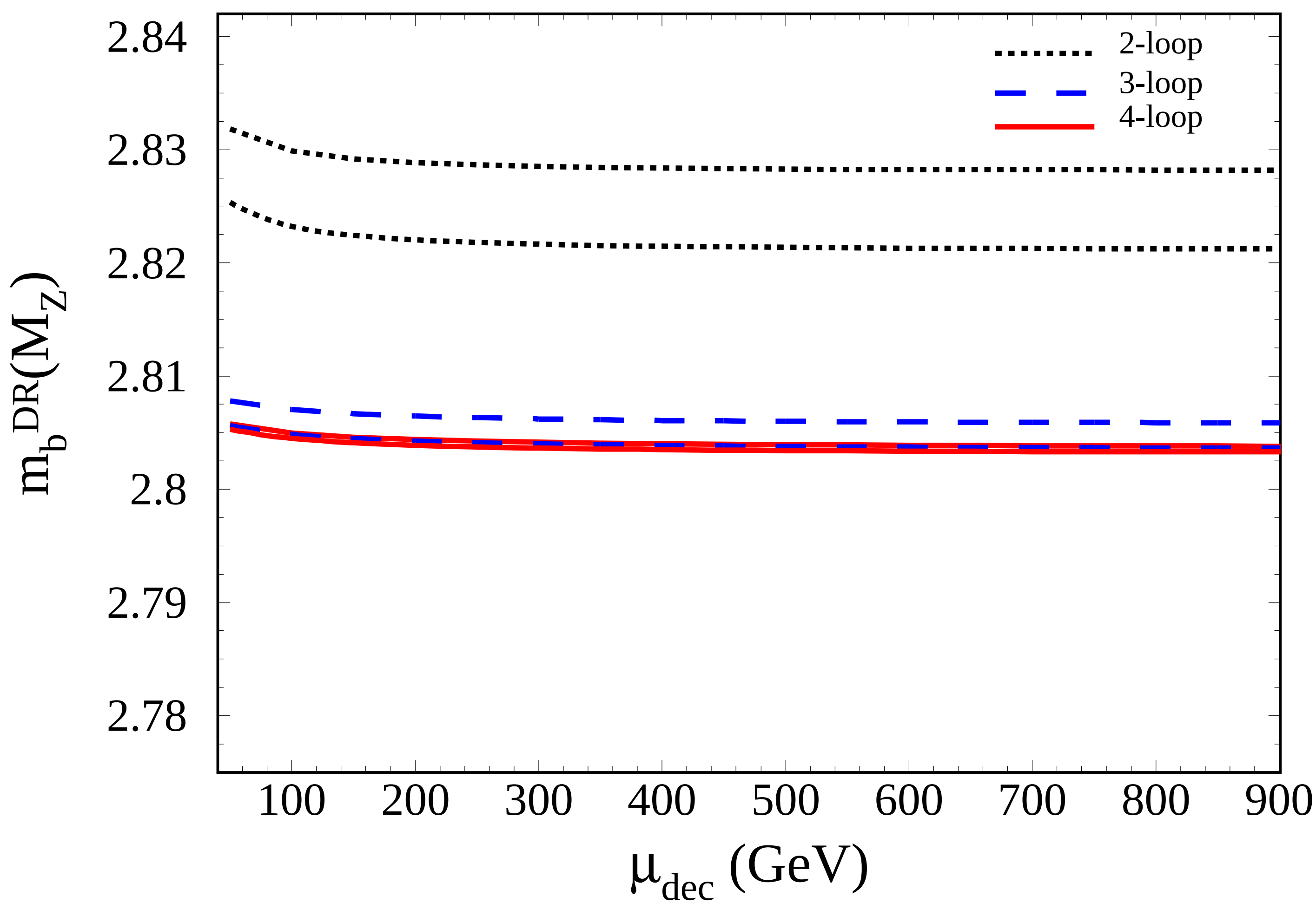


Aim : check 2 ways at $n = 1, 2, 3, 4$-loops

Input parameters:

$\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.1189 \pm 0.001$ *S. Bethke '06* (green band)

$m_b^{\overline{\text{MS}}}(m_b) = 4.164 \pm 0.025$ GeV *J. H. Kühn, M. Steinhauser, C. Sturm '07* (pink band)



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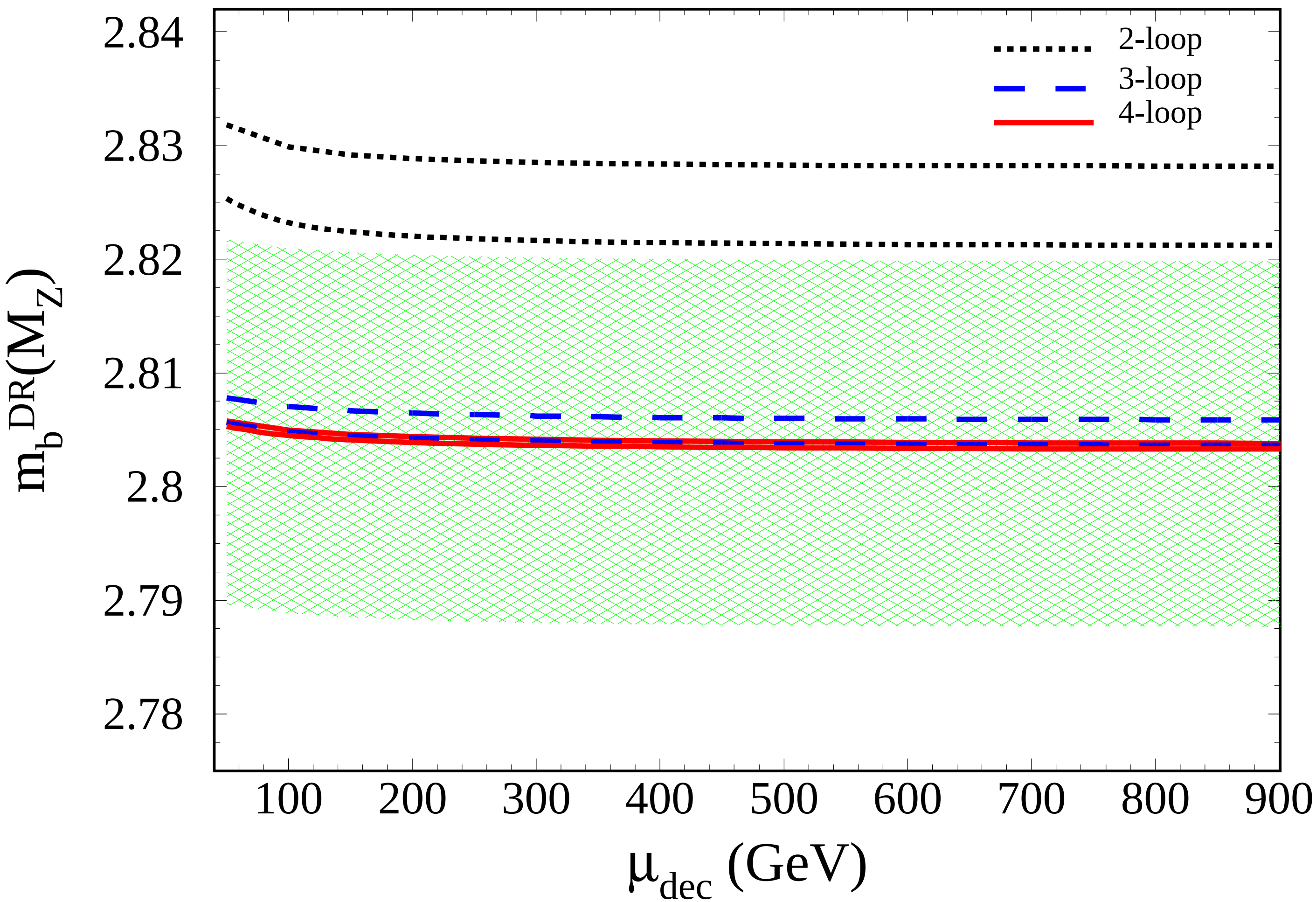

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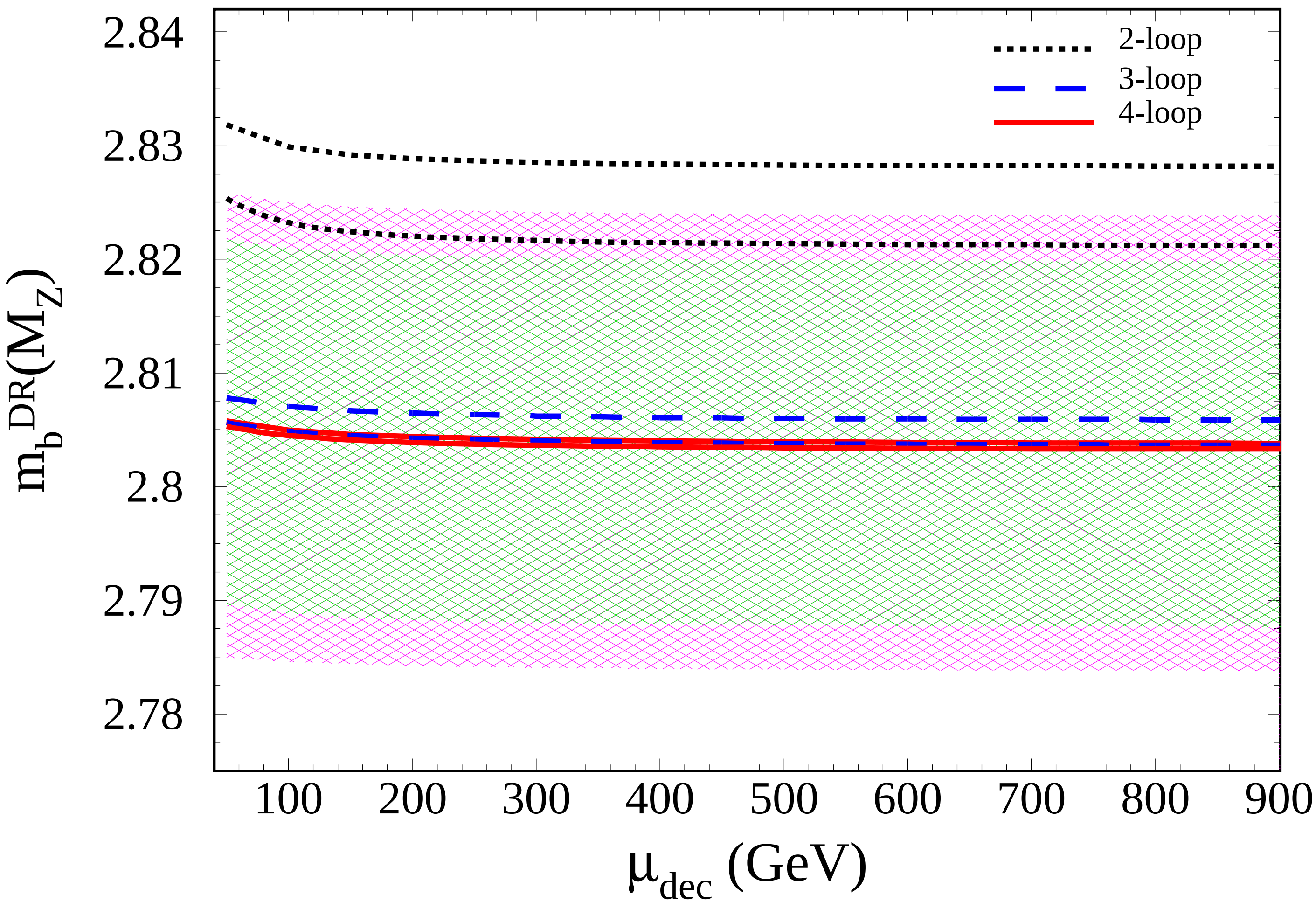
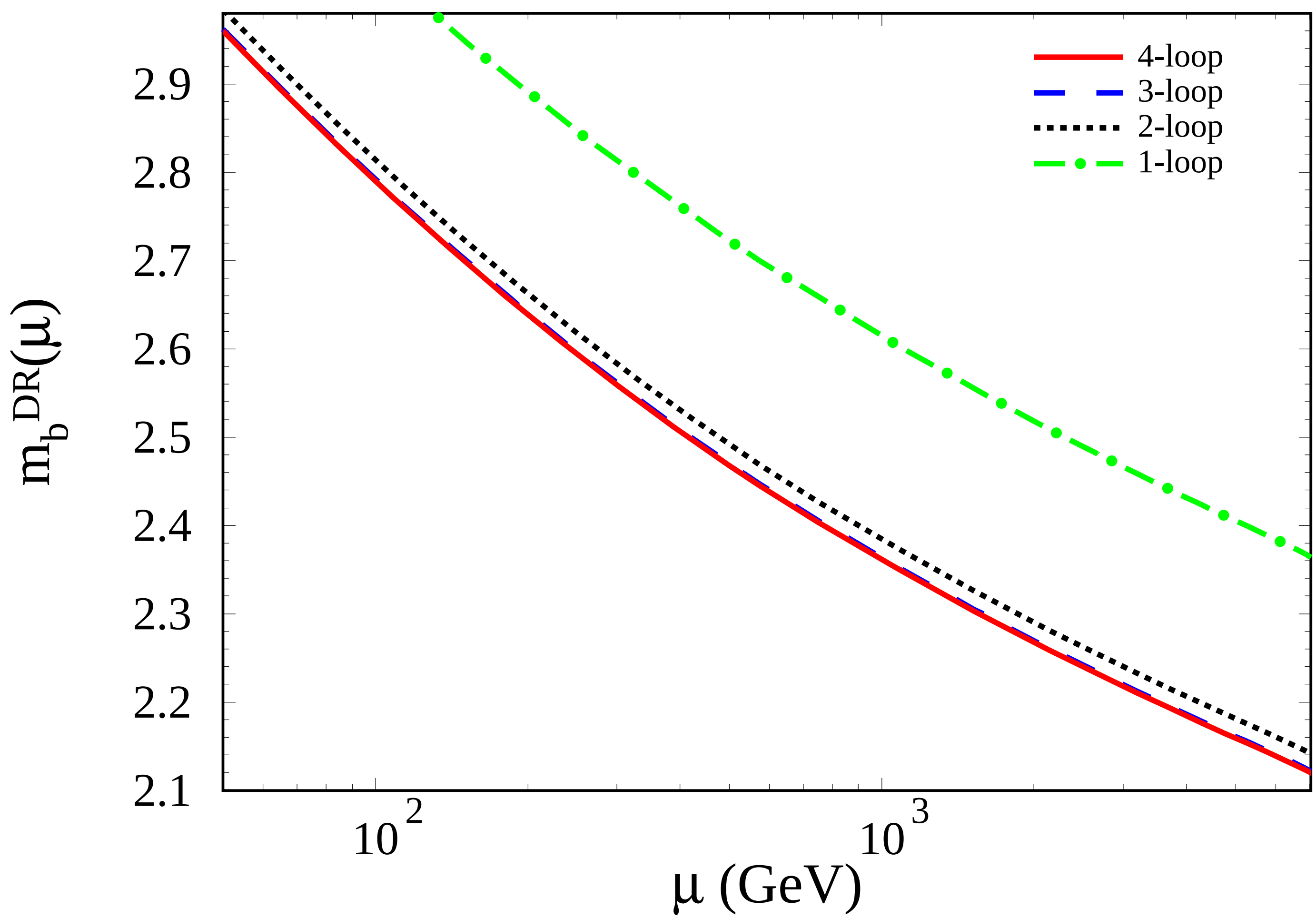


- Running of $m_b^{\overline{\rm DR}}(\mu)$ in QCD& $\overline{\rm DR}$ with 4-loop accuracy



Conclusions

- A consistent approach to compute $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ and $m_b^{\overline{\rm DR}}(M_{\rm SUSY})$ with 3- and 4-loop accuracy is proposed
- The 3-loop effects comparable with the experimental accuracy
- $\overline{\rm DR}$ & QCD tedious, but necessary
- 1-loop LL-approximation not adequate to precision analyses
- $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ very sensitive to SUSY-mass scale

To do:

- Common running analysis for the three couplings: $\alpha_e, \alpha_w, \alpha_s$
- Extend running analysis for (s)quark masses to MSSM

QCD β- and γ_m-functions within DRED

- Dimensional Reduction $\oplus$ Minimal Subtraction $\overline{\mathrm{DR}}$

$$\beta_s^{\overline{\mathrm{DR}}} = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi} \quad \cdots \qquad \gamma_m^{\overline{\mathrm{DR}}} = \frac{\mu^2}{m^{\overline{\mathrm{DR}}}} \frac{\mathrm{d}}{\mathrm{d}\mu^2} m^{\overline{\mathrm{DR}}}$$

$$\beta_s^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{\overline{\mathrm{DR}}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\beta_e(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{e} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\beta_{\eta_r}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \beta_{ijklm}^{\eta_r} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$

$$\gamma_m^{\overline{\mathrm{DR}}}(\alpha_s^{\overline{\mathrm{DR}}}, \alpha_e, \{\eta_r\}) = -\sum_{i,j,k,l,m} \gamma_{ijklm}^{\overline{\mathrm{DR}}} \left(\frac{\alpha_s^{\overline{\mathrm{DR}}}}{\pi}\right)^i \left(\frac{\alpha_e}{\pi}\right)^j \left(\frac{\eta_1}{\pi}\right)^k \left(\frac{\eta_2}{\pi}\right)^l \left(\frac{\eta_3}{\pi}\right)^m$$