Running of α_s in the MSSM with three-loop accuracy

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Abstract. The evolution of the strong coupling constant α_s from M_Z to the GUT scale is presented, involving three-loop running and two-loop decoupling. Accordingly, the two-loop transition from the $\overline{\text{MS}}$ to the $\overline{\text{DR}}$ scheme is properly taken into account. We find that the three-loop effects are comparable to the experimental uncertainty for α_s.

PACS. 11.30.Pb Supersymmetry – 12.38.-t Quantum chromodynamics

1 Introduction

Supersymmetry (SUSY) is currently believed to play an important role in physics beyond the Standard Model (SM). There are several reasons that point out the Minimal Supersymmetric Standard Model(MSSM) as a preferred theory describing new physics. The foremost ones are its milder divergency structure that solves the naturalness problem and the possibility to explain the electroweak supersymmetry breaking as a consequence of radiative corrections. Another compelling argument in favour of SUSY is the particle content of the MSSM, that leads in a natural way to the unification of the three gauge couplings at a high energy scale $\mu \simeq 10^{16}$ GeV, in agreement with Grand Unification Theories (GUT). This observation together with the consistent predictions made for SM parameters, such as the top quark mass and the ratio of the bottom quark to the tau lepton masses, using constraints on the Yukawa sector of SUSY-GUT models, brought SUSY in the center of the phenomenological studies.

Nevertheless, SUSY can only be an approximate symmetry in nature and several scenarios for the mechanism of SUSY breaking have been proposed. A possibility to constrain the type and scale of SUSY breaking is to study, with very high precision, the relations between the MSSM parameters evaluated at the electroweak and the GUT scales. The extrapolation over many orders of magnitude requires high-precision experimental data at the low energy scale. A first set of precision measurements is expected from the CERN Large Hadron Collider (LHC) with an accuracy at the percent level. A comprehensive high-precision analysis can be performed at the International Linear Collider (ILC), for which the estimated experimental accuracy is at the per mill level. In this respect, it is necessary that the same precision is reached also on the theory side in order to match with the data [1]. Running analyses based on full two-loop renormalization group equations (RGEs) [2,3] for all parameters and one-loop threshold corrections [4] are currently implemented in the public programs ISAJET [5], SOFTSUSY [6], SPHENO [7], SuSpect [8]. The agreement between the different codes is in general within one percent [9]. A first three-loop running analysis, based, however, only on one-loop threshold effects, was carried out in Ref. [10].

In this talk, we report on the evaluation of the strong coupling α_s in MSSM, based on three-loop RGEs [11] and two-loop threshold corrections [12]. On the one hand, the three-loop corrections reduce significantly the dependence on the scale at which heavy particles are integrated out [13]. On the other hand, they are essential for phenomenological studies, because they are as large as, or greater than, the effects induced by the current experimental accuracy of $\alpha_s(M_Z)$ [14]. Additionally, we compare the predictions obtained within the above mentioned approach with those based on the leading-logarithmic (LL) approximation suggested in Ref. [1].

2 Evaluation of $\alpha_s(\mu_{\rm GUT})$ from $\alpha_s(M_Z)$

The aim of this study is to compute α_s at a high-energy scale $\mu \simeq \mathcal{O}(\mu_{\rm GUT})$, starting from the strong coupling constant at the mass of the Z boson M_Z. We denote this parameter $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ to specify that the underlying theory is QCD with five active flavours and $\overline{\text{MS}}$ is the renormalization scheme. The value of $\alpha_s(\mu_{\rm GUT})$ is the strong coupling constant in the MSSM renormalized in the $\overline{\text{DR}}$-scheme, that we denote as $\alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm GUT})$. For the evaluation of $\alpha_s^{\overline{\rm DR},({\rm full})}$ from $\alpha_s^{\overline{\rm MS},(n_f)}$ we follow the "common scale approach" [15], which requires a unique scale for the matching between QCD and MSSM. More precisely, for mass independent renormalization schemes like $\overline{\text{MS}}$ or $\overline{\text{DR}}$, the decoupling of heavy particles has to be performed explicitely. In practice, this means that intermediate effective theories are introduced by integrating out the heavy degrees of freedom. One may separately integrate out every particle at its individual threshold ("step approximation"), a method suited for SUSY models with a severely split mass spectrum. But the intermediate effective theories with "smaller" symmetry raise the problem of introducing new couplings, each governed by its own RGE. To overcome this difficulty, for SUSY models with roughly degenerate mass spectrum at the scale $\tilde{M}$, one can consider the MSSM as the full theory that is valid from the GUT scale $\mu_{\rm GUT}$ down to $\tilde{M}$, which we assume to be around 1 TeV.

For the running analysis of the strong coupling constant, we can distinguish four individual steps that we detail below.

1. Running of $\alpha_s^{\overline{\rm MS},(n_f)}$ from $\mu = M_Z$ to $\mu = \mu_{\rm dec}$.
   The energy dependence of the strong coupling constant is governed by the RGEs
   $$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\alpha_s(\mu^2) = \beta(\alpha_s)\,,$$
   $$\beta(\alpha_s) = -\alpha_s^2 \sum_{n\geq 0} \beta_n \alpha_s^n\,. \quad (1)$$
   In QCD with n_f quark flavours, the β function is known through four loops both in the $\overline{\rm MS}$ [16,17] and the $\overline{\rm DR}$-scheme [18].
2. Conversion of $\alpha_s^{\overline{\rm MS},(n_f)}(\mu_{\rm dec})$ to $\alpha_s^{\overline{\rm DR},(n_f)}(\mu_{\rm dec})$.
   For the three-loop running analysis we are focusing on, one needs to evaluate the dependence of α_s values in the $\overline{\rm DR}$ scheme from those evaluated in $\overline{\rm MS}$ scheme with two loops accuracy [18]
   $$\alpha_s^{\overline{\rm MS}} = \alpha_s^{\overline{\rm DR}}\left[1 - \frac{\alpha_s^{\overline{\rm DR}}}{4\pi} - 5\frac{(\alpha_s^{\overline{\rm DR}})^2}{4\pi^2} + n_f\frac{\alpha_s^{\overline{\rm DR}}\alpha_e}{12\pi^2}\right] (2)$$
   Here, the following notations have been used $\alpha_s^{\overline{\rm DR}} \equiv \alpha_s^{\overline{\rm DR},(n_f)}(\mu)$ and $\alpha_s^{\overline{\rm MS}} \equiv \alpha_s^{\overline{\rm MS},(n_f)}(\mu)$. $\alpha_e \equiv \alpha_e^{(n_f)}(\mu)$ is one of the so-called evanescent coupling constants that occur when $\overline{\rm DR}$ is applied to non supersymmetric theories (QCD in this case). In particular, it describes the coupling of the 2ε-dimensional components (so-called ε-scalars) of the gluon to a quark. It is an unphysical parameter that must decouple from any prediction for physical observables. We also used this property as a consistency check for our method. As mentioned above, we assume that QCD is obtained by integrating out the heavy degrees of freedom (squarks and gluinos) from SUSY-QCD. In this case, the evanescent couplings are uniquely determined by the matching conditions between the two theories, that we discuss below.
3. Matching of $\alpha_s^{\overline{\rm DR},(n_f)}(\mu_{\rm dec})$ and $\alpha_s^{\overline{\rm DR},({\rm full})}(\mu_{\rm dec})$.
   Integrating out all SUSY particles at the common scale of SUSY mass spectrum, one directly obtains the SM as the effective theory, valid at low energies. The transition between the two theories can be done at an arbitrary decoupling scale μ:
   $$\alpha_s^{\overline{\rm DR},(n_f)}(\mu) = \zeta_s^{(n_f)}\,\alpha_s^{\overline{\rm DR},({\rm full})}(\mu)$$
   $$\alpha_e^{(n_f)}(\mu) = \zeta_e^{(n_f)}\alpha_e^{({\rm full})}(\mu)\,. \quad (3)$$
   ζ_s and ζ_e depend logarithmically on the scale μ, which is why one generally chooses $\mu \sim \tilde{M}$. In Eq. (3), $n_f = 6$ means that only the SUSY particles are integrated out, while for $n_f = 5$ at the same time the top quark is integrated out.
   In a supersymmetric theory, SUSY requires that $\alpha_e^{({\rm full})}(\mu) = \alpha_s^{\overline{\rm DR},({\rm full})}(\mu)$ at any scale. Let us remark that $\alpha_s^{\overline{\rm DR},(full)}(\mu_{\rm dec})$ is not known *a priori* and one cannot use Eq. (3) directly in order to derive $\alpha_e^{(5)}$. Rather, we start with a trial value for $\alpha_s^{\overline{\rm DR},(full)}(\mu_{\rm dec})$ and obtain the corresponding $\alpha_e^{(5)}(\mu_{\rm dec})$ as well as $\alpha_s^{\overline{\rm DR},(5)}(\mu_{\rm dec})$ through Eq. (3). Then we evaluate $\alpha_s^{\overline{\rm MS},(5)}(\mu_{\rm dec})$ through Eq. (2), and from that $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$. The trial value for $\alpha_s^{\overline{\rm DR},(full)}(\mu_{\rm dec})$ is systematically varied until the resulting $\alpha_s^{\overline{\rm MS},(5)}(M_Z)$ agrees with the experimental input.
4. Running of $\alpha_s^{\overline{\rm DR},({\rm full})}$ from $\mu = \mu_{\rm dec}$ to $\mu = \mu_{\rm GUT}$.
   The energy dependence of the strong coupling constant is in this case governed by the MSSM RGEs. In SUSY-QCD, the β function has been evaluated in the $\overline{\rm DR}$-scheme through three loops [11].

Assembling the above mentioned steps, we can predict the value of $\alpha_s(\mu_{\rm GUT})$ with up to three-loop accuracy. This procedure is implemented in most of the present codes computing the SUSY spectrum [7,6,8] by applying the one-loop approximation of Eq. (3) and setting $n_f = 5$ and $\mu = M_Z$. The advantage of this procedure as compared to a multi-scale approach is that the RGEs are only one-dimensional and that for α_e one can apply Eq. (3).

Let us note that in principle it is possible to decouple the top quark separately. The only new ingredient needed is the decoupling constant for going from five to six quark flavours in the $\overline{\rm MS}$ scheme. In any case, for a mass spectrum as given by the benchmark point SPS1a′ [1], for example, the separate decoupling of the top quark implies a numerically small effect. This can also be established by comparing "Scenario D" and "Scenario C" in Ref. [12].

The phenomenological significance of the three-loop order corrections is discussed in detail in the next section.

3 Numerical results

The result for $\alpha_s^{\overline{\rm DR}}(\mu_{\rm GUT} = 10^{16}$ GeV$)$, obtained using $M_Z = 91.1876$ GeV, $m_t = 170.9{\pm}1.9$ GeV , $\alpha_s^{\overline{\rm MS}}(M_Z) = 0.1189$, and $\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV as input parameters is shown if Figure 1. The dotted, dashed and solid line are based on the approach described above,

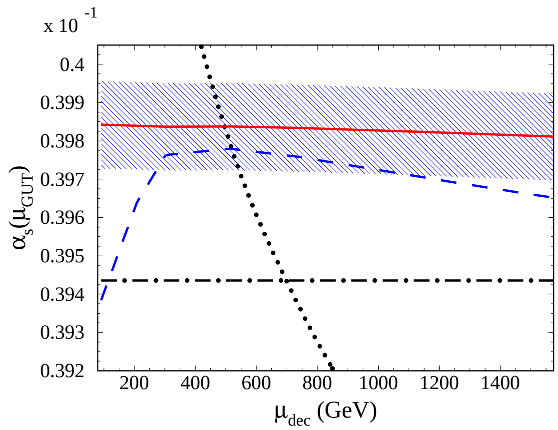

Fig. 1. $\alpha_s(\mu_{\rm GUT})$ as a function of $\mu_{\rm dec}$.

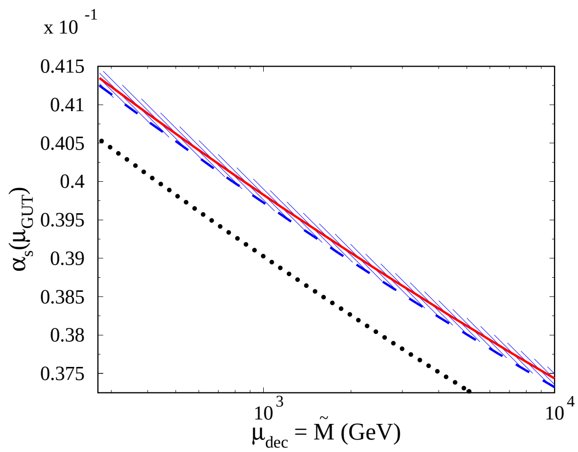

Fig. 2. $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$.

where n-loop running is combined with $(n-1)$-loop decoupling, as it is required for consistency ($n = 1, 2, 3$, respectively). We find a nice convergence when going from one to three loops, with a very weakly $\mu_{\rm dec}$–dependent result at three-loop order. For comparison, we show the result (the dash-dotted line) obtained from the formula given in Eq. (21) of Ref [1]. It corresponds to the resummed one-loop contributions originating from both the change of scheme and the decoupling of heavy particles. However, the difference between our three-loop result with two-loop decoupling (upper solid line) and the one-loop formula given in Ref. [1] exceeds the experimental uncertainty by almost a factor of four for sensible values of $\mu_{\rm dec}$. This uncertainty is indicated by the hatched band, derived from $\delta\alpha_s(M_Z) = \pm 0.001$ [14]. The formulae of Ref. [1] should therefore be taken only as rough estimates.

In Figure 2 we show $\alpha_s(\mu_{\rm GUT})$ as a function of $\tilde{M}$ where $\mu_{\rm dec} = \tilde{M}$ has been adopted. Dotted, dashed and full curve correspond again to the one-, two- and three-loop analysis and the uncertainty form $\alpha_s(M_Z)$ is indicated by the hatched band. One observes a variation of 10% as $\tilde{M}$ is varied between 100 GeV and 10 TeV. This shows that the actual SUSY scale can significantly influence the unification, respectively, the non-unification behaviour of the three couplings at the GUT scale.

4 Conclusions

We have used recent three- and two-loop results for the β functions and the decoupling coefficients, respectively, in order to derive $\alpha_s^{\overline{\rm DR}}(\mu_{\rm GUT})$ from $\alpha_s^{\overline{\rm MS}}(M_Z)$ at three-loop level.

It turns out that the three-loop terms are numerically significant. The dependence on where the SUSY spectrum is decoupled becomes particularly flat in this case. The theoretical uncertainty is expected to be negligible w.r.t. the uncertainty induced by the experimental input values.

Comparing our results and methods to the literature, we find that the issue of evanescent couplings has either been ignored (by assuming $\alpha_e = \alpha_s$) or circumvented by decoupling the SUSY spectrum at $\mu_{\rm dec} = M_Z$. We find that at one- and two-loop level, this choice does not allow for a good approximation of the higher order effects, if one assumes the SUSY partner masses to be of the order of 1 TeV.

In consequence, we recommend that phenomenological studies concerning the implications of low energy data on Grand Unification should be done at three-loop level.

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