

Stability and Leptogenesis in Left-Right Symmetric Seesaw Models

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Abstract. In left-right symmetric seesaw models an eight-fold degeneracy among the right-handed neutrino mass matrices is known to exist. We use stability and viability of leptogenesis as criteria in order to discriminate among the degenerate solutions and to partially lift the eight-fold degeneracy.

PACS. 98.80.Cq Particle-theory and field-theory models of the early Universe – 14.60.Pq Neutrino mass and mixing – 14.60.St Non-standard-model neutrinos, right-handed neutrinos

1 Introduction

It has become an established fact that neutrinos are massive, although very light. The Standard Model (SM) of particle physics is insufficient in explaining in a natural way why the neutrinos are so much lighter than the charged leptons. One attractive and elegant solution to this problem is provided by the seesaw mechanism [2], where the small masses of the neutrinos are induced by the presence of heavy particles, such as right-handed Majorana neutrinos or Higgs triplets. The seesaw mechanism has in addition the attractive feature of a built-in mechanism for generating the baryon asymmetry of the Universe (BAU) through the baryogenesis via leptogenesis mechanism [3], whereby the decays of the right-handed neutrinos produce a lepton asymmetry, which is then partly converted into a baryon asymmetry by sphaleron processes.

Recently, it was demonstrated [4,5] that in a certain class of left-right symmetric seesaw models, the seesaw mass formula can be inverted to yield eight (in the three-flavor case) possible mass matrices for the right-handed Majorana neutrinos, for a given light neutrino mass matrix and Dirac Yukawa couplings as input. Since the right-handed neutrino sector is not directly accessible by laboratory experiments, it is important to find ways of discriminating among the eight seesaw solutions. This was the objective of Refs.[1,6], where leptogenesis and stability of the solutions were used as selection criteria and it was shown that it is possible to partially lift the degeneracy in the right-handed neutrino sector.

2 The left-right symmetric seesaw model

We will consider the left-right symmetric seesaw model with gauge group $G = SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [7, 8]. The SM field content is extended to include three heavy right-handed Majorana neutrinos N_i and the Higgs sector contains an $SU(2)_L \times SU(2)_R$ bidoublet ϕ as well as $SU(2)_{L,R}$ triplets $\Delta_{L,R}$. When the neutral components of the Higgs fields acquire vacuum expectation values (VEVs) we obtain the type I+II seesaw mass formula

$$m_\nu = f v_L - \frac{v^2}{v_R} y f^{-1} y^T, \quad (1)$$

where v, v_L , and v_R denote the VEVs of the neutral components of ϕ and $\Delta_{L,R}$, y denotes the Dirac Yukawa coupling matrix and f the triplet Yukawa coupling matrix. In general, the triplet Yukawa couplings for $\Delta_{L,R}$ could be different, but we will restrict to the case where there is a discrete left-right symmetry such that $f_L = f_R \equiv f$ and y is symmetric. This case has much fewer parameters and is more predictive. We will assume that the Dirac Yukawa coupling matrix y equals the up-type quark Yukawa coupling matrix y_u , which is motivated by grand unified theories (GUTs). Many of the results depend mostly on the fact that y has hierarchical eigenvalues. The free parameters will then be the absolute neutrino mass scale m_0 , the ratio of the triplet VEVs, and the light neutrino mass hierarchy. The seesaw formula Eq. (1) can now be inverted to yield 2^N solutions (for N generations) for f , and hence 2^N possible right-handed Majorana mass matrices, which are given by $M_R = f v_R$. In the one-generation case, the two solutions admit a simple analytical form

$$f_\pm = \frac{m_\nu}{2v_L} \pm \frac{1}{v_L} \sqrt{\frac{m_\nu^2}{4} + \frac{y^2 v_L v^2}{v_R}}. \quad (2)$$

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^b This talk is based on Ref. [1].

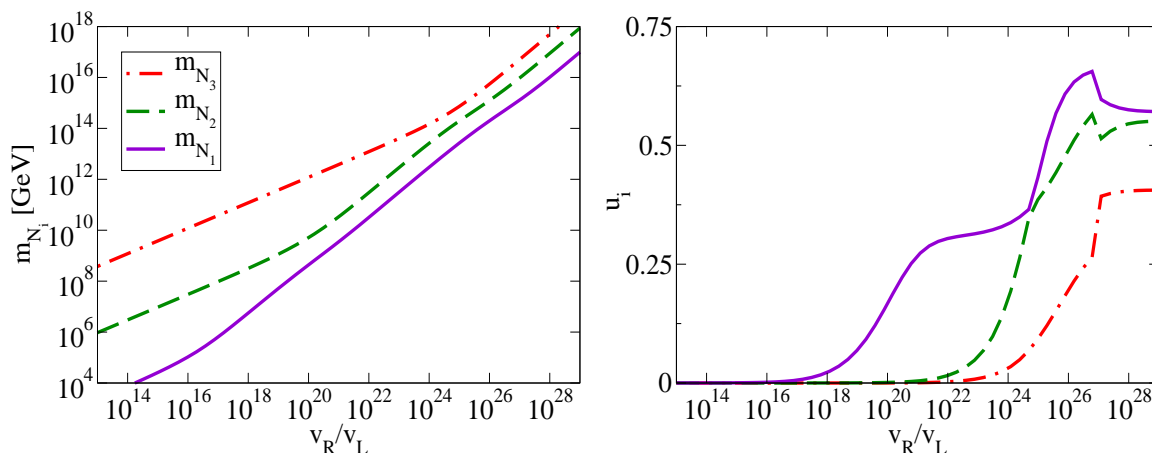


Fig. 1. The right-handed neutrino masses m_{N_i} and mixing parameters u_i as functions of v_R/v_L for the solution ‘+++’. Inverted mass hierarchy, $m_0 = 0.1$ eV. Figure from Ref. [1].

Analytical solutions exist also in the three-flavor case and we label the eight solutions as ‘ $\pm \pm \pm$ ’ where ‘-’(‘+’) denotes that the corresponding mass eigenvalue is type I (II) dominated in the large v_R/v_L regime. The labeling starts with the largest eigenvalue in the small v_R/v_L regime. Thus, there will be two solutions which are either pure type I or type II dominated for large v_R/v_L and six mixed solutions.

In Fig. 1 we show the eigenvalues and mixing for the solution ‘+++’ for $m_0 = 0.1$ eV and inverted light neutrino mass hierarchy. As a measure of mixing, we have introduced the parameters u_i which are related to the off-diagonal elements of the unitary matrix U which diagonalize f as:

$$\begin{aligned} u_1^2 &= \frac{1}{2}(|U_{12}|^2 + |U_{21}|^2), & u_2^2 &= \frac{1}{2}(|U_{13}|^2 + |U_{31}|^2), \\ u_3^2 &= \frac{1}{2}(|U_{23}|^2 + |U_{32}|^2). \end{aligned} \quad (3)$$

In the next section, we will consider ways of selecting among the eight solutions.

3 Discriminating among the degenerate solutions

3.1 Stability

First we will introduce a notion of naturalness, or stability, in order to select among the degenerate solutions. We pose the question if, for a given light neutrino mass matrix, the triplet Yukawa coupling matrix f has to be very special or fine-tuned in the sense that a marginally different f would give very different low-energy neutrino phenomenology. We consider such a situation unnatural. In order to quantify the stability of the solutions we introduce the following stability measure

$$Q = \left| \frac{\det f}{\det m_\nu} \right|^{1/3} \sqrt{\sum_{k,l=1}^{2N} \left(\frac{\partial m_l}{\partial f_k} \right)^2}. \quad (4)$$

The matrices f and m_ν are determined by the real coefficients f_k and m_l according to

$$f = \sum_k (f_k + i f_{k+N}) T_k, \quad m_\nu = \sum_k (m_k + i m_{k+N}) T_k, \quad (5)$$

where T_k , $k \in [1, 6]$, form a normalized basis of complex symmetric 3×3 matrices. This stability measure can be shown to be basis independent [1]. Fig. 2 shows the stability measure Q for the eight solutions, for $m_0 = 0.1$ eV and an inverted light neutrino mass hierarchy. First we note that all solutions are unstable for small v_R/v_L , which is due to the fact that in this regime there must be a very precise cancellation between the type I and type II contributions to the light neutrino mass matrix in Eq. (1). In addition, most of the solutions also become unstable for large v_R/v_L . This happens when there is a large spread in the eigenvalues, which imply suppressed mixing and fine-tuning between f and y (for a detailed discussion see Ref. [1]). The most stable solution is the purely type II dominated ‘+++’ solution. If one allows for a fine-tuning at the percent level, $Q \lesssim 10^3$, then the two solution ‘ $\pm \pm \pm$ ’, for $v_R/v_L \gtrsim 10^{18}$ and the solutions ‘ $\pm - +$ ’ with $v_R/v_L \simeq 10^{20}$ are favored. The stability results do not change qualitatively, when changing m_0 or the light neutrino mass hierarchy. Additional CP violating phases only change the results marginally.

3.2 Leptogenesis

The second selection criteria we use is viability of leptogenesis for a given solution. Thus, we ask whether a solution can reproduce the observed BAU, $\eta_B = (6.1 \pm 0.2) \times 10^{-10}$ [9]. The baryon asymmetry can be parametrized as

$$\eta_B \equiv \frac{n_B}{n_\gamma} = \eta \epsilon_{N_1}, \quad (6)$$

where η is the so-called efficiency factor that takes into account washout effects, the initial density of the right-handed neutrinos and the deviation from equilibrium

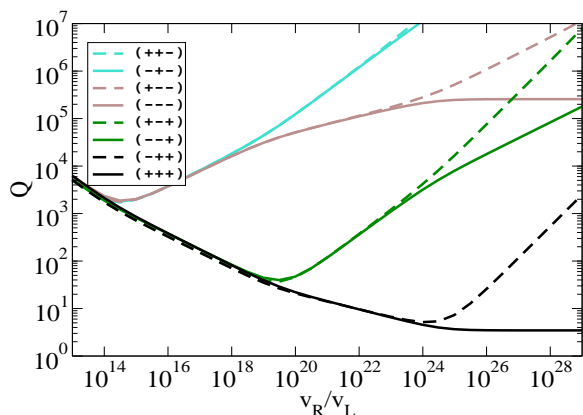


Fig. 2. The stability measure Q as a function of v_R/v_L for $m_0 = 0.1$ eV and inverted hierarchy. Figure from Ref. [1].

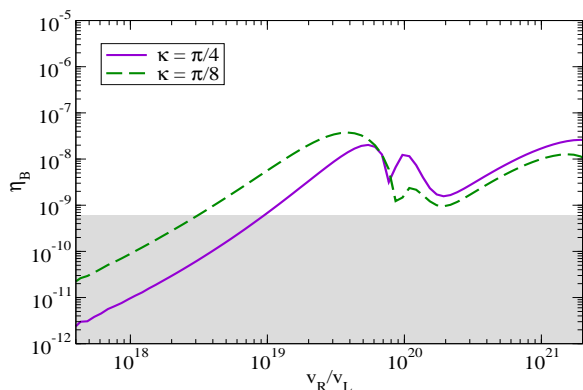


Fig. 3. The baryon-to-photon ratio η_B with an additional complex phase $\pi/8$ or $\pi/4$ attributed to the electron neutrino for the solution '+++'. The shaded area corresponds to values of η_B below the observed value. Inverted mass hierarchy, $m_0 = 0.1$ eV. Figure from Ref. [1].

in their decays and ϵ_{N_1} denotes the CP-asymmetry parameter, which measures the CP-asymmetry in the decay of the lightest right-handed neutrino. In Eq. (6) we have assumed that the lightest right-handed neutrino is much lighter than the other heavy neutrinos, such that it gives the dominating contribution to the lepton asymmetry. In Ref. [10] we study leptogenesis in the same framework, but where instead the decay of the triplet Δ_L is the source of the lepton asymmetry. The CP-asymmetry parameter is defined as

$$\epsilon_{N_i} = \frac{\Gamma(N_i \rightarrow l H) - \Gamma(N_i \rightarrow \bar{l} H^*)}{\Gamma(N_i \rightarrow l H) + \Gamma(N_i \rightarrow \bar{l} H^*)}. \quad (7)$$

From a numerical solution of the Boltzmann equations, it is possible to derive the following expression for the efficiency parameter

$$\eta = 1.45 \times 10^{-2} \frac{10^{-3} \text{ eV}}{\tilde{m}_1}, \quad (8)$$

assuming a mass hierarchy in the right-handed neutrino sector. Here we have also introduced the effective neutrino mass

$$\tilde{m}_i = \frac{v^2 (\hat{y}^\dagger \hat{y})_{ii}}{2m_{N_i}}. \quad (9)$$

The notation with a hat indicates that all matrices are evaluated in the basis where f is real and diagonal. The CP-asymmetry parameter receives contributions of type I (from the right-handed neutrinos) and type II (from the triplets). In the limit where the lightest right-handed neutrino is considerably lighter than the heavier neutrinos as well as the triplets, the CP-asymmetry can be written as [11]

$$\epsilon_{N_1} = \epsilon_{N_1}^I + \epsilon_{N_1}^{II} \rightarrow \frac{3}{16\pi} \frac{\hat{f}_{11} v_R}{v^2} \frac{\text{Im}[(\hat{y}^\dagger \hat{m}_\nu \hat{y}^*)_{11}]}{(\hat{y}^\dagger \hat{y})_{11}}. \quad (10)$$

Our numerical results indicate that the upper bound on the CP asymmetry found in Ref. [11] can be saturated. An important feature of the left-right symmetric model is the presence of additional physical Majorana phases, which can increase the CP asymmetry and improve the prospects for leptogenesis. As demonstrated in Ref. [1], this allows for leptogenesis even with only one right-handed neutrino. In the case of three generations there are more sources of CP violation, although mixing can increase the effective neutrino mass, thereby increasing the washout. Fig. 3 shows that baryon-to-photon ratio η_B for the solution '+++', when a Majorana phase κ is attributed to the electron neutrino. We have chosen $\kappa = \pi/4$ ($\pi/8$). The mass of the lightest right-handed neutrino required to reproduce the observed BAU is $m_{N_1} \gtrsim 1.4 \times 10^9$ GeV ($m_{N_1} \gtrsim 2.5 \times 10^8$ GeV). We find that leptogenesis is possible for four out of the eight solutions. For the other solutions, leptogenesis is not possible, since for these cases the mass of the lightest right-handed neutrino never exceeds 10^6 GeV. It should be noted that the results depend on the assumption that $y = y_u$. For a different choice of Yukawa couplings it is possible to relax the mass bounds. However, the choice $y = y_u$, as discussed before, is motivated by GUTs. Our results complements the results of the leptogenesis analysis in Ref. [6], since the washout is less severe for our specific choice of parameters.

4 Summary

We have studied the left-right symmetric type I+II seesaw mechanism with a hierarchical Dirac Yukawa coupling matrix motivated by GUTs. It has been shown previously that it is possible to invert the seesaw mass formula to obtain eight possible triplet Yukawa coupling matrices for a light neutrino mass matrix and Dirac Yukawa coupling matrix as input. Our goal was to discriminate among these degenerate solutions using stability properties and viability of leptogenesis as criteria. As a measure of stability we have introduced the parameter Q (see Eq. (4)) which quantifies the amount of fine-tuning.

Our results are summarized in Tab. 1. The stability criterion disfavors the four solutions of the type ' $\pm\pm-$ ' and favors the solutions ' $\pm-+$ ' provided that $v_R/v_L \simeq 10^{20}$. The remaining two solutions ' $\pm++$ ' are stable, for $v_R/v_L \gtrsim 10^{18}$. In addition, we find that successful leptogenesis is possible for the four solution of the

	$\pm + +$	$\pm - +$	$\pm \pm -$
Stability	$v_R/v_L > 10^{18}$	$v_R/v_L \simeq 10^{20}$	disfavored
Leptogenesis	$v_R/v_L > 10^{18}$	$v_R/v_L > 10^{18}$	excluded
Gravitinos	$v_R/v_L < 10^{21}$	unconstrained	unconstrained

Table 1. The allowed regions of the parameter v_R/v_L for the eight different types of solutions. Table from Ref. [1].

type ' $\pm \pm +$ ' if $v_R/v_L \gtrsim 10^{18}$. This requires the existence of additional Majorana-type phases, not present in the pure type I seesaw framework. Additional constraints would also be present in a supersymmetrized version, coming from the potentially dangerous overproduction of gravitinos in the early universe. For our specific choice of Yukawa couplings, $y = y_u$, only the solutions of the type ' $\pm + +$ ' are affected by this constraint. This leads to the requirement $v_R/v_L \lesssim 10^{21}$. Thus, we have shown, that it is possible, within the chosen framework, to partially lift the eight-fold degeneracy among the right-handed neutrino mass matrices in the left-right symmetric seesaw model.

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