Smooth Heterotic Compactifications and anomalous U(1)s

hep-th/0612030 by G.H. and Michele Trapletti

& hep-th/0602101 by G.H.

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Motivation

Different string theory model building strategies:

- Heterotic Abelian orbifolds of $E_8 \times E_8$ since $\sim$ '87

  see review talk by Nilles,

  also Groot Nibbelink, Kyae, Lebedev, Lüdeling, Wingerter
- Heterotic $E_8 \times E_8$ GUTs $SO(10)$ or $SU(5)$ on Calabi-Yaus with $SU(4)$ or $SU(5)$ bundles $\sim$ '99
- D-brane model building with intersecting or magnetised branes in type II orientifolds, Gepner models ... $\sim$ '99

  see Gmeiner, Suruliz

  Intersecting branes have description via *cycles* at *orbifold* point and on *Calabi-Yau's*

  Magnetic backgrounds are S-dual to heterotic $SO(32)$ compactifications with $U(1)$ *backgrounds*

Relations of heterotic orbifolds and CY_3 compactifications?

Motivation

This talk:
Consider heterotic $K3$ compactifications to $\mathcal{N}=1$ in 6D and their $T^4/\mathbb{Z}_N$ orbifold limits:

- $K3$ is unique
- check from $\mathrm{tr}R^4$, $\mathrm{tr}_{SU(N)}F^4$, $\mathrm{tr}_{SO(2M)}F^4$ anomaly cancellation in 6D: the massless spectrum with all singlets is known completely

For simplicity restrict to

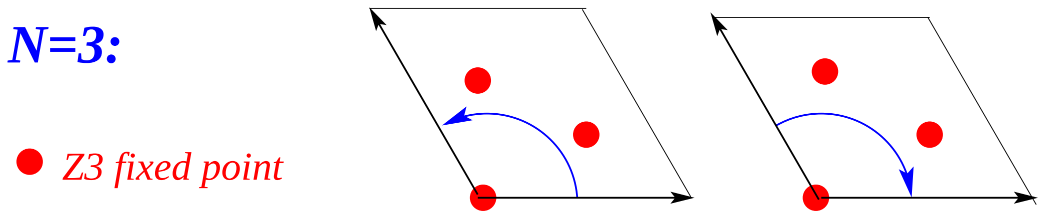
- perturbative vacua, no 5-branes

Het. $T^4/\mathbb{Z}_N$ orbifolds: consistency

- Geometry: singular $K3$ limit: $T^4/\mathbb{Z}_N$ with $N = 2, 3, 4, 6$

given by rotation:

$$\theta : z_k \to e^{2\pi v_k} z_k \quad \vec{v} = \frac{1}{N}(1, -1)$$



- Orbifold action is embedded into gauge d.o.f. via shift vector

$$\vec{V} = \frac{1}{N}(1, \dots, 1, 2, \dots, 2, \dots, 0, \dots, 0)$$

- *Quadratic* level matching condition:

$N\left(\sum_i V_i^2 - \sum_i v_i^2\right) = 0 \bmod 2$

- *Linear*: Spinors in the gauge bundle: $N \sum_i V_i = 0 \bmod 2$

Spectra & anomalies

Massless spectra are computed from the weight vectors $\vec{w} = (\underline{\pm 1, \pm 1, 0^{14}})$ of e.g. $SO(32)$ *($E_8 \times E_8$ discussion identical)*:

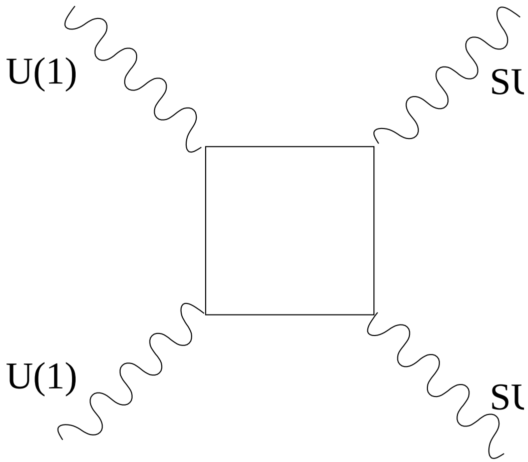
- gauge group: $\vec{w} \cdot \vec{V} \in \mathbb{Z}$, untwisted matter: $\vec{w} \cdot \vec{V} \notin \mathbb{Z}$
- normalise $U(1)$ charges to fit with untwisted sector of smooth case
- n^{th} twisted matter: $\vec{w} - n\vec{V}$ twisted ground state
  •count fixed points
  •identify oscillator states which lift tachyonic vacuum to massless level

⇒anomaly polynomial factorises as 4×4 G.H., Trapletti, 0612030

⇒absence of 2×6 term indicates that $U(1)$s are massless at the orbifold point

Anomalies

Anomaly arises via fermion loops (tensors also possible, but *perturbative* contributions cancel among universal tensor and SUGRA part)



- $\mathrm{tr}R^4$ and $\mathrm{tr}F^4$ anomalies are pathological - *absent*
- only *factorisable* anomalies can be cancelled
- on orbifolds: 4×4 factorisation only

$T^4/\mathbb{Z}_N$ anomaly polynomial

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Pert. anomaly polynomial at the orbifold point, e.g. $SO(32)$:

$$\left(\mathrm{tr}R^2 + \sum_i \alpha_i \mathrm{tr}_{SO(2M_i)} F^2 + \sum_j \beta_j \mathrm{tr}_{SU(N_j)} F^2 + \sum_k \gamma_k F^2_{U(1)_k} + \sum_{i<j} \delta_{ij}\, F_{U(1)_i}\, F_{U(1)_j}\right)$$
$$\times \left(\mathrm{tr}R^2 - \sum_i \mathrm{tr}_{SO(2M_i)} F^2 - 2\sum_j \mathrm{tr}_{SU(N_j)} F^2 + \sum_k \tilde{\gamma}_k F^2_{U(1)_k}\right) = I_8^{SO(3}$$

with

- at most two $SO(2M_i)$ factors
- regular case: $\alpha = 2$ and only *fundamental* reps. of $SO(2M)$ occur
- exceptions: $\alpha \leq 1$ and *spinor* reps. of $SO(2M)$
- $\beta \leq 1$

$\Rightarrow$Match coefficients with smooth $K3$ case

Heterotic $K3$ compactifications

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For both $SO(32)$ and $E_8 \times E_8$ compactifications:

- $\mathcal{N}=1$ SUSY in 6D $\Leftrightarrow \mathbb{R}^{1,5} \times K3$
- Switch on gauge background bundle along $K3$

  'standard embedding': spin = gauge connection: $SU(2)$ bundle on $K3$
- *Perturbative* gauge group is the *commutant* of the background bundle in $E_8 \times E_8$ or $SO(32)$
- Massless spectrum determined for given *bundle*
- No $\mathrm{tr}R^4$, $\mathrm{tr}_{SU(N)}F^4$, $\mathrm{tr}_{SO(2M)}F^4$ anomalies
- Factorisable field theory anomalies need to be cancelled by generalised Green Schwarz (GS) mechanism

$\Rightarrow$ massive $U(1)$s occur, geometric moduli frozen

Consistency conditions:

Background gauge bundle V with field strength $\overline{F}$ is SUSY:

- $\overline{F}$ is holomorphic (1,1) and primitive $\int_{K3} J \wedge \mathrm{tr}\overline{F} = 0$:

  J: Kähler form on $K3$

  $\Rightarrow$2+1 geometric moduli frozen

- $K3$: 20 neutral hyper multiplets:
  •19: 3 geometric + 1 B-field modulus
  •1: $K3$ volume + B-field triplet
  SUSY $\Rightarrow$massive mult. if B also frozen: GS couplings

- *Linear:* 'K-theory constraint' in order to have spinors on the worldsheet

  $N\sum_i V_i = 0 \bmod 2 \Leftrightarrow$

$$\frac{1}{2\pi}\mathrm{tr}\overline{F} \equiv c_1(W_{total}) \in H^2(K3, 2\mathbb{Z})$$

- *Quadratic:* Bianchi identity on $H_3 = dB - \frac{\alpha'}{4}(\omega_{YM} - \omega_L)$:

  $N\sum_i (V_i^2 - v_i^2) = 0 \bmod 2 \Leftrightarrow$

$$\mathrm{tr}\overline{F}^2 - \mathrm{tr}\overline{R}^2 = 0$$

Massless spectra

Massless spectrum contains:

- $\mathcal{N} = 1$ SUGRA multiplet
- 20 neutral hyper mults.
- 1 universal tensor mult. (dilaton + truncation of B to 6D)
- Gauge group & charged hyper mults. *model dependent*

Decomposition of the adjoint of $\mathbf{SO(32)}$ $\to SO(2M) \times \prod_{j=1}^{K} U(N_j\, n_j)$ leads to the assignment of bundles and matter reps:

$$\mathbf{496} \to \left(\begin{array}{c} (\mathbf{Anti}_{SO(2M)}, \mathbf{1}) + \sum_{j=1}^{K} (\mathbf{Adj}_{U(N_j)}; \mathbf{Adj}_{U(n_j)}) \\ \sum_{j=1}^{K} (\mathbf{Anti}_{U(N_j)}; \mathbf{Sym}_{U(n_j)}) + (\mathbf{Sym}_{U(N_j)}; \mathbf{Anti}_{U(n_j)}) + h \\ \sum_{i<j} (\mathbf{N}_i, \mathbf{N}_j; \mathbf{n}_i, \mathbf{n}_j) + (\mathbf{N}_i, \overline{\mathbf{N}}_j; \mathbf{n}_i, \overline{\mathbf{n}}_j) + h.c. \\ \sum_{j=1}^{K} (\mathbf{2M}, \mathbf{N}_j; \mathbf{n}_j) + h.c. \end{array} \right.$$

Massless spectra for $SO(32)$

Matter is counted by *cohomology classes* of $U(n_i)$ bundles:

reps.	$H = SO(2M) \times \prod_{i=1}^{K} SU(N_i) \times U(1)_i$
$(\mathbf{Adj}_{U(N_i)})_{0(i)}$	$H^*(K3, V_i \otimes V_i^*)$
$(\mathbf{Sym}_{U(N_i)})_{2(i)}$ $(\mathbf{Anti}_{U(N_i)})_{2(i)}$	$H^*(K3, \bigwedge^2 V_i)$ $H^*(K3, \bigotimes_s^2 V_i)$
$(\mathbf{N}_i, \mathbf{N}_j)_{1(i),1(j)}$ $(\mathbf{N}_i, \overline{\mathbf{N}}_j)_{1(i),-1(j)}$	$H^*(K3, V_i \otimes V_j)$ $H^*(K3, V_i \otimes V_j^*)$
$(\mathbf{Adj}_{SO(2M)})_0$ $(\mathbf{2M}, \mathbf{N}_i)_{1(i)}$	$H^*(K3, \mathcal{O})$ $H^*(K3, V_i)$

- *Net chirality* is counted by the index $\chi(W)$ of $H^*(K3, W)$
- 6D: gauginos and matter fermions have opposite chirality

$\Rightarrow \chi(W)_{K3} \equiv \mathrm{ch}_2(W) + 2\,\mathrm{rank}(W) \sim$ # Vector - # Hyper is sufficient to compute *massless* matter

Field theory anomalies

Anomaly polynomial factorises as $2 \times 6 + 4 \times 4$: G.H. 0602101

$$
\begin{aligned}
I_8^{SO(32)} \quad = \quad & \frac{1}{3}\left(\sum_i c_1(V_i)\mathrm{tr}_{U(N_i)}F\right) \times \left(\sum_j c_1(V_j)\left[\mathrm{tr}_{U(N_j)}F\,\mathrm{tr}R^2 - 16\mathrm{tr}_{U(N_j)}F^3\right]\right) \\
& + \left(\mathrm{tr}R^2 + 2\,\mathrm{tr}_{SO(2M)}F^2 + \sum_i 4\,(\mathrm{ch}_2(V_i) + n_i)\mathrm{tr}_{U(N_i)}F^2\right) \times \\
& \times \left(\mathrm{tr}R^2 - \mathrm{tr}_{SO(2M)}F^2 - 2\sum_i n_i\,\mathrm{tr}_{U(N_i)}F^2\right)
\end{aligned}
$$

Interpretation:

- 2×6 part is related to massive U(1)s
- 4×4 part will be matched with orbifold point

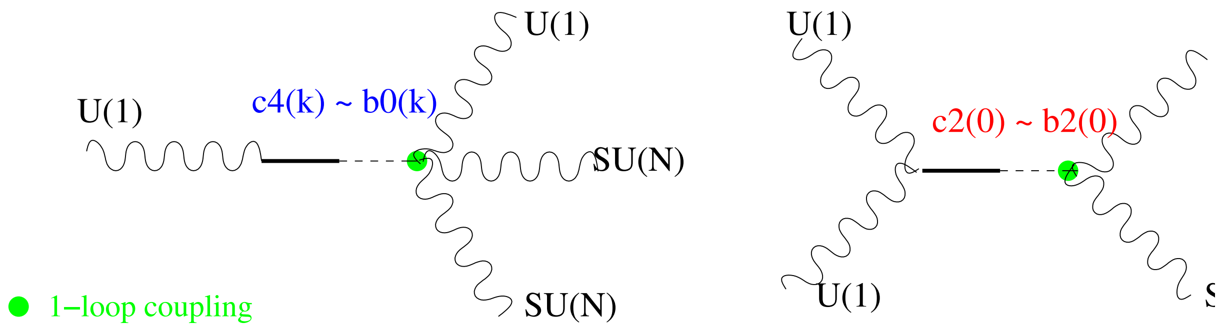
Green-Schwarz mechanism

•Counter terms arise from kinetic and 1-loop terms in D=10:

$$S_{tree} \supset \int_{\mathbb{R}^{1,9}} H_3 \wedge \star_{10} H_3 \qquad S_{1-loop} \supset \int_{\mathbb{R}^{1,9}} B \wedge X_8$$

•Combination of dual d.o.f. appearing in tree level and 1-loop terms leads to the correct counter terms

$$\mathcal{I}_{pert} \sim \int_{K3} \left(\mathrm{tr}(F\overline{F}) \wedge X_{\overline{2}+6} + \frac{1}{2}\left(\mathrm{tr}F^2 - \mathrm{tr}R^2\right) \wedge X_{\overline{4}+4} \right)$$



•First term: scalars and dual 4-forms
•Second term: 2-forms

Massive $U(1)$s

•U(1)s become massive via couplings to B field:

$$S_{mass} = \sum_k \int_{\mathbb{R}^{1,5}} c_k^{(4)} \wedge [\mathrm{tr}(F\overline{F})]^k$$

with $c_k^{(4)}$ dual to $b_k^{(0)} = \int_{k^{th} 2-\mathrm{cycle}} B$

⇒masses depend on $c_1(V)$ $\quad \mathrm{tr}_{SO(32)}(F\overline{F}) \sim \sum_i N_i\, c_1(V_i)\, F_{U(1)_i}$

⇒non-trivial mass matrix for several $U(1)$s

•$b_k^{(0)}$ belong to neutral hyper multiplets

•Dilaton is scalar d.o.f. in tensor mult. ⇒remains massless

Matching $K3$ and $T^4/\mathbb{Z}_N$: anomalies

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Comparison of 4×4 parts in anomaly polynomial:

$$I_8^{SO(32)} = \left(\mathrm{tr}R^2 + 2\,\mathrm{tr}_{SO(2M)}F^2 + \sum_i 4\,(\mathrm{ch}_2(V_i) + n_i)\mathrm{tr}_{SU(N_i)}F^2\right) \times$$
$$\times \left(\mathrm{tr}R^2 - \mathrm{tr}_{SO(2M)}F^2 - 2\sum_i n_i\,\mathrm{tr}_{SU(N_i)}F^2\right) + \cdots$$

$$\alpha_i \overset{!}{=} 2, \quad \beta_j \overset{!}{=} 4(\mathrm{ch}_2(V_j) + n_j), \quad -2 \overset{!}{=} -2n_j$$

$\Rightarrow$ $n_j = 1$ gives line ($U(1)$) bundles

$\Rightarrow$smooth instanton numbers $\mathrm{ch}_2(V_j)$ can be computed from orbifold point coefficients β_j

- Orbifold models can contain two $SO(2M_i)$ factors, smooth models only one
- $\alpha_i \neq 2$ $\Rightarrow$smooth model contains $SU(M_i)$ instead of $SO(2M_i)$

Matching spectra

•use $T^4/\mathbb{Z}_N$ shift vector to determine smooth embedding:

$$\frac{1}{N}(1_{n_1}, 2_{n_2}, 3_{n_3}, 0_{16-\sum_i n_i}) \to (L \ldots L, L^2 \ldots L^2, L^3 \ldots L^3, 0 \ldots)$$

•models independent of *signs* $\pm 1, \pm 2, \ldots$
•order N does not enter smooth solution

⇒smooth *non-Abelian* gauge group can:

- be identical to group at orbifold point
- have rank reduced by 1

•# matter reps. must match upon symmetry breaking

•symmetry beaking & blow-up via vevs of twisted scalars

Ex: $SO(32)$ 'standard embedding'

- Shift vector $V = \frac{1}{N}(1,1,0^{14})$ on $T^4/\mathbb{Z}_N$
- Gauge group: $SO(28) \times SU(2) \times \begin{cases} SU(2) & N=2 \\ U(1) & N=3,4,6 \end{cases}$

N	U	T	T^2
2	$(\mathbf{28},\mathbf{2},\mathbf{2}) + 4\,(\mathbf{1})$	$8\,(\mathbf{28},\mathbf{1},\mathbf{2}) + 32\,(\mathbf{1},\mathbf{2},\mathbf{1})$	–
3	$(\mathbf{28},\mathbf{2})_1 + 2\,(\mathbf{1})_0 + (1)_2$	$9\,(\mathbf{28},\mathbf{2})_{\frac{1}{3}} + 45\,(\mathbf{1})_{\frac{2}{3}} + 18\,(\mathbf{1})_{\frac{4}{3}}$	–
4	$(\mathbf{28},\mathbf{2})_1 + 2\,(\mathbf{1})_0 + (1)_2$	$4\,(\mathbf{28},\mathbf{2})_{\frac{1}{2}} + 24\,(\mathbf{1})_{\frac{1}{2}} + 8\,(\mathbf{1})_{\frac{3}{2}}$	$5\,(\mathbf{28},\mathbf{2})_0 + 32\,(\mathbf{1})_1$
6	$(\mathbf{28},\mathbf{2})_1 + 2\,(\mathbf{1})_0 + (1)_2$	$(\mathbf{28},\mathbf{2})_{\frac{2}{3}} + 8\,(\mathbf{1})_{\frac{1}{3}} + 2\,(\mathbf{1})_{\frac{5}{3}}$	$5\,(\mathbf{28},\mathbf{2})_{\frac{1}{3}} + 22\,(\mathbf{1})_{\frac{2}{3}} + 10\,($
			$3\,(\mathbf{28},\mathbf{2})_0 + 22\,(\mathbf{1})_1$

- Counting of non-Abelian d.o.f.: $10\,(\mathbf{28},\mathbf{2}) + 66\,(\mathbf{1})$:
- $U(1)$ charges in untwisted sector identical
- $U(1)$ charges in n^{th} twisted sectors: $1 + \frac{m}{n}$
- $U(1)$ ($SU(2)$) becomes massive in orbifold blow-up

Ex: $SO(28) \times SU(2) \times U(1)$

Two ways to obtain $SO(28) \times SU(2) \times U(1)_{massive}$:

- line bundle L in $U(2)$ or
- $L \oplus L^{-1}$ in $U(1) \times U(1)$
  $\Rightarrow U(1)_{diag}$ stays massless & is enhanced to $SU(2)$
- denote embeddings by (with $\mathrm{ch}_2(L) = -12$ for Bianchi identity)

$$(L, L, 0^{14}) \qquad\qquad (L, L^{-1}, 0^{14})$$

- Matter spectrum identical:

$$10\,(\mathbf{28}, \mathbf{2})_1 + 46\,(\mathbf{1}, \mathbf{1})_2 \qquad\qquad m^2_{U(1)} \sim c_1(L)^2$$

- adding the 20 neutral hypers on $K3$ gives agreement with orbifold 'standard embedding' in blow-up

difference: K-theory constraint: $c_1(W_{total}) = (1 \pm 1)\, c_1(L) \overset{!}{\in} H^2(K3, 2\mathbb{Z})$

Blow-up of orbifold singularities

The 6D scalar potential is determined by gauge interactions:

$$V = \sum_{a,\alpha} D^{a,\alpha} D^{a,\alpha} \quad \text{with} \quad D^{a,\alpha} = \Phi_i^\dagger \sigma^a t_{ij}^\alpha \Phi_j$$

containing the hyper multiplets Φ_j, Pauli matrices $\sigma^a_{a=1,2,3}$ and generators of gauge groups t_{ij}^α ($U(1)$ symmetry set $t_{ij}^\alpha \equiv 1$)

- standard 4D D-term arises from $\sigma^3 = \mathrm{Diag}(1,-1)$
- Blow-up breaks gauge group via vevs of twisted scalars
- vevs render gauge bosons massive via 6D kinetic terms
- D-flatness: at least 2 fixed points blown-up simultaneously

Discussion

Heterotic orbifolds

- Massless spectra complete for $T^4/\mathbb{Z}_N$, $N = 2, 3, 4$ - examples for $N = 6$ G.H. & M. Trapletti hep-th/0612030
- Systematics of anomaly polynomial: 4×4 only!

Heterotic $K3$ compactifications:

- Anomaly cancellation 6D fully understood G.H. '06
- Structure of massive $U(1)$s: depends on $c_1(V)$, non-trivial matrix

Comparison Heterotic on $K3 \Leftrightarrow T^4/\mathbb{Z}_N$

- Correspondence $U(1)$ embedding $\Leftrightarrow$ shift vector $(N = 2, 3)$
- Instanton numbers obtained from anomaly polynomial
- Spectra fit: $N = 4, 6$ with more line bundles $\Leftrightarrow$ twist sectors
- Blow-up via twisted scalars

Open questions

- List of $T^4/\mathbb{Z}_6$ spectra not complete
- Matching of remaining $N = 4, 6$ cases with several line bundles?
- Explicit realisation of line bundles in general case?

  Ansatz: democratic distribution among fixed points, toric geometry
- Generalisation to 4D? - see Groot Nibbelink
- Inclusion of Wilson lines? (- 5-branes straightforward)
- Same reasoning for non-Abelian orbifolds?