A natural route to near-flavour-conservation in SUSY: the Minimal Flavour Violating MSSM

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Outline

Introduction: impact of FCNC processes on generic SUSY corrections
⇒ New-Physics ‘flavour problem’

Approach 1: use exp. info to constrain the general MSSM
(general = with completely free soft terms)

Approach 2: implement a natural “near-flavour-conservation” mechanism
within the MSSM ⇒ MFV-MSSM

Application to meson mixings and discussion

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General parameterization of FCNC in SUSY

The “rotation” that makes quark masses diagonal does not need to diagonalize squark masses as well.

In the CKM basis for quarks the squark mass matrices are still off-diagonal in flavour (and ‘chirality’).

Then for squarks one chooses between:

1. Mass eigenstates
   \[ M_I \delta_{IJ} \]
   \[ q_I \xrightarrow{g} \tilde{q}_J \propto Z_{IJ} \]
   
   Mass Matrices

2. Flavour eigenstates
   \[ M_{IJ} \]
   \[ q_I \xrightarrow{\tilde{g}} \tilde{q}_J \propto \delta_{IJ} \]
   
   Interactions

Propagators diagonal ⇒ exact calculations
However, many parameters: \( M_J, Z_{IJ} \)

Propagators perturbatively diagonalized through “Mass Insertions”

\[ M \equiv \begin{pmatrix} M_{11} & \Delta_{12}^{LL} & \cdots \\ \Delta_{21}^{LL} & M_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \Rightarrow \bar{M} \begin{pmatrix} 1 \\ \vdots \\ \vdots \end{pmatrix} + \begin{pmatrix} 0 & \delta_{12}^{LL} & \cdots \\ \delta_{21}^{LL} & 0 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]

SUSY source of FCNC's

\[ \text{squark propagator} \quad (\text{flavor basis} + \text{MIA}) \]

\[ \Rightarrow \quad \text{diagonal} \quad + \quad \text{NON-diag.: Mass Insertion} \]

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Naïve assessment of SUSY effects

Example: $\Delta F = 2$ case

SUSY corrections $\sim \left( \frac{\delta}{M_{\text{SUSY}}} \right)^2 \times f(\text{SUSY mass ratios})$

Since mixing measurements check (within errors) with the SM, one has roughly:

$$|\text{SUSY corrections}| \leq \sqrt{\sigma_{\text{exp}}^2 + \sigma_{\text{th}}^2}$$

$\sigma_{\text{th}} > 10\%$ (!)

Sensitivity to $\mathcal{O}(\%)$ deviations from SM demands steady improvement on the non-pert. error

bounds on $\delta$
(or rather, on $\delta/M_{\text{SUSY}}$)
Analysing FCNCs in SUSY

The SUSY (and NP in general) flavour problem

\textbf{a)} assuming $M_{\text{SUSY}} \sim O(300 \text{ GeV})$ [if we want it to stabilize the EW scale]

$$\Rightarrow |\delta| < 10^{-2} \div 10^{-3}$$

in absence of a symmetry principle (e.g. GIM-like mechanism)

such small numbers are ugly

Let us write the \textbf{down quark} mass matrix, in the basis with diagonal interaction terms (the equivalent of the MIA basis) [values in GeV]

\[
\left[ \hat{m}_d \right]_{\text{MIA basis}} \equiv \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \cdot V_{\text{CKM}}^+ = \begin{pmatrix} 6.8 \times 10^{-3} & -1.6 \times 10^{-3} & (5.2 + 2.3 i) \times 10^{-5} \\ 2.3 \times 10^{-2} & 9.7 \times 10^{-2} & -4.2 \times 10^{-3} \\ (0.9 + 1.4 i) \times 10^{-2} & 1.8 \times 10^{-1} & 4.3 \end{pmatrix}
\]

( does this matrix “look better” ? )

\textbf{b)} assuming $|\delta| \sim O(1)$

$$\Rightarrow M_{\text{SUSY}} \gg O(\text{TeV})$$ back to “Separation-of-Scales” Problems

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Theoretical approaches to SUSY flavour effects

1 Constrain the ‘general’ MSSM (with completely free soft terms)
   not easy without simplifying assumptions:
   bulkiness of the parameter space

Take the δ bounds “as they come” (from measured FCNCs) and study allowed effects on still-to-measure quantities (e.g. $A_{CP}[B_s \rightarrow \psi \phi]$, ...)

2 Maybe FCNC effects in SUSY are small, because already those in the SM are.
   i.e. low-energy flavour breaking “building blocks” are the same in the SM and in the MSSM

Naturalness of “near-flavour-conservation” in SUSY: MFV-MSSM

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Constrain the general MSSM
Example: constraints from $b \to s$ FCNCs

y-axis: $\text{Im}[\delta^{23}_{xy}]$

x-axis: $\text{Re}[\delta^{23}_{xy}]$

**Constraints**

- $\Delta m_s$
- $b \to s \ y$
- $b \to s \ l^{+} \ l^{-}$
- All

**Examples**

- **LL only, $\tan \beta = 3$**
  
  $[-0.15, +0.15] + i [-0.25, 0.25]$

- **RR only, $\tan \beta = 3$**
  
  $[-0.4, +0.4] + i [-0.9, 0.9]$

- **LL=RR, $\tan \beta = 3$**
  
  $[-0.05, +0.05] + i [-0.03, 0.03]$

- **LL=RR, $\tan \beta = 10$**
  
  $[-0.03, +0.03] + i [-0.02, 0.02]$
Implication on the $B_s$ – mixing phase

✔ In the SM one has

$$\text{Arg } M_{12}^{SM} \equiv \text{Arg}\left\{ \langle B_s | H_{\text{eff},SM}^{\Delta B,S=2} | B_s \rangle \right\} = -2 \lambda^2 \eta \approx -0.04$$

✍ What is the allowed range for $\text{Arg } M_{12}^{\text{MSSM}}$ with the previous limits on the $\delta$’s?

- LR only, $\tan \beta = 3$
  - no sizable deviations from the SM

- LL only, $\tan \beta = 3$
  - $\sim 10 \times \text{SM value}$ are allowed

- RR only, $\tan \beta = 3$
  - $\sim 100 \times \text{SM value}$ are easy to get (but RR is still mildly constrained...)

- LL=RR, $\tan \beta = 3$
  - $\sim 100 \times \text{SM value}$ are again easy (yet LL=RR is severely constrained!)

The CP asymmetry in $B_s \rightarrow \psi \phi$

will provide a truly fantastic probe!

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Minimal Flavour Violation in the MSSM
**Minimal Flavour Violation (MFV)**

**MFV:**
In the SM, FCNC are small, because of the GIM mechanism. Can extensions of the SM incorporate a *similar* mechanism of near-flavour-(and CP)-conservation?

**Controversial issue on how to define MFV**

1. *‘pragmatic’ definition, Buras et al., ‘00:* in terms of allowed effective operators + explicit occurrence of the CKM

2. *EFT definition, D’Ambrosio et al., ‘02:* in terms of the SM Yukawa couplings

Def. 1 does not produce a consistent low-energy limit for the MSSM, even at low \( \tan \beta \)

- In fact, in extensions of the SM one has (by def.) **new**, a priori unrelated sources of flavour (and CP) violation.
- MFV can then only be defined as a ‘symmetry requirement’ for such **new** sources
- The set of allowed operators and FV structures is an **outcome** of such requirement

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MFV ‘principle’

- the SM Yukawa couplings are the only structures responsible for low-energy flavour and CP violation.

Every new source of flavour violation must be expressed as a function of the SM Yukawa couplings.

**Example:** soft mass term for ‘left-handed’ squarks

\[
L_{\text{soft}} = -\left(m_Q^{ij}\right)^2 \left(\tilde{u}_L^i \tilde{u}_L^j + \tilde{d}_L^i \tilde{d}_L^j\right) + \ldots
\]

This is an a priori new source of flavour violation.

**MFV expansion**

\[
\begin{bmatrix} m^2 \\ m^2 \end{bmatrix}^T = \begin{bmatrix} a_1 & 1 \\ b_1 K^+ Y_u^2 K & b_2 Y_d^2 & O(Y_u^2 Y_d^2) \end{bmatrix}
\]

**Strategy**

- After expansions, mass scales are only a few. Then:
  1. Fix them to scenarios
  2. Extract just the expansion coefficients (12 independent parameters)

**Dramatic increase in the predictivity and testability of the model**

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$\Delta F = 2$ example for mass scales chosen as

- $\bar{m} = 200$ GeV (squark scale)
- $M_g = 500$ GeV (gluino mass)
- $M_{1,2} = (100, 500)$ GeV (U(1)×SU(2) gaugino masses)
- $\mu = 1000$ GeV ($\mu$-parameter)

Due to MFV, the mixing phase is aligned with the SM value.

Comments:

- Distributions of values, due to the extraction of the expansion parameters, are quite narrow.
- Corrections are naturally small.
- Corrections are dominantly positive. Signature of the MFV-MSSM at low tan $\beta$.

Due to MFV, the mixing phase is aligned with the SM value.

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Large $\mu$ scenario

Notes
- When $\mu$ is large, LR entries in the squark mass matrices become relevant, even for low $\tan \beta$.
- They manifest dominantly in gluino contributions, which become competitive with chargino’s.

Example with (GeV):
- $m = 300$
- $M_g = 300$
- $M_{1,2} = (100, 500)$
- $\mu = 1000$

97.00% positive
$\Delta M^{\text{NP}}_g = (0.28 \pm 0.15) \text{ ps}^{-1}$
$\Delta M^{\text{NP}}_{\tilde{g}, \tilde{g}} > 0$ not included

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The case of $Q_1$-dominated MFV (so-called CMFV) can be tested by looking at the ratio

$$R_{\text{CMFV}} \equiv \frac{\text{contrib. to operators other than } Q_1}{\text{contrib. to } Q_1}$$

One can ‘define’ CMFV to hold when, e.g.

$$|R_{\text{CMFV}}| < 0.05$$

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The ratio $\Delta M_d / \Delta M_s$, usually included in the analysis of the Universal Unitarity Triangle, is not a good “constant” in generic MFV models.

The MFV – Unitarity Triangle can be constructed from

$$\sin 2\beta = \sin 2\beta_{\psi K_s}$$ and/or $|V_{ub}|$ and/or $\gamma$
Conclusions

In the general MSSM, SUSY effects are typically constrained to be small (exceptions: \(B_s \to \psi \phi\), ...) after imposing existing exp. input.

In the MFV-MSSM, SUSY effects are naturally small, due to a ‘built-in’ GIM-like mechanism.

In either case, to resolve such effects, one needs a better control, O(few %), of the effective operator matrix elements.
Back-up
Contributions to $\Delta F = 2$ (low $\tan \beta$)

\[
\Delta M_{s}^{MSSM} = \Delta M_{s}^{SM} + \Delta M_{s}^{H^{+}H^{+}} + \Delta M_{s}^{\tilde{\chi}^{+}\tilde{\chi}^{0}} + \Delta M_{s}^{\tilde{g}\tilde{g}} + \Delta M_{s}^{\tilde{g}\tilde{\chi}^{0}} + \Delta M_{s}^{\tilde{\chi}^{0}\tilde{\chi}^{0}}
\]

- **Higgses**
  - Depend on $|\mu|, m_{H_{u}}, m_{H_{d}}$, through the $H^{\pm}$ mass
  - Do not depend on any other SUSY scale and/or MFV coefficient

- **Charginos**
  - Depend on $\{\mu, M_{2}\} \rightarrow M_{\tilde{\chi}^{\pm}}$
  - Depend on $\{\bar{m}, A, \mu\} \rightarrow M_{\tilde{\mu}}$
  - Generically important

- **Gluinos**
  - Depend on $M_{3} \rightarrow M_{\tilde{g}}$
  - Depend on $\{\bar{m}, A, \mu\} \rightarrow M_{\tilde{d}}$
  - Important for large $\mu$

- **Neutralino-(gluino)**
  - Depend on $\{M_{1}, M_{2}, M_{3}\} \rightarrow M_{\tilde{\chi}^{0}}, M_{\tilde{g}}$
  - Depend on $\{\bar{m}, A, \mu\} \rightarrow M_{\tilde{d}}$
  - Generically unimportant (especially pure neutralino)

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Mass scenarios

Fixing mass scales

- Leaving aside Higgses, one has then to fix 6 mass scales: $\bar{m}, A, M_{1}, M_{2}, M_{3}, \mu$

Interesting cases

- **a)** $\bar{m}$ not large and $\mu$ small: $\{\text{charginos are light}\} \rightarrow \{\text{chargino dominated}\}$

- **b)** $\mu$ large: $\{\text{scalar operators become relevant}\} \rightarrow \{\text{chargino & gluino dominated}\}$

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Small $\mu$ scenario

Notes

- When $\mu$ is small, it governs the lightest chargino mass
- Scalar operator contributions from LR entries of the $d$-squark mass matrix are small, still because $\mu$ is small

Example with (GeV):

$m = 300$
$M_g = 300$
$M_{1,2} = (500,500)$
$\mu = 200$