The Minimal Flavor Violating MSSM: application to meson mixings

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Abstract. We provide a short account of the effects one expects on meson-antimeson oscillations in the context of the Minimal Supersymmetric Standard Model (MSSM). This issue is largely dependent on the assumptions made on the MSSM parameter space. In this respect, we consider in closer detail the cases of the general MSSM, with completely free soft terms, and the Minimal Flavor Violating limit of the MSSM, providing a natural mechanism of near-flavor-conservation. For the case of meson oscillations in the $\Delta B = 2$ sector, we show that this approach leads to a striking increase of the predictivity of the model. In particular, we find *(i)* SUSY corrections to be naturally small and always positive; *(ii)* if μ is not small, an increase in importance (even for low $\tan\beta$) of scalar operators due to gluino contributions. The last point signals that (V−A)×(V−A) dominated MFV is in general inconsistent with the MSSM. In this context, we also briefly discuss the MFV-Unitarity Triangle.

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1 Introduction

Within the Standard Model (SM) flavor-changing neutral current (FCNC) effects are forbidden at tree-level. Therefore FCNC observables are probes of the SM at the quantum level, allowing in principle to identify the need for additional degrees of freedom circulating in the loops. While the data on FCNC observables measured so far show no significant deviation from the SM expectations, in many extensions of the SM around the electroweak (EW) scale predicted new effects tend to be visible if not dominating. In the case of the MSSM, this happens because of its soft sector: the ignorance of symmetries underlying its structure compels to parameterize it most generally (general MSSM). However, a completely general parameterization of the soft Lagrangian, besides impairing the predictive power of the model, turns out to imply also way too large FCNC effects in 'most' [1] of the MSSM parameter space.

This puzzling circumstance is referred to as the 'flavor problem', in SUSY made acute by the bulkiness of the soft sector parameter space. To visualize this problem, one can consider the concrete FCNC example of $B_s - \bar{B}_s$ mixing. Within the general MSSM, one can focus on the leading order contributions from the strong interacting sector, represented by gluino-quark-squark vertices. Since flavor violation is driven by the flavor off-diagonal entries of the (down-)squark mass matrix and the relevant box diagrams feature two squark propagators, one expects the general structure: SUSY correction $\sim \delta^2/M^2_{\rm SUSY} \times f$(SUSY mass ratios), where δ indicates a 'mass-insertion', i.e. an off-diagonal entry in the squark squared-mass matrix normalized to the geometric average of the corresponding two diagonal entries. [2] Since mixing measurements are, within errors, in agreement with the SM, the total uncertainty – by far dominated by the theoretical error on the lattice matrix elements, still exceeding 10% – can be translated into bounds on the δ's, or rather on $\delta/M_{\rm SUSY}$. Now, assuming $M_{\rm SUSY} = {\rm O}(300\ {\rm GeV})$ entails $|\delta| \lesssim 10^{-2} \div 10^{-3}$, which calls for an explanation in terms of symmetries [3]. Assuming, on the other hand, $|\delta| = {\rm O}(1)$ implies $M_{\rm SUSY} \gg {\rm O(TeV)}$, posing again a problem of "separation of scales".

Two possible approaches to the SUSY flavor problem are the following: **(i)** Focus on the general MSSM and derive bounds on the δ's, however 'fine-tuned' they may turn out to be. Study effects allowed by these bounds on still to be measured observables; **(ii)** Implement symmetry requirements on the soft terms and study their implications by exploring the more manageable parameter space resulting from the symmetry.

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[1] This notion of course depends on the metric used to explore the parameter space.

[2] The name 'mass-insertions' is motivated by the fact that the squark propagator is diagonalized perturbatively, by taking δ's as interactions.

[3] This problem is actually present already in the quark mass matrices: they present disparate scales in the diagonal entries and (before the rotation to the CKM basis) small off-diagonal entries.

The prototype of this kind of approach is Minimal Flavor Violation (MFV) [1,2], in which FCNC effects in SUSY are small *because already those in the SM are.*

In the following two sections, I will provide just an example of the first approach and then dwell more in detail on the second one.

2 General MSSM

As an example, one can consider the bounds on the down-squark mass insertions imposed by the most precise $b \to s$ transitions, namely ΔM_s, $b \to s\gamma$ and $b \to s\ell^+\ell^-$. In the general MSSM, one can limit oneself to gluino-mediated contributions, and derive bounds on $(\delta^d_{23})_{AB}$, where 2,3 denote the external flavors and $A, B = L, R$ are the superfield chiralities. Bounds on a given δ are calculated by assuming its dominance, i.e. by setting all the other δ's as zero. This approach is justified a posteriori by noting that the bounds on different δ's are hierarchical. As an example, Fig. 1 (left) shows in a density plot the case $LL = RR$. It is evident that the combined constraint implies the quite severe bound $|(\delta^d_{23})_{LL=RR}| \lesssim 5\times10^{-2}$. One can now turn this bound into the maximum size of the predicted correction to $\arg(M_{12})$, with $M_{12} \equiv \langle \bar{B}_s | \mathcal{H}_{\rm eff}^{|\Delta B, S|=2} | B_s \rangle$. In the SM one has $\arg(M_{12}) \simeq 0.04$. In Fig. 1 (right) we report the corresponding profile of $\arg(M_{12})$ within the MSSM, upon variation of $(\delta^d_{23})_{LL=RR}$ within the bounds obtained from the left panel of the same figure. Notwithstanding the severe bound on the mass insertion, the phase of $B_s - \bar{B}_s$ mixing can still be enhanced by up to two orders of magnitude with respect to the tiny SM prediction. Access to this phase, via the measurement of the CP asymmetry in $B_s \to \psi\phi$, will then provide a further, extremely powerful probe into $b \to s$ transitions.

3 MFV-MSSM

From the example of the previous section, confronting the experimental data on $b \to s$ transitions with the corresponding predictions within the general MSSM, we are driven to the conclusion that 'generic' flavor violation, parameterized in terms of squark mass insertions, has to be very small. If one rejects fine-tuning as an explanation, this fact calls of course for ungrasped symmetries, underlying the SUSY soft terms' structure and implementing a mechanism of near-flavor-conservation [1]. In this respect, I turn now to discuss the case of the MFV limit of the MSSM. The starting observation is that, within the SM, FCNCs arise only because of the breaking of the flavor symmetry group due to the Yukawa couplings Y_u, Y_d [1,2]. In particular, the mechanism making FCNCs small within the SM is simply the specific misalignment Y_u and Y_d entail between the quark flavor eigenbases and the corresponding mass eigenbases. It is then interesting to address the question whether this specific mechanism can also be *embedded* in extensions of the SM, where *new* flavor violating structures arise, a priori *unrelated* to the SM Yukawa couplings. The assumption that the SM Yukawa couplings be, also in extensions of the SM, the *only* structures responsible for low-energy flavor and CP violation is known as MFV [2]. [4] The MFV assumption implies that new sources of flavor violation become functions of the SM Yukawa couplings. In order to identify the functional dependence, Yukawa couplings are promoted to spurion fields of the flavor group [2]. The resulting expansions for squark bilinear and trilinear soft terms are the following (for details on the formulae and on the notation see [5])

$$
\begin{aligned}
[m_Q^2]^T &= \overline{m}^2 \left(a_1 1\!\!1 + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger \right. \\
&\qquad \left. + b_3 (Y_d Y_d^\dagger Y_u Y_u^\dagger + Y_u Y_u^\dagger Y_d Y_d^\dagger) \right) , \\
m_U^2 &= \overline{m}^2 \left(a_2 1\!\!1 + b_4 Y_u^\dagger Y_u \right) , \\
m_D^2 &= \overline{m}^2 \left(a_3 1\!\!1 + b_5 Y_d^\dagger Y_d \right) , \\
A_u &= A \left(a_4 Y_u + b_6 Y_d Y_d^\dagger Y_u \right) , \\
A_d &= A \left(a_5 Y_d + b_7 Y_u Y_u^\dagger Y_d \right) .
\end{aligned} \tag{1}
$$

As expansions (1) show, the assumption of MFV dramatically simplifies the parametric dependence of the soft sector. In the case of meson mixings, to which we confine our attention here, mass terms to be considered are the bilinear and trilinear mass scales $\overline{m}$ and A, respectively, the gluino mass $M_{\tilde{g}}$, the EW gaugino masses $M_{1,2}$, the μ-parameter and the mass $M_{H^\pm}$ for the charged Higgs scalars $H^\pm$. The remaining parametric dependence is on the (real) parameters a_i, b_i ruling the MFV expansions.

Expansions (1) also explicitly show how, under the assumption of MFV, flavor violating effects generated by the squark soft terms are *naturally small.* As an example, after the rotation to the super-CKM basis, the flavor off-diagonal term $b_1 Y_u Y_u^\dagger$ becomes $b_1 K^\dagger \hat{Y}_u^2 K$, with K the CKM matrix and $\hat{Y}_u$ the diagonalized up-type Yukawa coupling. Considering b_1 an O(1) parameter, it is then clear that, in this approach, the mass insertions of Section 2 become $\delta = \mathrm{O}(1) \times f(\mathrm{CKM})$, showing the 'CKM-like' nature of MFV effects in SUSY.

A detailed study of meson mixings in the MFV-MSSM at low $\tan\beta$, adopting the above approach [2], has been recently reported in Ref. [5]. As already stated above, the main purpose was there to spell out the differences in the approaches [2] versus [3,4] to MFV, focusing on the benchmark case of meson mixings, where effects arising in MFV, and not reproducible in CMFV, are visible. [5] The strategy followed in this study was to fix mass scales to "scenarios" and, for each scenario, to study the distribution of corrections for $\Delta M_{d,s}$ as-

[4] This approach is more general than the so-called constrained MFV (CMFV) [3,4], in which one also imposes the dominance of the SM operators. The phenomenological differences between MFV and CMFV have been spelled out in [5].

[5] For an interesting MFV study in an instance where effects beyond CMFV are however not visible, see Ref. [6].

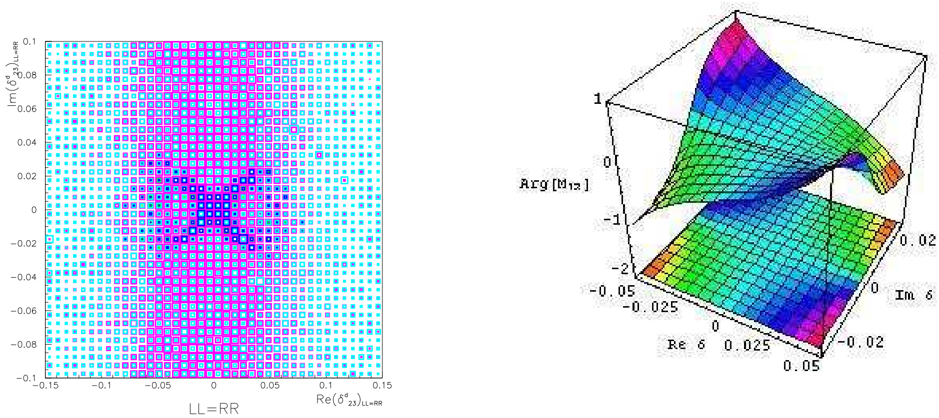

Fig. 1. Left: Constraints on $\mathrm{Re}(\delta^d_{23})_{LL=RR}$ for $\tan\beta = 3$, $M_{\tilde{q}} = M_{\tilde{g}} = 350$ GeV. Constraints are from ΔM_s (green), $b \to s\gamma$ (pink), $b \to s\ell^+\ell^-$ (cyan) and from the combined $b \to s$ transitions (blue). Right: $\arg(M_{12})$ profile upon variation of $(\delta^d_{23})_{LL=RR}$ within the bounds obtained from the left panel.

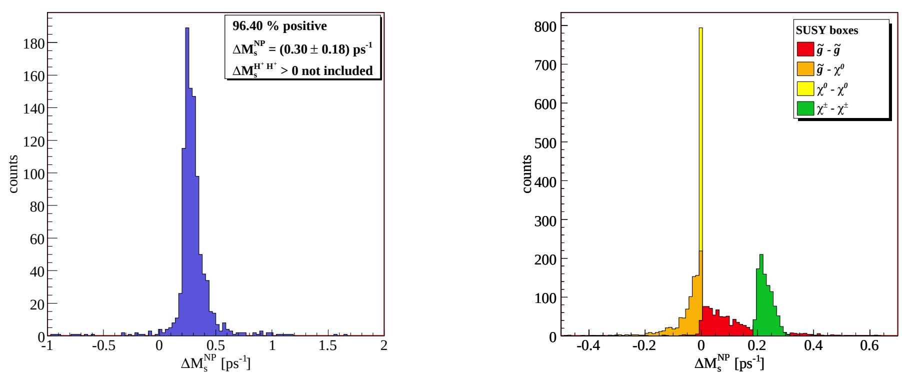

Fig. 2. Distribution of corrections to ΔM_s in the MFV-MSSM with $\tan\beta = 3$: sum of the SUSY contributions (left) and separate corrections (right). Mass scales are chosen as (GeV): $\mu = 1000$, $\overline{m} = 300$, $M_{\tilde{g}} = 300$, $M_1 = 100$, $M_2 = 500$.

suming the a_i, b_i coefficients (eq. (1)) to be flatly distributed in reasonable ranges (see [5] for details).

The main features exhibited by the resulting distributions are as follows: *(i)* corrections are naturally small, typically not exceeding a few percent of the SM central value; *(ii)* corrections are typically spread in a narrow range for each mass scenario: the standard deviation is smaller than the average correction; *(iii)* corrections are dominantly *positive*. This unexpected feature can then be considered a signature of the MFV-MSSM at low $\tan\beta$.

The above features are due to the interplay between chargino and gluino contributions, while Higgs contributions do not depend on the a_i, b_i and amount to just a further positive shift of the result, and neutralino contributions are always negligible. The mass scales ruling this interplay are the squark mass scale $\overline{m}$ and the parameter μ, with the other scales playing only a minor role. In particular, small values of μ with respect to $\overline{m}$ imply a small value for the lightest chargino mass and a correspondingly dominant (and positive [7]) chargino contribution. Conversely, large values of μ, around 1 TeV, with a smaller squark scale, imply enhanced LR contributions in the squark mass matrix and correspondingly enhanced gluino contributions. In addition, large values of μ also suppress the higgsino components of chargino contributions. An example of this scenario is shown in Fig. 2: here chargino contributions are still dominant, but gluino contributions amount to positive corrections in the range $30 \div 50\%$, relative to charginos. From the right panel, it is evident as well that gluino corrections also serve to compensate the negative contributions from gluino-neutralino boxes.

A final comment deserves the large $\tan\beta$ case. In this instance, it is well known [8] that, even in the MFV-MSSM, large negative corrections to ΔM_s are possible, due to double Higgs penguins, enhancing the contributions from scalar operators. Since the latter are sensitive to the external quark masses, the same enhancement is typically negligible for the ΔM_d case. However, even in the ΔM_s case, allowed corrections turn out to be more limited when taking into account the new combined bound on the $B_s \to \mu^+\mu^-$ decay mode from the CDF and DØ collaborations [9]. For positive μ, corrections exceeding -10% are basically excluded. In order to still observe a relatively large effect on ΔM_s, one needs negative values of μ and

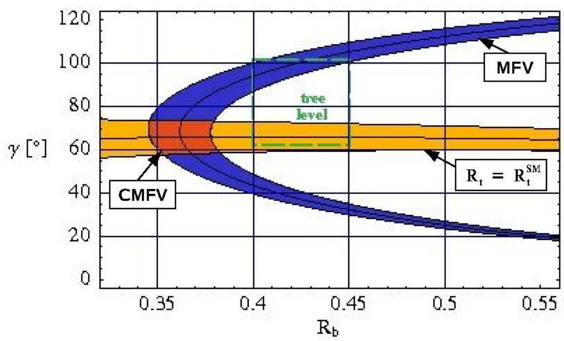

Fig. 3. Correlation between the UT angle γ and the side R_b in MFV models. See text for details on the areas.

typically large values for $M_A \gtrsim 500$ GeV, increasing with increasing $\tan\beta \gtrsim 30$ [10]. One should however also keep in mind that for $\mu < 0$ the MSSM worsens the $(g-2)_\mu$ discrepancy with respect to the SM [11].

4 The Unitarity Triangle in MFV

Basing on the discussion in the previous section, the ratio $\Delta M_d/\Delta M_s$ is not NP-independent in MFV, while it is equal to the SM ratio in CMFV, because of the *assumed* dominance of the SM operator in the latter framework. The unitarity triangle (UT) valid within CMFV is known as the Universal Unitarity Triangle (UUT) [3] and is determined from the mentioned ratio $\Delta M_d/\Delta M_s$, allowing access to the side R_t, as well as from observables dominated by SM tree-level contributions and from angle measurements. Among the tree-level dominated quantities one should mention in particular the value of $|V_{ub}|$, while the most precise angle determinations are β, measured by means of the $S_{\psi K_S}$ asymmetry, and to a lesser extent γ. Extensive analyses of the UUT have been performed by [12] (see e.g. [13]) through global fits to the abovementioned quantities.

However, when assuming MFV, the UT should be determined exclusively from tree-level observables and angle measurements. The side R_t should instead not be included, since the MFV NP contributions to ΔM_d and ΔM_s do not generally cancel in the ratio.

In the context of MFV, the known value $\beta_{\psi K_S} = (21.2 \pm 1.0)^\circ$ [14] establishes a correlation [5] between the side $R_b \propto |V_{ub}/V_{cb}|$ of the UT and the angle γ, that is valid for all models with MFV. This correlation is represented in Fig. 3 as a blue area, under the assumption $\beta = \beta_{\psi K_S}$. The orange area in the figure shows instead the region characterized by $R_t = R_t^{\rm SM}$, with an error dominated by that of the lattice quantity $\xi = 1.23(6)$. The intersection between the two areas, displayed in red, is then the one allowed to CMFV. Fig. 3 also shows the 1σ-allowed range from tree-level decays, namely $64^\circ \le \gamma \le 102^\circ$ and $0.40 \le R_b \le 0.46$, as a green dashed box. The latter overlaps with the higher branch of the MFV area but not with the CMFV one. This suggests that the slight tension [4] between the tree-level determination of R_b and the one favoured by CMFV (which includes the SM) disappears within MFV, provided $\gamma \gtrsim 80^\circ$. One should however also keep in mind that this effect is mainly driven by the discrepancy between the exclusive and the inclusive determinations of $|V_{ub}|$ – a not yet settled issue –, with the inclusive determination driving R_b to somehow too high values with respect to those favored by indirect SM fits [12].

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