Dimensional Reduction Applied to Non-SUSY Theories

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in collaboration with

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Outline

- introduction to Dimensional Reduction (DRED)
- DRED and non-SUSY theories
- results
  - anomalous dimensions of Yang-Mills theory in DRED
  - conversion to $\overline{\text{MS}}$ scheme
- checks in the SUSY case

an Alternative to Dimensional Regularisation

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Dimensional Reduction (DRED)

- modification of DREG designed to respect SUSY
  - no proof that DRED really preserves SUSY Ward-Identities
  - but so far, it worked

- phenomenology
  - Standard Model as low-energy effective theory of SUSY
  - relating SM observables to MSSM calculations
    $\Rightarrow$ relating the renormalisation schemes used
  - example: running of $\alpha_s^{\overline{\mathrm{MS}}}(M_Z) \to \alpha_s^{\overline{\mathrm{DR}}}(M_{GUT})$

see talk by Luminita Mihaila

DRED in a Non-SUSY Theory: Motivation

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    see talk by Luminita Mihaila
- more complicated

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Constraint: 4-dimensional space not *literally* 4-dimensional

- $\epsilon_{\mu\nu\rho\sigma}$ problematic, no Fierz transformations

[Stöckinger '05]

DRED Applied to a Yang-Mills Theory

Bare Lagrange density of Yang-Mills theory with fermions:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2\alpha} (\partial^\mu A_\mu)^2 + C^{a*} \partial^\mu D_\mu^{ab} C^b + i\bar{\psi}^\alpha \gamma^\mu D_\mu^{\alpha\beta} \psi^\beta$$

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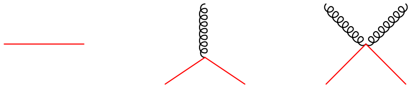
Propagator of 2ϵ scalar fields,
gauge interaction



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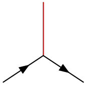
Yukawa-type interaction of fermion with scalars



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Quartic self-interaction of scalars

Evanescent Couplings

Reconsider ϵ-part of Yang-Mills Lagrange density:

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Each term separately invariant!

Gauge Transformations

$$\delta A^a_\mu = \partial_\mu \Lambda^a + g f^{abc} A^b_\mu \Lambda^c$$
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- Cannot expect identical couplings after Renormalization $\Rightarrow$ Evanescent Couplings
- colour structure in the 4ε-vertex: $f^{ace}f^{bde} \rightarrow H^{abcd}$ symmetric in (ab) and (cd)

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the Quartic ε-Scalar Self-Coupling

- tensors symmetric in (ab) and (cd)

$$H_1 = \frac{1}{2}\left(f^{ace}f^{bde} + f^{ade}f^{bce}\right)$$
$$H_2 = \frac{1}{2}\delta^{ab}\delta^{cd}$$
$$H_3 = \frac{1}{2}\left(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right)$$
$$H_4 = \frac{1}{2}\left(f^{aef}f^{bfg}f^{cgh}f^{dhe} + f^{aef}f^{bfg}f^{dgh}f^{che}\right)$$

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- sometimes not all linearly independent

$$SU(3): \quad H_4 = \tfrac{1}{2}H_1 + \tfrac{3}{2}\left(H_2 + H_3\right) \quad \Rightarrow p = 3$$
$$SU(2): \quad H_4 = 2H_2 + H_3,$$
$$H_1 = 2H_2 - H_3 \quad \Rightarrow p = 2$$

Consider α_s^{ph} in a Physical Renormalization Scheme

Relating DRED and DREG

Consider α_s^{ph} in a Physical Renormalization Scheme

- calculate α_s^{ph} using both regulators

$$\alpha_s^{0,\mathrm{DRED}} = \mu^{2\epsilon} Z_{\alpha_s,ph}^{\mathrm{DRED}} \alpha_s^{ph}$$
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physical: independent of Regularization employed

$$\Rightarrow \alpha_s^{\overline{\mathrm{DR}}} = \left(\frac{Z_{\alpha_s,ph}^{\mathrm{DRED}}}{Z_{\alpha_s,ph}^{\mathrm{DREG}}} \frac{Z_{\alpha_s}^{\overline{\mathrm{MS}}}}{Z_{\alpha_s}^{\overline{\mathrm{DR}}}} \right) \alpha_s^{\overline{\mathrm{MS}}}$$

Relating DRED and DREG

switch between schemes: finite shifts of renormalised parameters

$$\alpha_s^{\overline{\mathrm{DR}}} = \alpha_s^{\overline{\mathrm{MS}}} \left[1 + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} \frac{1}{4} + \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} \right)^2 \frac{11}{8} - \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} \frac{\alpha_e}{\pi} \frac{1}{12} n_f + \delta_\alpha^{(3)} + \ldots \right]$$

[Harlander, P. K., Mihaila, Steinhauser '06]

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[Harlander, P. K., Mihaila, Steinhauser '06]

$$\begin{aligned}
\delta_\alpha^{(3)} = & \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi} \right)^3 \left(\frac{3049}{384} - \frac{179}{864} n_f \right) \\
& + \frac{\left(\alpha_s^{\overline{\mathrm{MS}}} \right)^2}{\pi^3} \left(-\eta_1 \frac{9}{256} + \eta_2 \frac{15}{32} + \eta_3 \frac{3}{128} - \alpha_e \frac{887}{1152} n_f \right) \\
& + \frac{\alpha_s^{\overline{\mathrm{MS}}}}{\pi^3} \left[{\eta_1}^2 \frac{27}{256} - {\eta_2}^2 \frac{15}{16} - \eta_1 \eta_3 \frac{9}{64} + {\eta_3}^2 \frac{21}{128} + {\alpha_e}^2 \left(\frac{43}{864} n_f + \frac{19}{1152} n_f^2 \right) \right]
\end{aligned}$$

[Jones, Harlander, P. K., Mihaila, Steinhauser '06]

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[Jones, Harlander, P. K., Mihaila, Steinhauser '06]

general gauge group also known [Jack, Jones, P. K., Mihaila '07]

Anomalous Dimensions

calculated β functions and γ_m in $\overline{\mathrm{DR}}$-scheme

$$\beta_\alpha = \mu^2 \frac{d}{d\mu^2} \frac{\alpha}{\pi}$$

$$\gamma_m = \frac{\mu^2}{m} \frac{d}{d\mu^2} m$$

- β_{α_s} and γ_m to 4-loop order
- β_{α_e} to 3-loop order
- β_{η_i} to 1-loop order

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- checks for Super-Yang-Mills theory
  - $\beta_{\alpha_s} = \beta_{\alpha_e}$ through 3 loops
    $\Rightarrow$ invalidates claim that SUSY must be broken by DRED at 4-loop level [Avdeev '82]

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    $\Rightarrow$ invalidates claim that SUSY must be broken by DRED at 4-loop level [Avdeev '82]
  - β_{α_s} agrees with the literature through four loops

[Jack, Jones, Pickering 1998]

Trick: Bypass 4-Loop Diagrams

$$\beta_{\alpha_s}^{\overline{\mathrm{DR}}} = \frac{\mu^2}{\pi} \frac{d}{d\mu^2} \alpha_s^{\overline{\mathrm{DR}}} \left(\alpha_s^{\overline{\mathrm{MS}}}, \alpha_e, \{\eta_r\} \right)$$

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Ingredients for Calculating $\beta_{\alpha_s}^{\overline{\mathrm{DR}}}$

- $\alpha_s^{\overline{\mathrm{DR}}}$ as a function of $\alpha_s^{\overline{\mathrm{MS}}}$, α_e, $\{\eta_i\}$
- $\overline{\mathrm{MS}}$ result for $\beta_{\alpha_s}^{\overline{\mathrm{MS}}}$
- β functions of the evanescent couplings

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Conclusions

- DRED viable alternative to DREG for SUSY and non-SUSY theories
- without SUSY: evanescent couplings
  - cannot be identified with the gauge coupling
- conversion relation between $\overline{\text{MS}}$ and $\overline{\text{DR}}$ through 3 loops
- anomalous dimensions of fermion mass and gauge coupling through 4 loops
- checked explicitely that $\beta^3_{\alpha_s} = \beta^3_{\alpha_e}$ in a Super-Yang-Mills theory