GUT-like Superstring Standard Model from the Heterotic String

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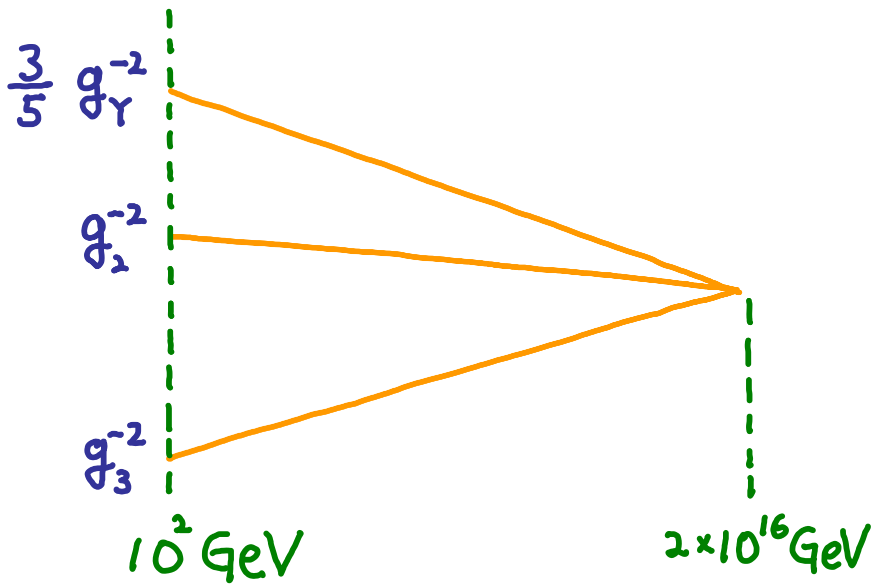
MSSM seems to be in GUT.

- $\{ g_3 , g_2 , \underbrace{\sqrt{\tfrac{5}{3}}}_{} g_Y \}$ unified at 10^{16} GeV

$\boxed{\text{GUT normalization}}$

SU(5), SO(10), ⋯

$$\longrightarrow \sin^2\theta_W^0 = \frac{g_Y^2}{g_2^2+g_Y^2} = \frac{3}{8}$$



• $\{Q, d^c, u^c, L, e^c, \nu^c\}$ Chiral MATTER

embeded in $\begin{cases} \mathbf{10}, \bar{\mathbf{5}}, \mathbf{1} & SU(5) \\ \mathbf{16} \text{ [spinor]} & SO(10) \end{cases}$

eigen values of 5 Cartan Σ_{ab} s

$\divideontimes$ $\mathbf{16} = |\pm\pm\pm\pm\pm\rangle$, with even # of "$-$"

$$\bar{\mathbf{5}}_{-3} = \begin{cases} |\underline{+--};--\rangle & d^c \\ |---;\underline{+-}\rangle & L \end{cases}$$

$$\mathbf{10}_1 = \begin{cases} |\underline{+--};++\rangle & u^c \\ |\underline{++-};\underline{+-}\rangle & Q \\ |+++;--\rangle & e^c \end{cases}$$

$$\mathbf{1}_5 = |+++++\rangle \quad \nu^c$$

$\divideontimes$ additional $^{\text{vec.-like}}$ SU(5) multiplets do NOT spoil gauge coupling unification.

MSSM seems NOT to be in GUT.

- Higgs Sector

$$\left.\begin{array}{l}\{H_d,\ \underset{\sim}{\bar{3}}\} \subset \mathbf{\bar{5}} \\ \{H_u,\ \underset{\sim}{3}\} \subset \mathbf{5}\end{array}\right\} \subset \mathbf{10}\ \text{[vector]}$$

SU(5) SO(10)

$\longrightarrow$ D/T splitting problem

- Difficult to avoid the relation

$$m_d = m_e .$$

Superstring Standard Model

toward

GUT-"**like**" MSSM

1. $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ SM

2. $\sin^2\theta_W = \frac{3}{8}$ GUT

3. $3 \times \{Q, d^c, u^c, L, e^c, \nu^c\} \subset 3 \times 16$ GUT

$\downarrow$ SO(10) spinors

4. $\{H_u, H_d\} \subset 10_h$ → SO(10) vector

but D/T split SM

5. All other Matter Fields are vector-like under G_{SM} → superheavy

6. $QH_d d^c + LH_d e^c \not\subset 10\,5\bar{5}\,\bar{5}_h$

→ $m_d \neq m_e$ or $16\,16\,10_h$

SM

7. R-parity for stable $\left\{ \begin{array}{l} \text{proton \&} \\ \text{LSP} \end{array} \right.$

Heterotic String Theory

- unification framework
- structure is rich enough to accomodate the MSSM

Orbifold Compactification

to reduce
space dim
SUSY
gauge sym.

- relatively simple, easy to analyze
- CFT is still useful [Yukawa coupl.]

The Model

$\mathbb{Z}_{12\text{-}I}$ orbifold compactification

$$\vec{\phi} = \left(\frac{-5}{12}, \frac{4}{12}, \frac{1}{12}\right)$$

: $SO(8)\times SU(3)$ lattice

$\sim \mathbb{Z}_3$ { Wilson line of order 3 can be set on this sub-lattice

leads to $N=1$ SUSY in 4d

Shift vector:

$$V = \left(\underbrace{\tfrac{1}{4}\ \tfrac{1}{4}\ \tfrac{1}{4}}_{SU(3)};\underbrace{\tfrac{1}{4}\ \tfrac{1}{4}}_{SU(2)};\ \tfrac{5}{12}\ \tfrac{5}{12}\ \tfrac{1}{12}\right)\left(\tfrac{1}{4}\ \tfrac{3}{4}\ 0;\ \underbrace{0^5}_{SO(10)'}\right)'$$

Wilson line

$$a_3 = \left(\tfrac{2}{3}\ \tfrac{2}{3}\ \tfrac{2}{3};\ \tfrac{-2}{3}\ \tfrac{-2}{3};\ \tfrac{2}{3}\ 0\ \tfrac{2}{3}\right)\left(0\ \tfrac{2}{3}\ \tfrac{-2}{3};\ 0^5\right)'$$

Modular Invariance

$$
\begin{cases}
12\cdot(V^2-\phi^2) = \text{even integer} = 12 \\
12\cdot a_3^2 = \text{even integer} = 48 \\
12\cdot V\cdot a_3 = \text{integer} = 12
\end{cases}
$$

Massless Conditions

$k = 0, 1, 2, \cdots, 11$

$$
\begin{cases}
\text{L-mover:} \quad \dfrac{|P+kV|^2}{2} + \sum_i N_i^L \tilde{\phi}_i - \tilde{c}_k = 0 \\
\text{R-mover:} \quad \dfrac{|\vec{r}+k\vec{\phi}|^2}{2} + \sum_i \underbrace{N_i^R}_{=0} \tilde{\phi}_i - c_k = 0
\end{cases}
$$

$$(P+kV)\cdot a_3 = \text{integer} \quad \text{for } k = 0, 3, 6, 9$$

Generalized GSO Projection

$$P_k(f) = \frac{1}{12\cdot 3} \sum_{l=0}^{N-1} \tilde{\chi}(\theta^k \theta^l)\, e^{2\pi i \Theta_f}$$

$$\Theta_f = \sum_i (N_i^L - N_i^R)\hat{\phi}_i - \frac{k}{2}(V_f^2 - \phi^2) + (P + kV_f)V_f - (\vec{r} + k\vec{\phi})\cdot\vec{\phi}$$

$$V_f \equiv V + m_f a_3$$

Untwisted Sector

◎ Gauge Sector :

$P\cdot V - \vec{r}\cdot\vec{\phi} = 0 \mod Z$
$P\cdot V = \vec{r}\cdot\vec{\phi} = 0 \mod Z$

The root vectors P of $E_8 \times E_8'$ satisfying

$$P\cdot V = P\cdot a_3 = \text{integer}$$

$(1\ -1\ 0;\ 0\ 0;\ 0^3)(0^8)'$: $SU(3)$

$(0\ 0\ 0;\ 1\ -1;\ 0^3)(0^8)'$: $SU(2)$

$(0^8)'(0^3;\ \pm1\ \pm1\ 0\ 0\ 0)'$: $SO(10)'$

$$\boxed{\begin{aligned} G = {} & SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)^4 \\ & \times [\,SO(10) \times U(1)^3\,]' \end{aligned}}$$

Hypercharges are defined with

$$Y = \left(\tfrac{1}{3}\,\tfrac{1}{3}\,\tfrac{1}{3};\,\tfrac{-1}{2}\,\tfrac{-1}{2};\,0^3\right)'\left(0^8\right)'$$

GUT

The current algebra in the heterotic string theory fixes the normalization of Y :

$$Z_{\text{string}} \equiv u \times Y = u \times \left[\sqrt{\tfrac{2}{3}}\,\frac{\vec{g}_3}{\sqrt{2}} - \frac{\vec{g}_2}{\sqrt{2}}\right]$$

$$\left.\begin{aligned}\vec{g}_3 &= \tfrac{1}{\sqrt{3}}(1\,1\,1;0\,0;0^3)(0^8)' \\ \vec{g}_2 &= \tfrac{1}{\sqrt{2}}(0\,0\,0;1\,1;0^3)(0^8)'\end{aligned}\right\}\ \text{orthonormal bases}$$

$$u^2\left(\tfrac{2}{3}+1\right) = 1 \quad \text{or} \quad u^2 = \tfrac{3}{5}$$

$$\therefore\ g_1^2 = \tfrac{5}{3}\, g_Y^2 \longrightarrow \sin^2\theta_W = \tfrac{3}{8}$$

SU(5) or SO(10)-like

Untwisted Sector

⊙ Matter Sector :

$P\cdot V - \vec{r}\cdot\vec{\phi} = 0 \mod Z$
$P\cdot V = \vec{r}\cdot\vec{\phi} \mod Z$

The root vectors P of $E_8 \times E_8'$ satisfying

$$P\cdot V = \underbrace{\left\{ \frac{-5}{12}, \frac{4}{12}, \frac{1}{12} \right\}}_{\text{mod } Z}, \quad P\cdot a_3 = \text{integer}$$

$\pm \equiv \pm\frac{1}{2}$

$\frac{-5}{12}$:

$(\underline{+\,+\,-};\underline{+\,-};+\,+\,+)\,(0^8)'$: Q

$(-\,-\,-;\underline{+\,-};+\,-\,-)\,(0^8)'$: L

$\frac{1}{12}$:

$(\underline{+\,-\,-};-\,-;+\,+\,+)\,(0^8)'$: d^c

$(+\,+\,+;+\,+;+\,+\,+)\,(0^8)'$: ν^c

$(\underline{+\,-\,-};+\,+;+\,-\,-)\,(0^8)'$: u^c

$(+\,+\,+;-\,-;-\,+\,-)\,(0^8)'$: e^c

$Q, L, d^c, \nu^c, u^c, e^c$: **16** Spinor

$\frac{4}{12}$:

$(0\,0\,0;\underline{1\,0};0\,0\,1)\,(0^8)'$: H_d

$(0\,0\,0;\underline{-1\,0};-1\,0\,0)\,(0^8)'$: H_u

$(0\,0\,0;0\,0;1\,0\,-1)\,(0^8)'$: 1_0

H_d, H_u: D/T splitting vec. type

T_4° Sector

$$\begin{cases} 4 \times \vec{\phi} = \left(\frac{-5}{3}\ \frac{4}{3}\ \frac{1}{3}\right) \\ 4 \times V^{I} \end{cases}$$

Massless modes of $P + 4V$

$$2\times\left\{\begin{array}{lcl}
(\underline{+--};--;\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(0^8)' & : & d^c \\
(---;\underline{+-};\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(\ ''\ )' & : & L \\
(\underline{+--};++;\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(\ ''\ )' & : & u^c \\
(\underline{++-};\underline{+-};\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(\ ''\ )' & : & Q \\
(+++;--;\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(\ ''\ )' & : & e^c \\
(+++;++;\ \frac{1}{6}\ \frac{1}{6}\ \frac{-1}{6})(\ ''\ )' & : & \nu^c
\end{array}\right\}\ \boxed{2\times\ \mathbf{16}\ \text{spinor}}$$

All the other matter fields in this model turn out to be exactly vector-like under G_{SM}.

[Anomalies have been checked out.]

REFER TO OUR PAPER !!

(hep-ph/0702278)

Phenomenologically desirable vacuum

SM singlet

If $\langle 1_0 \rangle_s \sim \Lambda_{string}$,

- G_{SM} is preserved, but U(1)s unobserved at low energies are broken
- unwanted vec.-like exotics achieve superheavy masses.
- SM Yukawa couplings are induced,

but R-parity violating terms need to be suppressed or absent.

Yukawa Couplings

A vertex op. $\langle \mathcal{O}_A \mathcal{O}_B \mathcal{O}_C \cdots \rangle$ should satisfy

1. Gauge Invariance

2. H-mom. Conservation $\sim$ discrete R-sym.

$$\sum_z R_1(z) = -1 \mod 12, \quad \sum_z R_2(z) = 1 \mod 3, \quad \sum_z R_3(z) = 1 \mod 12$$

where $R_i = (\vec{r} + k\vec{\phi})_i - (N_i^L - N_i^{\bar{L}})$

3. Space Group Selection Rules

$$\begin{cases} \sum_z k(z) = 0 \mod 12 \\ \sum_z [k m_f](z) = 0 \mod 3 \end{cases}$$

From the untwisted sector,

at renormalizable level,

only

QH_uU^c, $\rightarrow m_t$

LH_de^c, $LH_u\nu^c$,

$\downarrow$ m_τ $\downarrow$ $m^D_{\nu_\tau}$

are allowed.

No $\cancel{R}$ terms.

How $\langle 1_0 \rangle \sim \Lambda_{string}$?

There exist superpotential terms constructed purely with the neutral singlets

ex)

$$W = S_1 S_2 S_3 + S_4 S_5 S_6 + \cdots$$

In Z_{12-I} orbifold compactification,
if a superpotential term ω (eg. $S_1S_2S_3$) satisfies all the selection rules,
then ω^{12n+1} $(n = 1, 2, 3, \cdots)$ also does.

By including $\omega^{13}, \omega^{25}, \omega^{37}, \cdots$,
one can always find a vacuum where the singlets of interest develop VEVs of the string scale,
preserving F-flat conditions.

e.g.)

$$W \supset \omega \left[1 + \frac{\omega^{12}}{\Lambda_{string}^{36}} + \cdots \right]$$

In SUGRA, the SUSY condition

$$F_i = D_i W = 0, \quad D^a = G_i T^a_{ij} \phi_j \propto D_i W = 0.$$

$\partial_T W \neq 0$, $W \neq 0$

R-parity vs Superheavy Exotics

ex)

$$\begin{cases} (S_a S_b \cdots S_c) \times QHd^c & \{S_a, S_b, \cdots, S_c\} \neq 0 \\ (S_\alpha S_\beta \cdots S_\gamma) \times \overline{\Phi}_{Exo} \overline{\Phi}_{Exo} & S_\alpha \sim S_\beta \sim \cdots \sim S_\gamma \sim \Lambda_{str} \\ (S_1 S_2 \cdots S_n) \times QLd^c & \text{at least, one of } S = 0 \text{ or } S \ll \Lambda_{str} \end{cases}$$

vec.-like

Question : $S \in \{S_\alpha, S_\beta, \cdots\}$ or Not ?

In this model, it is possible to separate 1_0 s into 2 classes I & II.

1_0 s in I : $\langle 1_0 \rangle \sim \Lambda_{str}$

1_0 s in II : $\langle 1_0 \rangle \sim 0$ (or $\ll \Lambda_{str}$),

such that the above requirements are satisfied.

Conclusions

1. $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \quad [\otimes SO(10)]'$

2. $\sin^2\theta_W^0 = \frac{3}{8}$

3. $3 \times \{Q, d^c, u^c, L, e^c, \nu^c\} \subset 3 \times 16 \quad (+3\times 10')$

   $\hookrightarrow \begin{cases} 1 \text{ from } U \\ 2 \text{ " } T_4 \end{cases}$ $16 \to$ SO(10) spinors

4. $\{H_u, H_d\} \subset 10_h \to$ SO(10) vector

   but D/T split

5. All other Matter Fields (Exotics) are vector-like under G_{SM}. → shown to be superheavy

6. $QH_d d^c + LH_d e^c \not\subset 10\,\overline{5}\,\overline{5}_h$ or $16\,16\,10_h$

   $\rightarrow m_d \neq m_e$

7. (effective) R-parity can be consistent with superheavy EXOTICS on a vacuum.

Danke Schön!!