Toric Resolution of Heterotic Orbifolds

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based on collaborations with

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Contents

One of the aims of string phenomenology is to construct MSSM like models. There have been many approaches in this direction:

- free fermionic models Faraggi,Nanopoulos'90
- branes at angles models Blumenhagen et al'00
- Gepner models Dijkstra,Huiszoon,Schellekens'05
- heterotic orbifolds Kobayashi,Raby,Zhang'04 Lebedev, et al'06 Kim,Kyae'06
- heterotic string on eliptic fibered Calabi–Yaus Andreas,Curio,Klemm'99 Braun,He,Ovurt,Pantev'05

In this talk we focus on heterotic orbifolds and their blowups.

Field theory on orbifolds have some ambiguities:

- At the orbifold fixed points there are curvature singularities.
- At the 4D fixed points arbitrary new fields can be placed.

Heterotic strings on orbifolds Dixon,Harvey,Vaffa,Witten'85,Ibanez,Nilles,Quevedo'87

- String theory on orbifolds is perfectly well defined.
- Orbifolds are completely calculable in string theory.
- The spectrum contains twisted states localized at the orbifold fixed points.

Even though string theory on orbifolds is very powerful, it nevertheless just describes a special point in the full moduli space of the heterotic string on smooth Calabi–Yaus.

Also the analysis from the orbifold often forces one to go away from the orbifold point:

- When one of the twisted states get a VEV this means that the orbifold singularity will be (partly) blown up.
- This VEV cannot be avoided if a U(1) gauge symmetry is anomalous.
- Because these VEVs are generically of the string scale, one cannot trust the perturbative analysis at the orbifold point.

We would like to establish connections between orbifolds and smooth Calabi–Yaus. For this reason we consider resolutions of orbifold singularities. We have pursued this program using

- explicit blowup of $\mathbb{C}^n/\mathbb{Z}_n$ singularities,
- toric resolutions of generic orbifolds.

The motivations to construct explicit blowups of $\mathbb{C}^n/\mathbb{Z}_n$ are the following:

- $\mathbb{C}^2/\mathbb{Z}_2$ models the fixed points of $T^4/\mathbb{Z}_2$, which is a singular realization of K3.
- $\mathbb{C}^3/\mathbb{Z}_3$ models the fixed points of $T^6/\mathbb{Z}_3$, which is the prototype of 6D orbifolds.
- They are simple enough to obtain explicit results.

A Ricci-flat Kähler blowup of $\mathbb{C}^n/\mathbb{Z}_n$ is obtained when the determinant $\det G$ of the metric G is constant. Candelas,de la Ossa'90 This gives SGN,Trappletti,Walter'07

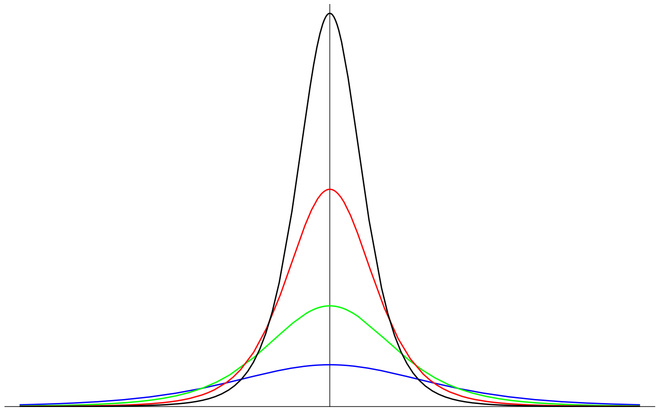
$$\mathcal{K}(X) = \int_1^X \frac{\mathrm{d}Y}{Y} M(Y), \qquad M(X) = \frac{1}{n}\big(r + X\big)^{\frac{1}{n}}, \qquad X = (\bar{z}z)^n .$$

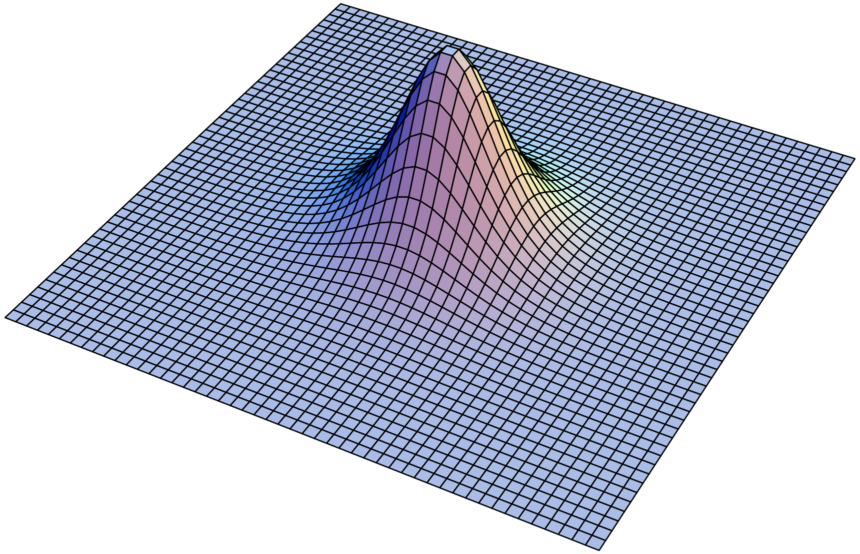
This is a generalization of the Eguchi-Hanson space. Eguchi-Hanson'78,Calabi'79

The gauge backgrounds on the blowup needs to satisfy the Hermitean Yang-Mills equations, Candelas,Horwitz,Strominger,Witten'85 fixes the field strength 2-form to Ganor,Sonnenschein'02 SGN,Trapletti,Walter'07

$$i\mathcal{F}_V = i\,\mathcal{F}\,V^I H_I, \quad i\mathcal{F} = \Big(\frac{r}{r+X}\Big)^{1-\frac{1}{n}}\Big(\bar{e}e - \frac{n-1}{n^2}\,\frac{1}{r+X}\,\bar{\epsilon}\epsilon\Big),$$

where V^I are either all integer or half-integer.





The curvature 2–form can be expressed as

$$\mathcal{R} = \frac{r}{r+X} \begin{pmatrix} e\,\bar{e} - \bar{e}\,e + \frac{1}{n}\,\frac{\bar{\epsilon}\,\epsilon}{r+X} & \frac{\bar{\epsilon}\,e}{\sqrt{r+X}} \\ \frac{\bar{e}\,\epsilon}{\sqrt{r+X}} & n\,\bar{e}\,e - \frac{n-1}{n}\,\frac{\bar{\epsilon}\,\epsilon}{r+X} \end{pmatrix},$$

where e is the vielbein 1-form of $\mathbb{CP}^{n-1}$ and ϵ is a 1-form associated with a complex line bundle over it.

Using the explicit geometry of the blowup of $\mathbb{C}^3/\mathbb{Z}_3$ with U(1) gauge bundle, we can construct compactifications of SO(32) SYM coupled to SUGRA.

The integrated Bianchi identity integrated over a compact 4–cycle has to vanish:

$$V^2 = 12 \ .$$

This condition selects 7 allowed models.

The spectra of these models can be compute using an index theorem. The multiplicities of the representations obtained from the branching of the adjoint of SO(32) via the multiplicities operator N_V which can take the values: $N_V = \frac{1}{9},\ 1,\ \frac{26}{9} = 3 - \frac{1}{9}$.

The multiplicity factors $\frac{1}{9} = \frac{3}{27}$ refer to untwisted (delocalized) states, while integral multiplicity factors correspond to states localized at the orbifold fixed point.

Gmeiner,SGN,Nilles,Olechowski,Walter'03

The six dimensional $\mathbb{C}^3/\mathbb{Z}_3$ blowup models are: SGN,Trapletti,Walter'07

Model	G_{blowup}	Representations
$(0^{12}, 1^3, 3)$	$\mathrm{SO}(24) \times \mathrm{U}(3) \times \mathrm{U}(1)$	$\frac{1}{9}(\mathbf{24}, \mathbf{3})_1 + \frac{2}{9}(\mathbf{1}, \mathbf{3})_{-2} + (\mathbf{24}, \mathbf{1})_{-3} + \frac{26}{9}(\mathbf{1}, \overline{\mathbf{3}})_{-4}$
$(0^{13}, 2^3)$	$\mathrm{SO}(26) \times \mathrm{U}(3)$	$\frac{1}{9}(\mathbf{26}, \overline{\mathbf{3}})_{-2} + \frac{26}{9}(\mathbf{1}, \mathbf{3})_{-4}$
$(0^{10}, 1^4, 2^2)$	$\mathrm{SO}(20) \times \mathrm{U}(4) \times \mathrm{U}(2)$	$\frac{1}{9}(\mathbf{1}, \overline{\mathbf{4}}, \mathbf{2})_1 + \frac{1}{9}(\mathbf{20}, \mathbf{4}, \mathbf{1})_1 + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{6}}, \mathbf{1})_{-2}$ $+\frac{1}{9}(\mathbf{20}, \mathbf{1}, \overline{\mathbf{2}})_{-2} + (\mathbf{1}, \overline{\mathbf{4}}, \overline{\mathbf{2}})_{-3} + \frac{26}{9}(\mathbf{1}, \mathbf{1}, \mathbf{1})_{-4}$
$(0^7, 1^8, 2)$	$\mathrm{SO}(14) \times \mathrm{U}(8) \times \mathrm{U}(1)$	$\frac{1}{9}(\mathbf{1}, \overline{\mathbf{8}})_1 + \frac{1}{9}(\mathbf{14}, \mathbf{8})_1 + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{28}})_{-2} + \frac{1}{9}(\mathbf{14}, \mathbf{1})_{-2} + (\mathbf{1}, \overline{\mathbf{8}})_{-3}$
$(0^4, 1^{12})$	$\mathrm{SO}(8) \times \mathrm{U}(12)$	$\frac{1}{9}(\mathbf{8}, \mathbf{12})_1 + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{66}})_{-2}$
$(\frac{1}{2}^{14}, \frac{3}{2}, -\frac{5}{2})$	$U(14) \times U(1) \times U(1)$	$\frac{1}{9}(\overline{\mathbf{14}})_1 + \frac{1}{9}(\mathbf{1})_1 + \frac{1}{9}(\mathbf{91})_1 + \frac{1}{9}(\mathbf{14})_{-2}$ $+\frac{1}{9}(\overline{\mathbf{14}})_{-2} + (\overline{\mathbf{14}})_{-3} + \frac{26}{9}(\mathbf{1})_{-4}$
$(\frac{1}{2}^{12}, \frac{3}{2}^4)$	$\mathrm{U}(4) \times \mathrm{U}(12)$	$\frac{1}{9}(\mathbf{4}, \overline{\mathbf{12}})_1 + \frac{1}{9}(\mathbf{1}, \mathbf{66})_1 + \frac{1}{9}(\overline{\mathbf{4}}, \overline{\mathbf{12}})_{-2} + (\overline{\mathbf{6}}, \mathbf{1})_{-3}$

In this table we list the gauge group in the blow down limit which is equal to the orbifold gauge group to identify the corresponding heterotic orbifold: SGN,Trapletti,Walter'07

Orbifold shift	Blowup shift	$G_{\text{orbifold}} = G_{\text{blow down}}$	Matter spectrum on the orbifold resolution	Additional twisted matter
$(0^{13}, 1^2, 2)$	$(0^{12}, 1^3, 3)$	$SO(26) \times U(3)$	$\frac{1}{9}(\mathbf{26}, \mathbf{3}) + \frac{26}{9}(\mathbf{1}, \overline{\mathbf{3}}) + (\mathbf{26}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1})$
	$(0^{13}, 2^3)$		$\frac{1}{9}(\mathbf{26}, \overline{\mathbf{3}}) + \frac{26}{9}(\mathbf{1}, \mathbf{3})$	$(\mathbf{1}, \mathbf{1}) + (\mathbf{26}, \mathbf{1})$
$(0^{10}, 1^4, 2^2)$	$(0^{10}, 1^4, 2^2)$	$SO(20) \times U(6)$	$\frac{10}{9}(\mathbf{1}, \overline{\mathbf{15}}) + \frac{1}{9}(\mathbf{20}, \mathbf{6}) + 3(\mathbf{1}, \mathbf{1})$	
$(0^7, 1^6, 2^3)$	$(0^7, 1^8, 2)$	$SO(14) \times U(9)$	$\frac{1}{9}(\mathbf{14}, \mathbf{9}) + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{36}}) + (\mathbf{1}, \overline{\mathbf{9}})$	
$(0^4, 1^8, 2^4)$	$(0^4, 1^{12})$	$SO(8) \times U(12)$	$\frac{1}{9}(\mathbf{8}, \mathbf{12}) + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{66}})$	$(\mathbf{1}, \mathbf{1}) + (\mathbf{8}_+, \mathbf{1})$
	$(\frac{1}{2}^{12}, \frac{3}{2}^4)$		$\frac{1}{9}(\mathbf{8}, \overline{\mathbf{12}}) + \frac{1}{9}(\mathbf{1}, \mathbf{66}) + (\mathbf{8}_+, \mathbf{1})$	$(\mathbf{1}, \mathbf{1})$
$(0^1, 1^{10}, 2^5)$	$(\frac{1}{2}^{14}, \frac{3}{2}, \text{-}\frac{5}{2})$	$SO(2) \times U(15)$	$\frac{11}{9}(\mathbf{15}) + \frac{1}{9}(\overline{\mathbf{105}}) + 3(\mathbf{1})$	

The last column of this table lists the twisted heterotic states that are not reproduced by the blowup model: They either got mass in blowup or are reinterpreted as the non–universal axion. SGN,Nilles,Trapletti'07

Notwithstanding the success of model building on the explicit blowup of $\mathbb{C}^3/\mathbb{Z}_3$, there are various questions that arise:

- Do the various integrals computed on the explicit blowup have a topological origin?
- Why do the vanishing integrated Bianchi identities on both compact and non–compact cycles give the same consistency condition?
- How can blowup models be investigated for which no explicit blowups are known?

We will see that using toric geometry we can answer all these questions.

For an introduction to toric geometry see the textbooks: Fulton, Oda, Hori et al.: Mirror symmetry. Discussion of orbifold resolutions using toric geometry can be found in Erler,Klemm'92, Lust,Reffert,Scheidegger,Stieberger'06.

The basic idea of toric resolutions is to replace the $\mathbb{Z}_n$ orbifold action by complex scalings:

$$\theta : \; (\tilde{Z}_1, \ldots) \;\to\; (e^{2\pi i \phi_1} \tilde{Z}_1, \ldots) \quad \longrightarrow \quad (z_1, \ldots; x_1, \ldots) \;\sim\; (\lambda^{p_1} z_1, \ldots; \lambda^{q_1} x_1, \ldots)$$

The additional homogeneous coordinates $x_1, \ldots$ ae introduced to keep the dimensionality the same as that of the orbifold.

Setting one of the homogeneous coordinates to zero gives complex codimension one hypersurface: Ordinary divisors $D_i = \big\{ z_i = 0 \big\}$ and exceptional divisors $E_\theta = \big\{ x_\theta = 0 \big\}$.

To each divisor we can associated a complex line bundle and interpreted it as a $(1,1)$–form. Because of so–called linear equivalence relations

$$\sum_i (v_i)_j \, D_i + \sum_\theta (w_\theta)_j \, E_\theta \;\sim\; 0$$

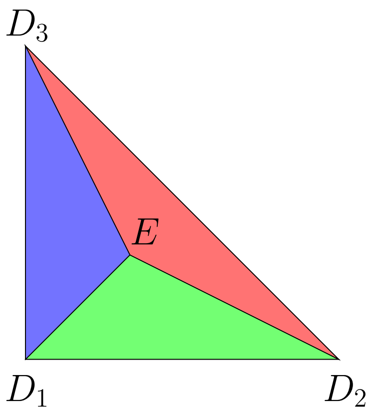
the exceptional divisors E_θ form a basis for the gauge background $i\mathcal{F}_V$.

The toric diagram encodes the intersections of divisors. Using the linear equivalences all other integrals can be computed.

The heterotic string on the orbifold $\mathbb{C}^3/\mathbb{Z}_3$ has only one twisted sector, hence we have, apart from the three ordinary divisors D_i, a single exception divisor E. They satisfy the linear equivalence relations:

$$D_i \sim D_j \;, \qquad 3\,D_i + E \sim 0 \;,$$

From the toric diagram we infer the basic integrals and intersections:



$$D_1 D_2 E = D_2 D_3 E = D_3 D_1 E = 1$$

$$\mathcal{F}_V = -\frac{1}{3}\, E\, H_V \;.$$

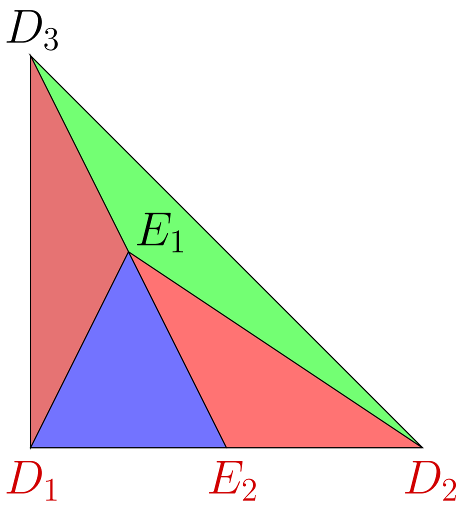
We obtained all the results of the explicit blowup. In particular, the Bianchi identity of the compact and non–compact cycles gave the same results:

$$V^2 = \int_E \mathrm{tr}(i\mathcal{F}_V)^2 = -3 \int_{D_i} \mathrm{tr}(i\mathcal{F}_V)^2 = -3 \int_{D_i} \mathrm{tr}\,\mathcal{R}^2 = \int_E \mathrm{tr}\,\mathcal{R}^2 = 12 \;.$$

As a non–trivial second example we consider the resolution of the $\mathbb{C}^3/\mathbb{Z}_4$ orbifold, with geometrical shift: $\phi = \frac{1}{4}(1,\,1,\,2)$. There are two exceptional divisors E_1 and E_2 which satisfy the linear equivalence relations

$$4\,D_1 + E_1 + 2\,E_2 \sim 0\,, \quad 4\,D_2 + E_1 + 2\,E_2 \sim 0\,, \quad 2\,D_3 + E_1 \sim 0\,.$$

To define the integrals on $\mathrm{Res}(\mathbb{C}^3/\mathbb{Z}_4)$ we use the toric diagram:



$$D_1\,E_1\,E_2 = D_2\,E_1\,E_2 = D_1\,D_3\,E_1 = D_2\,D_3\,E_1 = 1\,,$$

$$D_1\,D_2\,E_2 = D_3\,E_1\,E_2 = 0\,.$$

$\mathrm{Res}(\mathbb{C}^2/\mathbb{Z}_2)$Via the linear equivalences this implies:

$$E_1^2\,E_2 = 0\,,\quad E_2^2\,E_1 = -2\,,\quad E_1^3 = 8\,,\quad E_2^3 = 2\,.$$

$\Leftarrow$ This edge defines the toric diagram of $\mathrm{Res}(\mathbb{C}^2/\mathbb{Z}_2)$.

Writing $H_1 = V_1^I H_I$ and $H_2 = V_2^I H_I$ we expand the gauge background as:

$$\mathcal{F}_V = -\frac{1}{2}\,E_1\,H_1 - \frac{1}{4}\,(E_1 + 2\,E_2)H_2\,.$$

When all integrated Bianchi identities we can compute the spectrum:

$$E_1: \quad V_1^2 + V_1\cdot V_2 = 4\,,\quad E_2: \quad V_1\cdot V_2 = -2\,,\quad \mathrm{Res}(\mathbb{C}^2/\mathbb{Z}_2): \quad V_2^2 = 6\,.$$

We can compare the heterotic string orbifold models with the blowup models:

orbifold shift $4\,v$	blowup vector V_2	blowup vector V_1	Nr.
$(0^{13}, 1^2, 2)$	$(0^{13}, 1^2, 2)$	$(0^{13}, 1^2, \text{-}2)$	1a
	$(0^{13}, 1^2, 2)$	$(0^{12}, 2, \text{-}1^2, 0)$	1b
	$(0^{13}, 1^2, 2)$	$(0^{11}, 2, 1, 0^2, \text{-}1)$	1c
$(0^{11}, 1^2, 2^3)$	$(0^{13}, 1^2, 2)$	$(0^{10}, 1^4, \text{-}1^2)$	2a
	$(0^{13}, 1^2, 2)$	$(0^{11}, 1^2, \text{-}2, 0^2)$	2b
$(0^9, 1^2, 2^5)$	$(0^{13}, 1^2, 2)$	$(0^8, 1^5, 0^2, \text{-}1)$	3a
	$(0^{13}, 1^2, 2)$	$(0^9, 1^4, \text{-}1^2, 0)$	3b
$(0^7, 1^2, 2^7)$	–	–	4
$(0^{10}, 1^6)$	$(0^{10}, 1^6)$	$(0^{10}, 1^2, \text{-}1^4)$	5a
	$(0^{10}, 1^6)$	$(0^{13}, 1, \text{-}1, \text{-}2)$	5b
$(0^{10}, 1^5, 3)$	$(0^{10}, 1^6)$	$(0^9, 2, \text{-}1^2, 0^4)$	6
$(0^8, 1^6, 2^2)$	$(0^{10}, 1^6)$	$(0^8, 1^3, \text{-}1^3, 0^2)$	7a
	$(0^{10}, 1^6)$	$(0^8, 1^2, \text{-}2, 0^5)$	7b
$(0^6, 1^6, 2^4)$	$(0^{10}, 1^6)$	$(0^6, 1^4, \text{-}1^2, 0^4)$	8

orbifold shift $4\,v$	blowup vector V_2	blowup vector V_1	Nr.
$(0^5, 1^{10}, 2)$	$(0^{10}, 1^6)$	$\frac{1}{2}(\text{-}3, 1^{10}, \text{-}1^5)$	9
$(0^3, 1^{10}, 2^3)$	$(0^{10}, 1^6)$	$\frac{1}{2}(1^{12}, \text{-}1^3, \text{-}3)$	10
$(1^{14}, 2^2)$	$(0^{13}, \text{-}2, 1^2)$	$\frac{1}{2}(1^{15}, \text{-}3)$	11
$(1^{13}, \text{-}1, 2^2)$	$(0^{13}, 1^2, 2)$	$\frac{1}{2}(1^{15}, \text{-}3)$	12a
	$(0^{13}, 1^2, 2)$	$\text{-}\frac{1}{2}(\text{-}3, 1^{15})$	12b
$\frac{1}{2}(1^3, 3^{12}, \text{-}3)$	$\frac{1}{2}(\text{-}3, 1^{15})$	$\text{-}(0^{13}, 1^2, 2)$	13a
	$\frac{1}{2}(1^{15}, \text{-}3)$	$(0^{13}, 1^2, 2)$	13b
	$\frac{1}{2}(1^{15}, \text{-}3)$	$\frac{1}{2}(1^3, \text{-}1^{11}, 3, 1)$	13c
$\frac{1}{2}(1^7, 3^8, \text{-}3)$	$\frac{1}{2}(1^{15}, \text{-}3)$	$(\text{-}1^5, 1, 0^{10})$	14a
	$\frac{1}{2}(1^{15}, \text{-}3)$	$\frac{1}{2}(1^6, \text{-}1^8, \text{-}3, 1)$	14b
$\frac{1}{2}(1^{11}, 3^4, \text{-}3)$	$\frac{1}{2}(1^{15}, \text{-}3)$	$(0^{10}, 1^3, \text{-}1^3)$	15
$\frac{1}{2}(1^{15}, \text{-}3)$	$\frac{1}{2}(1^{15}, \text{-}3)$	$(0^{13}, \text{-}2, 1^2)$	16a
	$\frac{1}{2}(1^{15}, \text{-}3)$	$\frac{1}{2}(\text{-}1^{14}, 3, \text{-}1)$	16b

Only the model 4 cannot realized in blowup: This orbifold model has no 1st twisted sector, hence no blowup modes.

And we computed the resulting spectra:

Nr.	4D gauge group	$\frac{1}{8}\times$ "untwisted"	$\frac{1}{4}\times$ "2nd twisted"	"1st twisted"
1a	$SO(26)\times U(2)\times U(1)$	$(\mathbf{26},\mathbf{2})+2(\mathbf{1},\mathbf{2})$	$(\mathbf{26},\mathbf{1})+2(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1})$	$(\mathbf{26},\mathbf{1})+2(\mathbf{1},\mathbf{2})+3(\mathbf{1},\mathbf{1})$
1b	$SO(24)\times U(2)\times U(1)^2$	$(\mathbf{24},\mathbf{2})+4(\mathbf{1},\mathbf{2})$	$(\mathbf{24},\mathbf{1})+2(\mathbf{1},\mathbf{2})+3(\mathbf{1},\mathbf{1})$	$(\mathbf{24},\mathbf{1})+2(\mathbf{1},\mathbf{2})+5(\mathbf{1},\mathbf{1})$
1c	$SO(22)\times U(2)\times U(1)^3$	$(\mathbf{22},\mathbf{2})+6(\mathbf{1},\mathbf{2})$	$(\mathbf{22},\mathbf{1})+2(\mathbf{1},\mathbf{2})+5(\mathbf{1},\mathbf{1})$	$(\mathbf{22},\mathbf{1})+2(\mathbf{1},\mathbf{2})+5(\mathbf{1},\mathbf{1})$
2a	$SO(20)\times U(3)\times U(1)^3$	$2(\mathbf{20},\mathbf{1})+2(\mathbf{1},\mathbf{3})$ $2(\mathbf{1},\overline{\mathbf{3}})+4(\mathbf{1},\mathbf{1})$	$(\mathbf{20},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\overline{\mathbf{3}})+3(\mathbf{1},\mathbf{1})$	$2(\mathbf{1},\mathbf{3})+2(\mathbf{1},\overline{\mathbf{3}})+2(\mathbf{1},\mathbf{1})$
2b	$SO(22)\times U(2)\times U(1)^3$	$2(\mathbf{22},\mathbf{1})+4(\mathbf{1},\mathbf{2})+4(\mathbf{1},\mathbf{1})$	$(\mathbf{22},\mathbf{1})+2(\mathbf{1},\mathbf{2})+3(\mathbf{1},\mathbf{1})$	$2(\mathbf{1},\mathbf{2})+7(\mathbf{1},\mathbf{1})$
3a	$SO(16)\times U(2)\times U(5)\times U(1)$	$(\mathbf{16},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{5})$ $+(\mathbf{1},\mathbf{2},\overline{\mathbf{5}})+2(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{16},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{5})$ $+(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{10})+(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})$
3b	$SO(18)\times U(2)\times U(4)\times U(1)$	$(\mathbf{18},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2},\mathbf{4})$ $+(\mathbf{1},\mathbf{2},\overline{\mathbf{4}})+2(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{18},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{4})+(\mathbf{1},\mathbf{1},\overline{\mathbf{4}})$ $+(\mathbf{1},\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{6},\mathbf{1})$
5a	$SO(20)\times U(4)\times U(2)$	$(\mathbf{20},\mathbf{4},\mathbf{1})+(\mathbf{20},\mathbf{1},\mathbf{2})$	$(\mathbf{1},\overline{\mathbf{4}},\mathbf{2})+(\mathbf{1},\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\overline{\mathbf{4}},\mathbf{2})+(\mathbf{1},\mathbf{6},\mathbf{1})+3(\mathbf{1},\mathbf{1},\mathbf{1})$
5b	$SO(20)\times U(3)\times U(1)^3$	$3(\mathbf{20},\mathbf{1})+(\mathbf{20},\mathbf{3})$	$3(\mathbf{1},\overline{\mathbf{3}})+(\mathbf{1},\mathbf{3})+3(\mathbf{1},\mathbf{1},\mathbf{1})$	$2(\mathbf{1},\overline{\mathbf{3}})+5(\mathbf{1},\mathbf{1})$
6	$SO(18)\times U(4)\times U(2)\times U(1)$	$(\mathbf{18},\mathbf{4},\mathbf{1})+(\mathbf{18},\mathbf{1},\mathbf{2})$ $2(\mathbf{1},\mathbf{4},\mathbf{1})+2(\mathbf{1},\mathbf{1},\mathbf{2})$	$(\mathbf{1},\overline{\mathbf{4}},\mathbf{2})+(\mathbf{1},\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$2(\mathbf{1},\overline{\mathbf{4}},\mathbf{1})+(\mathbf{18},\mathbf{1},\mathbf{1})$ $+2(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1},\mathbf{1})$
7a	$SO(16)\times U(3)\times U(2)^2\times U(1)$	$(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{1})+(\mathbf{16},\mathbf{1},\mathbf{1},\mathbf{2})$ $+(\mathbf{16},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})+2(\mathbf{1},\mathbf{3},\mathbf{2},\mathbf{1})$ $+2(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2})+2(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{1})$ $+(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\mathbf{2})$ $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$2(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$
7b	$SO(16)\times U(2)\times U(5)\times U(1)$	$(\mathbf{16},\mathbf{1},\mathbf{5})+(\mathbf{16},\mathbf{1},\mathbf{1})$ $+2(\mathbf{1},\mathbf{2},\overline{\mathbf{5}})+2(\mathbf{1},\mathbf{2},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{10})+(\mathbf{1},\mathbf{1},\mathbf{5})$	$2(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})+(\mathbf{1},\mathbf{1},\mathbf{1})$
8	$SO(12)\times U(4)\times U(2)\times U(4)$	$(\mathbf{12},\mathbf{1},\mathbf{2},\mathbf{1})+(\mathbf{12},\mathbf{4},\mathbf{1},\mathbf{1})$ $+(\mathbf{1},\overline{\mathbf{4}},\mathbf{1},\mathbf{4})+(\mathbf{1},\overline{\mathbf{4}},\mathbf{1},\overline{\mathbf{4}})$ $+(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{4})+(\mathbf{1},\mathbf{1},\mathbf{2},\overline{\mathbf{4}})$	$(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})+(\mathbf{1},\overline{\mathbf{4}},\mathbf{2},\mathbf{1})$ $+(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{6})+(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$

and ...

Nr.	4D gauge group	$\frac{1}{8}\times$ "untwisted"	$\frac{1}{4}\times$ "2nd twisted"	"1st twisted"
9	$\mathrm{U}(5)\times\mathrm{U}(9)\times\mathrm{U}(1)^2$	$(\mathbf{5},\mathbf{9})+(\overline{\mathbf{5}},\mathbf{9})+(\mathbf{5},\mathbf{1})$ $+(\overline{\mathbf{5}},\mathbf{1})+2(\mathbf{1},\overline{\mathbf{9}})+2(\mathbf{1},\mathbf{1})$	$(\mathbf{10},\mathbf{1})+(\overline{\mathbf{5}},\mathbf{1})$	$(\mathbf{1},\overline{\mathbf{9}})+2(\mathbf{1},\mathbf{1})$
10	$\mathrm{U}(3)\times\mathrm{U}(10)\times\mathrm{U}(2)\times\mathrm{U}(1)$	$(\mathbf{3},\mathbf{10},\mathbf{1})+(\overline{\mathbf{3}},\mathbf{10},\mathbf{1})+$ $2(\mathbf{1},\overline{\mathbf{10}},\mathbf{2})+2(\mathbf{1},\overline{\mathbf{10}},\mathbf{1})$	$2(\overline{\mathbf{3}},\mathbf{1},\mathbf{1})+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2})$ $+(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{3},\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{2})$
11	$\mathrm{U}(13)\times\mathrm{U}(1)^3$	$4(\mathbf{13})+4(\mathbf{1})$	$2(\overline{\mathbf{13}})+5(\mathbf{1})$	$2(\mathbf{1})$
12a	$\mathrm{U}(13)\times\mathrm{U}(2)\times\mathrm{U}(1)$	$2(\mathbf{13},\mathbf{2})+2(\mathbf{1},\mathbf{2})$	$2(\mathbf{13},\mathbf{1})+2(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{13}},\mathbf{1})$
12b	$\mathrm{U}(12)\times\mathrm{U}(2)\times\mathrm{U}(1)^2$	$2(\mathbf{12},\mathbf{2})+4(\mathbf{1},\mathbf{2})$	$2(\mathbf{12},\mathbf{1})+2(\mathbf{1},\mathbf{2})+3(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{12}},\mathbf{1})+3(\mathbf{1},\mathbf{1})$
13a	$\mathrm{U}(12)\times\mathrm{U}(2)\times\mathrm{U}(1)^2$	$(\mathbf{66},\mathbf{1})+(\mathbf{12},\mathbf{1})+(\overline{\mathbf{12}},\mathbf{1})$ $+(\overline{\mathbf{12}},\mathbf{2})+2(\mathbf{1},\mathbf{2})+2(\mathbf{1},\mathbf{1})$	$(\mathbf{12},\mathbf{1})+(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{12}},\mathbf{1})+2(\mathbf{1},\mathbf{2})+3(\mathbf{1},\mathbf{1})$
13b	$\mathrm{U}(13)\times\mathrm{U}(2)\times\mathrm{U}(1)$	$(\mathbf{78},\mathbf{1})+(\overline{\mathbf{13}},\mathbf{2})$ $+(\overline{\mathbf{13}},\mathbf{1})+(\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1})$	$(\mathbf{13},\mathbf{1})+(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{13}},\mathbf{1})+2(\mathbf{1},\mathbf{2})+2(\mathbf{1},\mathbf{1})$
13c	$\mathrm{U}(11)\times\mathrm{U}(3)\times\mathrm{U}(1)^2$	$(\overline{\mathbf{55}},\mathbf{1})+(\mathbf{11},\mathbf{3})$ $+2(\overline{\mathbf{11}},\mathbf{1})+3(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{11}},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{1},\mathbf{1})$	$(\mathbf{11},\mathbf{1})+2(\mathbf{1},\overline{\mathbf{3}})$
14a	$\mathrm{U}(5)\times\mathrm{U}(9)\times\mathrm{U}(1)^2$	$(\mathbf{10},\mathbf{1})+2(\mathbf{5},\mathbf{1})+(\overline{\mathbf{5}},\overline{\mathbf{9}})$ $+2(\mathbf{1},\overline{\mathbf{9}})+(\mathbf{1},\mathbf{36})+(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{5}},\mathbf{1})+(\mathbf{1},\mathbf{9})+(\mathbf{1},\mathbf{1})$	$(\mathbf{5},\mathbf{1})$
14b	$\mathrm{U}(6)\times\mathrm{U}(8)\times\mathrm{U}(1)^2$	$(\overline{\mathbf{15}},\mathbf{1})+(\overline{\mathbf{6}},\mathbf{1})+(\mathbf{6},\mathbf{1})$ $+(\mathbf{6},\overline{\mathbf{8}})+(\mathbf{1},\mathbf{8})+(\mathbf{1},\overline{\mathbf{8}})$ $+(\mathbf{1},\mathbf{28})+(\mathbf{1},\mathbf{1})$	$(\mathbf{6},\mathbf{1})+(\mathbf{1},\mathbf{8})+(\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{6}},\mathbf{1})+(\mathbf{1},\mathbf{1})$
15	$\mathrm{U}(10)\times\mathrm{U}(3)\times\mathrm{U}(2)\times\mathrm{U}(1)$	$(\mathbf{45},\mathbf{1},\mathbf{1})+(\mathbf{10},\mathbf{1},\mathbf{1})$ $+(\overline{\mathbf{10}},\overline{\mathbf{3}},\mathbf{1})+(\overline{\mathbf{10}},\mathbf{1},\mathbf{2})$ $+2(\mathbf{1},\overline{\mathbf{3}},\mathbf{1})+(\mathbf{1},\mathbf{3},\mathbf{2})$ $+(\mathbf{1},\mathbf{1},\mathbf{2})+(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\overline{\mathbf{10}},\mathbf{1},\mathbf{1})$ $+(\mathbf{1},\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{1},\mathbf{2})$	$(\mathbf{1},\overline{\mathbf{3}},\mathbf{1})$ $+(\mathbf{1},\mathbf{3},\mathbf{2})+2(\mathbf{1},\mathbf{1},\mathbf{1})$
16a	$\mathrm{U}(13)\times\mathrm{U}(1)^3$	$(\mathbf{78})+2(\mathbf{13})+(\overline{\mathbf{13}})+3(\mathbf{1})$	$(\overline{\mathbf{13}})+2(\mathbf{1})$	$(\overline{\mathbf{13}})+4(\mathbf{1})$
16b	$\mathrm{U}(14)\times\mathrm{U}(1)^2$	$(\mathbf{91})+(\mathbf{14})+(\overline{\mathbf{14}})+(\mathbf{1})$	$(\overline{\mathbf{14}})+(\mathbf{1})$	$(\overline{\mathbf{14}})+3(\mathbf{1})$

Cancellation of the SU(12) anomaly in model 13a: $\frac{1}{8}\left(12\text{-}4-2\right)+\frac{1}{4}\cdot 1-1=0$.

We have constructed explicit blowups of $\mathbb{C}^n/\mathbb{Z}_n$ orbifolds, with U(1) gauge bundles:

- We found exact agreement between blowup and heterotic orbifold spectra on $\mathbb{C}^2/\mathbb{Z}_2$, consistent with Honecker,Trapletti'06
- We compared the resulting spectra with that of heterotic $\mathbb{C}^3/\mathbb{Z}_3$ orbifolds.
- We reproduce most of the twisted states; The "missing" states either got mass or are reinterpreted as non–universal axions. (Multiple anomalous U(1)s in blowup are possible, and explained that field redefinitions avoid contradictions with the orbifold picture. SGN,Nilles,Trapletti'07)

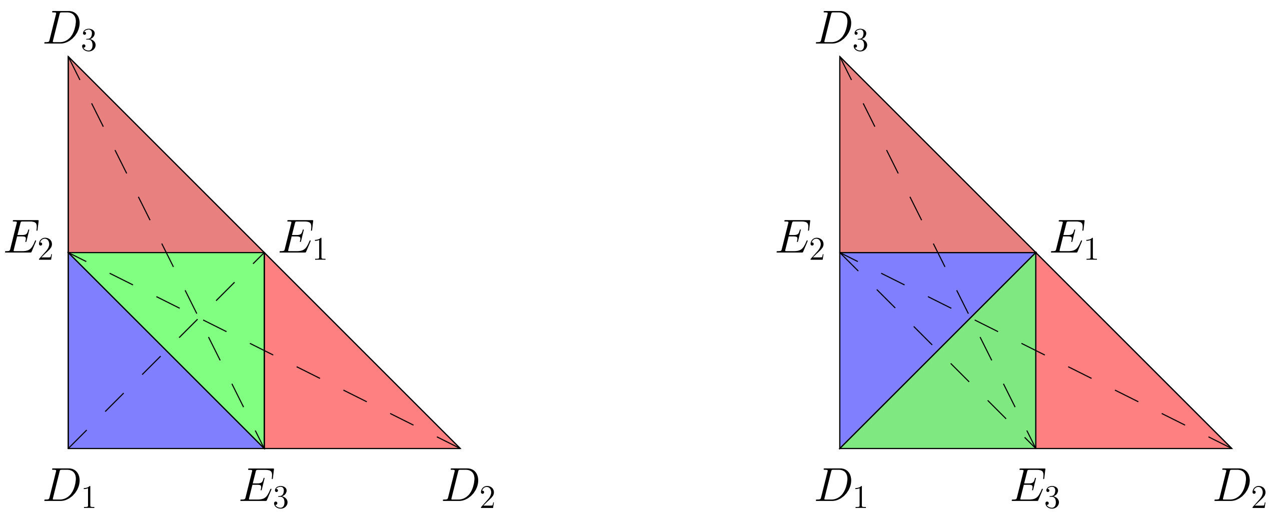
Heterotic models on orbifold resolutions can be obtained using toric geometry:

- We confirmed all results of the explicit blowups of $\mathbb{C}^2/\mathbb{Z}_2$ and $\mathbb{C}^3/\mathbb{Z}_3$.
- We showed that a similar analysis on more complicated orbifolds like $\mathbb{C}^3/\mathbb{Z}_4$ is doable.
- Again the matching with heterotic orbifold models is striking.
- Similar results we also found for $\mathbb{C}^2/\mathbb{Z}_3$ and $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orbifold resolutions: The latter allows for two inequivalent resolutions, but only the symmetric one is needed to reproduce the heterotic orbifold models.

Currently we are working on:

- Detailed investigation of blowups of heterotic $E_8 \times E_8$ $\mathbb{Z}_3$ orbifolds.
  SGN,Nilles,Plöger,Trapletti,Vandrevange
- Full classification of all possible gauge bundles on orbifold resolutions.
- Construction of resolution models of the orbifold $\mathbb{C}^3/\mathbb{Z}_6$-II.
- Construction of resolution models with localized torsion; i.e. with not all non–compact Bianchi's vanishing. SGN,Micu,Trapletti

The orbifold $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2'$ allows for two inequivalent blowups. On the level of the toric diagram this can be seen by the fact that it allows for two inequivalent triangulations:



Because in the different triangulations different basic cones are realized we have the following fundamental intersections:

$$D_1\, E_2\, E_3 = E_1\, E_2\, E_3 = 1, \qquad D_1\, E_1\, E_2 = D_1\, E_1\, E_3 = 1,$$

$$D_2\, E_3\, E_1 = D_3\, E_1\, E_2 = 1, \qquad D_2\, E_1\, E_3 = D_3\, E_1\, E_2 = 1,$$

$$D_1\, E_1\, E_2 = D_1\, E_1\, E_3 = D_2\, E_1\, E_2 = 0, \qquad D_1\, E_2\, E_3 = E_1\, E_2\, E_3 = D_2\, E_1\, E_2 = 0,$$

$$D_2\, E_2\, E_3 = D_3\, E_1\, E_3 = D_3\, E_2\, E_3 = 0, \qquad D_2\, E_2\, E_3 = D_3\, E_1\, E_3 = D_3\, E_2\, E_3 = 0.$$

From which the integrals of all exceptional divisors can be computed.

The gauge background can be expanded in the exceptional divisors as:

$$\frac{\mathcal{F}_V}{2\pi} = -\frac{1}{2}\big(H_1\, E_1 \;+\; H_2\, E_2 \;+\; H_3\, E_3 \big) \;,$$

where $H_1 = V_1^I H_I$, etc. The normalization and identification with the orbifold shifts are given by in 6D:

$$\int_{E_i} \frac{\mathcal{F}_V}{2\pi} \;=\; V_i^I\, H_i \;, \qquad v_i^I\, H_I \;\equiv\; \int_{D_2} \frac{\mathcal{F}_V}{2\pi} \;=\; -\,\frac{1}{2}\, V_i^I\, H_i \;.$$

The vanishing of the integrated Bianchi identities in 4D on the symmetric resolution and in 6D (on subresolutions) lead to many conditions:

$$6D \;\;:\quad V_1^2 = V_2^2 = V_3^2 = 6 \;, \qquad 4D \;\;:\quad V_1 \cdot V_2 = V_2 \cdot V_3 = V_1 \cdot V_3 = 1 \;.$$

The spectra can be computed using the formula:

$$N_V = \frac{1}{6}\,\big(H_1{+}H_2{+}H_3 \big)\Big[\frac{1}{2}\,\big(H_1 H_2{+}H_2 H_3{+}H_3 H_1 \big){-}\frac{1}{8}\big(H_1^2{+}H_2^2{+}H_3^2 \big){-}\frac{1}{4}\Big] - \frac{3}{8}\, H_1 H_2 H_3 .$$

We can reconstruct all orbifold models using the symmetric resolution only:

orbifold shift $2\,v_1$	orbifold shift $2\,v_2$	blowup vector V_1	blowup vector V_2	blowup vector V_3
$(1^2,0^{14})$	$(0,1^2,0^{13})$	$(1^2,0,2,0^{12})$ $(1^2,2,0^{13})$	$(0,1^2,0,2,0^{11})$ $(0,-1,1,2,0^{12})$	$(1,0,1,0,0,2,0^{10})$ $(-1,0,1,0,2,0^{11})$
$(1^2,0^{14})$	$(0,1^6,0^9)$	$(1^2,0^{13},2)$ $(1^2,2,0^{13})$	$(0,1^6,0^9)$ $(0,-1,1^5,0^9)$	$(1,0,1^3,-1^2,0^9)$ $(-1,0,1^3,-1^2,0^9)$
$(1^6,0^{10})$	$(0^3,1^6,0^7)$	$(1^6,0^{10})$	$(0^3,-1,1^5,0^7)$	$(1^2,-1,0^3,1^2,-1,0^7)$
$(1^6,0^{10})$	$(0^5,1^6,0^5)$	$(1^6,0^{10})$	$(0^5,1^6,0^5)$	$(0^5,1,0^5,1^5)$
$(1^2,0^{14})$	$\frac{1}{2}(1^{15},-3)$	$(-1,1,2,0^{13})$	$\frac{1}{2}(1^{15},-3)$	$\frac{1}{2}(-1^2,1^{12},-3,1)$
$(1^6,0^{10})$	$\frac{1}{2}(-3,1^{15})$	$(1^6,0^{10})$ $(1^4,-1^2,0^{10})$	$\frac{1}{2}(-3,1^{15})$ $\frac{1}{2}(1^{15},-3)$	$\frac{1}{2}(-3,1^5,-1^{10})$ $\frac{1}{2}(1^6,-1^8,3,-1)$