

Large $\tan\beta$ effects in flavour physics

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- ▶ Introduction: the SUSY flavour problem & the MFV hypothesis
- ▶ MFV at large $\tan\beta$: general considerations
- ▶ Large $\tan\beta$ effects in B (and K) physics
- ▶ Lepton Flavour violation and LF non-universality at large $\tan\beta$
- ▶ Conclusions

► The SUSY flavour problem & the MFV hypothesis

The flavour structure of the SM is quite constrained:

- a large global symmetry in the gauge sector

$$U(3)^5 = \text{SU}(3)_Q \times \text{SU}(3)_U \times \text{SU}(3)_D \times \dots$$

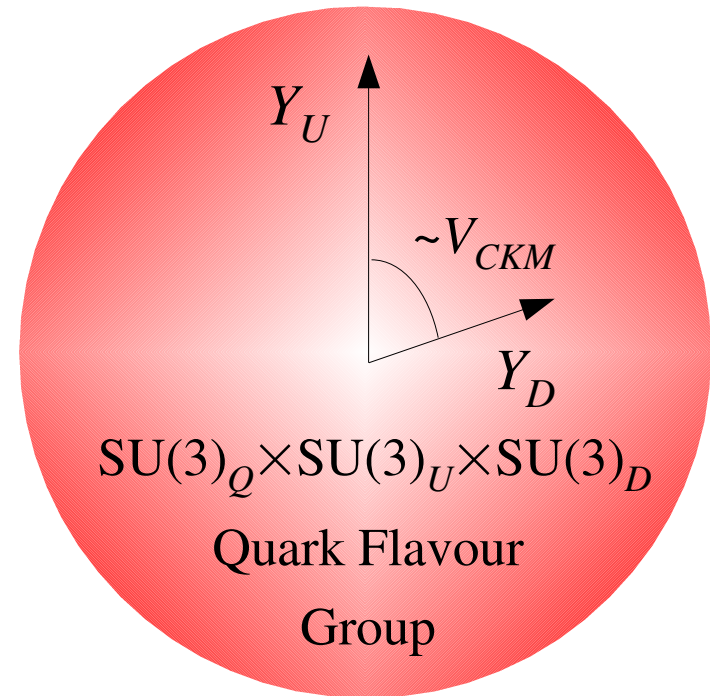
- broken only by the Yukawa couplings

$$Y_D \sim \bar{3}_Q \times 3_D \quad Y_U \sim \bar{3}_Q \times 3_U \quad (Y_E \sim \bar{3}_L \times 3_E)$$



This specific symmetry + symmetry-breaking pattern is responsible for the suppression of FCNCs, the suppression of CPV, etc...

The ugly (but highly successful...) part of the SM



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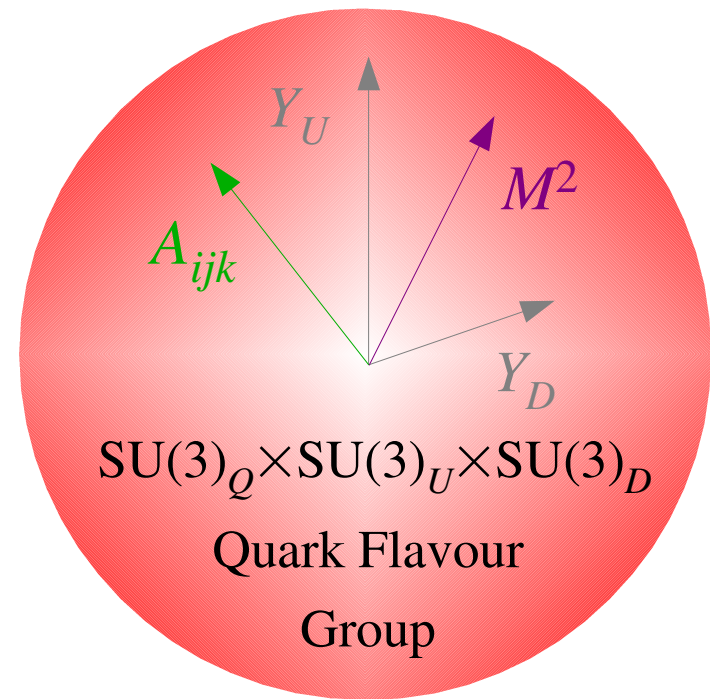
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In principle, the soft breaking terms of the MSSM allow a much richer symmetry-breaking structure:

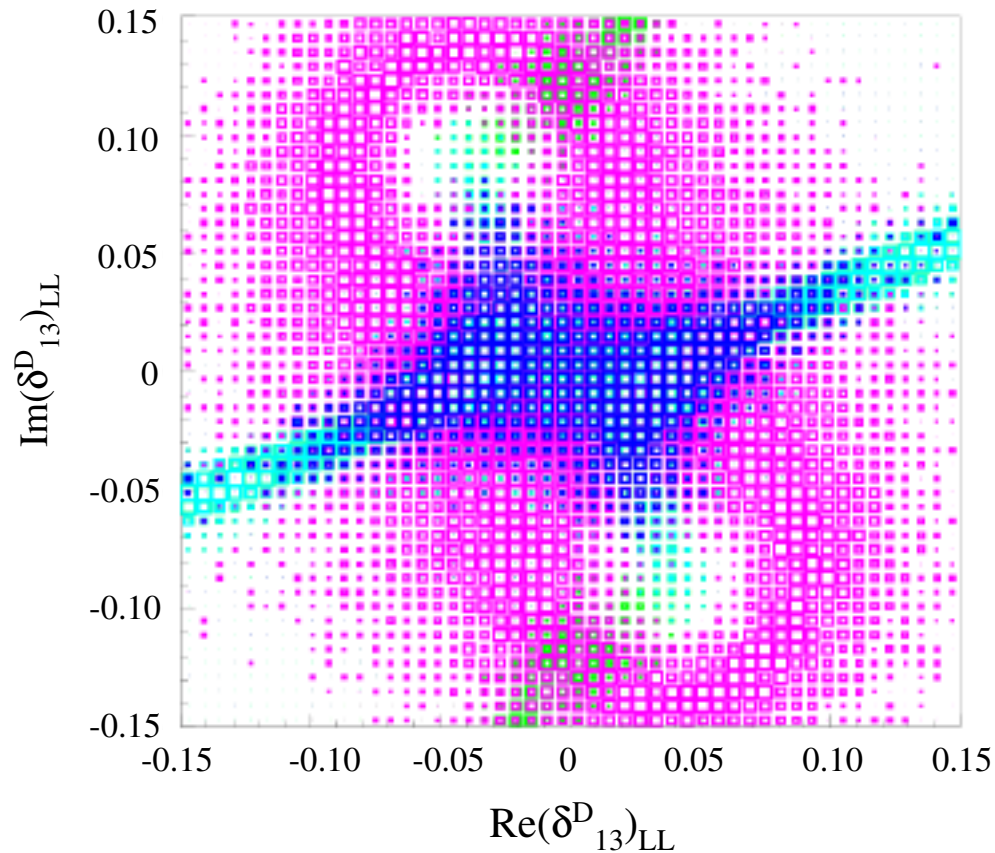
$$\mathcal{L}_{soft} \subset (M^2)_{ij} \phi_i \phi_j + A_{ijk} \phi_i \phi_j \phi_k$$

New flavour-breaking terms not necessarily related to the Yukawa couplings



In practice, the absence of deviations from the SM in rare processes implies *severe constraints* on flavour-symmetry breaking terms beyond the SM Yukawas (at least in the quark sector...)

E.g.: Constraints on $(\delta^D_{13})_{LL}$ from B_d -meson mixing [L. Silvestrini @ CKM 2006]

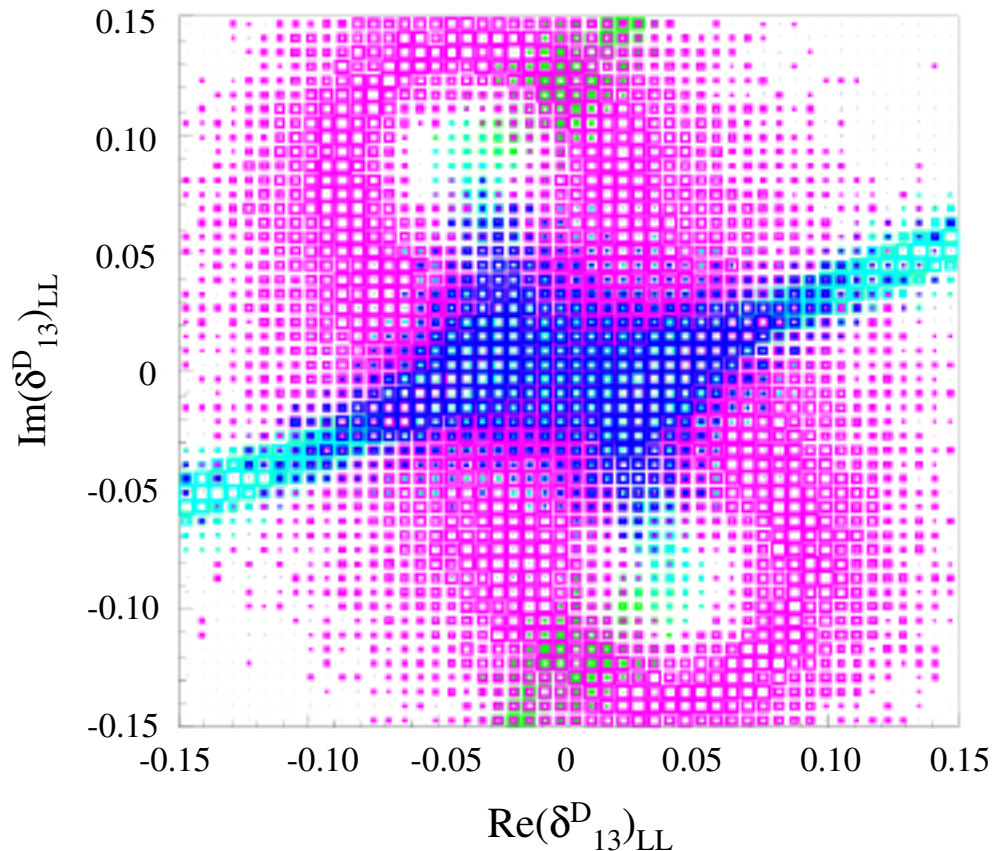


$$(\delta^D_{13})_{LL} = \frac{(M^D_{13})^2_{LL}}{\langle M^D_{11} \rangle \langle M^D_{33} \rangle}$$

Δm_B only
 $\sin 2\beta$ only
 $\sin 2\beta$ and $\cos 2\beta$
 All constraints

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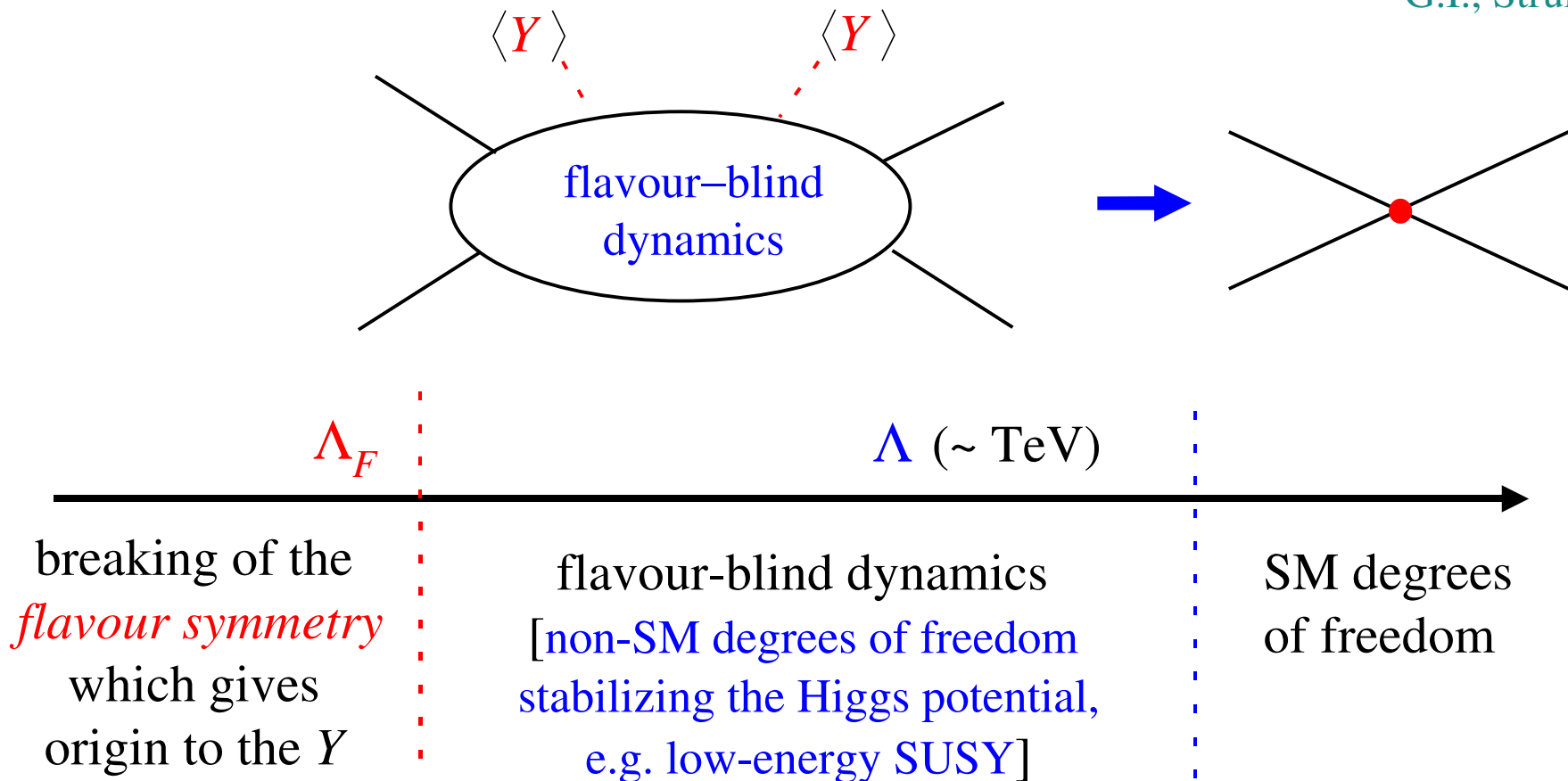
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Similar (even more stringent) bounds are obtained also on many other δ 's – taking into account ϵ_K , Δm_K , $K \rightarrow \pi\nu\nu$, $B \rightarrow X\gamma$, $B \rightarrow Xl^+l^-$, $B \rightarrow \mu\mu$, $\Delta m_{B_s}, \dots$

The most pessimistic (but also most natural) way out to this problem is the so-called Minimal Flavour Violation [MFV] hypothesis: *the Yukawa couplings are the only irreducible sources of flavour symmetry breaking*

General principle (symmetry + symmetry-breaking structure) which can be formulated for any (TeV-scale) SM extension:

D'Ambrosio, Giudice,
G.I., Strumia '02



Within the MSSM, the MFV hypothesis implies a strong restriction on the flavour-structure of the soft breaking terms:

$$\text{E.g.: } M_Q^2 \tilde{Q}_L^+ \tilde{Q}_L \longrightarrow M_Q^2 \propto \sum a_n (Y_U Y_U^+)^n \sim a_0 I + a_1 Y_U Y_U^+$$

- More general than universality [$M_Q^2 \propto I$]
- RGE invariant structure [with the a_n linked together by RGE]
- Perfectly compatible with present data without fine-tuning on the a_n

$$(\delta_{ij}^D)_{LL} \propto y_t^2 (V_{CKM})_{3i}^* (V_{CKM})_{3j}$$

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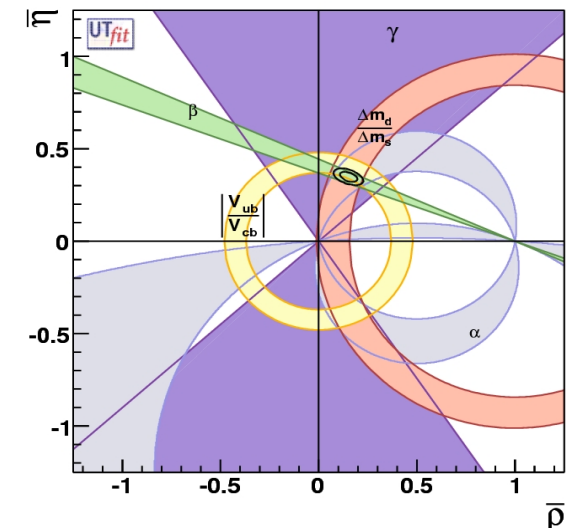
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$$(\delta^D_{ij})_{LL} \propto y_t^2 (V_{CKM})_{3i}^* (V_{CKM})_{3j}$$

\Rightarrow Same CKM factors as in SM:
only the flavour-independent magnitude of FCNC amplitudes can be modified
[$A(b \rightarrow s\gamma) \propto V_{tb} V_{ts}$, $\Delta M_{Bd} \propto (V_{tb} V_{td})^2$, ...]

\Rightarrow very efficient suppression of NP effects in flavour physics, especially in the standard CKM fits...



► Minimal Flavour Violation at large $\tan\beta$: general considerations

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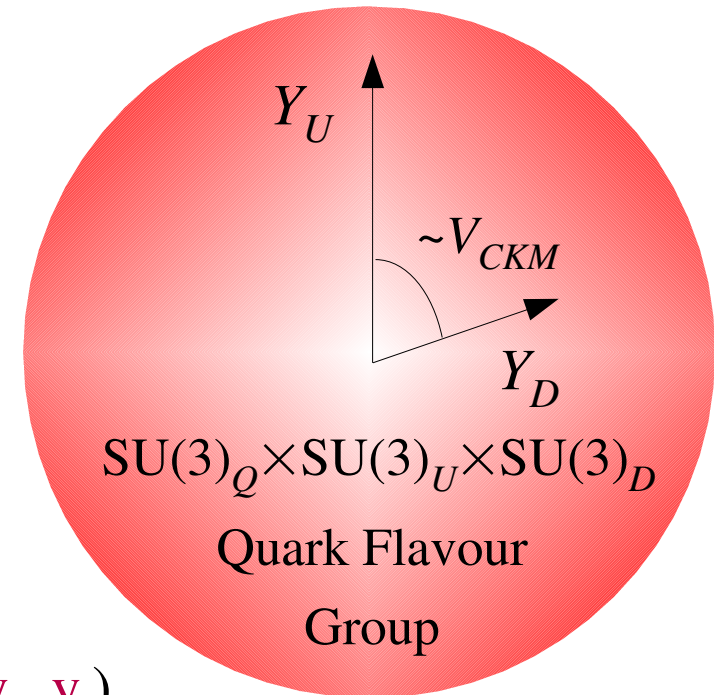
...however, with two Higgs doublets and a large ratio of vevs, interesting effects in rare decays can occur also under the pessimistic MFV hypothesis

$$\mathcal{L}_{\text{q-Yukawa}} = \bar{Q}_L Y_D D_R H_D + \bar{Q}_L Y_U U_R H_U + \text{h.c.}$$

Y_D & Y_U are still the only irreducible breaking sources of $SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

negligible non-standard effects
in the standard CKM fits

$$Y_D = \text{diag}(y_d, y_s, y_b) \quad Y_U = (V_{\text{ckm}})^{\dagger} \times \text{diag}(y_u, y_c, y_t)$$



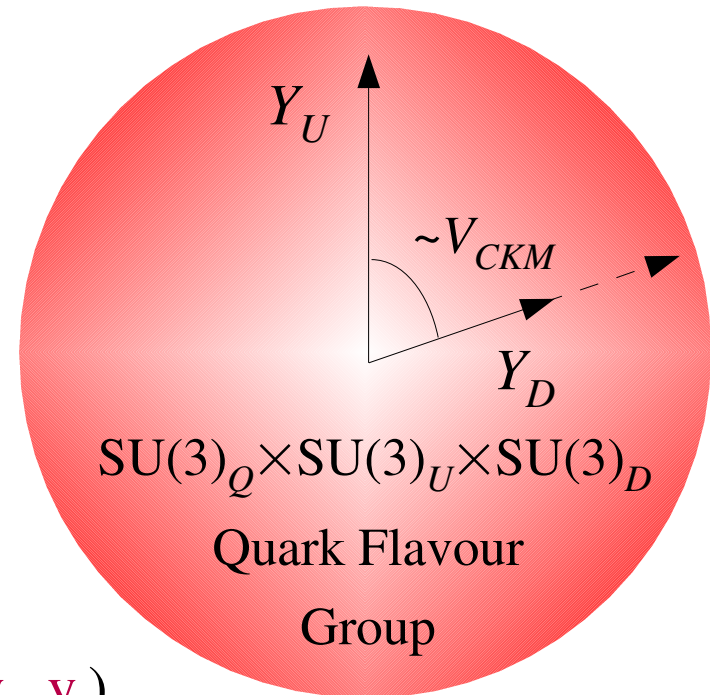
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However, we are free to change their overall normalization

$$y_u = m_u / \langle H_U \rangle \quad y_d = m_d / \langle H_D \rangle = \tan\beta m_d / \langle H_U \rangle$$

sizable phenomenological consequences in helicity-suppressed
processes if $\tan\beta \gg 1$

► Motivations

Large $\tan\beta$ values are motivated by interesting theoretical considerations:

- $\tan\beta \sim 40\text{--}50$ allows the unification of top & bottom Yukawa couplings
[\Rightarrow SO(10) GUTs] wide literature

- $\tan\beta \gg 1$ is a natural prediction of the Minimal Gauge-Mediated SUSY-breaking scenario [$\langle H_D \rangle = 0$ at the tree level],
which also provide a natural dynamical motivation for MFV

Dine, Nir, Shirman '96
Rattazzi, Sarid '97

Hisano, Shimizu '07

It's not a scenario ad hoc for flavour physics !

► Motivations

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+

Intriguing phenomenological observations:

- Natural explanation of the $\sim 3\sigma$ discrepancy in the anomalous magnetic moment of the muon:

$$\Delta a_{\mu}^{\text{exp}} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (29 \pm 9)10^{-10} \quad (\sim 2 \times a_{\mu}^{\text{ew-SM}}) \quad \text{Hertzog et al. '07}$$

$$\Delta a_{\mu}^{\text{SUSY}} \sim \tan\beta \times (m_W/M_{\text{SUSY}})^2 \times (a_{\mu}^{\text{ew-SM}}) \times \text{sgn}(\mu)$$

► Motivations

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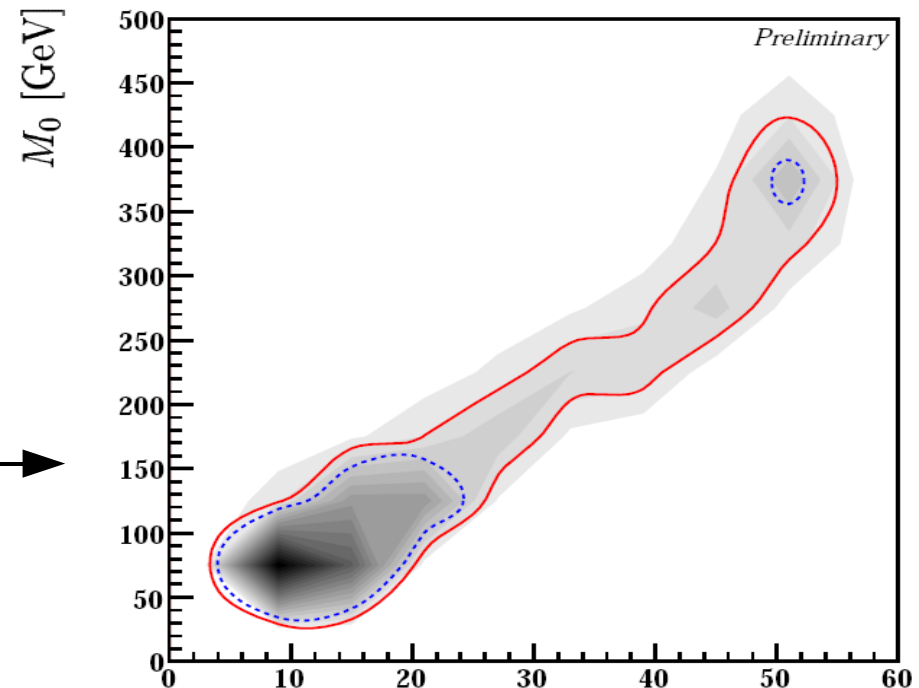
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- Good consistency with e.w. & dark-matter constraints even in constrained scenarios

→ CMSSM [[Buchmuller et al. '07](#)]

→ NUHM: large $\tan\beta$ allowed even with (relatively) light M_H
[[Ellis, Heinemeyer, Olive, Weiglein '07](#)]



⇒ talk by Buchmuller & Ronga in the Collider session $\tan\beta$

► MSSM vs. Two-Higgs doublet models

Warning: the *effective* Yukawa interaction of the MSSM can be very different with respect to the non-supersymmetric Two-Higgs Doublet Model of type-II, even in the limit of light Higgses & heavy squarks

$$\mathcal{L}_{\text{tree}} = \bar{Q}_L Y_D D_R H_D + \bar{Q}_L Y_U U_R H_U$$

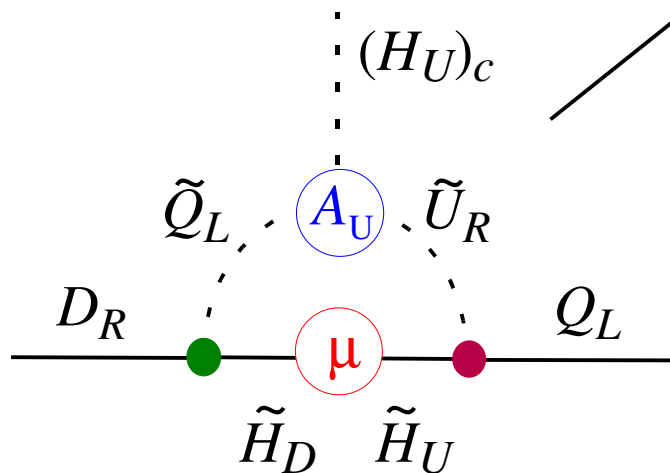
invariant under $U(1)_{\text{PQ}} \dots$

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$$\mathcal{L}_{\text{eff}} = \bar{Q}_L Y_D D_R H_D + \bar{Q}_L Y_U U_R H_U + \epsilon_1 \bar{Q}_L Y_U Y_U^+ Y_D D_R (H_U)_c + \dots$$

...possible large $U(1)_{\text{PQ}}$ breaking
induced by the μ term



Even if $\epsilon_i \sim (16\pi^2)^{-1}$ these (*non-holomorphic*) terms are a potential large destabilization of the tree-level Yukawa structure:

- $\epsilon_i \times \tan\beta \sim 1$
- **dim-4 ops.** \Rightarrow **non-decoupling effects**

Hall, Rattazzi, Sarid, '94

Blazek, Raby, Pokorski, '95

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$$\mathcal{L}_{\text{eff}} = \bar{Q}_L (Y_D) D_R H_D + \bar{Q}_L Y_U U_R H_U + \epsilon_1 \bar{Q}_L (Y_U Y_U^\dagger Y_D) D_R (H_U)_c + \dots$$

⇒ non-decoupling effects, include Higgs-mediated FCNC's

A complete re-diagonalization of all the effective dim-four Yukawa terms is necessary in order to re-sum the large $\epsilon_i \times \tan\beta$ terms beyond ordinary perturbation theory [correct identification of the ground state]

Babu & Kolda '00; G.I. & Retico '02; Dedes & Pilaftsis '03,...

No further large quantum corrections if

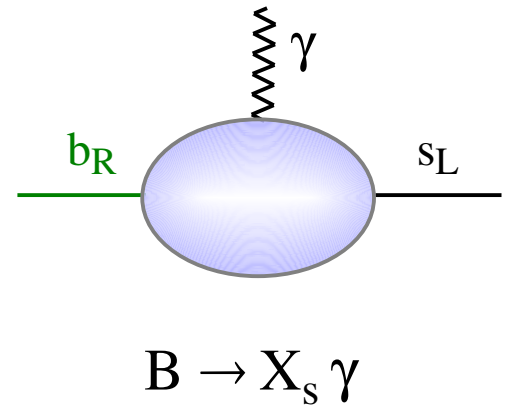
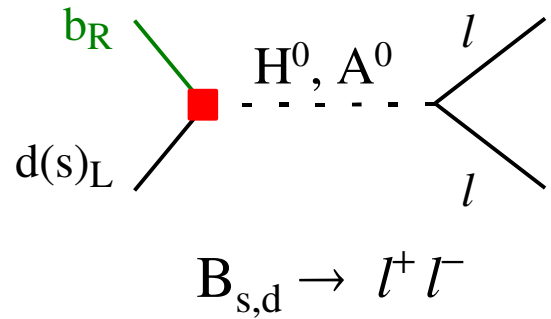
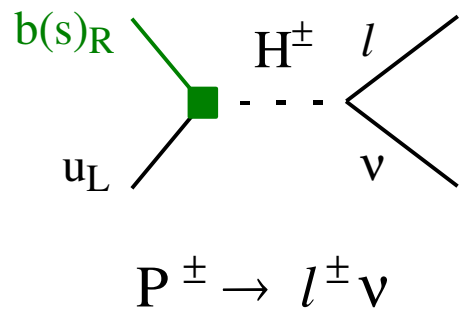
- $\tilde{M}_Q \gg M_A \gg m_W$
- $\tan\beta$ defined as $\langle H_U \rangle / \langle H_D \rangle$

Alternative hierarchies and/or different $\tan\beta$ definitions can be more problematic

Freitar, Gasser, Haisch, '07
 ⇒ talk by S.Trine in the Flavour Session

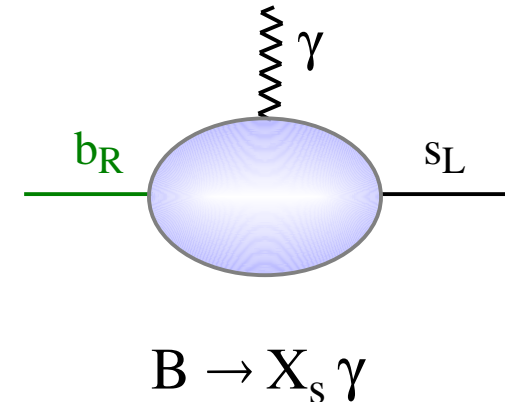
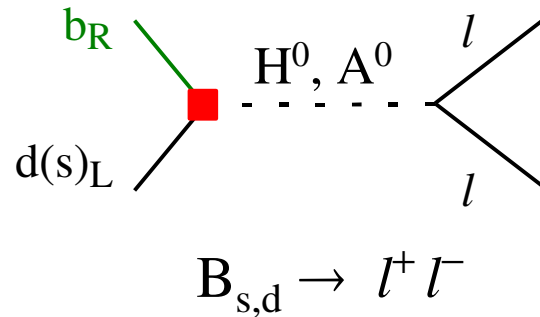
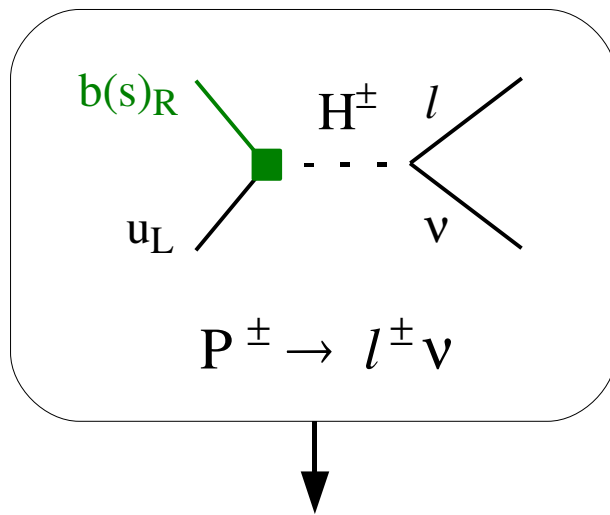
► Large $\tan\beta$ effects in B (and K) decays

Three most interesting sets of observables:



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Simplest M_H & $\tan\beta$ dependence [[mild dependence on other parameters](#)]

$$BR = BR_{SM} \times \left(1 - \frac{m_p^2 \tan^2\beta}{M_H^2 (1 + \epsilon_0 \tan\beta)} \right)^2$$

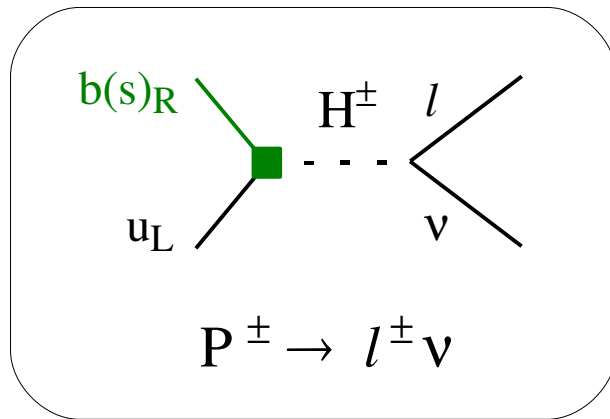
- O(100%)–O(10%) in $B^\pm \rightarrow l^\pm \nu$
[[most likely \$BR_{SUSY} < BR_{SM}\$](#)]

- O(1%)–O(0.1%) in $K^\pm \rightarrow l^\pm \nu$
[[necessarily \$BR_{SUSY} < BR_{SM}\$](#)]

G. Hou, '93; Ackeroid, Recksiegel, '03

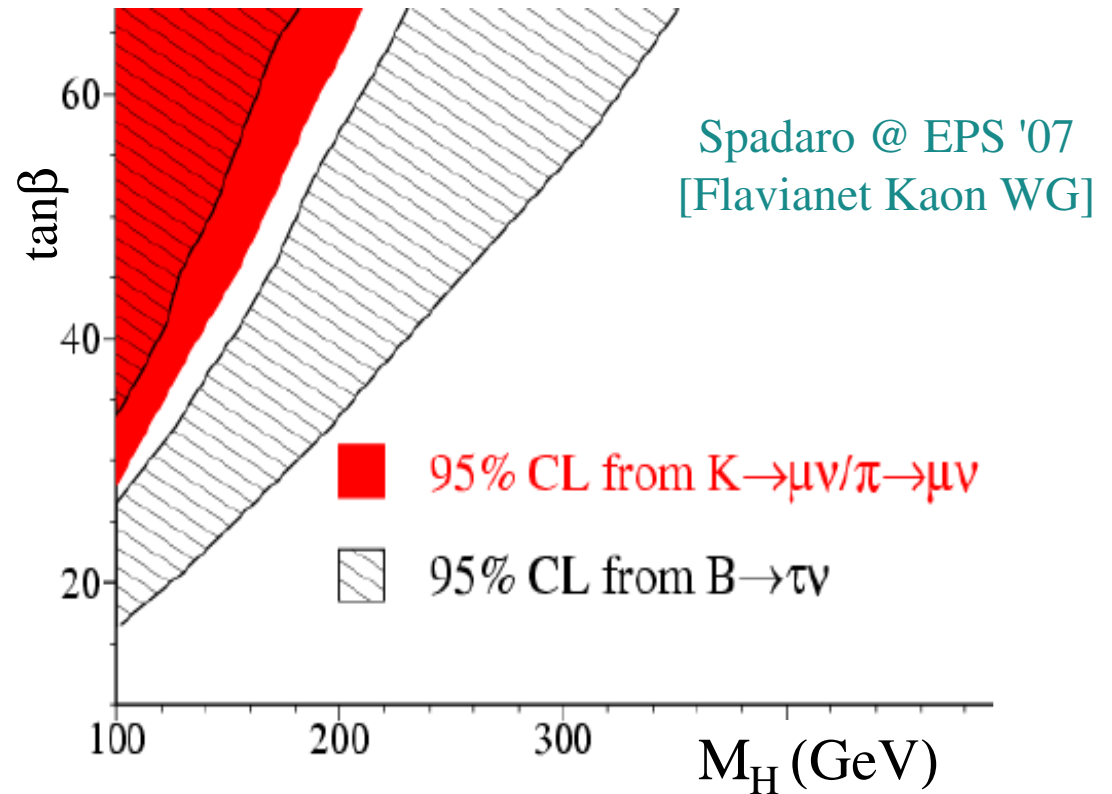
G.I. Paradisi '06

► Large $\tan\beta$ effects in B (and K) decays



$$B(B \rightarrow \tau \nu) = (1.43 \pm 0.43) \times 10^{-4}$$

$$[B_{\text{SM}} \approx 1.2 \times 10^{-4}] \quad [\text{Babar+Belle '07}]$$



$$B(K \rightarrow \mu \nu (\gamma)) = (63.66 \pm 0.17)\% \quad [\text{KLOE}]$$

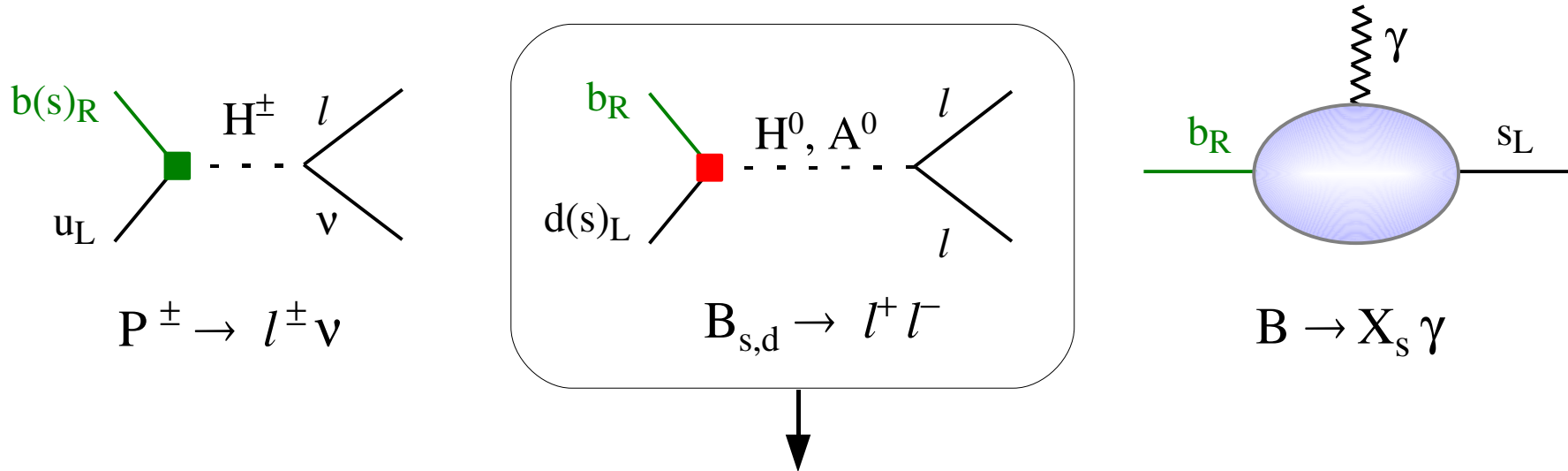
$$+ f_K/f_\pi @ 0.7\% \quad [\text{MILC/UKQCD '07}]$$

$$+ V_{us} @ 0.5\% \quad [\text{KLOE/NA48/KTeV + Theory}]$$

Improving the measure of $B(B \rightarrow l \nu)$ provide very valuable infos on M_H & $\tan\beta$!

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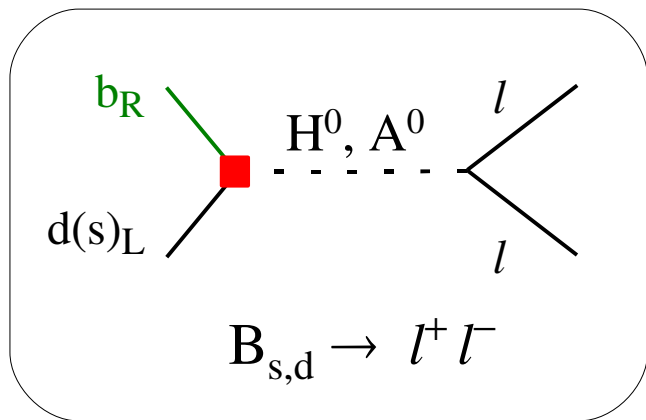
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Crucial dependence on μ and A_U [in addition to M_H & $\tan\beta$]

$$A(B \rightarrow ll)_H \sim \frac{m_b m_l}{M_A^2} \frac{\mu A_U}{\tilde{M}_q^2} \tan^3\beta$$

Possible large enhancement over the SM
 but size (and magnitude) of the effect can change
 substantially in different SUSY-breaking scenarios

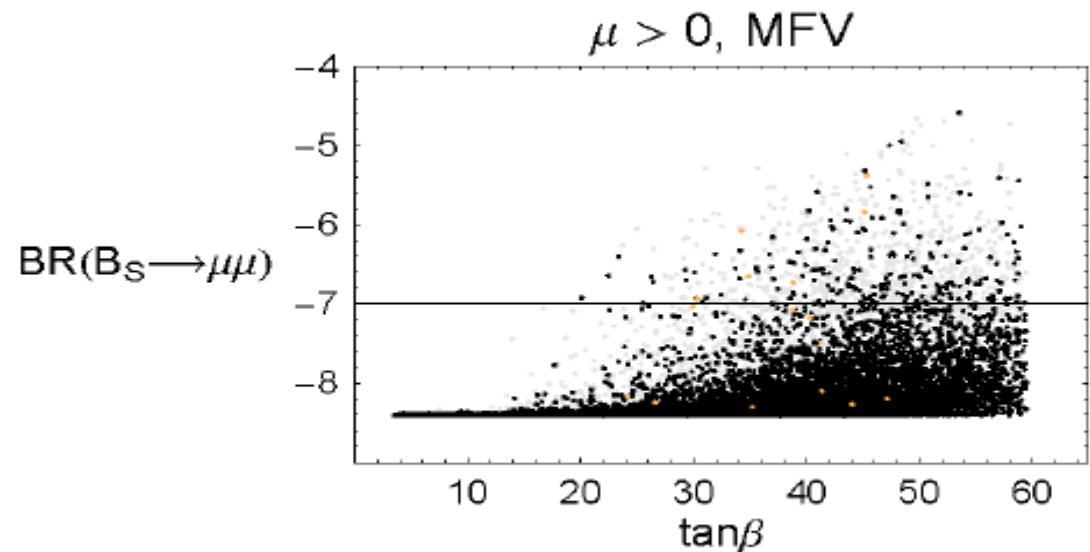
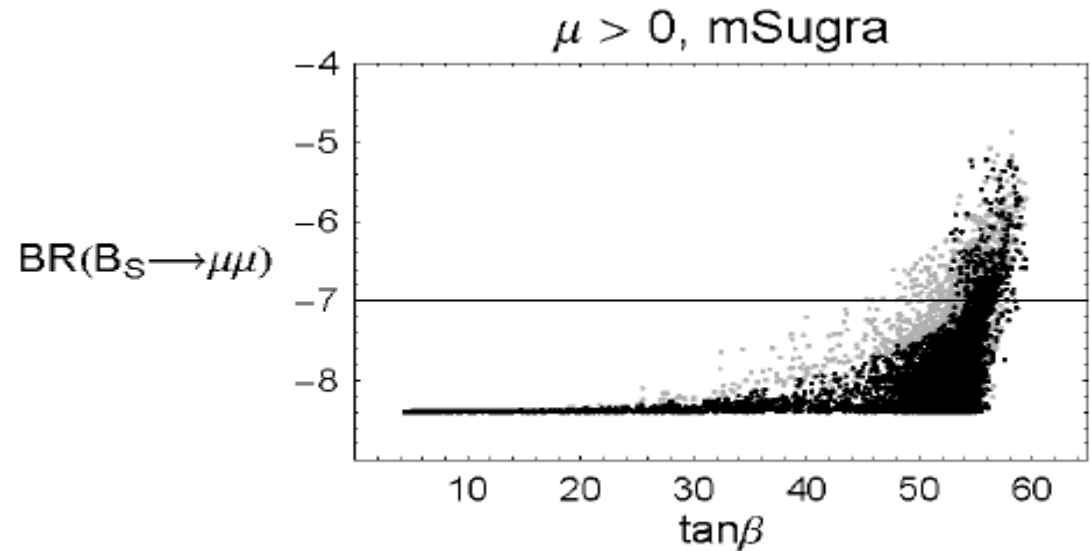


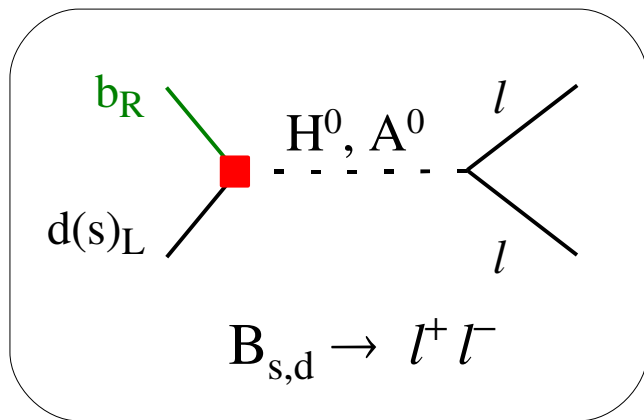
$$B(B_s \rightarrow \mu\mu) < 5.8 \times 10^{-8} \text{ (95\%CL)}$$

$$[B_{\text{SM}} \sim 3 \times 10^{-9}]$$

non-official CDF+D0 combined
limit [EPS '07]

Significant constraint
but a good fraction of the
parameter space is still allowed





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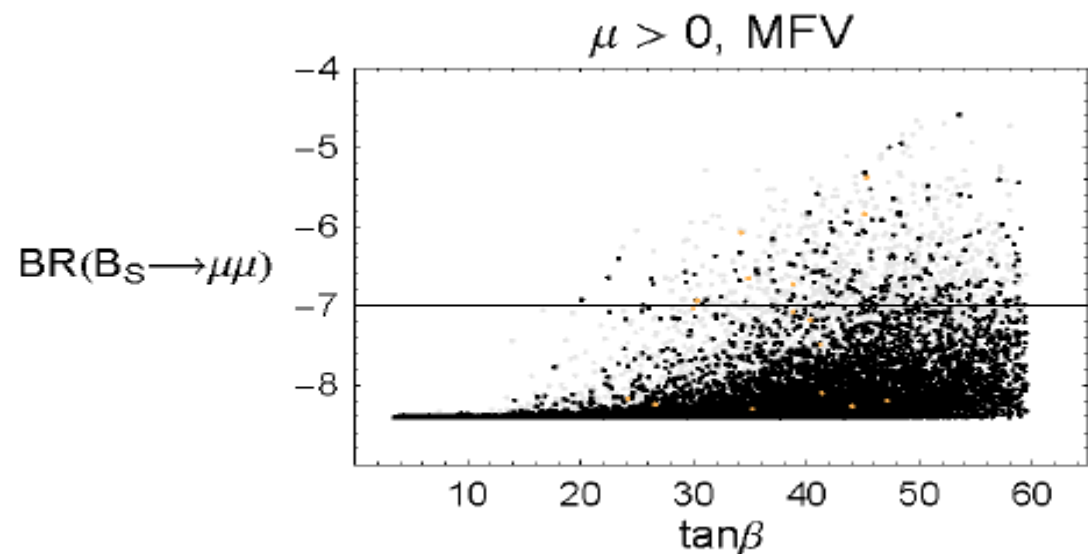
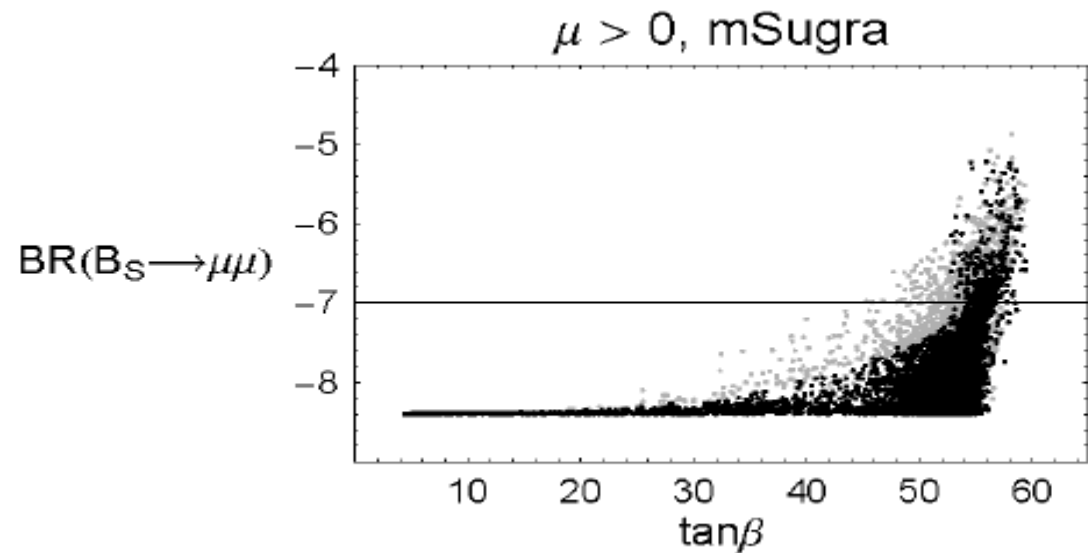
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important phenomenological implication:

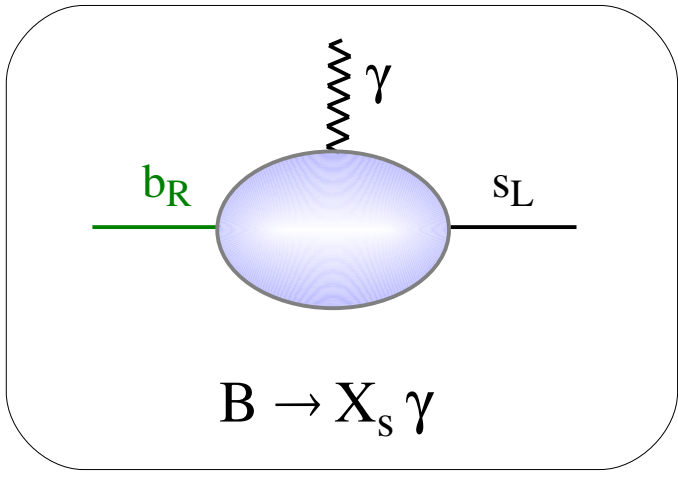
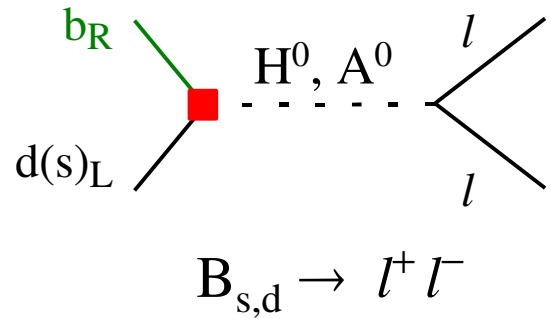
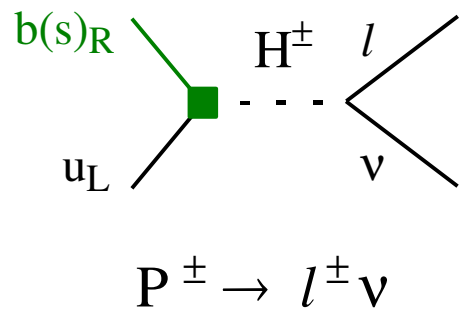
negligible double-Higgs penguin effect [Buras *et al.* '01] in ΔM_{B_s}



Lunghi, Porod, Vives '06

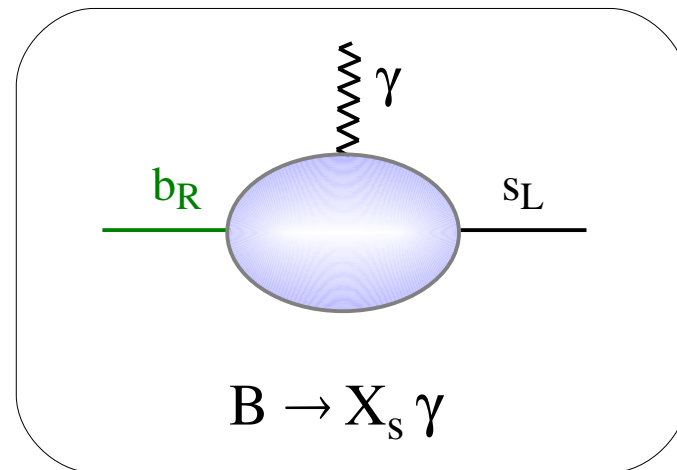
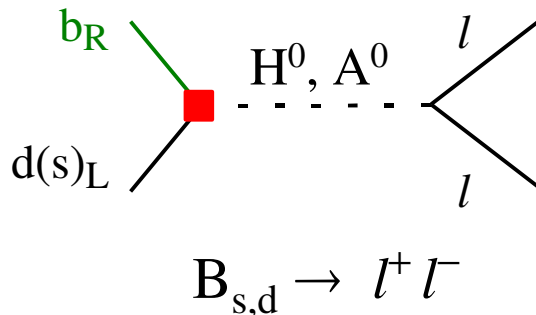
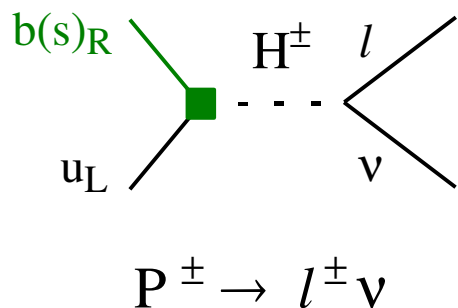
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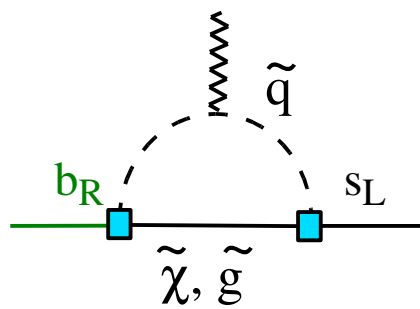
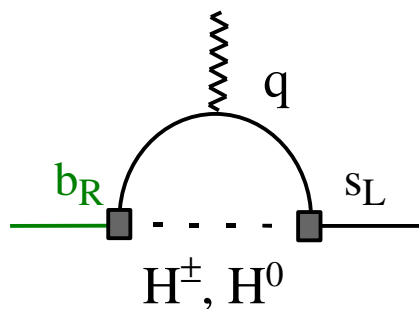


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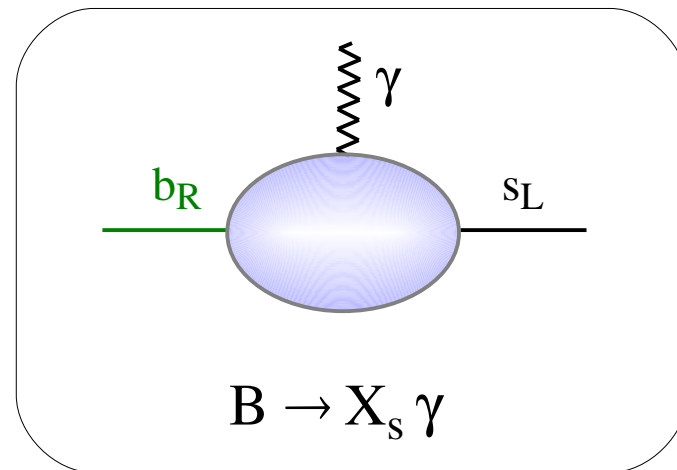
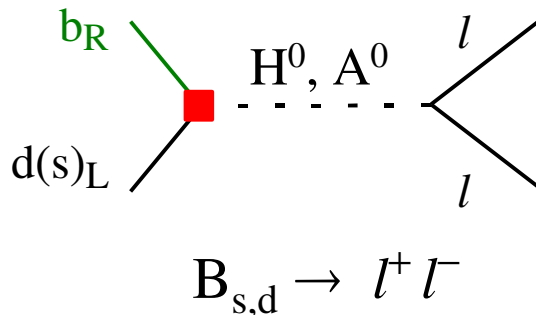
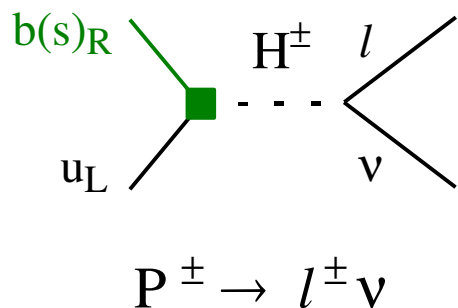
Most complicated observable with several, naturally competitive, contributions:



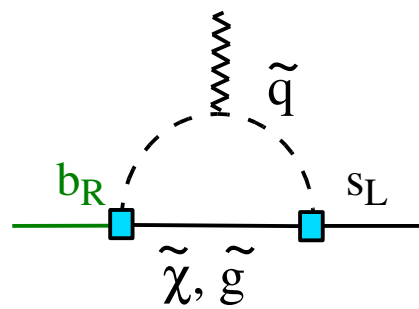
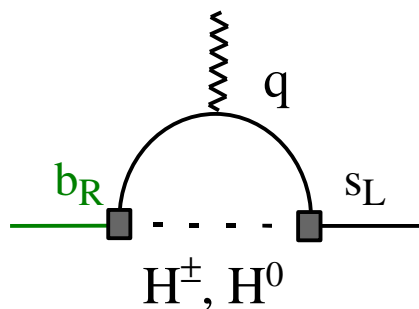
- positive
- decreasing with $\tan\beta$
- sign $\sim \text{sgn}(\mu, A)$
- increasing with $\tan\beta$

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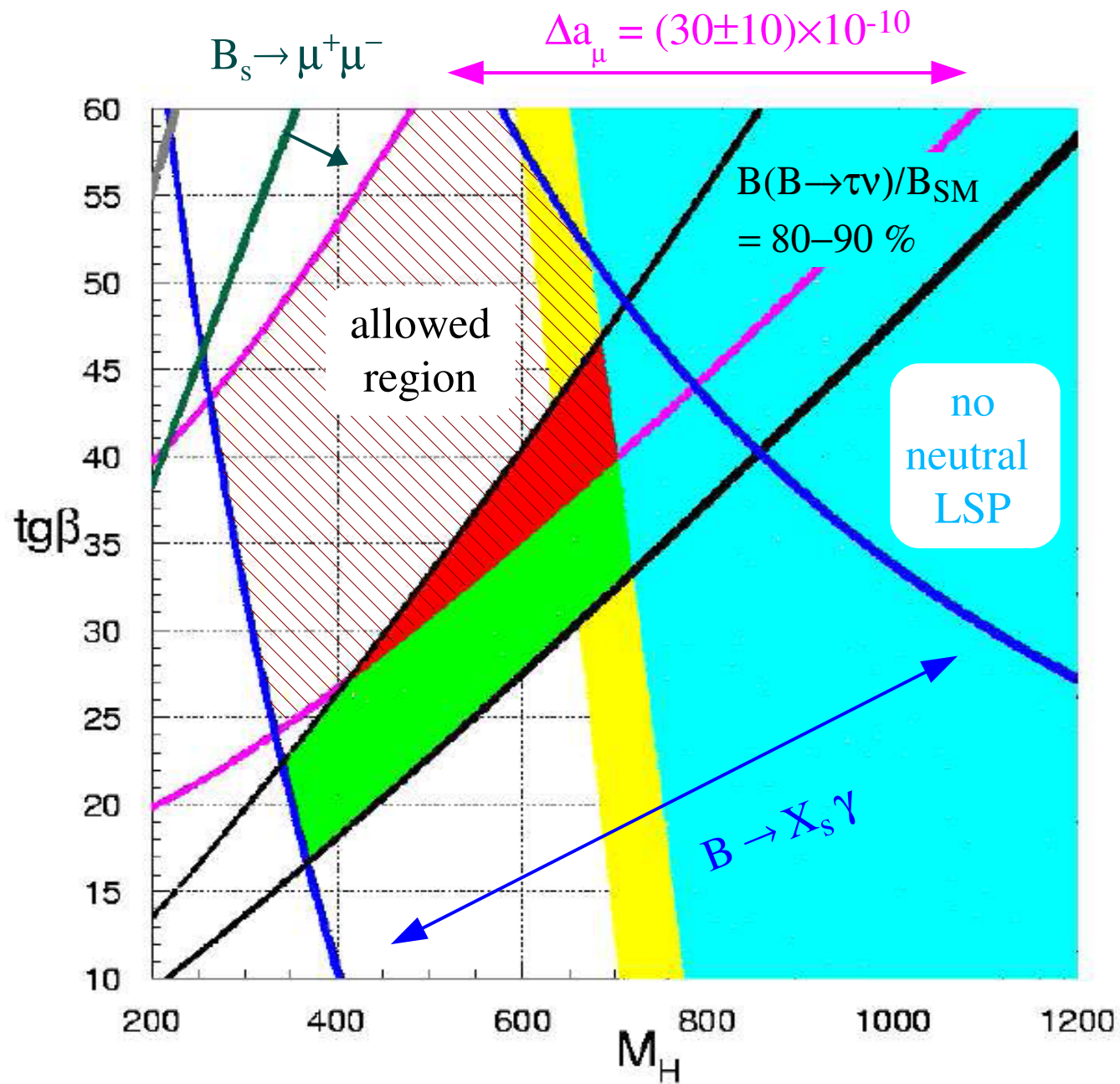
One of the most significant constraint of the MSSM:

$$B(B \rightarrow X_s \gamma)^{\text{exp}} = (3.55 \pm 0.26) \times 10^{-4} \quad [\text{HFAG '06}]$$

$$B(B \rightarrow X_s \gamma)^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4} \quad [\text{Misiak et al. '06}]$$

- positive
- decreasing with $\tan\beta$
- sign $\sim \text{sgn}(\mu, A)$
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- E.g.: combined low-energy constraints
 + dark matter (A-funnel region)
 + heavy squarks



$$M_{sq} = 1.5 \text{ TeV} \quad M_{sl} = 0.5 \text{ TeV}$$

$$A_u = -1.0 \text{ TeV} \quad \mu = 0.5 \text{ TeV}$$

► Lepton Flavour violation and LF non-universality at large $\tan\beta$

Large $\tan\beta$ values naturally enhance various LFV rates [e.g. $\Gamma(\mu \rightarrow e\gamma) \sim \tan\beta^2, \dots$], but interesting phenomena can show up also in the quark sector under specific circumstances:

If the model has sizable **non-MFV** sources of breaking in the **lepton sector** (non-minimal LFV) \Rightarrow possible visible violations of LF universality in $B(K) \rightarrow l\nu$

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Large $\tan\beta$ values naturally enhance various LFV rates [e.g. $\Gamma(\mu \rightarrow e\gamma) \sim \tan\beta^2, \dots$], but interesting phenomena can show up also in the quark sector under specific circumstances:

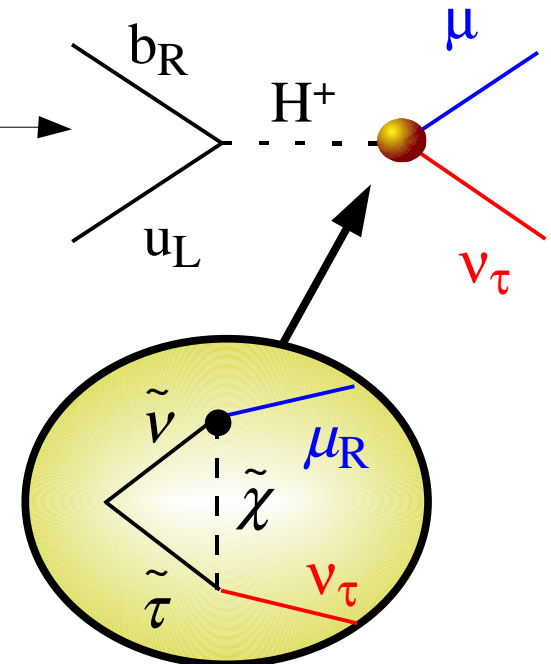
If the model has sizable **non-MFV** sources of breaking in the **lepton sector** (non-minimal LFV) \Rightarrow possible visible violations of LF universality in $B(K) \rightarrow l\nu$

$$\Gamma(B \rightarrow \mu\nu)^{\text{exp}} = \underbrace{\Gamma(B \rightarrow \mu\nu_\mu)}_{\text{SM}} + \underbrace{\Gamma(B \rightarrow \mu\nu_e)}_0 + \Gamma(B \rightarrow \mu\nu_\tau)$$

sizable one-loop eff. coupl.
because of 3rd generation
& large mixing
in the lepton sector

Masiero, Paradisi,
Petronzio, '05

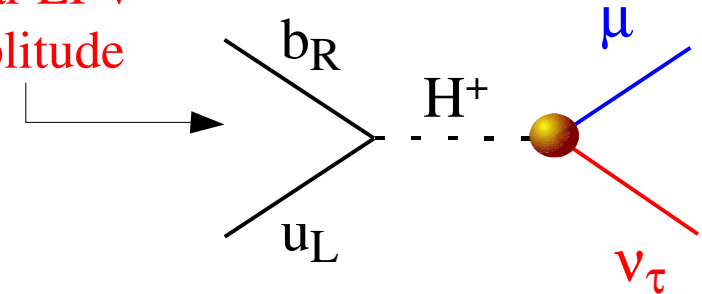
scalar LFV
amplitude



► Lepton Flavour violation and LF non-universality at large $\tan\beta$

$$\Gamma(B \rightarrow \mu\nu)^{\text{exp}} = \underbrace{\Gamma(B \rightarrow \mu\nu_\mu)}_{\text{SM}} + \underbrace{\Gamma(B \rightarrow \mu\nu_e)}_0 + \Gamma(B \rightarrow \mu\nu_\tau)$$

scalar LFV
amplitude



Possible probe of this effect in the ratios:

$$R_{\mu\tau}^B = \frac{\Gamma(B^+ \rightarrow \mu^+\nu)}{\Gamma(B^+ \rightarrow \tau^+\nu)} \quad R_{e\tau}^B = \frac{\Gamma(B^+ \rightarrow e^+\nu)}{\Gamma(B^+ \rightarrow \tau^+\nu)}$$

deviations from the SM

$$\sim (\tan\beta)^4 \times (\delta^{\text{slept}})_{23}$$

up to +10% $(R_{\mu\tau}^B)^{\text{SM}}$

up to +10³ $(R_{e\tau}^B)^{\text{SM}}$

Interesting correlation with the same phenomenon in the Kaon system, which so far set the best upper limit:

$$R_{\mu e}^K = \frac{\Gamma(K^+ \rightarrow \mu^+\nu)}{\Gamma(K^+ \rightarrow e^+\nu)} \rightarrow \text{up to 1\% diff. from SM}$$

2007 dedicated run by NA48 to reach 0.3% accuracy

► Conclusions

The MSSM with **MFV** + **large $\tan\beta$** is a quite interesting scenario:

- well motivated (not *ad hoc* for flavour physics)
- consistent with present data
(no more fine-tuned than other MSSM scenarios)
- predictive, with enhanced correlations between low- and high-energy data which could possibly be falsified/verified in the (**near ?**) future

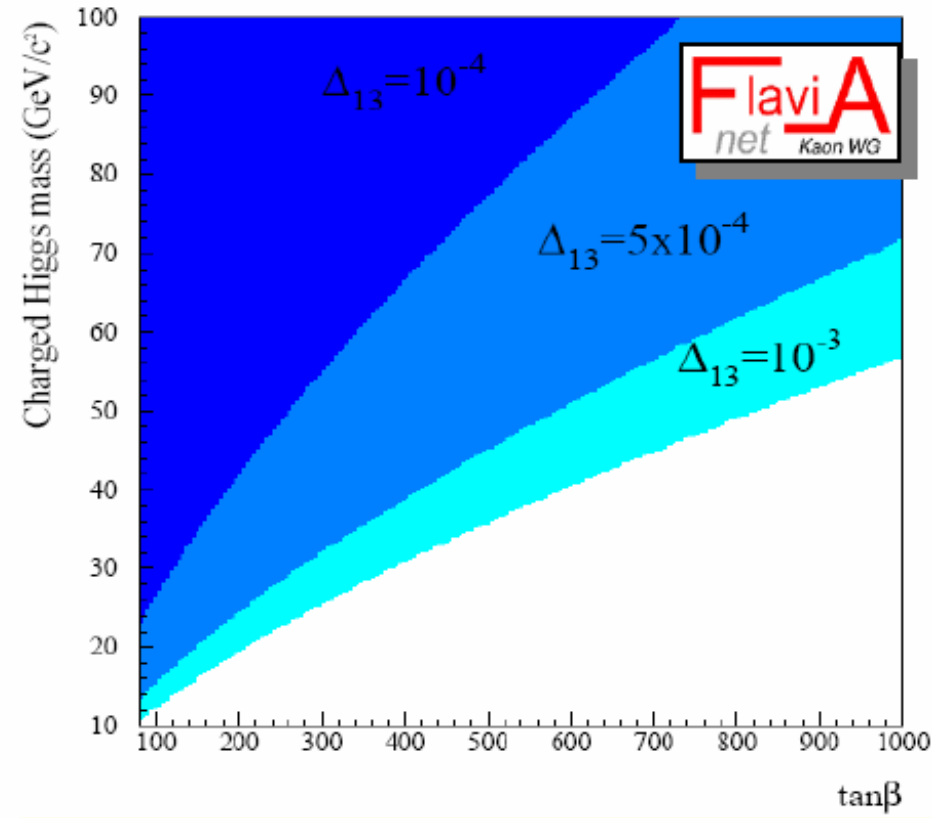
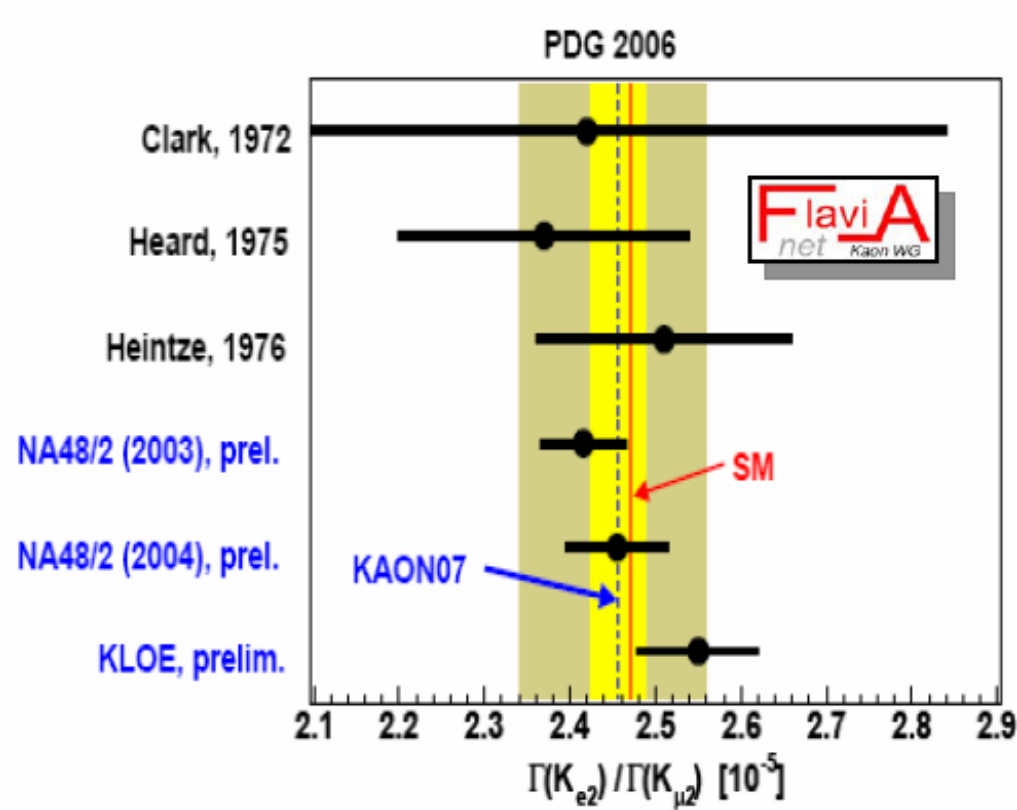


Key role played by future improvements on the helicity-suppressed modes **$B(K) \rightarrow lv$** , **$B \rightarrow ll$** , and **$\mu \rightarrow e\gamma$**

► Backup

► Backup

Limit on LFV in H^+ coupling \Rightarrow $B(K \rightarrow e\nu) / B(K \rightarrow \mu\nu)$



$$\Delta_{13} \sim (\alpha/4\pi) (\delta_{RR})_{13}$$