Flavor violation in a SUSY GUT

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Work in progress
in collaboration with
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Supersymmetrizing the Standard Model

- attractively offers a natural candidate of dark matter
- stabilizes hierarchy between distant scales
- helps gauge coupling unification

⇒ has an intimate relation with grand unification
Supersymmetry in the Standard Model

**attractively**
- offers a natural candidate of dark matter
- stabilizes hierarchy between distant scales
- helps gauge coupling unification

⇒ has an intimate relation with grand unification

**however**
- may cause cosmological problems (gravitino, moduli)
- introduces many potential sources of flavor and $CP$ violations in the soft SUSY breaking terms

⇒ may serve as a probe of SUSY
Flavor violation as a probe of SUSY

- Down-type scalar quark mass term:

\[- \mathcal{L}_{\text{soft}} \ni \tilde{d}^*_{Ai} (M^2_{d,AB})_{ij} \tilde{d}_{Bj}\]

\[A, B = L, R \text{ and } i, j = 1, 2, 3\]

- may lead to an interaction

\[\tilde{d}_{Bj} \times \tilde{d}_{Ai} = -i (M^2_{d,AB})_{ij}\]

if \(M^2_d\) is non-diagonal in the basis where \(m_d\) is diagonal and a \(d-\tilde{d}-\tilde{g}\) vertex preserves flavor

- For almost diagonal \(M^2_d\), use a mass insertion parameter

\[(\delta^d_{ij})_{AB} \equiv (\Delta^d_{ij})_{AB} / m^2_d\]

\[(M^2_{d,AB})_{ij} = m^2_d \delta_{ij} + (\Delta^d_{ij})_{AB}\]
Hadronic and leptonic flavor violations

- $K^0 - \overline{K^0}$ mixing

$$\propto (\delta^{d}_{12})_{BA} \times (\delta^{d}_{12})_{DC}$$

- $\mu \rightarrow e\gamma$

$$\propto (\delta^{l}_{12})_{LL}$$
SUSY GUT and flavor physics

- Superpotential of SUSY SU(5) model

\[ W_{\text{GUT}} = \left(\frac{1}{8}\right) T^T \lambda_U T H + T^T \lambda_D \overline{F} H + \nu_R^T \lambda_v \overline{F} H + \left(\frac{1}{2}\right) \nu_R^T M_R \nu_R \]

\[ T \left[10\right] = \{ Q, V_Q^\dagger \Theta_Q^\dagger U, \Theta_L E \}, \quad \overline{F} \left[5\right] = \{ D, \Theta_L^\dagger L \} \]

- Part of \( W_{\text{GUT}} \) in terms of MSSM fields, in a particular basis

\[ W_{\text{SSM}} = Q^T [V_Q^T \hat{Y}_U] \overline{U} H_u + Q^T [\hat{Y}_D] \overline{D} H_d + \overline{E}^T [\hat{Y}_D] L H_d + \nu_R^T [\hat{Y}_V] L H_u + \left(\frac{1}{2}\right) \nu_R^T M_R \nu_R \]

- Soft scalar mass terms

\[ -\mathcal{L}_{\text{soft}} \supset \tilde{Q}^\dagger m_{10}^2 \tilde{Q} + \tilde{E}^\dagger \Theta_L^\dagger m_{10}^2 \Theta_L \tilde{E} + \overline{U} \Theta_Q V_Q m_{10}^2 V_Q^\dagger \Theta_Q^\dagger \overline{U} + \tilde{D}^\dagger m_5^2 \tilde{D} + \tilde{L}^\dagger \Theta_L m_5^2 \Theta_L \tilde{L} \]

- Ignore phase factor \( \Theta_L \) not relevant to \( B(l_i \rightarrow l_j \gamma) \); then, \( \begin{align*} (\delta_{ij}^d)_{LL} &= (\delta_{ij}^l)_{RR}^*, \quad (\delta_{ij}^d)_{RR} = (\delta_{ij}^l)_{LL}^* \text{ at } M_{\text{GUT}} \end{align*} \)
Grand unification of flavor violations

Off-diagonal entries of scalar mass matrix are not renormalized very much if between $M_{\text{GUT}}$ and $M_{\text{SUSY}}$ is a desert

$$ (\delta_{ij}^d)_{RR} = (\delta_{ij}^l)^*_{LL} \quad \text{at} \quad M_{\text{GUT}} \quad \implies \quad (\delta_{ij}^d)_{RR} \approx \frac{m_{\tilde{L}}^2}{m_{\tilde{d}}^2} (\delta_{ij}^l)^*_{LL} \quad \text{at} \quad M_{\text{SUSY}} $$

Large couplings and large mixing in $Y_\nu$ may modify the relation at $M_{\text{SUSY}}$; still, a footprint of the GUT relation remains

Naturally expected correlations among flavor transitions

<table>
<thead>
<tr>
<th>Transitions of $L$</th>
<th>Transitions of $\bar{D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu \rightarrow e$</td>
<td>$s \rightarrow d$</td>
</tr>
<tr>
<td>$\tau \rightarrow e$</td>
<td>$b \rightarrow d$</td>
</tr>
<tr>
<td>$\tau \rightarrow \mu$</td>
<td>$b \rightarrow s$</td>
</tr>
</tbody>
</table>
Caveat: $Y_D \neq Y_E^T$ at $M_{\text{GUT}}$

- Fermion mass relations from $W_{\text{GUT}}$,

$$m_e = m_d, \quad m_\mu = m_s, \quad m_\tau = m_b \quad \text{at } M_{\text{GUT}},$$

are inconsistent with data

- Solutions: add non-renormalizable terms or 45 Higgs

- In the basis where $Y_D$ is diagonal,

$$Y_U = V_Q^T \hat{Y}_U U_Q^*, \quad Y_D = \hat{Y}_D, \quad Y_E = U_L^T \hat{Y}_E U_R^*, \quad Y_N = U_L^T V_L^T \hat{Y}_N$$

- GUT relations are modified to

$$\delta_{LL}^l = U_L \delta_{RR}^{d*} U_L^*, \quad \delta_{RR}^l = U_R \delta_{LL}^{d*} U_R^*$$

- Suppose $U_L = U_R = 1$ for numerical analysis (a general case is discussed later)
A natural question
A natural question

- Which constraint is stronger, hadronic or leptonic?
A natural question

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- Or, how do they interplay?

For recent analyses, see e.g.
Ciuchini, Masiero, Paradisi, Silvestrini, Vempati, Vives, hep-ph/0702144
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For recent analyses, see e.g. Cheung, Kang, Kim, Lee, hep-ph/0702050
Ciuchini, Masiero, Paradisi, Silvestrini, Vempati, Vives, hep-ph/0702144

This talk deals with cases with more than one mass insertions.
Consider $\left( \delta_{23}^d \right)_{RR} = \left( \delta_{23}^l \right)_{LL}^\ast$ at $M_{GUT}$

- Well motivated from large neutrino mixing within SUSY GUT + $\nu_R$ with O(1) neutrino Yukawa couplings
  $\leftarrow$ GUT partner of $\left( \delta_{23}^l \right)_{LL}^\ast$

  Moroi, JHEP(2000)
  Baek, Goto, Okada, Okumura, PRD(2001)

- Recently has attracted interests regarding $b \rightarrow s$ transitions such as $B \rightarrow \phi K$ and $B_s$ mixing

  With Kane, Ko, Kolda, Wang x 2
  Hisano, Shimizu
  With Ko, Masiero, PRD(2005)
  Endo, Mishima, PLB(2006)
Constraints on \((\delta_{23}^d)_{RR}\) in SUSY GUT

- Linked to \(\tau \rightarrow \mu \gamma\)

Constraints on $(\delta_{23}^d)_{RR}$ in SUSY GUT

- Linked to $\tau \rightarrow \mu \gamma$
  

- Radiatively generated $(\delta_{13}^l)_{RR}$ included,
Constraints on \((\delta^d_{23})_{RR}\) in SUSY GUT

- Linked to \(\tau \rightarrow \mu \gamma\)
  

- Radiatively generated \((\delta^l_{13})_{RR}\) included,

  also related to \(\mu \rightarrow e \gamma\)

Radiatively generated \((\delta^l_{13})_{RR}\) included, also related to \(\mu \rightarrow e \gamma\)

Enhanced by a factor \(m_{\tau}/m_{\mu}\)

Hisano, Moroi, Tobe, Yamaguchi, PRD(1996)

Baek, Goto, Okada, Okumura, PRD(2001)
Constraints on \((\delta_{23}^d)^{RR}\) in SUSY GUT

- Linked to \(\tau \rightarrow \mu \gamma\)
  

- Radiatively generated \((\delta_{13}^l)^{RR}\) included,
  
  also related to \(\mu \rightarrow e \gamma\)

Enhanced by a factor \(m_\tau/m_\mu\)

Hisano, Moroi, Tobe, Yamaguchi, PRD(1996)
Baek, Goto, Okada, Okumura, PRD(2001)
Procedure of numerical analysis

1. Run $Y_u, Y_d, Y_e$ from $M_Z$ to $M_{GUT}$
2. Go to the basis where $Y_d$ and $Y_e$ are diagonal
3. Set $Y_v, m^2_f$ at $M_{GUT}$ by hand regarding $(\delta^d_{23})_{RR}$ and $(\delta^d_{13})_{LL}$ as free parameters $A(M_{GUT}) = 0$
4. Run down to $M_R$
5. Remove $\nu_R$
6. Run down to $M_Z$
7. Calculate $\mu$ from EWSB condition
8. Compute flavor violations
Mixings and constraints

- Scan over \((\delta^d_{23})_{RR} = (\delta^l_{23})^{*}_{LL}\) and \((\delta^d_{13})_{LL} = (\delta^l_{13})^{*}_{RR}\)

- Leptonic constraints

<table>
<thead>
<tr>
<th>Process</th>
<th>Present upper bound</th>
<th>Future upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B(\mu \to e\gamma))</td>
<td>(1.2 \times 10^{-11})</td>
<td>(1 \times 10^{-13})</td>
</tr>
<tr>
<td>(B(\tau \to e\gamma))</td>
<td>(3.1 \times 10^{-7})</td>
<td>(1 \times 10^{-8})</td>
</tr>
<tr>
<td>(B(\tau \to \mu\gamma))</td>
<td>(6.8 \times 10^{-8})</td>
<td>(1 \times 10^{-8})</td>
</tr>
</tbody>
</table>

- Hadronic constraints

<table>
<thead>
<tr>
<th>Observable</th>
<th>Range</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta M_s)</td>
<td>[12.5, 23] ps(^{-1})</td>
<td>CDF central value ± 30%</td>
</tr>
<tr>
<td>(B(B \to X_s\gamma))</td>
<td>[2.0, 4.5] \times 10^{-4}</td>
<td>NLO uncertainty</td>
</tr>
<tr>
<td>(B(B \to X_d\gamma))</td>
<td>[1, 10] \times 10^{-6}</td>
<td></td>
</tr>
</tbody>
</table>

Also, \(\Delta M_d, \sin 2\beta, \cos 2\beta, \phi_s, S_{\phi K}, \varepsilon_K, \varepsilon' / \varepsilon_K, d_n\)
Constraints on \( [(\delta^d_{13})_{LL}, (\delta^d_{23})_{RR}] \) plane

- **Parameters**
  
  \[
  \mu > 0, \quad \tan \beta = 3
  \]
  
  \[
  m_0(M_{\text{GUT}}) = 210 \text{ GeV}
  \]
  
  \[
  M_2(M_Z) = 150 \text{ GeV}
  \]
  
  \[
  [m_\tilde{d}(M_Z) = M_3(M_Z) = 500 \text{ GeV}]
  \]
  
  \[
  [(\delta^d_{23})_{LL}](M_{\text{GUT}}) = 0.014
  \]
  
  No \( v_R \) below \( M_{\text{GUT}} \)

- **Effective constraints**
  
  \[
  (\delta^d_{13})_{LL} : B \to X_d \gamma
  \]
  
  \[
  (\delta^d_{23})_{RR} : \Delta M_s, \tau \to \mu \gamma
  \]
  
  \[
  (\delta^d_{13})_{LL} \times (\delta^d_{23})_{RR} : \mu \to e \gamma
  \]
Flavor violation in SUSY GUT

Constraints on \[ (\delta^d_{13})_{LL}, (\delta^d_{23})_{RR} \] plane

\( (\delta^d_{13})_{LL} \) from RGE between \( M_* \) and \( M_{GUT} \)

- Parameters
  \[ \mu > 0, \quad \tan \beta = 3 \]
  \[ m_0(M_{GUT}) = 210 \text{ GeV} \]
  \[ M_2(M_Z) = 150 \text{ GeV} \]
  \[ [m_{\tilde{d}}(M_Z) = M_3(M_Z) = 500 \text{ GeV} ] \]
  \[ (\delta^d_{23})_{LL}(M_{GUT}) = 0.014 \]
  No \( \nu_R \) below \( M_{GUT} \)

- Effective constraints
  \( (\delta^d_{13})_{LL} : B \rightarrow X_d \gamma \)
  \( (\delta^d_{23})_{RR} : \Delta M_s, \tau \rightarrow \mu \gamma \)
  \( (\delta^d_{13})_{LL} \times (\delta^d_{23})_{RR} : \mu \rightarrow e \gamma \)
Constraints on $[\delta_{13}^d]_{LL}, (\delta_{23}^d)_{RR}$ plane

- **Parameters**
  
  $\mu > 0, \tan \beta = 3$
  
  $m_0(M_{GUT}) = 210$ GeV
  
  $M_2(M_Z) = 150$ GeV
  
  $[m_{\tilde{d}}(M_Z) = M_3(M_Z) = 500$ GeV $]$
  
  $[(\delta_{23}^d)_{LL}]_{M_{GUT}} = 0.014$
  
  No $\nu_R$ below $M_{GUT}$

- **Effective constraints**
  
  $(\delta_{13}^d)_{LL} : B \rightarrow X_d \gamma$
  
  $(\delta_{23}^d)_{RR} : \Delta M_s, \tau \rightarrow \mu \gamma$

  $((\delta_{13}^d)_{LL} \times (\delta_{23}^d)_{RR}) : \mu \rightarrow e \gamma$
Constraints on \( [\text{Re}(\delta^d_{23})_{RR}, \text{Im}(\delta^d_{23})_{RR}] \) plane

- **Parameters**
  \[
  \mu > 0, \quad \tan \beta = 3
  \]
  \[
  m_0(M_{\text{GUT}}) = 210 \text{ GeV}
  \]
  \[
  M_2(M_Z) = 150 \text{ GeV}
  \]
  \[
  [(\delta^d_{13})_{LL}](M_{\text{GUT}}) = 0.0028
  \]
  \[
  [(\delta^d_{23})_{LL}](M_{\text{GUT}}) = 0.014
  \]
  No \( \nu_R \) below \( M_{\text{GUT}} \)

- **Constraints**
  - **Present**
    \[ \mu \rightarrow e \gamma \] comparable to \( \tau \rightarrow \mu \gamma \)
  - **Future**
    \[ \mu \rightarrow e \gamma \] stronger than \( \tau \rightarrow \mu \gamma \)
Flavor violation in SUSY GUT

Constraints on $[\text{Re}(\delta_{23}^d)_{RR}, \text{Im}(\delta_{23}^d)_{RR}]$ plane

- Parameters
  
  \[
  \mu > 0, \quad \tan \beta = 3 \\
  m_0(M_{\text{GUT}}) = 400 \text{ GeV} \\
  M_2(M_Z) = 150 \text{ GeV} \\
  [(\delta_{13}^d)_{LL}](M_{\text{GUT}}) = 0.0028 \\
  [(\delta_{23}^d)_{LL}](M_{\text{GUT}}) = 0.014 \\
  \text{No } \nu_R \text{ below } M_{\text{GUT}}
  \]

- Constraints
  
  LFV
  got weaker

  $\Delta M_s$
  got stronger
Same exercise on \([\text{Re}(\delta^d_{13})_{RR}, \text{Im}(\delta^d_{13})_{RR}]\) plane

- Parameters
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  \text{No } \nu_R \text{ below } M_{\text{GUT}}
  \end{align*}
  \]

- Constraints
  - Present
    \(\mu \to e\gamma\) stronger than any other constraint
  - Future
    \(\mu \to e\gamma\) even stronger
Same exercise on \([\text{Re}(\delta_{13}^d)_{RR}, \text{Im}(\delta_{13}^d)_{RR}]\) plane

- **Parameters**

  \[
  \begin{align*}
  \mu &> 0, \quad \tan \beta = 3 \\
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  \text{No } \nu_R \text{ below } M_{\text{GUT}}
  \end{align*}
  \]

- **Constraints**

  - LFV got weaker
  - hadronic got stronger

\[
\begin{align*}
\mu & \rightarrow e \gamma \\
\tau & \rightarrow e \gamma \\
\Delta M_d \\
sin 2\beta \\
cos 2\beta \\
b & \rightarrow d \gamma \\
\epsilon_K \\
\epsilon'/\epsilon_K \\
\epsilon_n
\end{align*}
\]
Effects of non-renormalizable terms on an \( RR \) mixing

- **Key equation**

\[
(\delta_{a3}^l)_{LL} = [U_L]_{ab} (\delta_{b3}^d)_{RR}^* [U_L]_{33}^* + \mathcal{O}(\cos^2 \beta \delta_{RR}^d), \quad a, b = 1, 2
\]

- A tau decay is dominated by chargino loop

\[
B(\tau \to (e + \mu)\gamma) \propto |(\delta_{13}^l)_{LL}|^2 + |(\delta_{23}^l)_{LL}|^2
\]

\[
\approx |(\delta_{13}^d)_{RR}|^2 + |(\delta_{23}^d)_{RR}|^2 + \mathcal{O}[\cos^2 \beta (\delta_{RR}^d)^2]
\]

\[\Rightarrow \text{Constraint on } \delta_{RR}^d \text{ roughly independent of } U_L \]

- \( \mu \to e\gamma \) is dominated by neutralino loop

\[
B(\mu \to e\gamma) \propto |(\delta_{13}^l)_{RR}(\delta_{32}^l)_{LL}|^2 + |(\delta_{13}^l)_{LL}(\delta_{32}^l)_{RR}|^2
\]

\[
\approx |(\delta_{13}^l)_{RR}|^2 \times |[U_L]_{21}(\delta_{13}^d)_{RR}^* + [U_L]_{22}(\delta_{23}^d)_{RR}^*|^2
\]

\[
+ |(\delta_{23}^l)_{RR}|^2 \times |[U_L]_{11}(\delta_{13}^d)_{RR}^* + [U_L]_{12}(\delta_{23}^d)_{RR}^*|^2
\]

\[
+ \mathcal{O}[\cos^2 \beta (\delta_{RR}^l)^2 (\delta_{RR}^d)^2]
\]

\[\Rightarrow \text{Constraint on } \delta_{RR}^d \text{ generically remains strong} \]
### Summary

[δ’s at $M_{\text{GUT}}$; $\tan \beta = 3$, $m_0 = 210$ GeV, $M_2 = 150$ GeV by default]

<table>
<thead>
<tr>
<th>Constraints on $(\delta_{23}^d)<em>{RR}$, with fixed $(\delta</em>{13}^d)_{LL} = 0.0028$</th>
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<td>• At present, $\mu \rightarrow e\gamma \sim \tau \rightarrow \mu\gamma$ stronger than $\Delta M_s$</td>
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<tr>
<td>• In future, $\mu \rightarrow e\gamma$ stronger than $\tau \rightarrow \mu\gamma$ stronger than $\Delta M_s$, unless hadronic uncertainty is reduced</td>
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</tbody>
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**In either case**

- For higher $\tan \beta$, LFV constraints become stronger
- For $m_0 = 400$ GeV, LFV constraints become weaker, hadronic constraints become stronger
Summary
[δ's at $M_{\text{GUT}}$; $\tan\beta = 3$, $m_0 = 210$ GeV, $M_2 = 150$ GeV by default]

Constraints on $(\delta^d_{23})_{RR}$, with fixed $(\delta^d_{13})_{LL} = 0.0028$

- At present, $\mu \rightarrow e\gamma \sim \tau \rightarrow \mu\gamma$ stronger than $\Delta M_s$
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- Wait for MEG result!
Summary

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In either case

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- **Wait for MEG result! (and SuperB)**
Constraints on $[\text{Re}(\delta^d_{13})_{LL}, \text{Im}(\delta^d_{13})_{LL}]$ plane

- Parameters

\[\mu > 0, \ \tan \beta = 3\]
\[m_0(M_{\text{GUT}}) = 210 \text{ GeV}\]
\[M_2(M_Z) = 150 \text{ GeV}\]
\[\left[\delta^d_{23}\right]_{LL}(M_{\text{GUT}}) = 0.014\]

No $\nu_R$ below $M_{\text{GUT}}$
Constraints on $[\text{Re}(\delta_{13}^d)_{LL}, \text{Im}(\delta_{13}^d)_{LL}]$ plane

- **Parameters**
  
  $\mu > 0, \quad \tan \beta = 3$

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  \]
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- Parameters

  $\mu > 0, \quad \tan \beta = 3$
  $m_0(M_{\text{GUT}}) = 400 \text{ GeV}$
  $M_2(M_Z) = 150 \text{ GeV}$
  $[(\delta_{13}^d)_{LL}](M_{\text{GUT}}) = 0.0028$
  No $\nu_R$ below $M_{\text{GUT}}$
‘Mass insertion’ parameter at $M_{\text{GUT}}$

- An off-diagonal entry of a soft scalar mass matrix divided by a diagonal entry

$$m^2_q = m_0^2 \begin{pmatrix} 1 & 0 & (\delta^d_{13})_{LL} \\ 0 & 1 & (\delta^d_{23})_{LL} \\ (\delta^d_{13})^*_{LL} & (\delta^d_{23})^*_{LL} & 1 \end{pmatrix},$$

$$m^2_d = m_0^2 \begin{pmatrix} 1 & 0 & (\delta^d_{13})_{RR} \\ 0 & 1 & (\delta^d_{23})_{RR} \\ (\delta^d_{13})^*_{RR} & (\delta^d_{23})^*_{RR} & 1 \end{pmatrix},$$
List of hadronic constraints

<table>
<thead>
<tr>
<th>Observable</th>
<th>Measured value</th>
<th>Imposed constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta M_{B_d}$</td>
<td>$0.507 \pm 0.004$ ps$^{-1}$</td>
<td>$[0.355, 0.659]$ ps$^{-1}$</td>
</tr>
<tr>
<td>$\sin 2\beta$</td>
<td>$0.678 \pm 0.025$</td>
<td>$2\sigma$</td>
</tr>
<tr>
<td>$\cos 2\beta$</td>
<td>$&gt; -0.4$</td>
<td></td>
</tr>
<tr>
<td>$B(B \to X_d\gamma)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta M_{B_s}$</td>
<td>$17.77 \pm 0.12$ ps$^{-1}$</td>
<td>$[12.5, 23]$ ps$^{-1}$</td>
</tr>
<tr>
<td>$\phi_{B_s}$</td>
<td>$-0.70^{+0.47}_{-0.39}$</td>
<td>$[-1.48, 0.24]$</td>
</tr>
<tr>
<td>$B(B \to X_s\gamma)$</td>
<td>$(355 \pm 24^{+9}_{-10} \pm 3) \times 10^{-6}$</td>
<td>$[2.0, 4.5] \times 10^{-4}$</td>
</tr>
<tr>
<td>$S_{\phi K}$</td>
<td>$0.39 \pm 0.18$</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>\epsilon_K</td>
<td>$</td>
</tr>
<tr>
<td>$\epsilon'/\epsilon_K$</td>
<td>$(1.66 \pm 0.26) \times 10^{-3}$</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>d_n</td>
<td>$</td>
</tr>
</tbody>
</table>
$B \rightarrow X_s \gamma$?

- Parameters

  \[
  \begin{align*}
  \mu & > 0 \\
  \tan \beta & = 25 \\
  m_0 & = 400 \text{ GeV} \\
  M_2 & = 150 \text{ GeV} \\
  \text{No } \nu_R \text{ below } M_{\text{GUT}} \\
  \left[ (\delta_{23}^d)_{LL} (M_{\text{GUT}}) \right] & = 0.014
  \end{align*}
  \]
Extra slides

$\tau \rightarrow e\gamma$?

- **Parameters**
  - $\mu > 0$
  - $\tan \beta = 25$
  - $m_0 = 210$ GeV
  - $M_2 = 150$ GeV
  - No $\nu_R$ below $M_{GUT}$

$$[(\delta_{23}^d)_{LL}] (M_{GUT}) = 0.014$$
Large neutrino Yukawa couplings

No $\nu_R$ below $M_{\text{GUT}}$
Large neutrino Yukawa couplings

\[ Y_\nu = Y_u \text{ at } M_{\text{GUT}} \text{ (CKM-like mixing)} \]

\[ \text{No } \nu_R \text{ below } M_{\text{GUT}} \]
Large neutrino Yukawa couplings

\[ Y_\nu = Y_u \text{ at } M_{\text{GUT}} \] (CKM-like mixing)

\[ Y_\nu = U_{\text{PMNS}}^* Y_u^{\text{diag}} \text{ at } M_{\text{GUT}} \] (large mixing)