

Lepton Flavour Violating Effects on Chargino Production at the ILC

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Abstract. We review the influence of lepton flavour violation (LFV) on the production processes $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ at the International Linear Collider (ILC) with longitudinal e^+ and e^- beam polarizations in the framework of the Minimal Supersymmetric Standard Model (MSSM). The t -channel sneutrino exchange contribution to the processes $e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-$ is modified in the case of LFV, as the sneutrino mass eigenstates have no definite flavour, and therefore more than one sneutrino can contribute. This influence can alter the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ by a factor of 2 or more when varying the LFV mixing angles, in accordance with the restrictions due to the current limits on rare lepton decays. Hence, the inclusion of LFV parameters can be important when deducing the underlying model parameters from measured observables such as $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$.

PACS. 11.30.Pb Supersymmetry – 14.80.Ly Supersymmetric partners of known particles

1 Introduction

The MSSM includes the spin-1/2 partners of the W^\pm bosons and the charged Higgs bosons H^\pm . These states mix and form the charginos $\tilde{\chi}_k^\pm$, $k = 1, 2$, as the mass eigenstates. The charginos are of particular interest, as they will presumably be among the lightest supersymmetric (SUSY) particles. Therefore the study of chargino production

$$e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-, \quad i, j = 1, 2, \quad (1)$$

will play an important role at the ILC. This process has been studied extensively in the literature, see e.g. [2, 3, 4]. Procedures have been developed [3] to determine the underlying parameters $\tan \beta$, M_2 and $|\mu|$, including the cosine of the phase of μ , $\cos \phi_\mu$, through a measurement of a set of suitable observables in the processes (1). These studies assume that individual lepton flavour is conserved, which means that only one sneutrino ($\tilde{\nu}_e$) contributes to the processes (1) via t -channel exchange.

In [5] we have dropped this assumption, and have studied the influence of LFV parameters on the production cross sections $\sigma(e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^-)$. In general, the sizes of the SUSY LFV parameters are restricted as they give rise to LFV rare lepton decays at 1-loop level, which have not been observed so far. The current experimental upper bounds on the branching ratios of LFV muon decays are $\text{BR}(\mu^- \rightarrow e^- \gamma) < 1.2 \cdot 10^{-11}$, $\text{BR}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \cdot 10^{-12}$ and for the rate of $\mu^- - e^-$ conversion the best limit so far is $R_{\mu e} < 7.0 \cdot 10^{-13}$, with $R_{\mu e} = \Gamma[\mu^- + N(Z, A) \rightarrow$

$e^- + N(Z, A)]/\Gamma[\mu^- + N(Z, A) \rightarrow \nu_\mu + N(Z - 1, A)]$ [6]. The sensitivities on LFV tau decays, are smaller but have been improved substantially during the last years, where the current limits are $\text{BR}(\tau^- \rightarrow e^- \gamma) < 1.1 \cdot 10^{-7}$, $\text{BR}(\tau^- \rightarrow \mu^- \gamma) < 6.8 \cdot 10^{-8}$, $\text{BR}(\tau^- \rightarrow e^- e^+ e^-) < 2.0 \cdot 10^{-7}$ and $\text{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 1.9 \cdot 10^{-7}$ [6].

We have demonstrated in [5] that in spite of the restrictions due to LFV rare lepton decays the production cross section $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ can change by a factor of 2 and more in the presence of LFV. This can be the case even if the present bounds on LFV rare lepton decays improve by three orders of magnitude. If LFV effects of this size occur, then the minimal sets [3] of observables may not be sufficient to determine the parameters in the chargino sector and have to be extended appropriately.

2 Sneutrino mixing

The sneutrino mass matrix in the MSSM including lepton flavour violation, in the basis $(\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$, is given by

$$M_{\tilde{\nu}, \alpha\beta}^2 = M_{L, \alpha\beta}^2 + \frac{1}{2} m_Z^2 \cos 2\beta \delta_{\alpha\beta}. \quad (2)$$

The indices $\alpha, \beta, \gamma = 1, 2, 3$ characterize the flavours e, μ, τ , respectively. M_L^2 is the hermitean soft SUSY breaking mass matrix for the left sleptons, m_Z is the mass of the Z boson and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the Higgs fields. The physical mass eigenstates are given by

$$\tilde{\nu}_i = R_{i\alpha}^{\tilde{\nu}} \tilde{\nu}'_\alpha \quad (i = 1, 2, 3), \quad (3)$$

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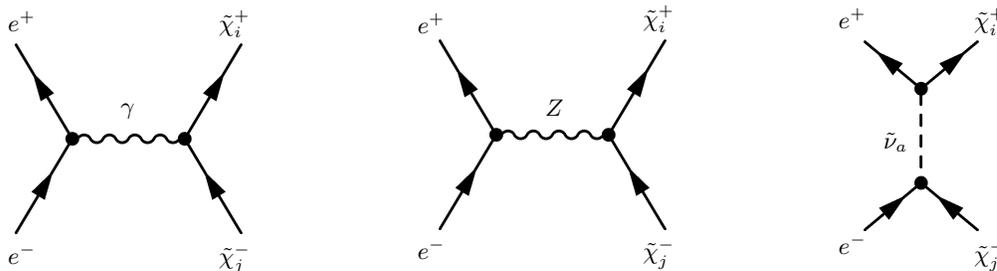


Fig. 1. Feynman diagrams for chargino production in e^+e^- -collisions.

with $\tilde{\nu}'_\alpha = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$. The mixing matrix and the physical mass eigenvalues are obtained by an unitary transformation

$$R^{\tilde{\nu}} \cdot M_{\tilde{\nu}}^2 \cdot R^{\tilde{\nu}\dagger} = \text{diag}(m_{\tilde{\nu}_1}^2, m_{\tilde{\nu}_2}^2, m_{\tilde{\nu}_3}^2), \quad (4)$$

where $m_{\tilde{\nu}_1} < m_{\tilde{\nu}_2} < m_{\tilde{\nu}_3}$. Clearly, for $M_{L,\alpha\neq\beta} \neq 0$ the mass eigenstates, Eq. (3), are not flavour eigenstates.

The Feynman diagrams contributing to the processes (1) are pictured in Fig. 1. In the case of LFV the sneutrino contribution has to be modified, as now more than one sneutrino couples to the electron and positron (unless LFV arises solely due to the parameter $M_{L,23}^2$). This can be seen from the part of the interaction Lagrangian which gives rise to the t -channel sneutrino contribution [1]:

$$\mathcal{L}_{\ell\tilde{\nu}\tilde{\chi}^\pm} = -g V_{j1}^* R_{a1}^{\tilde{\nu}*} \overline{\tilde{\chi}_j^-} P_L e \tilde{\nu}_a^\dagger - g V_{j1} R_{a1}^{\tilde{\nu}} \bar{e} P_R \tilde{\chi}_j^- \tilde{\nu}_a, \quad (5)$$

where $P_{L,R} = 1/2(1 \mp \gamma_5)$, g is the weak coupling constant and the unitary 2×2 mixing matrices U and V diagonalize the chargino mass matrix \mathcal{M}_C , $U^* \mathcal{M}_C V^{-1} = \text{diag}(m_{\chi_1}, m_{\chi_2})$.

3 Numerical analysis

In the following we analyze numerically the influence of LFV on the production cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$. The analysis is carried out for the ILC with a cms energy of $\sqrt{s} = 500$ GeV, and we assume that a degree of beam polarization of -90% for the electron beam and of 60% for the positron beam is feasible.

3.1 $\tilde{\nu}_e - \tilde{\nu}_\tau$ mixing case

We start the discussion assuming LFV through non-vanishing $M_{L,13}^2$. The size of $M_{L,13}^2$ is restricted by the experimental upper bounds on the LFV processes $\tau^- \rightarrow e^- \gamma$ and $\tau^- \rightarrow e^- e^+ e^-$ to which it contributes at loop level. The formulae for the decay widths of these reactions can be found in [7]. For a complete 1-loop calculation of the LFV leptonic three-body decays see [8]. Furthermore, we require that the MSSM parameters have to respect the experimental limits of the anomalous magnetic moments of the leptons, in

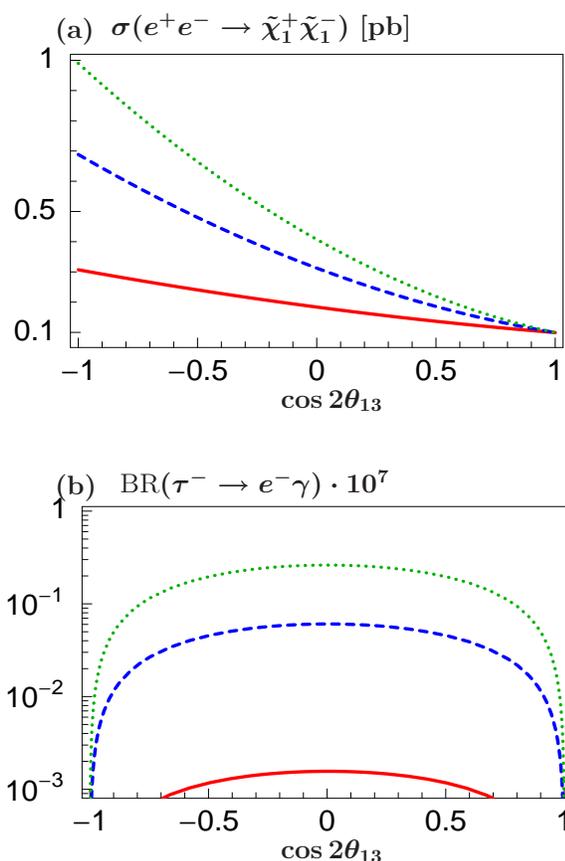


Fig. 2. (a) Cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ and (b) branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ as a function of $\cos 2\theta_{13}$. The three lines correspond to $m_{\tilde{\nu}_3} = 400$ GeV (solid line), 600 GeV (dashed line) and 900 GeV (dotted line). The other parameters are as specified in the text.

particular that one of the muon, where the difference between experiment and Standard Model (SM) prediction is $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29 \pm 9) \cdot 10^{-10}$ [9]. We impose that the SUSY contributions to a_μ must be positive and below $38 \cdot 10^{-10}$.

The MSSM parameters on which the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ depends are the parameters in the chargino sector μ , M_2 and $\tan \beta$, and the soft SUSY breaking mass parameters in the sneutrino sector $M_{L,11}$, $M_{L,22}$, $M_{L,33}$ and $M_{L,13}$ ($M_{L,12} = M_{L,23} =$

0 in this subsection). By employing the eigenvalue equations, Eq. (4), we treat the sneutrino masses $m_{\tilde{\nu}_1}$, $m_{\tilde{\nu}_2}$, $m_{\tilde{\nu}_3}$ and the LFV mixing angle $\cos 2\theta_{13}$ (with $\tan 2\theta_{13} = 2M_{L,13}^2/(M_{L,11}^2 - M_{L,33}^2)$) instead of the SUSY parameters in the sneutrino sector as our input parameters.

In addition to the MSSM parameters listed above the decay widths of the rare lepton decays depend also on other MSSM parameters, which we fix throughout this study. These are the soft SUSY breaking parameters in the charged slepton sector, which we take as $M_{E,11} = 700$ GeV, $M_{E,22} = 800$ GeV, $M_{E,33} = 900$ GeV, $M_{E,\alpha\neq\beta} = 0$, $A_{\alpha\beta} = 0$, $\alpha, \beta = 1, 2, 3$, (for the convention see e.g. [10]), and the parameter M_1 of the neutralino sector, where we assume the GUT inspired relation $|M_1| = (5/3) \tan^2 \Theta_W M_2$, with $M_1 < 0$.

In Fig. 2a we show the $\cos 2\theta_{13}$ dependence of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for three values of $m_{\tilde{\nu}_3} = (400, 600, 900)$ GeV with $m_{\tilde{\nu}_1} = 300$ GeV, $m_{\tilde{\nu}_2} = 350$ GeV, $\mu = 1500$ GeV, $M_2 = 240$ GeV and $\tan \beta = 5$. The resulting chargino masses are $m_{\chi_1} = 238$ GeV and $m_{\chi_2} = 1505$ GeV. Fig. 2b shows the appropriate dependence of the branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ for the same parameters. As can be seen in Fig. 2b, the LFV mixing angle $\cos 2\theta_{13}$ is not restricted and can have any value in the range $[-1, 1]$. $\cos 2\theta_{13} = -1, 1$ are the cases where lepton flavour is conserved, while for $\cos 2\theta_{13} = 0$ LFV is maximal, and the mass eigenstates $\tilde{\nu}_1$ and $\tilde{\nu}_3$ are mixtures containing an equal amount of $\tilde{\nu}_e$ and $\tilde{\nu}_\tau$.

Furthermore, we can see in Fig. 2 that even if the present bound on the rare decay $\tau^- \rightarrow e^- \gamma$ improves by a factor of thousand the cross section for $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ can change by a factor two when comparing the cross section for the lepton flavour conserving (LFC) case $\cos 2\theta_{13} = 1$ with the one for which LFV is maximal ($\cos 2\theta_{13} = 0$). We note that the branching ratio $\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$ is 1–2 orders of magnitude smaller than $\text{BR}(\tau^- \rightarrow e^- \gamma)$. We find that although the size of the cross section strongly depends on the choice of the beam polarizations, the relative size of the cross section with and without LFV is almost independent of it.

In Fig. 3 we plot the contours of the branching ratio $10^7 \cdot \text{BR}(\tau^- \rightarrow e^- \gamma)$ (dashed lines) and the contours of the ratio $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ (solid lines) in the μ/M_2 - $\tan \beta$ plane, where we have used the abbreviations $\sigma_{11}^{\text{LFV}} \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for maximal LFV ($\cos 2\theta_{13} = 0$) and $\sigma_{11}^{\text{LFC}} \equiv \sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for the lepton flavour conserving case ($\cos 2\theta_{13} = 1$). The other MSSM parameters are the same as in Fig. 2. In Fig. 3a we show the result for $m_{\tilde{\nu}_3} = 400$ GeV where the contours of $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ are 1.5, 1.7, 1.8, 1.85 and 1.9 for increasing μ/M_2 . In Fig. 3b we have chosen $m_{\tilde{\nu}_3} = 900$ GeV and the contours for $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ in this case are 4, 4.1, 4.2, 4.3 and 4.35 for increasing μ/M_2 . As can be seen in Fig. 3a and b there is a region in the μ/M_2 - $\tan \beta$ plane where the branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma)$ is two to three orders of magnitude below its present experimental bound and the values of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ in the LFV case can be about a factor 2 and 4 larger than in the LFC case.

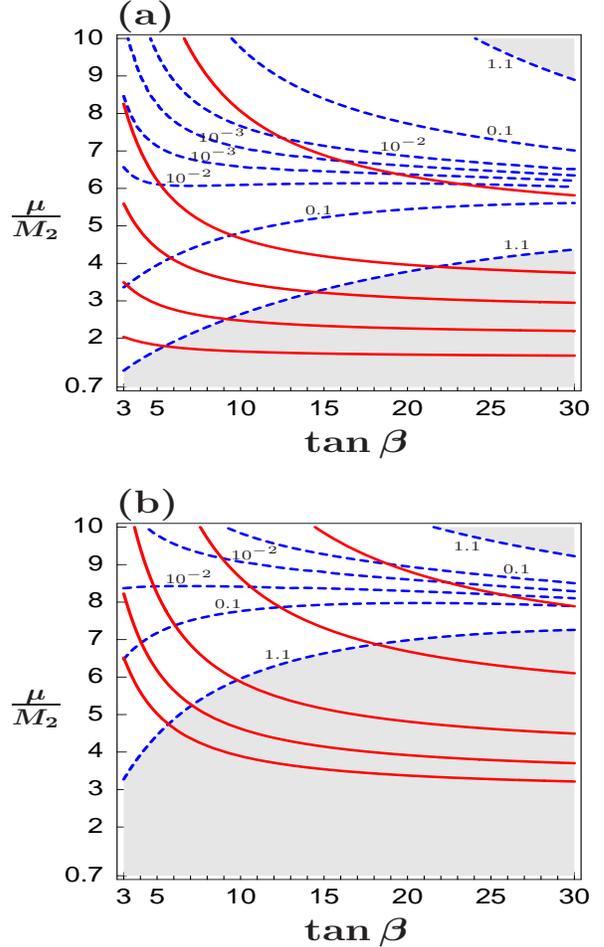


Fig. 3. Contours of $10^7 \cdot \text{BR}(\tau^- \rightarrow e^- \gamma)$ (dashed lines) and $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}}$ (solid lines) in the μ/M_2 - $\tan \beta$ plane. In (a) we have $m_{\tilde{\nu}_3} = 400$ GeV with the contours $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}} = (1.5, 1.7, 1.8, 1.85, 1.9)$ (bottom-up), and in (b) we have $m_{\tilde{\nu}_3} = 900$ GeV with the contours $\sigma_{11}^{\text{LFV}}/\sigma_{11}^{\text{LFC}} = (4, 4.1, 4.2, 4.3, 4.35)$ (bottom-up). The shaded areas in (a) and (b) mark the regions excluded by the present experimental limit $\text{BR}(\tau^- \rightarrow e^- \gamma) < 1.1 \cdot 10^{-7}$.

3.2 $\tilde{\nu}_e$ - $\tilde{\nu}_\mu$ mixing case

Now we consider the case of a non-vanishing $M_{L,12}^2$, putting $M_{L,13}^2$ and $M_{L,23}^2$ to zero. The size of $M_{L,12}^2$ is strongly restricted by the experimental upper bounds on the LFV processes $\mu^- \rightarrow e^- \gamma$ and $\mu^- \rightarrow e^- e^+ e^-$ whose sensitivities are about four orders of magnitude larger than those on LFV tau decays and will improve substantially in the near future [11]. Similarly as in the previous subsection we take as our input parameters the sneutrino masses $m_{\tilde{\nu}_1}$, $m_{\tilde{\nu}_2}$, $m_{\tilde{\nu}_3}$ and the LFV mixing angle $\cos 2\theta_{12}$ instead of the soft SUSY breaking parameters in the sneutrino mass matrix, Eq. (2).

In Fig. 4a we show the $\cos 2\theta_{12}$ dependence of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ for three values of $m_{\tilde{\nu}_2} = (305, 310, 315)$ GeV with $m_{\tilde{\nu}_1} = 300$ GeV, $m_{\tilde{\nu}_3} = 500$ GeV, $\mu = 1350$ GeV and the other parameters as defined in Fig. 2. The chargino masses

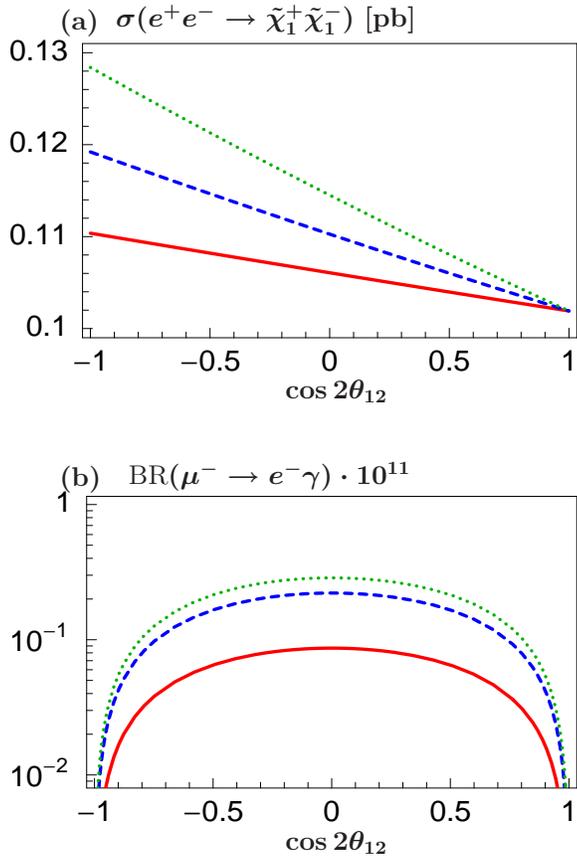


Fig. 4. (a) Cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ and (b) branching ratio $\text{BR}(\mu^- \rightarrow e^- \gamma)$ as a function of $\cos 2\theta_{12}$. The three lines correspond to $m_{\tilde{\nu}_2} = 305$ GeV (solid line), 310 GeV (dashed line) and 315 GeV (dotted line). The other parameters are as specified in the text.

are $m_{\chi_1} = 237$ GeV and $m_{\chi_2} = 1355$ GeV. Fig. 2b shows the corresponding dependence of the branching ratio $\text{BR}(\mu^- \rightarrow e^- \gamma)$ for the same parameters. The LFV mixing angle $\cos 2\theta_{12}$ is not restricted and can have any value in the whole range $[-1, 1]$, where for the values $\cos 2\theta_{12} = -1, 1$ lepton flavour is conserved. Once $\cos 2\theta_{12} \neq -1, 1$ the sneutrinos $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are mixtures of the flavour states $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$. For $\cos 2\theta_{12} = 0$ they are a mixture containing an equal amount of $\tilde{\nu}_e$ and $\tilde{\nu}_\mu$, corresponding to the case of maximal LFV. By comparing the cross sections of the LFC case with $\cos 2\theta_{12} = 1$ and the case where LFV is maximal ($\cos 2\theta_{12} = 0$), we see from Fig. 4a that the difference can be about 12%. We find that the branching ratio $\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$ is 1–2 orders of magnitude below its present bound in this scenario.

4 Conclusions

In conclusion, we have pointed out that the influence of LFV can enormously change the predicted values of the chargino production cross sections at the ILC. Studying the production cross section for the reaction $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$, we have shown that it can change by a

factor of 2 or more through non-vanishing LFV parameters which are consistent at the same time with the present limits on LFV rare lepton decays. Moreover, we have pointed out that this statement holds even in the case where the limit on $\text{BR}(\tau^- \rightarrow e^- \gamma)$ improves by a factor of thousand. In the effort of reconstructing the underlying model parameters from measurements of chargino production cross sections, one inevitably has to take the LFV parameters into account. This can be done by measurements of lepton flavour violating production and decay rates of SUSY particles at the ILC. For example, a measurement of the event rates for the reaction $e^+e^- \rightarrow \tilde{\nu} \tilde{\nu} \rightarrow \tau^+ e^- \tilde{\chi}_1^+ \tilde{\chi}_1^-$ may allow us to determine the LFV mixing angle $\cos 2\theta_{13}$ in the sneutrino sector [12, 13].

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References

1. D. J. H. Chung *et al.*, Phys. Rept. **407** (2005) 1.
2. A. Bartl, H. Fraas and W. Majerotto, Z. Phys. C **30** (1986) 441.
3. S. Y. Choi *et al.*, Eur. Phys. J. C **7** (1999) 123; Eur. Phys. J. C **8** (1999) 669; Phys. Lett. B **479** (2000) 235; Eur. Phys. J. C **14** (2000) 535.
4. G. Moortgat-Pick *et al.*, Eur. Phys. J. C **7** (1999) 113.
5. K. Hohenwarter-Sodek and T. Kernreiter, JHEP **0706** (2007) 071.
6. W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
7. J. Hisano *et al.*, Phys. Rev. D **53** (1996) 2442.
8. E. Arganda and M. J. Herrero, Phys. Rev. D **73** (2006) 055003.
9. F. Jegerlehner, arXiv:hep-ph/0703125.
10. A. Bartl *et al.*, Eur. Phys. J. C **46** (2006) 783.
11. L. M. Barkov *et al.*, a research proposal to PSI search for $\mu^+ \rightarrow e^+ \gamma$ down to 10^{-14} branching ratio (1999).
12. D. Nomura, Phys. Rev. D **64** (2001) 075001.
13. A. Bartl *et al.*, arXiv:0709.1157 [hep-ph].