Low Energy Physics and Cosmology in Nonlinear Supersymmetric General Relativity

Kazunari Shima and Motomu Tsuda
(Saitama Institute of Technology)

July. 31, 2007 @SUSY07

Abstract The basic ideas and the physical meanings of nonlinear supersymmetric(NLSUSY) general relativity(GR) in the form of vacuum Einstein-Hilbert(EH) action are discussed.

NLSUSY GR gives new insights into the origin of mass and the observed mysterious relations between the (low energy) particle physics and the cosmology.

OUTLINE

1. Introduction

SUSY and its spontaneous breakdown are profound notions related to the space-time symmetry and are essential for understanding the rationale of space-time and matter, therefore necessarily to be studied not only from the low energy particle physics but also from the cosmology.

We have found group theoretically that the NLSUSY invariant coupling of the spin $\frac{1}{2}$ fermion with graviton is crucial for reproducing the low energy standard model(SM) with the self-contained spontaneous breakdown of SUSY.

A breakthrough to

@All observed particles in the single irreducible representation of SO(10) superPoincaré !!

@Detour of NO-GO Theorem on massless high spin elementary fields !?

@SUSY Composites as eigenstates of spacetime ?

@SUSY and the spontaneous breakdown encoded in ultimate spacetime

And we are tempted to imagine that there may be a certain composite structure (far) beyond the SM and/or that they should be attributed to the geometrical structure of particular space-time which unifies the two notions:

the object and the background space-time manifold.

2. Nonlinear Supersymmetric General Relativity (NLSUSY GR)

We extend the geometrical arguments of Einstein general relativity(EGR) on Riemann spacetime to new (called SGM hereafter) spacetime just inspired by NLSUSY :

The tangent space-time of SGM space-time is specified by the SL(2,C) Grassman coordinates ψ_α of NLSUSY besides the ordinary SO(3,1) Minkowski coordinate x^a,

i.e. the local NLSUSY degrees of freedom(d.o.f) ψ_α turning subsequently to the NG fermion d.o.f. (called $superon$ hereafter) of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ and x^a are attached at every curved spacetime point.

(Note that locally homomorphic SO(3,1) and SL(2,C) are non-compact groups for spacetime d.o.f, which are analogous to SO(3) and SU(2) compact groups for gauge d.o.f. of 't Hooft-Polyakov monopole.)

We have found that the parallel arguments to the Einstein genaral relativity(EG) theory on Riemann spacetime is possible on new (SGM) spacetime.

We have obtained the following NLSUSY GR(N=1 SGM) action of the vacuum EH-type in SGM spacetime,

$$L(w) = \frac{c^4}{16\pi G}|w|(\Omega(w) - \Lambda), \tag{1}$$

$$|w| = \det w^a{}_\mu = \det(e^a{}_\mu + t^a{}_\mu(\psi)), \quad t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar\psi\gamma^a\partial_\mu\psi - \partial_\mu\bar\psi\gamma^a\psi), \tag{2}$$

where $w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi)$, $e^a{}_\mu$ for the local SO(3,1), $t^a{}_\mu(\psi)$ for the local SL(2,C) and $\Omega(w)$ are the invertible unified vierbein of SGM spacetime, the ordinary vierbein of EGR, the mimic vierbein composed of the stress-energy-momentum of superons $\psi(x)$ and the the unified scalar curvature of SGM spacetime, respectively.

$s_{\mu\nu} \equiv w^a{}_\mu \eta_{ab} w^b{}_\nu$ and $s^{\mu\nu}(x) \equiv w^\mu{}_a(x) w^{\nu a}(x)$ are unified metric tensors of SGM spacetime.

The explicit expression of the scalar curvature $\Omega(w)$ of new spacetime is obtained by just replacing $e^a{}_\mu(x)$ by $w^a{}_\mu(x)$ in Ricci scalar R of EGR.

G is the Newton gravitational constant and Λ is a ($small$) cosmological term.

Remarkably the arbitrary constatnt κ^2 of NLSUSY VA model with the dimension $(length)^4$ is now fixed to

$\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$.

Also the signature of the cosmological term is now fixed uniuely to give the correct signature to the kinetic term of $\psi(x)$,

which allows the the dark energy for the acceleration.

SYMMETRIES OF NLSUSY GR(N=1 SGM ACTION)

NLSUSY GR action (1) is invariant at least under the following spacetime symmetries which is locally isomorophic to super-Poincaré(SP):

$$[\mathrm{NLSUSY}]\otimes[\mathrm{local\ GL}(4,\mathrm{R})]\otimes[\mathrm{local\ Lorentz}]\otimes[\mathrm{local\ spinor\ translation}] \tag{3}$$

and the following internal symmetries for N-extended NLSUSY GR (with N-superons ψ^j $(j=1,2,..N)$) :

$$[\mathrm{global\ SO(N)}]\otimes[\mathrm{local\ U(1)^N}]\otimes[\mathrm{chiral}]. \tag{4}$$

For example,

NLSUSY GR action (1) is invariant under the new NLSUSY transformation;

$$\delta_\zeta \psi = \frac{1}{\kappa}\zeta - i\kappa\bar{\zeta}\gamma^\rho\psi\partial_\rho\psi, \quad \delta_\zeta e^a{}_\mu = i\kappa\bar{\zeta}\gamma^\rho\psi\partial_{[\mu}e^a{}_{\rho]}, \tag{5}$$

which induce remarkably GL(4,R) transformations on $w^a{}_\mu$ and the unified metric $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{6}$$

where ζ is a constant spinor parameter, $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$ and $\xi^\rho = -i\kappa\bar{\zeta}\gamma^\rho\psi)$.

The commutators of two new NLSUSY transformations (5) on ψ and $e^a{}_\mu$ are GL(4,R),

i.e. new NLSUSY (5) is the square-root of GL(4,R);

$$[\delta_{\zeta_1}, \delta_{\zeta_2}]\psi = \Xi^\mu \partial_\mu \psi, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a{}_\mu = \Xi^\rho \partial_\rho e^a{}_\mu + e^a{}_\rho \partial_\mu \Xi^\rho, \tag{7}$$

where $\Xi^\mu = 2i\bar{\zeta}_1 \gamma^\mu \zeta_2 - \xi_1^\rho \xi_2^\sigma e_a{}^\mu \partial_{[\rho} e^a{}_{\sigma]}$**.** $Q.E.D.$

c.f. SUGRA

$$[\delta_1, \delta_2] = \delta_P + \delta_L + \delta_g$$

And the local Lorentz transformation on $w^a{}_\mu$

$$\delta_L w^a{}_\mu = \epsilon^a{}_b w^b{}_\mu \tag{8}$$

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$

or equivalently on ψ and $e^a{}_\mu$

$$\delta_L \psi(x) = -\frac{i}{2}\epsilon_{ab}\sigma^{ab}\psi, \quad \delta_L e^a{}_\mu(x) = \epsilon^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\epsilon_{bc}). \tag{9}$$

The local Lorentz transformation forms a closed algebra, for example, on $e^a{}_\mu(x)$

$$[\delta_{L_1}, \delta_{L_2}]e^a{}_\mu = \beta^a{}_b e^b{}_\mu + \frac{\kappa^4}{4}\varepsilon^{abcd}\bar{\psi}^j\gamma_5\gamma_d\psi^j(\partial_\mu\beta_{bc}), \tag{10}$$

where $\beta_{ab} = -\beta_{ba}$ is defined by $\beta_{ab} = \epsilon_{2ac}\epsilon_1{}^c{}_b - \epsilon_{2bc}\epsilon_1{}^c{}_a$. $Q.E.D.$

Note that the NG fermion d.o.f. ψ can be transformed away neither by the local spinor translation(LST) nor by the local GL(4R).

For LST with the local spinor paremeter $\theta(x)$;
$\delta\psi = \theta$, $\delta e^a{}_\mu = -i\kappa^2\bar\theta\gamma^a\partial_\mu\psi + \bar\psi\gamma^a\partial_\mu\theta)$, $[\delta_1, \delta_2] = 0$
$w(e+\delta e, t(\psi+\delta\psi)) = w(e+t(\psi), 0) = w(e, t(\psi))$ **under** $\theta(x) = -\psi(x)$
as indicated $\delta w^a{}_\mu(x) = 0$

and

for GL(4R), once GL(4R) is performed for $L(w) \rightarrow L(e)$, then the subsequent ordinary GL(4R) on the consequent EH action $L(e)$ woul bocome pathological(restricted) and the physical graviton as well.

SGM action $L(w)$ (1) on SGM spacetime is unstable due to the global NLSUSY structure of tangent spacetime and breakes down spontaneously to Riemann spacetime(EH action) with the superon(NG fermion matter) depicted as follows :

$$L(e,\psi) = \frac{c^4}{16\pi G}|e|\{R(e) - \Lambda + \tilde{T}(e,\psi)\}, \tag{11}$$

where $R(e)$ and Λ are the scalar curvature and the cosmological term of EH action and $\tilde{T}(e,\psi)$ represents the kinetic term and the gravitational interaction of superon.

The second and the third terms produces N-extended NLSUSY VA action with $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$ in Riemann-flat($e^a{}_\mu(x) \to \delta^a{}_\mu$) spacetime.

3. Physical Meaning of Linearization of NL SUSY

For obtaining an equivalent and renormalizable field theory we should linearize the highly nonlinear SGM action (11), where NLSUSY is recasted as the broken LSUSY on the LSUSY supermultiplet-fields of the irreducible representation of SO(N) SP.

Toward SGM linearization we have shown in flat spacetime that the N=2 NLSUSY VA model (expanded in κ),

$$L_{\rm VA} = -\frac{1}{2\kappa^2}|w| = -\frac{1}{2\kappa^2}\left[1 + t^a{}_a + \frac{1}{2}(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) + \cdots\right], \qquad (12)$$

where, $|w| = \det w^a{}_b = \det(\delta^a_b + t^a{}_b)$**,**
$t^a{}_b = -i\kappa^2\bar\psi_{Lj}\gamma^a\partial_b\psi^j_L - \bar\psi_{Rj}\gamma^a\partial_b\psi^j_R)$**(j=1,2),**
which is invariant under N=2 NLSUSY transformation,
$\delta_\zeta\psi^j_L = \frac{1}{\kappa}\zeta^j_L - i\kappa\bar\zeta_{Lk}\gamma^a\psi^k_L - \bar\zeta_{Rk}\gamma^a\psi^k_R)\partial_a\psi^j_L, (j = 1, 2)$**,**

is (algebraicly) equivalent to the following action of the spontaneouly broken N=2 LSUSY defined on the supermultiplet:
$(\lambda_R^j, \phi, A_a, D^I)$,

$$L_{\rm lin} = \partial_a \phi \partial^a \phi^* - \frac{1}{4} F_{ab}^2 + i \bar{\lambda}_{Rj} \partial\!\!\!/ \lambda_R^j + \frac{1}{2} (D^I)^2 - \frac{1}{\kappa} \xi^I D^I, \qquad (13)$$

ϕ **is a complex scalar field,**
λ_R^j **are two right-handed Weyl spinor fields,**
A_a **is a U(1) gauge field with** $F_{ab} = \partial_a A_b - \partial_b A_a$,
D^I $(I = 1, 2, 3)$ **are three auxiliary real scalar fields and the** ξ^I **are arbitrary real parameters for the induced global SU(2)(SO(3)) rotation satisfying** $(\xi^I)^2 = 1$.

Note that the last two terms in (13) are the Fayet-Iliopoulos(FI) D **terms indicating spontaneous SUSY breaking with the vacuum expectation value** $< D^I >= \frac{1}{\kappa} \xi^I$.

In these arguments of the linearization, all fields of LSUSY supermultiplet can be constructed uniquly by the symmetry as the composites of superons ψ_{Lj} as follows,

$$
\begin{aligned}
v^a &= -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma^a\psi^j|w|, \\
\lambda^i &= \xi\left[\psi^i|w| - \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^i\bar{\psi}^j\psi^j(1 - i\kappa^2\bar{\psi}^k\partial\!\!\!/\psi^k)\}\right], \\
A &= \frac{1}{2}\xi\kappa\bar{\psi}^i\psi^i|w|, \\
\phi &= -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar{\psi}^i\gamma_5\psi^j|w|, \\
D &= \frac{\xi}{\kappa}|w| - \frac{1}{8}\xi\kappa^3\partial_a\partial^a(\bar{\psi}^i\psi^i\bar{\psi}^j\psi^j).
\end{aligned}
\tag{14}
$$

These relations are called SUSY invariant relations in the sence that the familiar LSUSY transformations on the component fields of the supermultiplet are reproduced in terms of the NLSUSY transformations on the superons ψ_{Lj} contained.

$\xi^1 = \xi^3 = 0$ **produces the ordinary vector supermultiplet.**
Note that the global SU(2) emerges in Riemann-flat spacetime of N=2 SGM (12).

Remarkably these supertransformations satisfy a new off-shell commutator algebra which closes on a translation

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v), \qquad (15)$$

where $\delta_P(v)$ is a translation with a parameter

$$v^a = 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}) \qquad (16)$$

Note that the commutator algebra does not induce the U(1) gauge transformation, which is different from the ordinary LSUSY.

The arguments are free case so far.
Recently we have found in two dimensional spacetime(d=2) that

N=2 NLSUSY VA action is equivalent to N=2 SUSY QED action for two scalar supermultiplets: (χ, B^i, ν, F^i; i=1,2),

where (χ, ν) for two (Majorana) fermions, B^i for doublet scalar fields and F^i for auxiliary scalar fields.

The interacting equivalent renormalizable action is the following one.

• $N=2$ **SUSYQED action in** $d=2$**,**

$$L \ = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} + L_{Vm} \quad (= L_{\rm VA} + [\text{suface terms}]), \quad (17)$$

where

$$L_{V0} = -\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar{\lambda}^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a \phi)^2 + \frac{1}{2}D^2 - \frac{\xi}{\kappa}D,$$

$$L'_{\Phi 0} = \frac{i}{2}\bar{\chi}\partial\!\!\!/ \chi + \frac{1}{2}(\partial_a B^i)^2 + \frac{i}{2}\bar{\nu}\partial\!\!\!/ \nu + \frac{1}{2}(F^i)^2,$$

$$L_e = e\left\{ i v_a \bar{\chi}\gamma^a \nu - \epsilon^{ij} v^a B^i \partial_a B^j + \frac{1}{2}A(\bar{\chi}\chi + \bar{\nu}\nu) - \phi\bar{\chi}\gamma_5\nu \right.$$

$$\left. + B^i(\bar{\lambda}^i \chi - \epsilon^{ij}\bar{\lambda}^j \nu) - \frac{1}{2}(B^i)^2 D \right\} + \frac{1}{2}e^2({v_a}^2 - A^2 - \phi^2)(B^i)^2,$$

$$L_{Vf} = f\{A\bar{\lambda}^i\lambda^i + \epsilon^{ij}\phi\bar{\lambda}^i\gamma_5\lambda^j + (A^2-\phi^2)D - \epsilon^{ab}A\phi F_{ab}\},$$

$$L_{Vm} = -\frac{1}{2} m \ (\bar{\lambda}^i \lambda^i - 2AD + \epsilon^{ab} \phi F_{ab}). \tag{18}$$

To see explicitly the local gauge invariance of the action, we define a complex (Dirac) spinor field χ_D and complex scalar fields (B^i, F^i) by

$$\chi_D = \frac{1}{\sqrt{2}}(\chi + i\nu), \quad B = \frac{1}{\sqrt{2}}(B^1 + iB^2), \quad F = \frac{1}{\sqrt{2}}(F^1 - iF^2), \quad (19)$$

and substitute them into $S'_{\Phi 0} + S_e$ in the action we obtain

$$S'_{\Phi 0} + S_e = \int d^2x \{ i\bar{\chi}_D \not{\mathcal{D}} \chi_D + |\mathcal{D}_a B|^2 + |F|^2 + e(\bar{\chi}_D \lambda B + \bar{\lambda}\chi_D B^* - D|B|^2 + \bar{\chi}_D \chi_D A + i\bar{\chi}_D \gamma_5 \chi_D \phi) - e^2(A^2 + \phi^2)|B|^2 \} + [\text{ surface term }], \quad (20)$$

with the covariant derivative $\mathcal{D}_a = \partial_a - iev_a$ and $\lambda = \frac{1}{\sqrt{2}}(\lambda^1 - i\lambda^2)$.

We can see the action is invariant under the ordinary local $U(1)$ gauge transformations,

$$
\begin{aligned}
(\chi_D, B, F) \quad &\rightarrow \quad (\chi'_D, B', F')(x) = e^{i\theta(x)}(\chi_D, B, F)(x), \\
v_a \quad &\rightarrow \quad v'_a(x) = v_a(x) + \frac{1}{e}\partial_a \theta(x).
\end{aligned}
\tag{21}
$$

The commutor algebra is also rewritten for the fields (19) as

$$
[\delta_{Q1}, \delta_{Q2}] = \delta_g(\mathcal{D}),
\tag{22}
$$

where $\delta_g(\mathcal{D})$ means a gauge covariant transformation according to $\mathcal{D} = \Xi^a \partial_a + ie\theta$.

SUSY invariant relations(relaxed) for the scalar multiplet are

$$\chi = \xi^i \left[\psi^i |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^i \bar{\psi}^j \psi^j (1 - i\kappa^2 \bar{\psi}^k \partial\!\!\!/ \psi^k) \} \right],$$

$$B^i = -\kappa \left(\frac{1}{2} \xi^i \bar{\psi}^j \psi^j - \xi^j \bar{\psi}^i \psi^j \right) |w|,$$

$$\nu = \xi^i \epsilon^{ij} \left[\psi^j |w| + \frac{i}{2}\kappa^2 \partial_a \{ \gamma^a \psi^j \bar{\psi}^k \psi^k (1 - i\kappa^2 \bar{\psi}^l \partial\!\!\!/ \psi^l) \} \right],$$

$$\tilde{F}^i = \frac{1}{\kappa} \xi^i \left\{ |w| + \frac{1}{8}\kappa^3 \partial_a \partial^a (\bar{\psi}^j \psi^j \bar{\psi}^k \psi^k) \right\} - i\kappa \xi^j \partial_a (\bar{\psi}^i \gamma^a \psi^j |w|)$$
$$-\frac{1}{4} e \kappa^2 \xi \xi^i \bar{\psi}^j \psi^j \bar{\psi}^k \psi^k. \qquad (23)$$

It is interesting that the four-fermion self-interaction term (i.e. the condensation of ψ^i) appearing in only the auxiliary field F^i is the

origin of the familiar local $U(1)$ gauge symmetry of LSUSY theory.

Is the condensation of superons the origin of the local gauge interaction?

4. Cosmology and Low Energy Particle Physics in NLSUSY GR

SGM spacetime is unstable due to NLSUSY structure at asymptotic flat spacetime and spontaneously breaks down, we call $Big\ Decay$**, to Riemann spacetime with superon(massless NG fermion) matter described by EH action with matter, which may be the birth of the present universe by the quantum effect in advance of the inflation and/or the Big Bang.**

The variation of with respect to ${e^a}_\mu$ **gives the equation of motion for** ${e^a}_\mu$ **in Riemann spacetime:**

$$R_{\mu\nu}(e) - \frac{1}{2} g_{\mu\nu} R(e) = \frac{8\pi G}{c^4} \{ \tilde{T}_{\mu\nu}(e, \psi) - g_{\mu\nu} \frac{c^4 \Lambda}{8\pi G} \}, \qquad (24)$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) matter including the gravitational interactions.

Note that $-\frac{c^4\Lambda}{8\pi G}$ can be interpreted as *the negative energy density of empty spacetime*, **i.e.** *the dark energy density* ρ_D.
(The negative sign is fixed relatively to the kinetic term.)

While, on tangent spacetime, the low energy theorem of the particle physics gives the following superon(massless NG fermion matter)-vacuum coupling

$$< \psi^j{}_\alpha(q)|J^{k\mu}{}_\beta|0 >= i\sqrt{\frac{c^4\Lambda}{8\pi G}}(\gamma^\mu)_{\alpha\beta}\delta^{jk}e^{iqx} + \cdots, \qquad (25)$$

where $J^{k\mu} = i\sqrt{\frac{c^4\Lambda}{8\pi G}}\gamma^\mu\psi^k + \cdots$ **is the conserved supercurrent of NLSUSY VA action.**

$\sqrt{\frac{c^4\Lambda}{8\pi G}}$ **is** *the coupling constant* g_{sv} *of superon with the vacuum*.

Further we have seen in the preceding section that the right hand side of (24) for N=2 is essentially N=2 NLSUSY VA action and it is equivalent to the broken LSUSY action with the vacuum expectation value of the auxiliary field(FI D-term) which gives the SUSY breaking mass

$$M_{SUSY}{}^2 \sim < D^I > \sim \sqrt{\frac{c^4\Lambda}{8\pi G}}\xi^I, \tag{26}$$

to the component fields of the (massless) LSUSY supermultiplet. We find NL SUSY GR(SGM) scenario gives interesting relations among the important quantities of the cosmology and the low energy particle physics, i.e.,

$$\rho_D \sim \frac{c^4\Lambda}{8\pi G} \sim < D >^2, \quad g_{sv} \sim \sqrt{\frac{c^4\Lambda}{8\pi G}} \sim < D > . \tag{27}$$

It may be natural to suppose that among the LSUSY supermultiplet the stable and the lightest particle retains the mass of the order of the spontaneous SUSY breaking.

And if the neutrino $\lambda(x)$ is such a particle, i.e.

$$ {m_\nu}^2 \sim \sqrt{\frac{c^4 \Lambda}{8\pi G}}, \tag{28} $$

then SGM predicts the observed value of the (dark) energy density of the universe and naturally explains the mysterious coincidence between m_ν and ρ_D^{obs}

$$ \rho_D^{obs} \sim (10^{-12} GeV)^4 \sim {m_\nu}^4. \tag{29} $$

The tiny neutrino mass is the direct evidence of SUSY (breaking), i.e., the spontaneous phase transition of SGM spacetime.

NLSUSY GR gives in general

$$\Lambda \sim {M_{SUSY}}^2 (\frac{M_{SUSY}}{M_{Planck}})^2. \tag{30}$$

The large mass scales and the compact gauge d.o.f. necessary for reproducing the realistic and interacting broken LSUSY model will appear through the linearization of $\tilde{T}_{\mu\nu}(e, \psi)$ which contains the mass scale Λ^{-1} and the higher order with ψ.

NLSUSY GR(or SGM) can be easily generalized to spacetime with extra dimensions, which allows to consider the unification in terms of the elementary fields $a\ la$ Kaluza-Klein.

To obtain the low energy particle physics in more realistic case of NLSUSY GR, we study the right hand side of (24) for N=2 in d=2 asymptotic flat spacetime which is essentially N=2 NLSUSY VA action and equivalent to N=2 SUSYQED action

$$L = L_{V0} + L'_{\Phi 0} + L_e + L_{Vf} + L_{Vm} \quad (= L_{\rm VA} + [\text{suface terms}]). \quad (31)$$

The vacuum is determined by the minimum of the potential of L

- **Potential in L,**

$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{ \frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e(B^i)^2 - mA \right\} D. \quad (32)$$

$\Longrightarrow$

$$V(A', \phi, B^i) = V(A, \phi, B^i, D)|_{\frac{\delta L}{\delta D}=0} = \frac{1}{2}D(A, \phi, B^i)^2$$

$$= \frac{1}{2} f^2 \left\{ A'^2 - \phi^2 - \frac{e}{2f}(B^i)^2 - \left(\frac{\xi}{f\kappa} + \frac{m^2}{4f^2} \right) \right\}^2 \geq 0, \quad (33)$$

where $A' = A + \frac{m}{2f}$**.**

• **The conditions for** $V = V_{\min}(A', \phi, B^i) = 0$ in (A', ϕ, B^i)**-space,**

(1) $ef > 0$**,** $\quad \frac{\xi}{f\kappa} + \frac{m^2}{4f^2} > 0$ **case,**

$$A'^2 - \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = \frac{\xi}{f\kappa} + \frac{m^2}{4f^2}. \right) \tag{34}$$

(2) $ef < 0$**,** $\quad \frac{\xi}{f\kappa} + \frac{m^2}{4f^2} > 0$ **case,**

$$A'^2 - \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = \frac{\xi}{f\kappa} + \frac{m^2}{4f^2}. \right) \tag{35}$$

(3) $ef > 0$**,** $\quad \frac{\xi}{f\kappa} + \frac{m^2}{4f^2} < 0$ **case,**

$$-A'^2 + \phi^2 + (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} - \frac{m^2}{4f^2}. \right) \tag{36}$$

(4) $ef < 0, \quad \frac{\xi}{f\kappa} + \frac{m^2}{4f^2} < 0$ **case,**

$$-A'^2 + \phi^2 - (\tilde{B}^i)^2 = k^2. \quad \left(\tilde{B}^i = \sqrt{-\frac{e}{2f}} B^i, \quad k^2 = -\frac{\xi}{f\kappa} - \frac{m^2}{4f^2}.\right)] \quad (37)$$

We will show that cases (2) and (3) give the correct kinetic terms to LSUSY supermultiplets.

• **The particle spectrum in the vacuum is specified by the following parametrization for** (A', ϕ, B^i) **in** (A', ϕ, B^i)**-space,**
For example, for case (2)we write

$$
\begin{aligned}
A' &= \pm(k+\rho)\cos\theta\cos\varphi\cosh\omega \simeq \pm(k+\rho), \\
\phi &= (k+\rho)\sinh\omega \qquad\qquad\qquad\quad \simeq (k+\rho)\omega, \\
\tilde{B}^1 &= (k+\rho)\sin\theta\cosh\omega \qquad\qquad\quad \simeq (k+\rho)\theta, \\
\tilde{B}^2 &= (k+\rho)\cos\theta\sin\varphi\cosh\omega \qquad \simeq (k+\rho)\varphi.
\end{aligned}
$$

Substituting them in to $V(A', \phi, B^i)$ **and expanding the action around the vacuum configuration we obtain for (2)-II [**$\rightarrow A \simeq -(k+\rho)$**],**

$$
\begin{aligned}
L|_{\frac{\delta L}{\delta D}=0} \simeq & \frac{1}{2}\{(\partial_a\rho)^2 - 4f^2k^2\rho^2\} \\
& +\frac{1}{2}\{(\partial_a\theta)^2 + (\partial_a\varphi)^2 - e^2k^2(\theta^2+\varphi^2)\} \\
& +\frac{1}{2}(\partial_a\omega)^2 \\
& -\frac{1}{4}(F_{ab})^2 \\
& +\frac{1}{2}(i\bar{\lambda}^i \partial\!\!\!/ \lambda^i - 2fk\bar{\lambda}^i\lambda^i) \\
& +\frac{1}{2}\{i(\bar{\chi}\partial\!\!\!/\chi + \bar{\nu}\partial\!\!\!/\nu) - ek(\bar{\chi}\chi + \bar{\nu}\nu)\} + \cdots .
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&m_\rho^2 = m_{\lambda^i}^2 = 4f^2k^2|_{m=0} = \frac{4\xi f}{\kappa}, \\
&m_\theta^2 = m_\varphi^2 = m_\chi^2 = m_\nu^2 = e^2k^2|_{m=0} = \frac{\xi e^2}{\kappa f}, \\
&(\omega, v_a) \rightarrow \text{massless.}
\end{aligned}
\tag{39}
$$

We see that another new mass scale appears, which can produce mass hierarchy by the factor $\left(\frac{e}{f}\right)^2$.

4. Summary

NLSUSY(SGM) scenario:

New unstable space-time as ultimate entity
$[x^a, \psi^N; x^\mu; L_{EH}(w) - \Lambda]$
Big Decay (phase transition)

$\Longrightarrow$

Riemann space-time and massless matter
$[x^a; x^\mu; L_{EH}(e) - \Lambda + T(\psi.e)]$
Big Bang, Inflation

$\Longrightarrow$

SM, new physics !?
(doubly charge spin 1/2 particle E^{2+}, lepton-type doublet for spin 3/2 with mass below 1Tev, .. for N=10)

Many open questions !

d=4 case, large N case(especially N=5 and N=10), $\cdots$.

Direct linearization of SGM action in curved space-time.

What is the equivalent LSUSY theory?

Systematic linearization for interacting and large N case.