Towards Reconstructing the SUSY Lagrangian with Fittino

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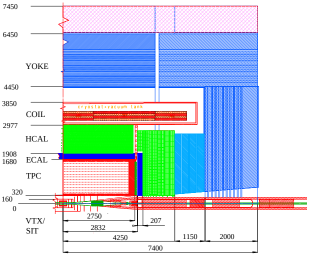
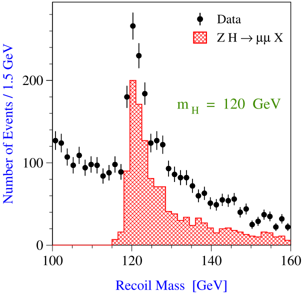
Outline

Observables and Parameters





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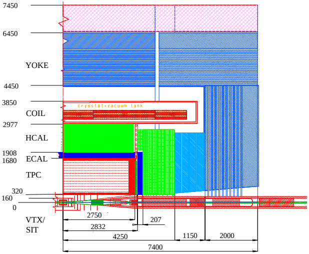
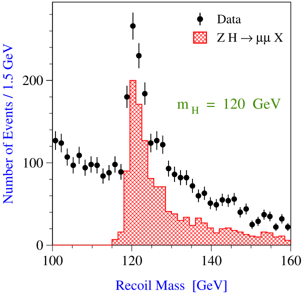
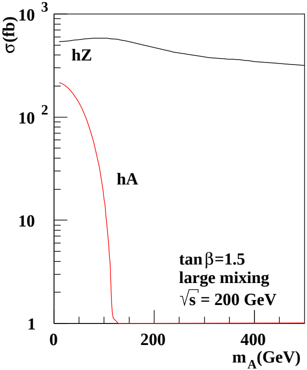
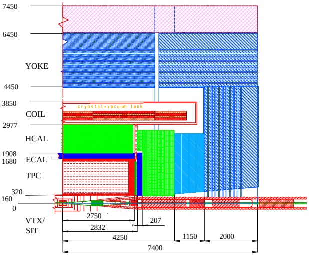


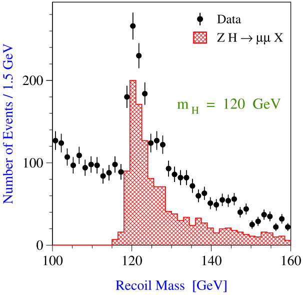


$$V_{\mathrm{Higgs}} = m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2|H_1^* H_2|^2$$



Observables and Parameters

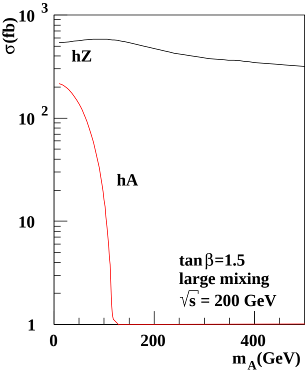




Link:

Parameter Determination

$$
\begin{aligned}
V_{\mathrm{Higgs}} \quad = \quad & m_{1H}^2|H_1|^2 + m_{2H}^2|H_2|^2 \\
& - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + h.c.) \\
& + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - \\
& |H_2|^2)^2 + \frac{1}{2}g^2|H_1^* H_2|^2
\end{aligned}
$$



Aims of Fittino

- Fit the MSSM parameters to the observables from all possible sources: LHC, ILC, Tevatron, B-factories, etc.
- Fit high-scale (mSUGRA, GMSB, AMSB) and low-scale models (MSSM,NMSSM)
- Bottom-up approach
- To be unbiased: Use no prior knowledge of the parameters at any step
- Provide easy user interface for measurements, parameter definitions and output
- Goals:
  - Show that unambiguous parameter determination without human bias is possible
  - Determine precision of parameter measurements
  - Test the necessary experimental and theoretical precision
  - Study comparisons of models: MSSM vs. NMSSM etc.
- See SFitter (next talk) for another example

Observables and other Inputs

Observables:

- Fittino can fit to any combination of the following observables:
  - Masses of SM and MSSM particles
  - Edges in mass spectra
  - Particle widths
  - Branching fractions
  - Cross-sections
  - Any product of cross-sections and/or branching fractions
  - Any ratio of the above
  - LE-Observables: $b \to s\gamma$, $(g-2)_\mu$, relic density, $\Delta m_s/\Delta m_d$, etc.
- Correlations among observables can be specified
- Limits on masses of unobserved particles can be specified

Calculations:

- Use SLHA as an interface to theory codes
- Here: Use SPheno (W. Porod) and micrOmegas (Bélanger et al.)

Very Short Example Input File

```
################################################
###          Fittino input file              ###
################################################
massh0                   115.237 GeV +- 0.05 GeV +- 0.5 GeV # comment
massTop                  178.0   GeV +- 0.3  GeV
correlationCoefficient   massh0 massTop 0.05                # quatsch
# etc
edge  1 massNeutralino1 massNeutralino2      263.5 GeV +- 1.2 GeV alias 1
sigma ( ee -> Neutralino1 Neutralino2, 1000.,0.8,0.6 ) 7.678 fb +- 2.0 fb
BR ( h0 -> Bottom Bottom~ )   0.8033 +- 0.01 +-(lumiErr) 0.05
BR ( h0 -> Charm Charm~   )   0.05   +- 0.02 +-(lumiErr) 0.01
nofit cos2PhiL                                         0.62865 +- 0.0005
# etc
fitParameter  TanBeta          10.0
fixParameter  Mu               358.6 GeV
universality  MSelectronR MSmuR
# etc
LoopCorrections       on
CalcPullDist          off
```

Technologies

- To find the χ^2 minimum
  - Tree-level estimate + MIGRAD in MINUIT:
    fast but unreliable, if not started very close to the true minimum
  - Tree-level estimate + Simulated Annealing:
    not too slow, very reliable detection of the global minimum
- To map the parameter space
  - 1D or 2D scans:
    fast, but no correct treatment of the correlations to fixed parameters
  - Markov Chain:
    n-dim. Probability map of the available parameter space
- To determine the uncertainties
  - MINOS in MINUIT:
    Slow and not very reliable
  - Automatic generation of pull fits:
    Very reliable and (at least on a farm) not slower than MINOS
- To get a feeling for the parameters and observables
  - Visualize the effect of variations of each parameter on observable χ^2's

Markov Chains

- Build a chain of parameter points $\vec{x}_i$ using the Metropolis-Hastings algorithm
- Calculate a likelihood $\mathcal{L}(\vec{x}_i)$ for each point. Here: $\mathcal{L}(\vec{x}_i) = e^{-\chi^2}$
- Randomly pick a new point $\vec{x}_{i+1}$ near $\vec{x}_i$, using a proposal distribution $Q(\vec{x}_{i+1}; \vec{x}_i)$
- Calculate $\rho = \mathcal{L}(\vec{x}_{i+1})/\mathcal{L}(\vec{x}_i)$ for symmetrical $Q(\vec{x}_{i+1}; \vec{x}_i)$
- If $\rho > 1$, accept the new point and add it to the end of the chain
- If $\rho < 1$, accept it with probability ρ, else, add $\vec{x}_i$ again to the chain
- For the efficiency of the algorithm: Optimize $Q(\vec{x}_{i+1}; \vec{x}_i)$, e.g. gaussian distribution around $\vec{x}_i$ with widthes $\vec{\sigma}_i$
- Result: Point density in the chain is proportional to the probability distribution
- Extremely effective sampling: needed number of steps scales with the number of dimensions D instead of n^D

Outline

SPS1a' MSSM Scenario Fit with ILC and LHC Observables

- Observables:
  - SM observables $m_Z, m_W, G_F, m_t, \ldots$
  - Higgs sector masses from 500 GeV and 1 TeV LC
  - All accessible sparticle and gaugino masses from LHC and LC with realistic uncertainties from hep-ph/0410364
  - LC cross sections at 400,500,1000 GeV, polarisation LR, RL, LL and RR
  - h and largest $\tilde{\mathrm{t}}_1$ BR's
- Assumptions for this test:
  - Unification in the first two generations
- Two fits:
  - Theory uncertainty only on m_{h}
  - Theory uncertainty on all masses (hep-ph/0511344) and $2\times$ larger σ uncertainties

ILC + LHC MSSM Fit Results

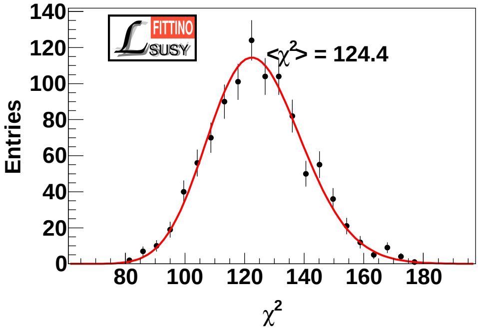
Parameter	"True" value	Fit value	Uncertainty (exp.)	Uncertainty (exp.+theor.)
$\tan\beta$	10.00	10.00	0.11	0.15
μ	400.4 GeV	400.4 GeV	1.2 GeV	1.3 GeV
X_τ	−4449. GeV	−4449. GeV	20. GeV	30. GeV
$M_{\tilde{e}_R}$	115.60 GeV	115.60 GeV	0.27 GeV	0.50 GeV
$M_{\tilde{\tau}_R}$	109.89 GeV	109.89 GeV	0.41 GeV	0.60 GeV
$M_{\tilde{e}_L}$	181.30 GeV	181.30 GeV	0.10 GeV	0.12 GeV
$M_{\tilde{\tau}_L}$	179.54 GeV	179.54 GeV	0.14 GeV	0.19 GeV
X_t	−565.7 GeV	−565.7 GeV	3.1 GeV	15.4 GeV
X_b	−4935. GeV	−4935. GeV	1284. GeV	1825. GeV
$M_{\tilde{u}_R}$	503. GeV	503. GeV	24. GeV	27. GeV
$M_{\tilde{b}_R}$	497. GeV	497. GeV	8. GeV	15. GeV
$M_{\tilde{t}_R}$	380.9 GeV	380.9 GeV	2.5 GeV	3.9 GeV
$M_{\tilde{u}_L}$	523. GeV	523. GeV	10. GeV	15. GeV
$M_{\tilde{t}_L}$	467.7 GeV	467.7 GeV	3.1 GeV	5.1 GeV
M_1	103.27 GeV	103.27 GeV	0.06 GeV	0.14 GeV
M_2	193.45 GeV	193.45 GeV	0.10 GeV	0.15 GeV
M_3	569. GeV	569. GeV	7. GeV	7. GeV
$m_{A_{run}}$	312.0 GeV	311.9 GeV	4.6 GeV	6.9 GeV
m_t	178.00 GeV	178.00 GeV	0.050 GeV	0.108 GeV

χ^2 for unsmeared observables: 5.3×10^{-5}

Don't read all numbers!

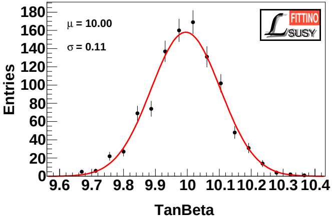
Checking the Results

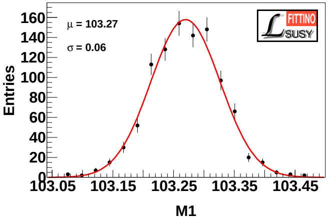
χ^2 distribution:



for n.d.f. $= 126$

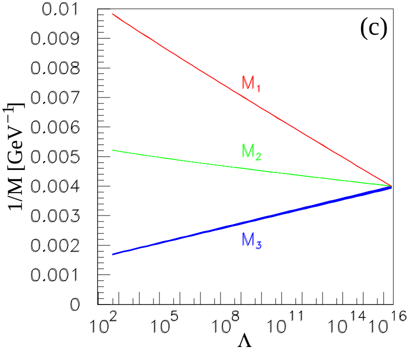
Toy Fits:

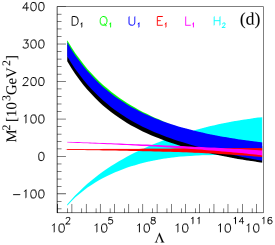


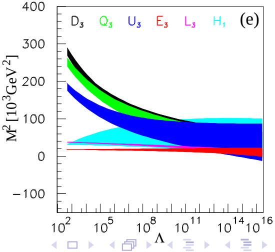


Evolution to the GUT Scale

- Based on the results of the low-energy parameter fit:







Outline

Available Measurements

- $\mathcal{B}(b \to s\gamma) = 3.26799 \times 10^{-4} \pm 4.2 \times 10^{-5}$ HFAG Summer 2006
- $\mathcal{B}(b \to s\ell\ell) = 1.89357 \times 10^{-6} \pm 1.0 \times 10^{-6}$ HFAG Summer 2006
- $\mathcal{B}(B_s \to \mu\mu) < 7 \times 10^{-8}$ HFAG Summer 2006
- $\mathcal{B}(B_u \to \tau\nu) = 1.25338 \times 10^{-4} \pm 4.8 \times 10^{-5}$ HFAG Summer 2006
- $\Delta m_d = 0.281021 \pm 0.004\,\mathrm{ps}$ HFAG Winter 2006
- $\Delta m_s = 24.2673 \pm 0.13\,\mathrm{ps}$ CDP Sep. 2006
- $(g-2)_\mu = 1.0011659208 \pm 0.0000000006$ PDG 2006
- $\Delta\rho = 8.64968 \times 10^{-3} \pm 0.0007$ PDG 2006
- $\Omega = 0.101501 \pm 0.006$ PDG 2006

Here: Central values from SPS1a'

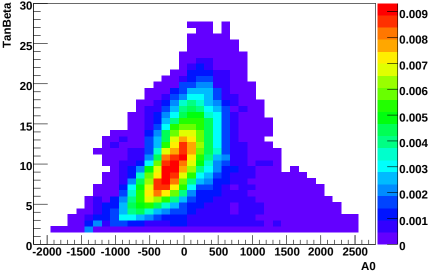
Additional 'first' LHC Results

- $m_h = 110.936 GeV \pm 0.5 GeV$
- $m_{\tilde{e}_L} = 189.442 GeV \pm 3 GeV \pm 6 GeV$
- $m_{\chi_1^+} = 183.795 GeV \pm 25 GeV$
- $m_{\tilde{g}} = 605.759 GeV \pm 4 GeV \pm 10 GeV$
- $m_{\tilde{u}_R} = 545.83 GeV \pm 3.6 GeV \pm 10 GeV$
- edge type 3 $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\tilde{e}_R}) = 83.7527 GeV \pm 0.05 GeV \pm 0.08 GeV$
- edge type 3 $(m_{\chi_1^0}, m_{\tilde{u}_L}, m_{\chi_2^0}) = 450.329 GeV \pm 2.4 GeV \pm 4.3 GeV$
- edge type 3 $(m_{\tilde{e}_R}, m_{\tilde{u}_L}, m_{\chi_2^0}) = 390.12 GeV \pm 1.5 GeV \pm 3.0 GeV$
- edge type 4 $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\tilde{e}_R}, m_{\tilde{u}_L}) = 330.597 GeV \pm 1.8 GeV \pm 3.8 GeV$
- edge type 5 $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\tilde{e}_R}, m_{\tilde{u}_L}) = 218.578 GeV \pm 2.8 GeV \pm 2.0 GeV$
- edge type 5 $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\tilde{e}_R}, m_{\tilde{b}_1}) = 195.219 GeV \pm 6.3 GeV \pm 1.8 GeV$
- edge type 3 $(m_{\chi_1^0}, m_{\chi_2^0}, m_{\tilde{\tau}_1}) = 64.5068 GeV \pm 9.0 GeV \pm 6.0 GeV$
- edge type 3 $(m_{\chi_1^0}, m_{\chi_4^0}, m_{\tilde{e}_L}) = 313.768 GeV \pm 4 GeV$
- edge type 6 $(m_t, m_{\tilde{t}_1}, m_{\chi_1^+}, m_{\tilde{g}}) = 408.112 GeV \pm 4.8 GeV$

Here: Central values from SPS1a', uncertainties for $\int \mathcal{L} \leq 100\,\mathrm{fb}^{-1}$
See hep-ph/0412012 for the definition of the mass edges

Markov Chain Probability Maps

Fit SPS1a' mSUGRA with 5 parameters (4 continous)

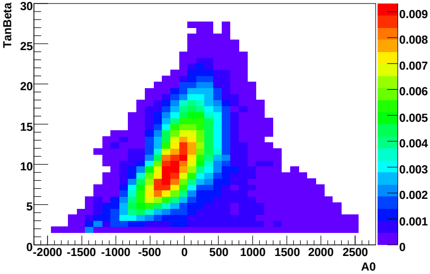


Available LE-Observables with real uncertainties

Central values taken from SPS1a'

Markov Chain Probability Maps

Fit SPS1a' mSUGRA with 5 parameters (4 continous)



Available LE-Observables with real uncertainties

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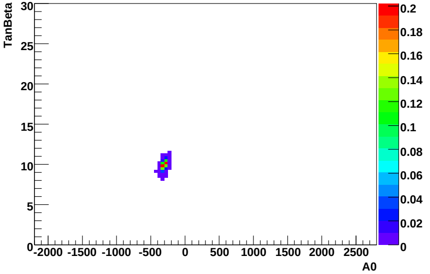
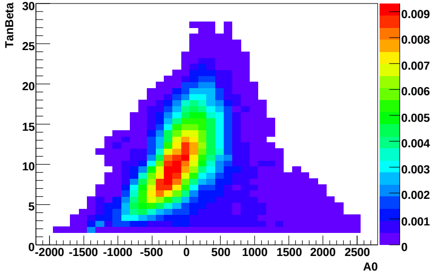


Additionally expected LHC-results from $100\,\mathrm{fb}^{-1}$

Central values taken from SPS1a'

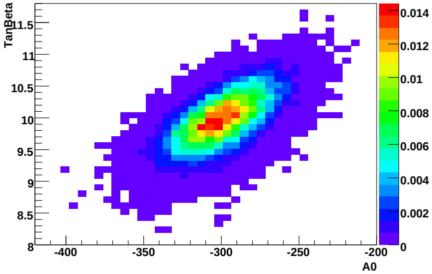
Markov Chain Probability Maps

Fit SPS1a' mSUGRA with 5 parameters (4 continous)



Available LE-Observables with real uncertainties

Central values taken from SPS1a'



Additionally expected LHC-results from $100\,\mathrm{fb}^{-1}$

Central values taken from SPS1a'

Summary and Outlook

- Fittino provides a flexible and powerful framework for parameter determination studies
- Parameter determination is possible (non-trivial!)
- Precise analysis of LE SUSY parameters requires high ILC precision
- Theoretical Uncertainties can strongly affect parameter uncertainties
- Fittino can be used to identify regions with need of theoretical or experimental improvements
- Things to come:
- Exploration of the SUSY parameter space with real LE measurements
- Explorations of the general MSSM parameter space with several simplifications with LHC and LE observables
- ILC: NMSSM vs. MSSM comparisons, but: need studies for precision of measurements in the NMSSM
- Better implementation of theory uncertainties (if provided by theory codes – important)
- More precise implementation of the LE observable calculations

More Information

- Fittino:
  http://www-flc.desy.de/fittino/
  hep-ph/0412012 Comp. Phys. Comm. 174, Issue 1, (2006), 47-70
  hep-ph/0511006 Eur.Phys.J.C46:533-544,2006
- SFitter:
  http://sfitter.web.cern.ch/SFITTER/
- SPA:
  http://spa.desy.de/spa/
- SPheno:
  http://www-theorie.physik.unizh.ch/~porod/SPheno.html
- More Results from Fittino:
  DESY-THESIS-2004-040

Backup Slides

Theory Input: The SPA Project

- Supersymmetry Parameter Analysis Project
- LHC + ILC: Comprehensive picture of SUSY particles
- Experimental accuracies of the ILC at the per-mille level
- Must be matched by equally precise
  - Theoretical predictions, relevant higher orders + uncertainties
  - Interpretation framework ($\rightarrow$ Fittino)
- SPA provides:
  - Well defined framework for the calculation of
    - masses, mixings
    - couplings
    - branching ratios, production cross-sections
    - widths
  - Framework for parameter extraction and evolution to high scale

The SPA Project

- The SPA convention specifies the framework, on which information is transferred between codes:
  - The masses of the particles are defined as pole masses
  - All parameters are given in $\overline{DR}$ scheme at $\tilde{M} = 1$ TeV
  - ...
- A program base is created, including
  - Scheme translation tools, spectrum calculators, event generators, ...
  - RGE programs
  - Analysis codes, e.g. Fittino, SFitter
  - ...
- A reference point for analyses is chosen based on SPS1a: SPS1a'

The SPA Project

- SUSY parameter measurement
  - Parameter = Observable

    m_{A}

    running or pole-parameter!
  - Parameter measurement on tree-level

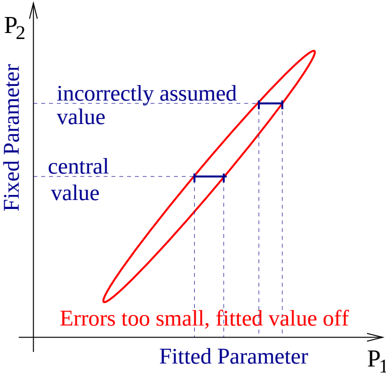
    $M_{\tilde{t}_L} = f(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \cos\theta_{\mathrm{t}})$

    Caution: loop corrections are not included!
  - Full precision and correlation

    Observable

    $O_i = f(\text{all parameters } P_i)$

- Mutual influence:



- Loop corrections are much larger than experimental uncertainties:
  maximal $m_{\mathrm{h,tree}} = m_{\mathrm{Z}} \Rightarrow$ maximal $m_{\mathrm{h,loop}} \approx 135$ GeV,
  $\Delta m_{\mathrm{hexp}} = 50\,\mathrm{MeV}$

The Parameters of the 'MSSM24'

- General parametrization of minimal SUSY and SUSY breaking: $\mathcal{L}_{\mathrm{soft}}$
- 105 free SUSY parameters
- Assume:
  - No complex phases
  - No mixing between generations (and between q and ℓ)
  - No mixing in first and second generation
- 24 additional parameters are left:
  - Higgs sector: $\tan\beta$, $m_{\mathrm{A}_{\mathrm{run}}}$
  - Gaugino sector: μ, M_1, M_2, M_3
  - Squark sector: A_q, M_{u_L}, M_{u_R}, M_{d_R}
  - Slepton sector: A_ℓ, M_{ℓ_L}, M_{ℓ_R}
- Understand theory and observables $\Leftrightarrow$ Measure parameters
- Understand SUSY breaking?