Detectable Gravitational Waves in Multi M5-Brane Inflation

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Outline

1. Multi Brane Inflation
   - General Idea
   - Multi M5-Brane Inflation

2. Gravitational Waves
   - Tensor Fraction
   - Two Puzzles and Their Resolution

3. Cascade Inflation
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MBI from DBI [A.K. & D. Lüst, in progress]

- choose $N$ 4d space-filling branes (resp. antibranes) and distribute them over the internal compact space
- provide mechanism for susy-breaking, allowing branes to interact with each other (e.g. anti-branes, fluxes, non-perturbative effects)
- kinetic terms for inflaton components from multi-brane DBI action

\[
S = \sum_{i=1}^{N(+1)} T_\rho \int d^4 x \int d^{p-3} y \sqrt{-\det P_i[G]}
\]
General Case

- simplest cases = symmetric cases: underlying symmetry of brane arrangement allows for identification (can be generalized)
  \[ \Delta X_1^q = \ldots = \Delta X_N^q \equiv \Delta X \]
  where \( q \) counts internal directions along which the MBI branes are distributed
- the latter identification means we have both a multi-inflaton and an effective single inflaton description
- split the non-dynamical CM position from the dynamical inflaton field \( \Delta X \)
  \[ X_i^q = X_{CM}^q + f_i^q \Delta X, \]
  with constant coefficients \( f_i^q \) capturing brane distribution
General Case

- working in static gauge, we have for the 4d pullback

\[ P_i[G]_{\mu\nu} = g_{\mu\nu} + \left( \sum_{q,s} f_i^q f_i^s g_{qs} \right) \partial_\mu \Delta X \partial_\nu \Delta X \]

Without loss of generality, take internal metric \( g_{qs} \) to be diagonal.

- Expand DBI action up to quadratic order in \( \partial_\mu \Delta X \) and extract inflaton’s kinetic term

\[ \mathcal{L}_{kin} = -\frac{1}{2} T_p V_\parallel \left( \sum_q \sum_{i=1}^N (f_i^q)^2 g_{qq} \right) \partial_\mu \Delta X \partial^\mu \Delta X \]
Multi Brane Inflation
Gravitational Waves
Cascade Inflation

General Idea

General Case

- read off proper normalization of inflaton $\varphi$

$$\varphi = c_N \Delta X$$

$$c_N = \left( T_p V_\parallel \sum_q \sum_{i=1}^N (f_q^i)^2 g_{qq} \right)^{1/2} \sim N^b$$

- stronger N-scaling than in 4d assisted inflation possible!

$$\epsilon_N = \frac{M_{Pl}^2}{2} \left( \frac{dU/d\varphi}{U} \right)^2 \sim \frac{1}{N^{2b}}$$

$$|\eta_N| = M_{Pl}^2 \left| \frac{d^2 U/d\varphi^2}{U} \right| \sim \frac{1}{N^{2b}}$$

- N-dependence of $U$ drops out: N-scaling of slow-roll parameters depends only on geometry of brane distribution!
Towards “Observing” Brane Distribution

We will now show how exponent $b$ (if measured) contains information about the distribution geometry of the branes.

1. Hypercubic Arrangements

Consider $d$-dimensional hypercubic lattice arrangements for $N = n^d$ branes ($q = 1, \ldots, d$). For instance, $N = n^d \text{ } Dp - D\bar{p}$ pairs with $n$ pairs along each row or column.
Multi Brane Inflation

Gravitational Waves

Cascade Inflation

General Idea

Hypercubic Geometrical Factor

- choose $X_{CM}^q = 0$; individual brane positions are given by ($n$ even)

$$X_i^q = f_i^q \Delta X, \quad f_i^q = i_q = -\frac{n}{2}, \ldots, \frac{n}{2}.$$ 

⇒ reduction to effective single field model due to discrete translational symmetry

- geometrical factor becomes (flat background)

$$\sum_q \sum_{i_q = -n/2}^{n/2} (f_i^q)^2 = d \frac{(n+2)(n+1)n}{12} \sim N^{2b}$$

- scales like $n^3 = N^3/d$, hence scaling exponent

$$b = \frac{3}{2d}$$
Significance of Linear “Throats”

- maximal scaling exponent when $d = 1$

  $$d = 1 : \quad b_{\text{max}} = \frac{3}{2},$$

  that means for linear brane arrangements. In this case we achieve maximal suppression of the slow-roll parameters

  $$d = 1 : \quad \epsilon_{N} \sim \eta_{N} \sim \frac{1}{N^{2b_{\text{max}}}} = \frac{1}{N^{3}}.$$

- One-dimensional distributions of brane/antibranes (“throats”) are thus most effective in generating a prolonged period of inflation (catalyzers for inflation)
Tadpole cancellation generically requires to add $N$ M5-branes to heterotic M-theory setup.

$N - 1$ moduli $\Delta X_i$ measure the nearest-neighbor distances between adjacent M5-branes. Identification

$$\Delta X_1 = \ldots = \Delta X_{N-1} \equiv \Delta X$$

turns out to be stable attractor solution!
M5-Brane MBI

- open membrane instantons generate repulsive potential between neighboring M5-branes and break supersymmetry

\[ \Sigma_2 \times S^1 / \mathbb{Z}_2 \]

- exponential superpotentials for \( \Delta X_i \)

\[ W_i(\Delta X_i) \sim e^{-\Delta X_i} \]
Multi M5-Brane Inflation

M5-Brane MBI

- leads to exponential potential for canonically normalized inflaton
  \[ U(\varphi) \sim (N - 1)^2 e^{-\sqrt{\frac{2}{p_N}} \frac{\varphi}{M_{Pl}}} \]

- linear brane configuration, hence \( b = b_{max} = 3/2 \) and
  \[ \varphi \sim N^{3/2} \Delta X \]

- implies parametrically most suppressed slow-roll parameters
  \[ \epsilon_N = \frac{1}{p_N} \sim \frac{1}{N^3} \]
  \[ \eta_N = \frac{2}{p_N} \sim \frac{1}{N^3} \]
M5-Brane MBI

- in fact, 4d effective FRW cosmology with this potential has exact solution power-law inflation [Lucchin & Matarrese '95]

\[ a(t) = a_0 t^{p_N} \]

with inflaton evolution

\[ \varphi(t) = \sqrt{2p_N} M_{Pl} \ln \left( \frac{t}{t_i} \right) \]

- solution is valid for \( p_N > 1/3 \) and inflation arises only if \( p_N > 1 \) which is satisfied

\[ p_N \sim N^3 \gg 1 \]
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Scalar and Tensor Perturbations

- **negative spectral tilt** \( P_s(k) = \text{curvature perturbation spectrum} \)

\[
\begin{align*}
n_s &= 1 + \left. \frac{d \ln P_s}{d \ln k} \right|_{k=aH=\text{horizon exit}} = 1 - 6\epsilon_N + 2\eta_N \\
&= 1 - \frac{2}{p_N} < 1
\end{align*}
\]

- **tensor fraction** \( P_t(k) = \text{primordial tensor perturbation spectrum} \)

\[
r = \left. \frac{P_t}{P_s} \right|_{k=aH} = 16\epsilon_N = \frac{16}{p_N}
\]

- hence **predicted relation**

\[
r = 8(1 - n_s)
\]
M5-Brane MBI Gravitational Waves

![Graph showing tensor fraction as a function of n_S.](image)
1. Large Energy Scale of Inflation

- amplitude of gravitational wave CMB anisotropy fixes energy-scale of slow-roll inflation [Lyth, ’84]
  \[ U^{1/4} \approx 3.3 \times 10^{16} r^{1/4} \text{ GeV} \]

- detectable gravitational waves with \( r > 0.01 \) imply large inflation energy-scale \( U^{1/4} > 10^{16} \) GeV

- seemingly difficult to reconcile with particle theory models

- in M5 MBI
  \[ U = (N - 1)^2 \tilde{U}(\varphi) \]

- thus true inflationary energy-scale \( \tilde{U} \) parametrically smaller
  \[ \tilde{U}^{1/4} \approx 3.3 \times 10^{16} \frac{r^{1/4}}{(N - 1)^{1/2}} \text{ GeV} \]
2. The Lyth Bound

- slow-roll inflation provides a lower bound on inflaton variation during inflation [Lyth, ’96] (Efstathiou and Mack ’05 find even tighter relation $\sim r^{1/4}$ from stochastic analysis)

$$\frac{\Delta \varphi}{M_{Pl}} \gtrsim \sqrt{2r}$$

- hence large field models with $\Delta \varphi \geq M_{Pl}$ give detectable tensor modes but effective field theory description

$$V = \left( \text{const.} + \frac{1}{2} m^2 \varphi^2 + \lambda \varphi^4 \right) + \sum_{i=3,4,\ldots} \frac{\lambda_{2i}}{M_{Pl}^{2i-4}} \varphi^{2i}$$

becomes unreliable if non-renormalizable couplings $\lambda_{2i}$ are of $O(1)$

- way out: small couplings $\lambda_{2i}$ as in chaotic inflation [Linde, ’04]
2. M5 MBI – the Lyth Bound

- relation between canonically normalized inflaton $\varphi$ and microscopically relevant modulus $\Delta X$

\[
\frac{\varphi}{M_{Pl}} = \mathcal{V}_{M2} \sqrt{2p_N} \frac{\Delta X}{L}
\]

$\mathcal{V}_{M2}$ is open M2-instanton volume, $L$ is $S^1/Z_2$ size

- Lyth bound translates into $(\Delta X_f - \Delta X_i \approx \Delta X_f)$

\[
\frac{\Delta X_f}{L} \gtrsim \frac{\sqrt{r}}{\sqrt{p_N} \mathcal{V}_{M2}}
\]

- thus Lyth bound becomes parametrically suppressed

$\sqrt{p_N} \sim N^{3/2}$

- E.g. $\Delta X_f/L \gtrsim \sqrt{r}/70$ for concrete data, which is perfectly consistent with geometric constraint $\Delta X_f \leq L$
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Since M5-branes repel each other, they will ultimately hit the boundaries.

The outermost M5-branes are absorbed by the boundaries and change their topological data (number of chiral families!)

Cascade Inflation [Ashoorioon & A.K. ’06]
Power Spectrum

- results in jumps in inflaton potential causing damped oscillations in the power spectrum

- left graph: dependence of log $P_s(k)$ on log $k$
- right graph: zoom in on the first transition.
Spectral Index

- running of spectral index

- left graph: dependence of $n_s$ on log $k$ for the first five inflationary bouts
- right graph: zoom in on the first transition.