Bulk-brane supergravity

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Branes appeared in SUGRA as solitonic solutions.

p-branes are d=p+1 dimensional surfaces in D dimensional space.

String Theory branes (D=10):

p=0,2,4,6,8 in Type IIA, p=-1,1,3,5,7,9 in Type IIB, p=1,5,9 in Type I

M-theory branes (D=11): p=2,5,9

String/M-theory on orbifolds: fixed planes as branes (?)

EFT $\Rightarrow$ back to branes in SUGRA

How worldvolume brane fields couple to bulk SUGRA fields?

Our subset of branes

Co-dimension one branes (d=D-1): hypersurfaces or boundaries

Examples: p=8 in Type IIA, p=9 in M-theory

Orbifolds: String/M-theory/SUGRA on $\mathbb{R}/Z_2$ or S^1/Z_2

Horava-Witten (HW):

- D=11 SUGRA $\oplus$ (two) d=10 branes with E_8 SYM
- M-theory on S^1/Z_2 [Horava, Witten 1996(a)]
- strongly coupled $E_8 \times E_8$ heterotic string

(SUSY) Randall-Sundrum (RS):

- D=5 (gauged) SUGRA $\oplus$ (two) d=4 branes with SUSY Standard Model
- compactification of D3-branes in Type IIB (?) or HW (?)
- LHC hopeful: extra dimensions within reach (!?)

Is EFT description at all possible?

Horava-Witten [1996(b)]:

- D=11 SUGRA coupled to d=10 E_8 SYM
- to lowest order in bulk-brane coupling constant $\kappa^{2/3}$
- have to cancel $\delta(0) \equiv \delta(z)^2$ terms to next order in $\kappa^{2/3}$
- "It is hard to believe that the classical discussion can usefully be continued to higher order…One would very likely find $\delta(0)$ terms in the supergravity transformation laws and $\delta(0)^2$ terms in the Lagrangian."

Moss [2003, 2004, 2005]

- HW in the "downstairs picture" (on a manifold with boundary)
- no products of distributions
- SUSY using boundary conditions
- SUSY to all orders in $\kappa^{2/3}$ up to variation of Green-Schwarz terms (!?)

Randall-Sundrum [1999]:

$$\int d^5x e_5 (R_5 - k^2) + \int d^4x e_4 k \quad \Rightarrow \quad ds_5^2 = e^{-2kz} ds^2_{\mathrm{Mink}_4} + dz^2$$

Altendorfer, *Bagger*, Nemeschansky [2000/1]

Gherghetta, Pomarol [2000]

Falkowski, Lalak, Pokorski [2000]

$$\int d^5x e_5 (\text{gauged 5D SUGRA}) + \int d^4x e_4 (k + \alpha \psi_m \sigma^{mn} \psi_n)$$

Bagger, DB [2002]

$$\text{brane tensions } k_{1,2} \neq k \quad \Rightarrow \quad ds_5^2 = a^2(z) ds^2_{\mathrm{AdS}_4} + dz^2$$

Various approaches to construct SUSY bulk-brane coupling:

1. Noether procedure
   - tedious; boundary conditions (b.c.) are coupling-dependent
2. Tensor calculus [D=5 to d=4]

   [Zucker 2000; Kugo, Ohashi 2002]
   - not completely off-shell (harmonic superspace: ∞ of auxiliary fields)
   - identification of (boundary) submultiplets uses "odd=0" b.c.
   - EL variation of bulk-plus-brane action $\Rightarrow$ "odd=$J \neq 0$" b.c.
   - inconsistent?... neglects backreaction!...
   - coupling to lowest order in bulk-brane coupling constant (cf. HW)
3. Superfield formulation

   [Pacceti Correia, Schmidt, Tavartkiladze 2004/5]

   [Abe, Sakamura 2004]
   - all problems of the tensor calculus remain (?)

Where does "odd=0" come from?

String Theory on orbifolds (e.g. D=10 on T^6/Z_3 gives d=4):

- untwisted sector (in the bulk) and twisted sectors (on the fixed planes)
- untwisted sector is projected onto invariant states
  $\Rightarrow$ keep "even" (invariant), remove "odd" (non-invariant)
- gives a consistent truncation of String Theory, from D to d dimensions
- cf. M-theory on S^1/Z_2 would give d=10 theory (HW is not this truncation)

Field Theory on S^1/Z_2

[Mirabelli, Peskin 1997]
[Bergshoeff, Kallosh, van Proeyen 2000]

$$\Phi_{\rm odd}(-z) = -\Phi_{\rm odd}(z) \quad \Rightarrow \quad \Phi_{\rm odd}(0) = -\Phi_{\rm odd}(0) \quad \Rightarrow \quad \Phi_{\rm odd}(0) = 0$$

- assumption: fields are continuous
- reality: fields are discontinuous because of brane sources

Goldstone fermion on the brane: [Bagger, DB 2004]

$$S = \int d^5x e_5 \mathcal{L}_5 + \int_{z \equiv x^5 = 0} d^4x e_4 \mathcal{L}_4$$

$$\begin{aligned}
\mathcal{L}_5 &= R - k^2 + i\widetilde{\Psi}_M{}^i \gamma^{MNK} D_N \Psi_{Ki} + (F_{MN})^2 + \ldots \\
\mathcal{L}_4 &= k_1 + \alpha \psi_{m1} \sigma^{mn} \psi_{n1} \\
&\quad + \overline{\chi} \overline{\sigma}^m D_m \chi + \beta \psi_{m1} \sigma^m \overline{\chi} + e_5{}^{\hat{5}} F^{m5} \chi \sigma_m \overline{\chi} + \ldots
\end{aligned}$$

Euler-Lagrange variation gives "odd=J" b.c.

$$\begin{aligned}
\omega_{ma\hat{5}} &\overset{+0}{=} k_1 e_{ma} \\
\psi_{m2} &\overset{+0}{=} \alpha \psi_{m1} + \beta \sigma_m \overline{\chi} \\
B_m &\overset{+0}{=} \chi \sigma_m \overline{\chi}
\end{aligned}$$

$$\Psi_m \sim \begin{pmatrix} \psi_{m1} \\ \psi_{m2} \end{pmatrix} \sim \begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix}$$

Bulk-plus-brane action is SUSY using these b.c. (not "odd=0").

SUSY with(out) boundary conditions

Changing brane action changes boundary conditions.

This complicates construction of SUSY bulk-plus-brane actions.

Analogy: without auxiliary fields SUSY transformations depend on $\mathcal{L}_{\text{int}}$

Conclusion/Hints/Motivation:

- a formulation with "SUSY without b.c." may exist
- it is likely to require additional fields

Look ahead:

- works in rigid susy; non-WZ auxiliary fields become important
- only some progress in local SUSY ...

Boundary picture construction: [DB 2005]

$$S = \int_{\mathcal{M}} d^5x e_5 \mathcal{L}_5 + \int_{\partial\mathcal{M}} d^4x e_4 Y + \int_{\partial\mathcal{M}} d^4x e_4 \frac{1}{2}\mathcal{L}_4$$

$$Y = K + \psi_{m1}\sigma^{mn}\psi_{n2} + e_5{}^{\hat{5}} F^{m5} B_m$$

- York term is an extension of Gibbons-Hawking term $K = e^{ma}\omega_{ma\hat{5}}$
- S is SUSY without using b.c. (except B_m b.c.) (only to 2-Fermi order . . .)

SUSY without b.c. can be achieved on orbifold as well:

- need "different sign functions" for different odd fields

$$\eta_2 \sim \epsilon(z), \quad \omega_{ma\hat{5}} \sim \frac{1}{\epsilon(z)}, \quad \psi_{m2} \sim \frac{1}{\epsilon(z)}, \quad B_m \sim \frac{1}{\epsilon(z)}$$

- fancy calculus . . .

$$\epsilon^2\delta(z) = \frac{1}{3}\delta(z), \quad \epsilon^{-2}\delta(z) = -\delta(z); \qquad \epsilon'(z) = 2\delta(z)$$

Toy model: (odd) Mirabelli-Peskin

Mirabelli, Peskin [1997]: [DB 2005,2006]

$$(A_M,\ \Phi,\ \Lambda_i,\ \vec{X}); \qquad [\delta_\xi, \delta_\eta] = \bar{\xi}\gamma^M \eta \partial_M + \delta_{U(1)}(u \sim \bar{\xi}\gamma^m \eta A_m)$$

(missing 5D auxiliary fields . . .)

$$\mathbf{V} = (0,\ 0,\ 0;\ A_m,\ \lambda_1,\ X_3 - \partial_5 \Phi), \qquad \mathbf{\Phi} = (\Phi + iA_5,\ \lambda_2,\ X_1 + iX_2)$$

(missing even 4D auxiliary fields . . .)

$$\mathbf{W} = \overline{D}^2 D\mathbf{V}, \quad \mathbf{Z} = \partial_5 \mathbf{V} - (\mathbf{\Phi} + \mathbf{\Phi}^\dagger)$$

$$S = \int_{\mathcal{M}} \mathbf{W}^2 + \mathbf{Z}^2 \quad \Rightarrow \quad Y = \Phi(X_3 - \partial_5 \Phi) + \lambda_1 \lambda_2$$

$$S' = \int_{\mathcal{M}} \mathbf{W}^2 + \mathbf{Z}^2 + \int_{\partial\mathcal{M}} \mathbf{Z}\mathbf{V} \quad \Rightarrow \quad Y' = F_{m5} A^m - \lambda_1 \lambda_2$$

$$\mathbf{V} = (C,\ \chi,\ M;\ \text{the same}) \quad \Rightarrow \quad Y' = C(\ldots) + \chi(\ldots) + M X_{12} + \text{the same}$$

SUSY without b.c.! Eliminating C and χ forces A_m b.c.!

Add "boundary current" superfield $\mathbf{J}$:

$$S = \int_{\mathcal{M}} \mathbf{W}^2 + \mathbf{Z}^2 + \int_{\partial\mathcal{M}} \mathbf{Z}\mathbf{V} - \int_{\partial\mathcal{M}} \mathbf{Z}\mathbf{J}$$

$$\delta S = \int_{\mathcal{M}} (\text{EOM}) + \int_{\partial\mathcal{M}} (\mathbf{V} - \mathbf{J})\delta\mathbf{Z} \quad \Rightarrow \quad \mathbf{V} \overset{+0}{=} \mathbf{J}$$

$$\mathbf{J} = (C_J,\ \chi_J,\ M_J;\ J_m,\ \lambda_J,\ D_J) \quad \Rightarrow \quad A_m \overset{+0}{=} J_m \quad (\text{cf. } B_m \overset{+0}{=} \chi\sigma_m\overline{\chi})$$

But . . . SUSY (algebra) requires $U(1)$ gauge invariance!
Solution: add a compensator (superfield $\mathbf{K}$)

$$\mathbf{J} = \mathbf{G} + \mathbf{K} + \mathbf{K}^\dagger, \quad \delta_u\mathbf{K} = \mathbf{\Lambda}, \quad \delta_u\mathbf{V} = \mathbf{\Lambda} + \mathbf{\Lambda}^\dagger$$

Gauge fixing C, χ and M in $\mathbf{V}$ leaves only a single scalar compensator K:

$$C_J = \chi_J = M_J = 0 \quad \Rightarrow \quad \mathbf{K} = (C_G + iK,\ \chi_G,\ M_G)$$

$$J_m = G_m + \partial_m K; \quad \delta_u K = u, \quad \delta_\eta K = \eta\chi_G + h.c.$$

Note: in the orbifold picture, $A_5 = K\delta(z) + A_5^{\text{non-sing.}}$

Example: $\mathbf{G} = \phi\phi^\dagger$ with $\phi = (\phi,\ \psi,\ F)$ gives

$$G_m = i(\phi\partial_m\phi^* - \phi^*\partial_m\phi) + \psi\sigma_m\overline{\psi}$$

$$\delta_\eta K = i\phi^*(\eta\psi) + h.c. = i(\phi^*\delta_\eta\phi - \phi\delta_\eta\phi^*)$$

Matches (surprisingly well) RS with chiral brane matter [Falkowski 2005]

$$\mathcal{F}_{m5} = F_{m5} + G_m\delta(z), \quad \delta' A_5 = (\delta_\eta K)\delta(z)$$

Indicates that a compensator is present in HW as well [HW 1996(b)]

$$\delta' C_{11BC} = \kappa^{2/3}\mathrm{tr}(A_B\delta A_C - A_C\delta A_B)\delta(x^{11})$$

The compensator is unavoidable in boundary picture;
it arises as a singular part of an even field in orbifold picture.

Summary

Co-dimension one branes can be better understood in boundary picture.

A generalization of Gibbons-Hawking term arises (Y-term).

Bulk-brane coupling forces "odd=$J \neq 0$" boundary conditions.

SUGRA tensor calculus approach uses "odd=0" boundary conditions,
not fully consistent (brane backreaction is neglected).

There is more to the story of HW and RS!…

SUSY without b.c. would be helpful.

In rigid SUSY, this is easy to do in superfields (keep C,χ,M !).

In SUGRA?… [DB, van Nieuwenhuizen – 2007(soon)]