

Bulk-brane supergravity

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Abstract. We point out a limitation of the existing supergravity tensor calculus on the S^1/\mathbb{Z}_2 orbifold that prevents its use for constructing general supersymmetric bulk-plus-brane actions. We report on the progress achieved in removing this limitation via the development of “supersymmetry without boundary conditions.”

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1 Introduction

Supergravity serves as a bridge between a more fundamental string/M-theory on the (very) high-energy side and a variety of “beyond the Standard Model” extensions on the low-energy side. If it is the only bridge, then any effective low-energy theory of relevance to the real world should be possible to fit into the supergravity context.

Branes and orbifolds have proven to be important (useful) in string/M-theory, and they also found very interesting realizations at low energy. Among the latter we note the Randall-Sundrum scenario and orbifold GUT models. These models have led to exciting research in the past decade and are expected to be tested at the LHC. The question of fitting these models into supergravity has also been addressed, but only partial success has been achieved. The main purpose of this talk is to point out some of the difficulties and indicate a possible resolution of the related problems.

2 Orbifold brane action

We will concentrate on a particular setting of one-dimensional orbifolds: S^1/\mathbb{Z}_2 or \mathbb{R}/\mathbb{Z}_2 , leading to co-dimension one fixed planes (that we, perhaps loosely, will call “branes”). As our main interest is in *local* properties (local supersymmetry and boundary conditions), for most of the discussion we need to consider only one brane. The well-known constructions of Horava-Witten [1] and Randall-Sundrum [2] belong to this class. In both cases, when one looks at the supergravity realization of these constructions, there is a D-dimensional bulk supergravity and some (D-1)-dimensional brane-localized matter.

The goal is to construct a bulk-plus-brane action that is (locally) supersymmetric under a *half* of bulk supersymmetry (the other half being spontaneously

broken by the presence of the brane). In fact, as the bulk action is already known (it is one of the standard D-dimensional supergravity actions), all we need to find is the brane action.

The brane action, in general, should include interaction between two types of fields,

- induced fields (bulk supergravity fields evaluated at the location of the brane), and
- brane-localized fields (living *only* on the brane).

These fields should combine into multiplets (representations) of the *induced supersymmetry algebra*. As the latter may, a priori, be *different* from the standard (D-1)-dimensional supersymmetry algebra (since the brane is *embedded in the bulk* and does not represent a closed system), the standard methods of constructing supersymmetric (D-1)-dimensional actions may not be applicable to this problem.

To complicate the matters even more, the brane action, in general, is *not even separately supersymmetric*, because the supersymmetry variation of the bulk supergravity action may produce a brane-localized contribution which must then be canceled by the variation of the brane action.

3 Upstairs and downstairs pictures

Let us consider the \mathbb{R}/\mathbb{Z}_2 orbifold and choose the D-dimensional coordinates (x, z) so that the brane (fixed plane) is at $z = 0$. Orbifolding makes fields on one side of the brane be mirror images of the fields on the other side. More precisely, bulk fields $\Phi(x, z)$ get subdivided into two classes of “even” and “odd” fields,

$$\begin{aligned}\Phi_{\text{even}}(x, -z) &= +\Phi_{\text{even}}(x, z) \\ \Phi_{\text{odd}}(x, -z) &= -\Phi_{\text{odd}}(x, z).\end{aligned}\tag{1}$$

Therefore, the dynamics of such a system can be completely specified by writing a bulk-plus-*boundary* action for the fundamental domain $z \in [0, +\infty)$ which

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is, geometrically, a manifold \mathcal{M} with boundary $\partial\mathcal{M}$. This approach is called “downstairs picture” [1].

Alternatively, one can keep working on the total space, $z \in (-\infty, +\infty)$ (which is a manifold *without* boundary), with an additional requirement of symmetry under the \mathbb{Z}_2 reflection. This approach is called “upstairs picture” [1].

The two approaches are physically equivalent, but very different technically. In the upstairs picture, one has to deal with the fact that odd fields are, in general, *discontinuous*:

$$\Phi_{\text{odd}}(x, -0) \neq \Phi_{\text{odd}}(x, +0). \quad (2)$$

As we will see, *on-shell*, the discontinuities (or “jumps”) of the odd fields are related to brane-localized sources, so that

$$\Phi_{\text{odd}}(x, +0) = \text{brane sources}. \quad (3)$$

The brane-orthogonal derivative ∂_z acting on the discontinuous fields produces a brane-localized delta function, $\delta(z)$. With supergravity being a highly non-linear theory, one then finds various products of distributions, such as

$$\delta(z)^2, \quad \epsilon^2(z)\delta(z), \quad \text{etc.} \quad (4)$$

in the supersymmetry transformation laws and the supersymmetry variation of the bulk-plus-brane action. Here $\epsilon(z)$ is a “sign function,”

$$\epsilon(z) = \begin{cases} +1, & z > 0 \\ -1, & z < 0 \end{cases} \quad (5)$$

that arises as a *profile function* for odd fields:

$$\Phi_{\text{odd}}(x, z) = \epsilon(z)\Phi_{\text{odd}}(x, |z|). \quad (6)$$

The products of distributions are, in general, not well-defined. One way to make sense of them is to impose certain relations between the distributions involved. For example, demanding $\partial_z\epsilon(z) = 2\delta(z)$ gives

$$\epsilon(z)^2\delta(z) = \frac{1}{3}\delta(z). \quad (7)$$

These kind of relations are indeed important for constructing supersymmetric bulk-plus-brane actions in the upstairs picture [3, 4, 5].

On the other hand, none of this fancy mathematics is needed in the downstairs picture, because on a manifold \mathcal{M} with boundary $\partial\mathcal{M}$ all fields are *continuous*.

In the downstairs picture, it is still instructive to use the subdivision of bulk fields into even and odd ones. They all have now well-defined boundary-induced values,

$$\Phi_{\text{even}}(x, 0), \quad \Phi_{\text{odd}}(x, +0), \quad (8)$$

so that there is no conceptual difficulty in putting them all in the boundary action. The equivalence with the upstairs picture indicates only that it should be possible to find a boundary action that gives the same boundary condition (3) via the variational principle, so that, *on-shell*, $\Phi_{\text{odd}}(x, +0)$ are fixed in terms of other fields, whereas $\Phi_{\text{even}}(x, 0)$ are independent.

4 Natural boundary conditions

In the downstairs picture, the bulk-plus-boundary action has the following general form,

$$\int_{\mathcal{M}} \mathcal{L}_{\text{bulk}}(\Phi) + \int_{\partial\mathcal{M}} [Y(\Phi) + \mathcal{L}_{\text{brane}}(\Phi, \phi)], \quad (9)$$

where Φ and ϕ denote the bulk and brane-localized fields, respectively. The general (Euler-Lagrange) variation of the action gives

$$\int_{\mathcal{M}} (\text{EOM})\delta\Phi + \int_{\partial\mathcal{M}} (\text{BC})\delta\Phi + \int_{\partial\mathcal{M}} (\text{eom})\delta\phi. \quad (10)$$

Requiring this variation to vanish for *arbitrary* $\delta\Phi$ and $\delta\phi$, gives bulk and boundary equations of motion as well as “natural” boundary conditions [6]. For this derivation of boundary conditions to make sense, the $Y(\Phi)$ term has to be chosen appropriately [7, 8]. Its role is to bring the boundary variation of the bulk action to the “ $p\delta q$ ” form (removing possible “ $q\delta p$ ” terms). For example, the York-Gibbons-Hawking prescription,

$$\mathcal{L}_{\text{bulk}}(\Phi) = R \quad \Rightarrow \quad Y(\Phi) = K, \quad (11)$$

brings the boundary variation to the form

$$\int_{\partial\mathcal{M}} (K_{mn} - Kg_{mn})\delta g^{mn}, \quad (12)$$

where K_{mn} is the extrinsic curvature and K is its trace. The brane-localized matter adds to this variation a term $-T_{mn}\delta g^{mn}$, with T_{mn} being the energy-momentum tensor, and therefore leads to the following natural boundary conditions,

$$K_{mn} - Kg_{mn} \stackrel{\partial\mathcal{M}}{=} T_{mn}, \quad (13)$$

which is the downstairs picture version of the Israel matching conditions. Here g_{mn} is the induced (D-1)-dimensional metric obtained from the bulk D-dimensional metric g_{MN} . In supergravity, there are more fields than just the metric. Accordingly, the $Y(\Phi)$ term has to be extended and boundary conditions for other fields have to be understood.

Note that this derivation of boundary conditions puts them on the same footing as the equations of motion. On the other hand, supersymmetry variation of a supersymmetric action must vanish *identically*, without using equations of motion. Putting these two facts together we are led to conjecture that, if the bulk-plus-boundary supersymmetry makes sense, it should be possible to construct bulk-plus-boundary actions that are supersymmetric *without* using boundary conditions. Achieving this is what we refer to as the “*supersymmetry without boundary conditions*” program.

5 Induced supergravity multiplet

The field content of the supergravity multiplet depends on the space-time dimension D. However, the vielbein

e_M^A and the gravitino ψ_M are always present. So, we write the D-dimensional supergravity multiplet as

$$(e_M^A, \psi_M, \dots). \quad (14)$$

The supersymmetry transformations read

$$\begin{aligned} \delta e_M^A &= \bar{\epsilon} \gamma^A \psi_M \\ \delta \psi_M &= \partial_M \epsilon + \hat{\omega}_{MAB} \gamma^{AB} \epsilon + \dots, \end{aligned} \quad (15)$$

where $\hat{\omega}_{MAB}$ is the supercovariant spin connection,

$$\begin{aligned} \hat{\omega}_{MAB} &= \omega(e)_{MAB} + \kappa_{MAB} \\ \kappa_{MAB} &= \bar{\psi}_M \gamma_A \psi_B - \bar{\psi}_M \gamma_B \psi_A + \bar{\psi}_A \gamma_M \psi_B \end{aligned} \quad (16)$$

(all numerical coefficients are omitted). Splitting the D-dimensional indices into the (D-1)-dimensional ones as $M = (m, z)$, $A = (a, \hat{z})$, we can identify even and odd fields as follows,

$$\begin{aligned} \text{even: } & e_m^a, e_z^{\hat{z}}, \omega_{mab}, \omega_{za\hat{z}}, \psi_{m+}, \psi_{z-}, \epsilon_+ \\ \text{odd: } & e_m^{\hat{z}}, e_z^a, \omega_{ma\hat{z}}, \omega_{zab}, \psi_{m-}, \psi_{z+}, \epsilon_- \end{aligned} \quad (17)$$

where $\psi_{\pm} = \frac{1}{2}(1 \pm \gamma^{\hat{z}})\psi$. For the following, we will impose a gauge

$$e_m^{\hat{z}} = 0 \quad (18)$$

(using the $\lambda^{a\hat{z}}$ part of the D-dimensional Lorentz transformation) that is very convenient [8] in the bulk-plus-boundary setting. Then, in particular, the extrinsic curvature is related to the spin connection as

$$K_{mn} = e_n^a \omega(e)_{ma\hat{z}}; \quad (19)$$

e_m^a is the induced vielbein, and $\omega(e)_{mab}$ is the corresponding torsion-free connection.

Assuming that the unbroken half of supersymmetry is described by ϵ_+ , the variation

$$\delta e_m^a = \bar{\epsilon}_+ \gamma^a \psi_{m+} \quad (20)$$

tells us that

$$(e_m^a, \psi_{m+}, \dots) \quad (21)$$

should be the induced supergravity multiplet.

6 The key point

However, for (21) to be the standard (D-1)-dimensional supergravity multiplet, the variation of ψ_{m+} should have the standard form,

$$\delta \psi_{m+} = \partial_m \epsilon_+ + \hat{\omega}_{mab}^+ \gamma^{ab} \epsilon_+ + \dots, \quad (22)$$

where $\hat{\omega}_{mab}^+ = \omega(e)_{mab} + \kappa_{mab}^+$ with

$$\kappa_{mab}^+ = \bar{\psi}_{m+} \gamma_a \psi_{b+} - \bar{\psi}_{m+} \gamma_b \psi_{a+} + \bar{\psi}_{a+} \gamma_m \psi_{b+}. \quad (23)$$

At the same time, $\hat{\omega}_{mab} = \hat{\omega}_{mab}^+ + \kappa_{mab}^-$ with

$$\kappa_{mab}^- = \bar{\psi}_{m-} \gamma_a \psi_{b-} - \bar{\psi}_{m-} \gamma_b \psi_{a-} + \bar{\psi}_{a-} \gamma_m \psi_{b-}, \quad (24)$$

so that (15) gives

$$\delta \psi_{m+} = (\text{standard}) + \kappa_{mab}^- \gamma^{ab} \epsilon_+ + \dots \quad (25)$$

Unless the κ_{mab}^- term is removed, (21) is *not* the correct (D-1)-dimensional supergravity multiplet.

This problem was resolved in Refs. [9,10] simply by imposing the following boundary condition,

$$\psi_{m-} \stackrel{\partial \mathcal{M}}{=} 0, \quad (26)$$

which comes naturally with the commonly accepted ideology that ‘‘odd fields vanish’’ at the fixed point. However, as we will see shortly, this approach makes it impossible to construct consistent coupling of bulk supergravity to brane-localized matter.

7 Do odd fields vanish?

The standard way to argue that odd fields vanish at the fixed point [11,12] is to use both the parity condition (1) that implies

$$\Phi_{\text{odd}}(x, -0) = -\Phi_{\text{odd}}(x, +0), \quad (27)$$

and *the assumption of continuity of fields* that gives

$$\Phi_{\text{odd}}(x, -0) = +\Phi_{\text{odd}}(x, +0), \quad (28)$$

from which $\Phi_{\text{odd}}(x, +0) = 0$ does follow. However, this argument becomes invalid in the presence of brane-localized sources, which, in the upstairs picture, require odd fields to be *discontinuous*. Consistency with equations of motion leads to (on-shell) boundary conditions given in Eq. (3).

8 Boundary conditions in supergravity

Boundary conditions must follow from (or, at least, be consistent with) the variational principle. With the standard kinetic terms for e_M^A and ψ_M being

$$\mathcal{L}_{\text{bulk}}(\Phi) = R + \bar{\psi}_M \gamma^{MNK} \partial_N \psi_K + \dots, \quad (29)$$

obtaining boundary conditions from the variational principle requires the following Y-term [7,8],

$$Y(\Phi) = K + \bar{\psi}_{m+} \gamma^{mn} \psi_{n-} + \dots \quad (30)$$

(note that it has *odd* parity). This puts the boundary part of the variation into the ‘‘ $p\delta q$ ’’ form,

$$\int_{\partial \mathcal{M}} (K_{ma} - K e_{ma}) \delta e^{ma} + \delta \bar{\psi}_{m+} \gamma^{mn} \psi_{n-}. \quad (31)$$

Brane-localized matter couples to bulk supergravity via the induced supergravity multiplet (21). Therefore, the variation of $\mathcal{L}_{\text{brane}}(\Phi, \phi)$ gives

$$- \int_{\partial \mathcal{M}} T_{ma} \delta e^{ma} + \delta \bar{\psi}_{m+} J^m, \quad (32)$$

where T_{ma} and J^m are the brane-localized energy-momentum tensor and the supercurrent, respectively. This gives the following boundary conditions,

$$K_{ma} - Ke_{ma} \stackrel{\partial\mathcal{M}}{=} T_{ma}, \quad \gamma^{mn}\psi_{n-} \stackrel{\partial\mathcal{M}}{=} J^m, \quad (33)$$

which is Eq. (3) for the case at hand. This makes it obvious that the boundary condition (26) is allowed only when $J^m = 0$. (A more strict application of the “odd fields vanish” rule would require $T_{ma} = 0$ as well, which would “kill” even the bosonic Randall-Sundrum scenario.)

We conclude that the orbifold supergravity tensor calculus of Refs. [9,10] does not allow (consistent) construction of supersymmetric bulk-plus-brane actions, because the actions it leads to are supersymmetric using the “odd=0” boundary conditions which are incompatible with the “odd=sources” boundary conditions following from the variational principle applied to these actions.

9 Supersymmetry with(out) boundary conditions

One can try to construct bulk-plus-brane actions that are supersymmetric using the (natural) boundary conditions (33). This approach was used in Refs. [5,13] to supersymmetrize the Randall-Sundrum scenario *with detuned brane tensions*. However, this road becomes very steep very soon. The difficulty lies in the fact that as one changes the brane action to achieve supersymmetry of the bulk-plus-brane system, the boundary conditions (33) change as well.

This is very similar to the necessity of adjusting supersymmetry transformation laws when attempting to couple supergravity to matter in the absence of auxiliary fields. Accordingly, one can speculate that the proper procedure for constructing supersymmetric bulk-plus-brane actions may include new kind of auxiliary fields, not present in the standard (Wess-Zumino gauge-fixed) supergravity. For example, the appearance of “boundary compensators” discussed in Ref. [14] is expected.

The program of constructing bulk-plus-brane actions that are (*locally*) *supersymmetric without the use of any boundary conditions* was started in Ref. [8]. There it was shown that the action for the detuned supersymmetric Randall-Sundrum scenario of Ref. [5] can be written in an alternative form so that one does not need to use boundary conditions (33) to prove supersymmetry of the action. However, this statement was proven only to two-fermi order, whereas an analysis of the supersymmetry algebra appeared to indicate that the use of (at least) the gravitino boundary condition would be required in the next fermi order.

Recent progress in this direction [15] indicates that the program of “supersymmetry without boundary conditions” should be realizable to the full extent. In a simpler setting of 3D supergravity, we resolved all the problems indicated above. We found that

- supersymmetry algebra does not impose any boundary conditions on fields;
- it is possible to identify co-dimension one submultiplets, such as the induced supergravity multiplet (21), without imposing any boundary conditions on fields.

Extending this analysis to the 5D case would improve the orbifold supergravity tensor calculus of Refs. [9,10] allowing its use for constructing supersymmetric bulk-plus-brane actions. The basic structure of multiplets is expected to remain unchanged and only be augmented by terms involving odd fields that so far have been “consistently” set to zero. But even this “minor modification” would lead to very significant changes in the structure of the bulk-plus-brane actions.

References

1. P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996); Nucl. Phys. B **475**, 94 (1996).
2. L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); Phys. Rev. Lett. **83**, 4690 (1999).
3. J. O. Conrad, Phys. Lett. B **421**, 119 (1998).
4. R. Altendorfer, J. Bagger and D. Nemeschansky, Phys. Rev. D **63**, 125025 (2001).
5. J. Bagger and D. V. Belyaev, Phys. Rev. D **67**, 025004 (2003).
6. N. H. Barth, Class. Quant. Grav. **2**, 497 (1985).
7. I. G. Moss, Phys. Lett. B **577**, 71 (2003); Nucl. Phys. B **729**, 179 (2005).
8. D. V. Belyaev, JHEP **0601**, 047 (2006).
9. M. Zucker, Phys. Rev. D **64**, 024024 (2001); Fortsch. Phys. **51**, 899 (2003).
10. T. Kugo and K. Ohashi, Prog. Theor. Phys. **108**, 203 (2002).
11. E. A. Mirabella and M. E. Peskin, Phys. Rev. D **58**, 065002 (1998).
12. E. Bergshoeff, R. Kallosh and A. Van Proeyen, JHEP **0010**, 033 (2000).
13. J. A. Bagger and D. V. Belyaev, Phys. Rev. D **72**, 065007 (2005).
14. D. V. Belyaev, JHEP **0608**, 032 (2006).
15. D. V. Belyaev and P. van Nieuwenhuizen, to appear.