Radius stabilization in 5D SUGRA models on orbifold

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Introduction

Models with extra dimensions:

- Randall-Sundrum model $\longrightarrow$ $\dfrac{M_{\text{Planck}}}{M_{\text{weak}}}$ hierarchy

  (Randall & Sundrum, 1999)

- Split fermions $\longrightarrow$ Yukawa hierarchy

  (Arkani-Hamed & Schmaltz, 2000)

- Gauge symmetry breaking by B.C.

  (Kawamura, 2000; Csaki, et.al, 2003,…)

- SUSY breaking by B.C.

  (Scherk-Schwarz, 1979)

⋮

5D SUGRA on S^1/Z_2 :

the simplest setup for extra-dimensional model

e.g.,

- SUSY Randall-Sundrum model

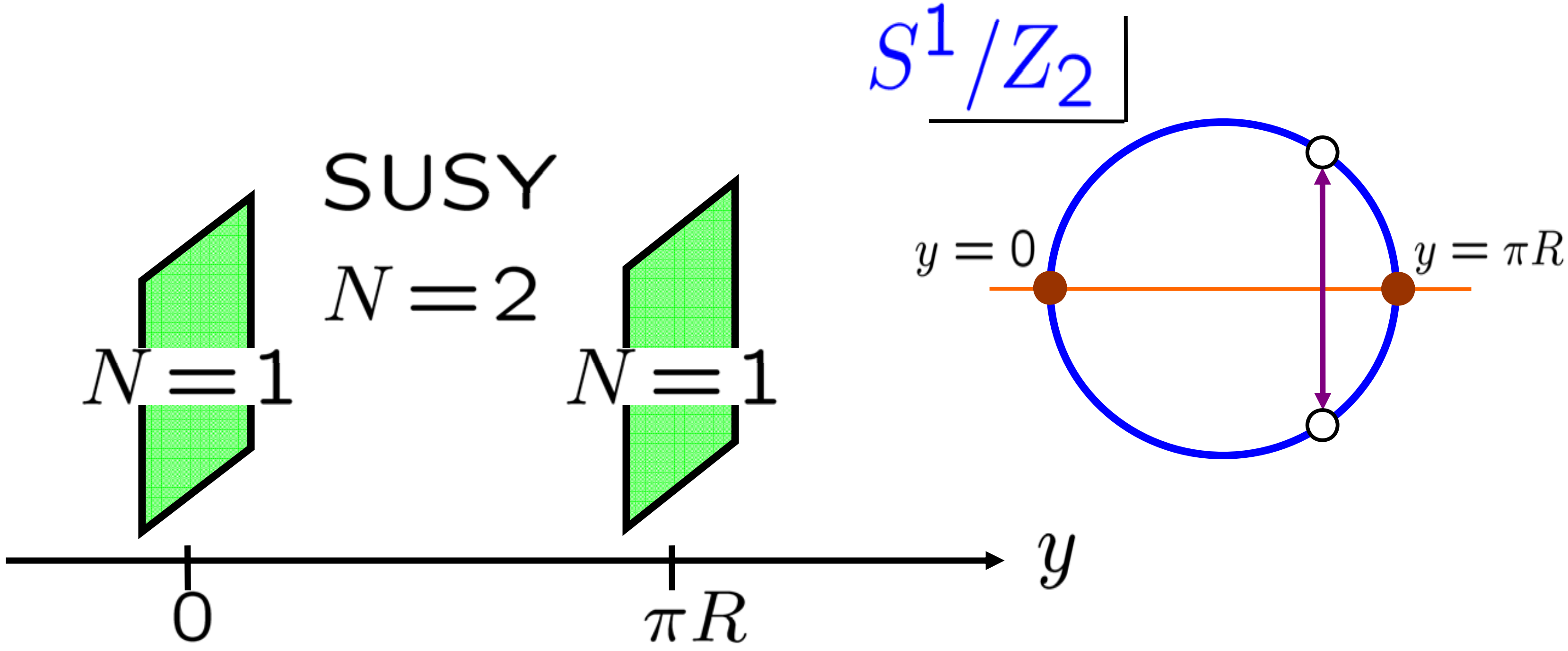
  (Gherghetta & Pomarol, NPB586 (2000) 141;
  Falkowski et.al, PLB491 (2000) 172;
  Altendorfer et.al, PRD63 (2001) 125025.)

- 5D effective theory of the heterotic M theory

  (compactified on 6D Calabi-Yau manifold)

  (Horava & Witten, NPB460 (1996) 506;
  Lukas et.al, PRD59 (1999) 086001)

Spacetime



Background metric:

$$ds^2 = e^{\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Randall-Sundrum model

$$e^{\sigma(y)} = e^{-\beta y}, \quad (0 \leq y \leq \pi R)$$

5D effective theory of the heterotic M theory

$$e^{\sigma(y)} = \{1 + 2\alpha y\}^{1/6}, \qquad (0 \leq y \leq \pi R)$$

These backgrounds are realized in the 5D **gauged** SUGRA.

We introduce a hypermultiplet $\mathcal{H}$ whose manifold is SU(2,1)/U(2).
(the "universal hypermultiplet" in 5D heterotic M theory)

Isometry group: SU(2,1)

We obtain both RS model & 5D heterotic M theory
by gaging different isometries.

$$\mathcal{H} = (S, \xi), \quad \begin{cases} \mathbf{S}\text{: } Z_2\text{-even} \\ \xi\text{: } Z_2\text{-odd} \end{cases}$$

(N=1 chiral multiplets)

[In 5D heterotic M theory,
Re S: Calabi-Yau volume]

We consider the following gaugings (by the graviphoton).

(Falkowski, Lalak, Pokorski, PLB491 (2000) 172)

- **α-gauging**

$$\begin{cases} S \to S + 2i\alpha \\ \xi \to \xi \end{cases}$$

→ 5D heterotic M theory

$$e^{\sigma(y)} = \{1 + 2\alpha y\}^{1/6}, \qquad (0 \leq y \leq \pi R)$$

- **β-gauging**

$$\begin{cases} S \to S, \\ \xi \to e^{i\beta}\xi \end{cases}$$

→ SUSY RS model

$$e^{\sigma(y)} = e^{-\beta y}, \quad (0 \leq y \leq \pi R)$$

The radius ***R*** must be stabilized at some finite value.

No boundary terms ⇒ Radius ***R*** is undetermined (at tree level)

We introduce the boundary superpotentials.

In the RS background, ***R*** can be stabilized by
(Maru & Okada, PRD70 (2004) 025002)

W= (constant)+(linear term for S) at y=0, πR.

Thus we consider this type of boundary superpotentials.

We analyze the scalar potential in the 4D effective theory.

Derivation of 4D effective theory

[Paccetti Correia, Schmidt, Tavartkiladze, NPB751 (2006) 222;
Abe & Sakamura, PRD75 (2007) 025018]

N=1 superfield description of off-shell 5D SUGRA action

keeping N=1 off-shell structure

⬇ Integrating out Z_2-odd superfields
& Dimensional reduction

N=1 superfield description of off-shell 4D SUGRA action

Physical fields: (S, T) + (4D gravitational multiplet)

↑ (T): radion multiplet

By this method, we obtain

$$K^{(4)}(S,T), \qquad W^{(4)}(S,T).$$

Scalar potential

$$V = \exp\left(K^{(4)}\right)\left\{K^{(4)I\bar{J}} D_I W^4 D_{\bar{J}} \bar{W}^{(4)} - 3\left|W^{(4)}\right|^2\right\},$$

where

$$D_I W^{(4)} \equiv W_I^{(4)} + K_I^{(4)} W^{(4)}.$$

SUSY condition

$$D_I W^{(4)} = 0, \qquad (I = S, T)$$

SUSY point is a stationary point of *V*,
but not always a local minimum.

α-gauging

$$K^{(4)} = -3 \ln \left[\frac{3}{8\alpha} \left\{ (\mathrm{Re} S_\pi)^{4/3} - (\mathrm{Re} S_0)^{4/3} \right\} \right],$$
$$W^{(4)} = a + b_0 S_0 - b_\pi S_\pi,$$

where

$$S_0 \equiv S,$$
$$S_\pi \equiv S + 2\alpha\pi T.$$

For simplicity, we assume that

$$\alpha > 0, \qquad a, b_0, b_\pi : \ \text{real}.$$

SUSY condition:

$$D_I W^{(4)} \equiv W_I^{(4)} + K_I^{(4)} W^{(4)} = 0, \quad (I = S_0, S_\pi)$$

SUSY point

$$(S_0, S_\pi) = \left(\frac{2ab_0^3}{b_\pi^4 - b_0^4}, \frac{2ab_\pi^3}{b_\pi^4 - b_0^4} \right)$$

This is a saddle point. $(V(S_0, S_\pi) < 0)$

non-SUSY point (when $b_\pi \gg b_0$,)

$$(S_0, S_\pi) \simeq \left(\frac{16ab_0^3}{b_\pi^4}, \frac{2ab_\pi^3}{b_\pi^4} \right)$$

This is a local minimum. $(V(S_0, S_\pi) < 0)$

β-gauging

(SUSY RS model)

$$K^{(4)} = -3\ln\left(\frac{1-|\Omega|^2}{2\beta}\right) - \ln(\mathrm{Re}S),$$

$$W^{(4)} = (a_0 + b_0 S) - (a_\pi + b_\pi S)\Omega^3,$$

where

$$\Omega \equiv e^{-\beta\pi T}.$$ (warp factor superfield)

Here we assume that

$$\beta > 0$$

SUSY point

$$(S, \Omega) = \left(-\frac{a_0}{b_0}, \left(\frac{b_0}{b_\pi} \right)^{1/3} \right)$$

This is a local minimum.

Minkowski vacuum if $a_0 b_\pi - b_0 a_\pi = 0.$

(Maru & Okada, PRD70 (2004) 025002)

non-SUSY point

$$(S, \Omega) \simeq \left(-\frac{a_0}{b_0}, \left(\frac{2b_0^2}{3b_\pi^2} \right)^{1/4} \right)$$

This is a saddle point. $(V(S, \Omega) > 0)$

(α,β)-gauging

$$K^{(4)} = -3 \ln \left[\frac{1}{2\alpha} \mathcal{F}(\mathrm{Re} S_0, \mathrm{Re} S_\pi) \right],$$

$$W^{(4)} = \exp\left\{ -\frac{3\beta}{2\alpha} S_0 \right\} (a_0 + b_0 S_0) - \exp\left\{ -\frac{3\beta}{2\alpha} S_\pi \right\} (a_\pi + b_\pi S_\pi),$$

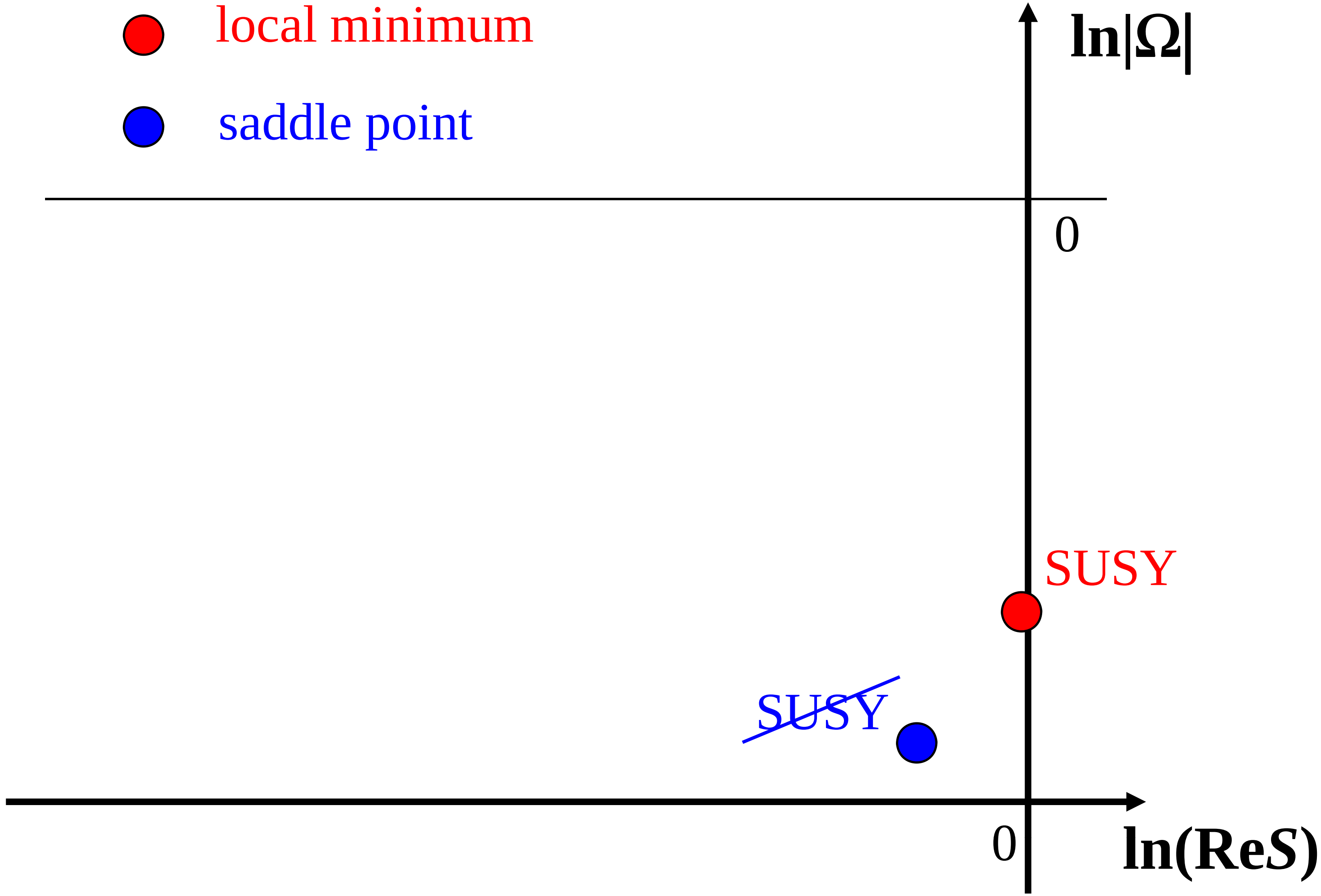
where

$$\mathcal{F}(\mathrm{Re} S_0, \mathrm{Re} S_\pi) \equiv \left(\frac{\alpha}{\beta} \right)^{4/3} \left\{ \Gamma\left(\frac{4}{3}, \frac{\beta}{\alpha} \mathrm{Re} S_0 \right) - \Gamma\left(\frac{4}{3}, \frac{\beta}{\alpha} \mathrm{Re} S_\pi \right) \right\}.$$

incomplete gamma function

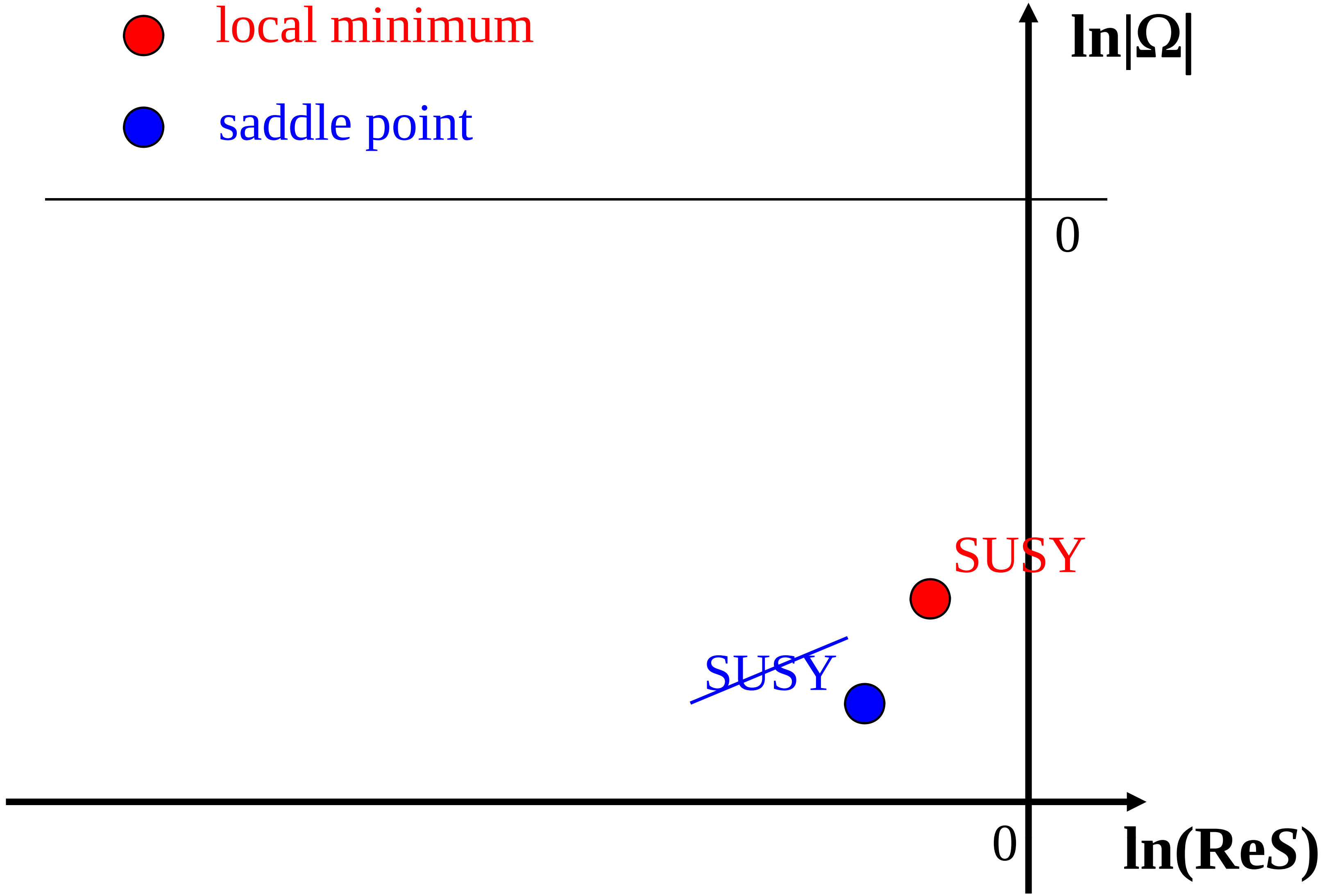
$$\begin{aligned} S_0 &= S, \\ S_\pi &= S + 2\alpha\pi T \\ &= S - \frac{2\alpha}{\beta} \ln \Omega. \end{aligned}$$

Stationary points $\left(\frac{\beta}{\alpha} \simeq 50\right)$



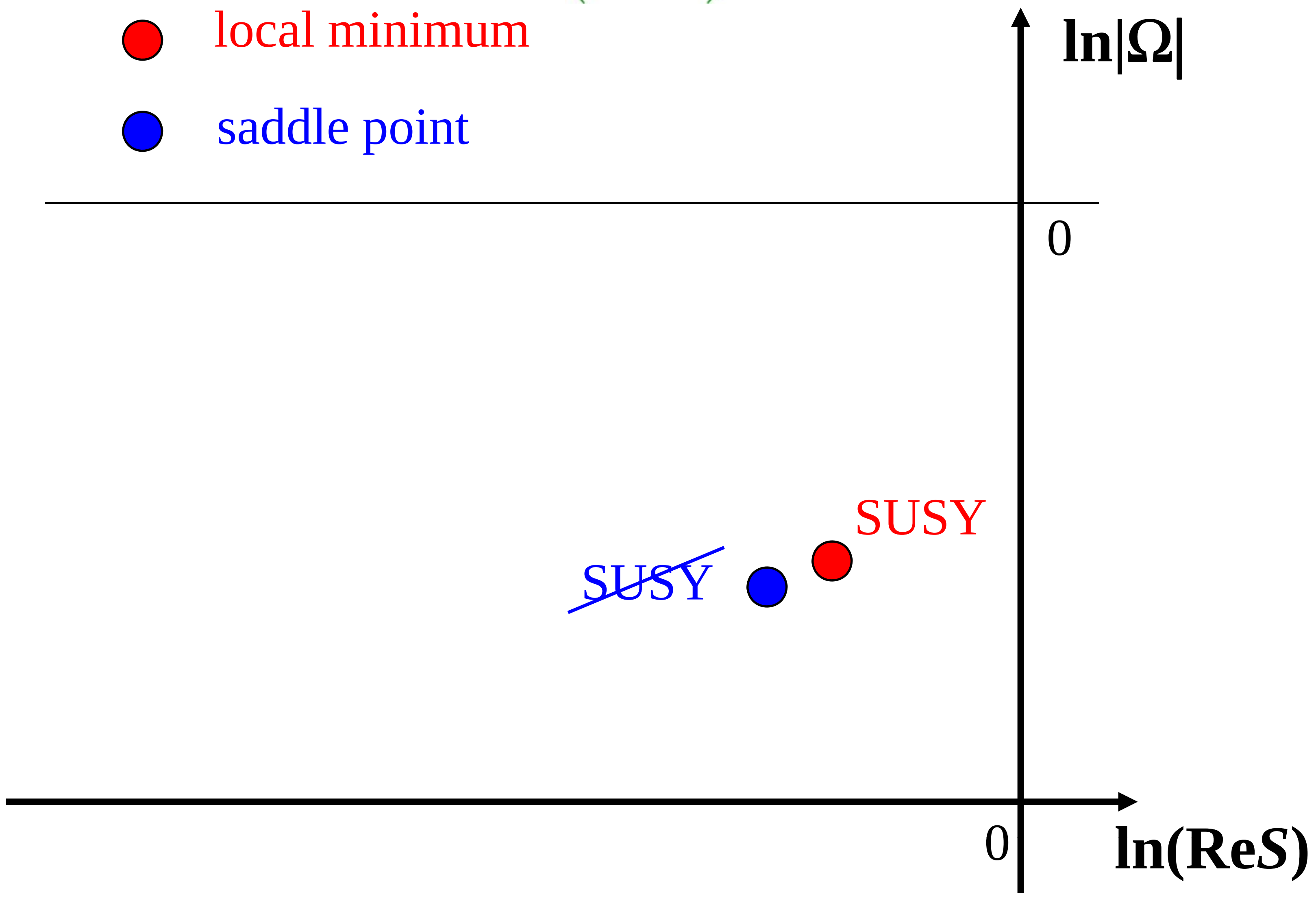
Stationary points $\left(\frac{\beta}{\alpha} \simeq 10\right)$

- local minimum (red dot)
- saddle point (blue dot)

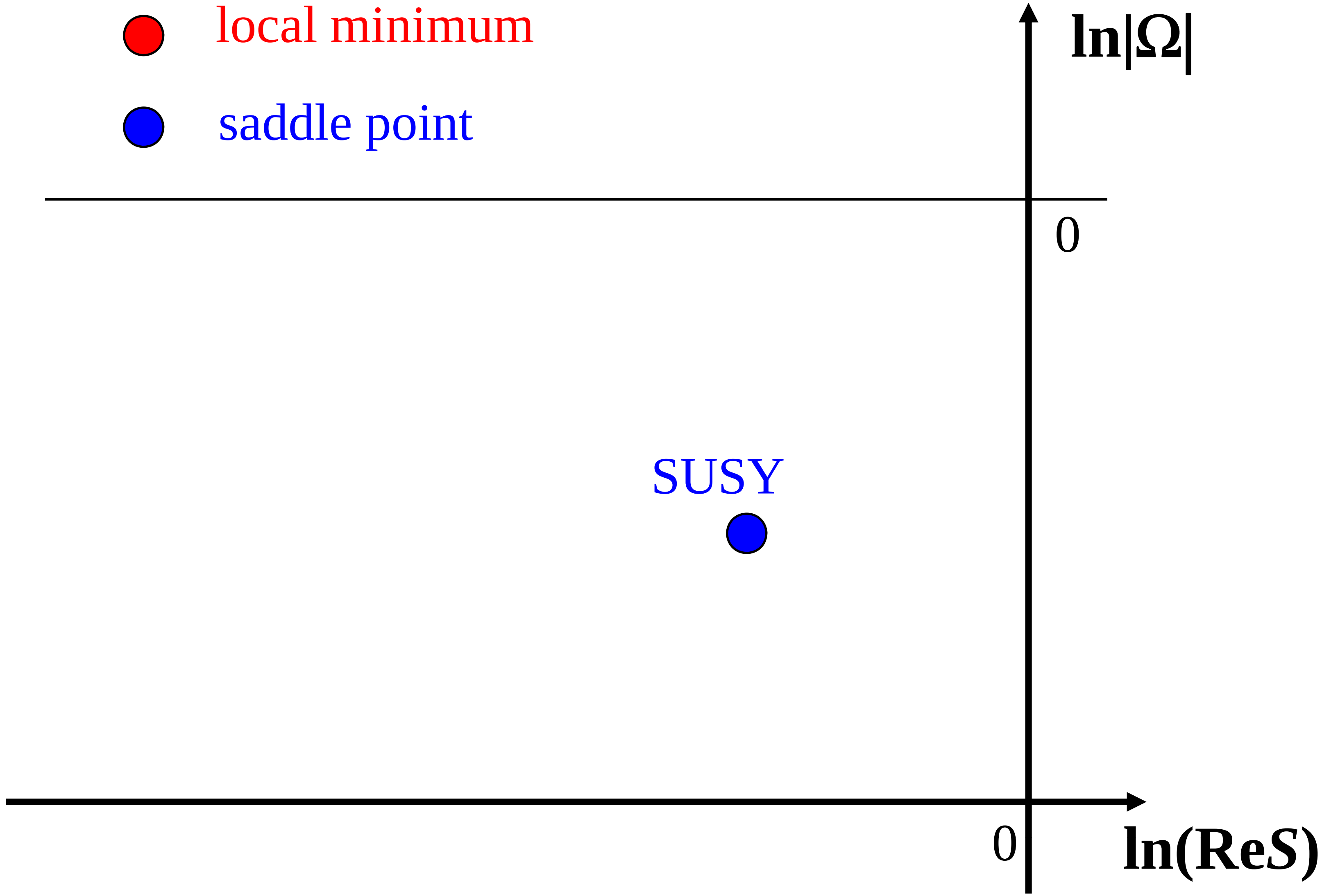


Stationary points $\left(\dfrac{\beta}{\alpha} \simeq 5\right)$

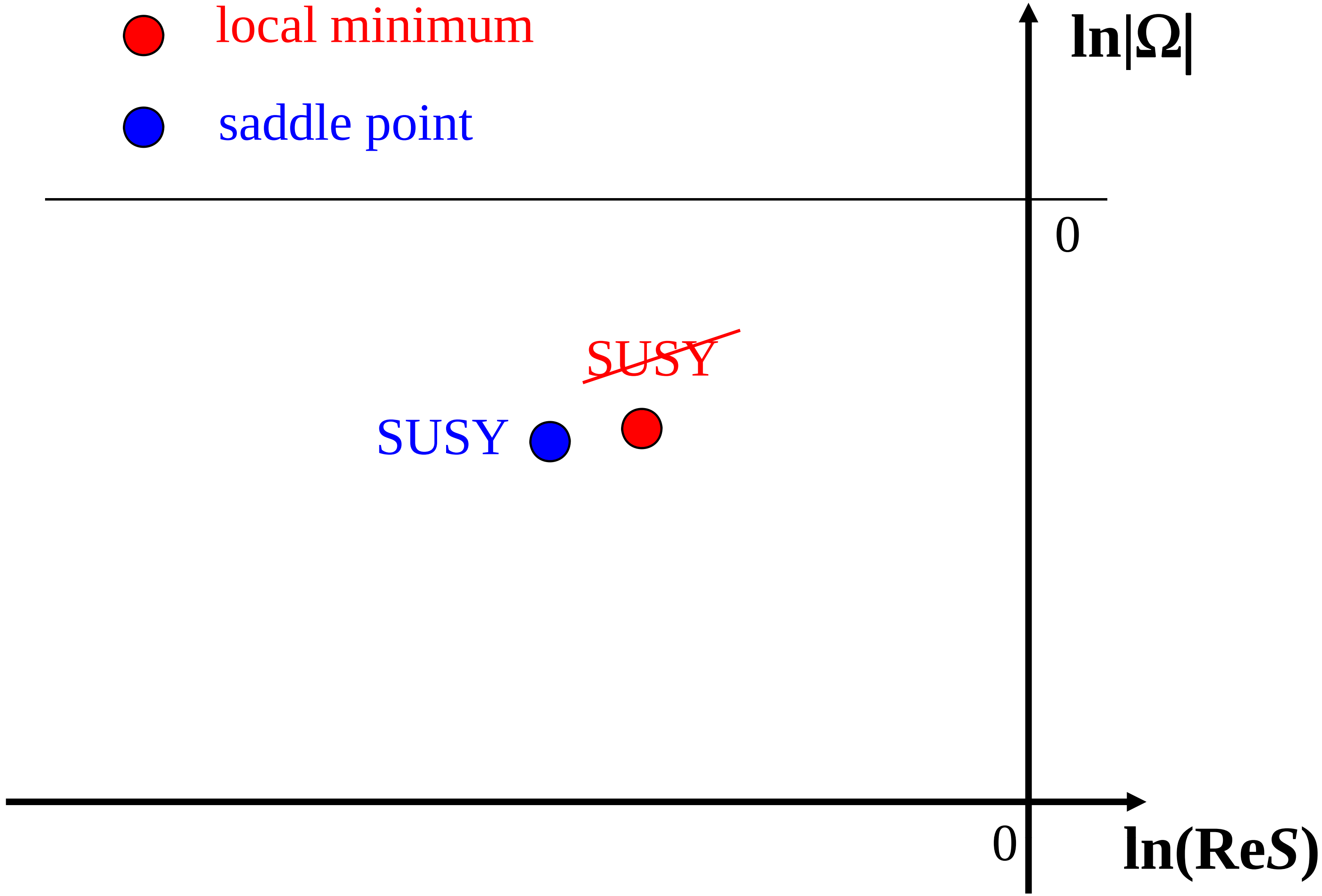
- local minimum (red)
- saddle point (blue)



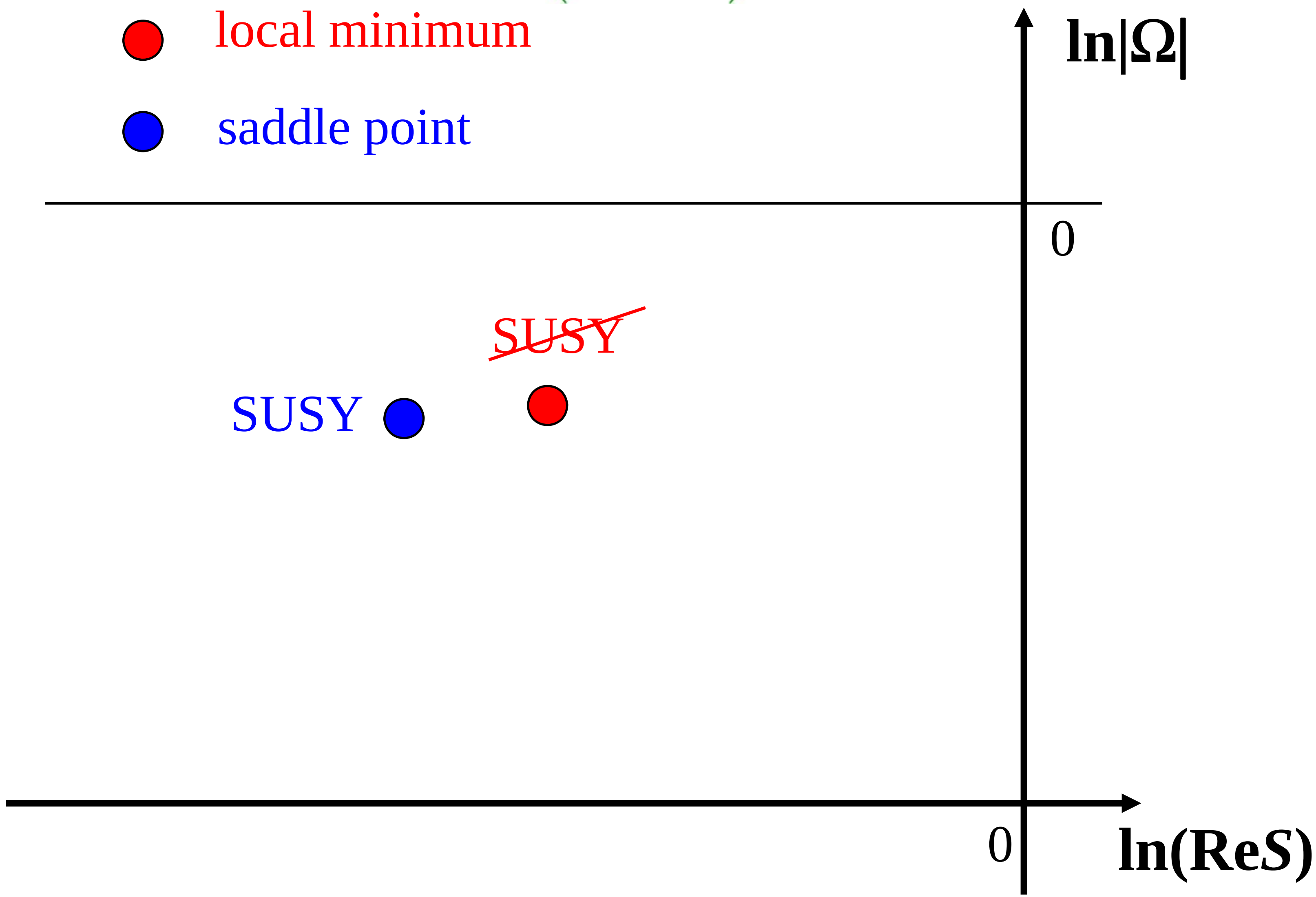
Stationary points $\left(\frac{\beta}{\alpha} \simeq 1\right)$



Stationary points $\left(\frac{\beta}{\alpha} \simeq 0.2\right)$



Stationary points $\left(\frac{\beta}{\alpha} \simeq 0.1\right)$



gauging **vacuum**

pure β → SUSY Minkowski

$\beta > \alpha$ → SUSY AdS_4

$\beta < \alpha$ → ~~SUSY~~ AdS_4

Comment 1

SUSY mass at boundaries

So far, we consider

W= (constant)+(linear term for *S*)   at boundaries.

mass terms in ***W*** may change a saddle point to a local minimum.

Comment 2

Uplift of AdS vacuum

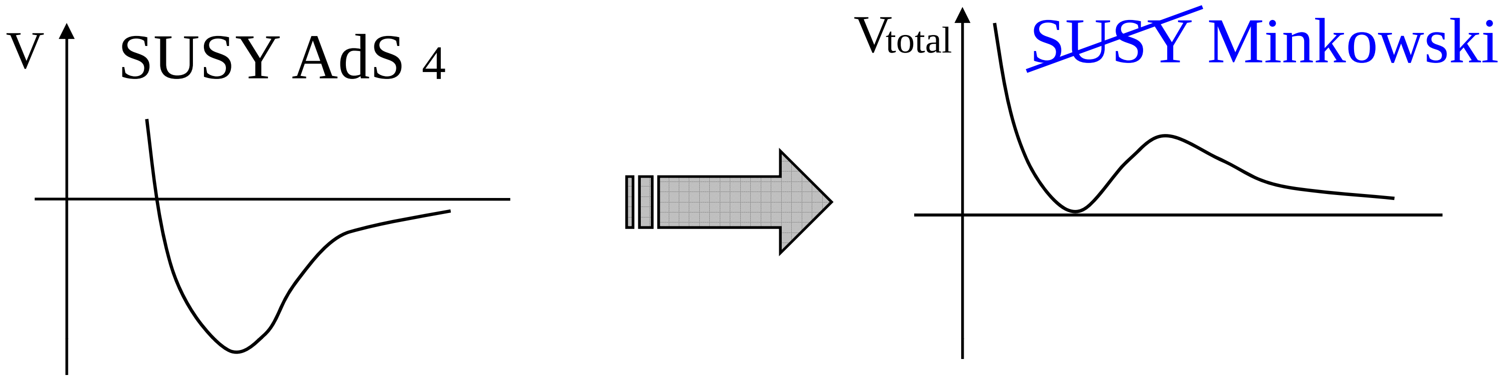
Assume SUSY sector that decouples from (S,T).

$$V_{\text{uplift}} = \zeta \exp\left(\frac{nK^{(4)}}{3}\right) \qquad \text{(typically, } n = 2\text{)}$$

(K.Choi, arXiv:0705.3330)

Total potential: $V_{\text{total}} = V(S,T) + V_{\text{uplift}}$

For β > α , by fine-tuning ζ ,



Summary

- Radius stabilization in 5D gauged SUGRA with the universal hypermultiplet.
- Boundary superpotential:

  W= (constant)+(linear term for S)   at y=0, πR.

gauging		vacuum
pure β	⇒	SUSY Minkowski
$\beta > \alpha$	⇒	SUSY AdS_4
$\beta < \alpha$	⇒	~~SUSY~~ AdS_4

Future Plan

- More general superpotential at boundaries
- Gauging other isometries
  (e.g., SUSY bulk mass)
- ~~SUSY~~ spectrum after uplifting AdS vacuum