Unitarity constraints on MSSM trilinear couplings

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Abstract. For MSSM phenomenology, soft SUSY breaking dimension-three operators are important, in particular the couplings between Higgs bosons and squarks. In scattering processes, perturbative unitarity is violated at modest center-of-mass energy if these couplings are much larger than the masses of the scalar particles involved. Assuming perturbative unitarity, constraints on the trilinear couplings can be determined using a computer program that we have developed.

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1 Introduction

For phenomenology in the Minimal Supersymmetric Standard Model (MSSM), the soft supersymmetry (SUSY) breaking dimension-three trilinear couplings between Higgs bosons and squarks play an important role. They determine the mass splitting in the stop and sbottom sector and can have a profound influence on Higgs physics. Unfortunately, these SUSY breaking couplings are essentially free parameters. Thus, any constraints that can be placed by theoretical considerations are useful. Such constraints can be obtained, for example, by considering the symmetry breaking of the scalar sector and requiring that there are no color or charge breaking minima [1]. Requiring that the MSSM allows for a perturbation theory treatment, we here consider a different approach: perturbative unitarity in $2 \to 2$ scattering of scalar particles limits the size of the trilinear couplings.

The approach is similar to the consideration of longitudinal weak boson scattering in the Standard Model (SM), where the absence of perturbative unitarity violation at high energies provides an upper bound on the Higgs quartic coupling and thus on the Higgs boson mass [2,3]. For trilinear couplings in the MSSM scalar sector, these unitarity violations arise at intermediate energies, somewhat above pair production thresholds, if trilinear couplings are chosen much larger than the masses of the scalar particles in the scattering process. The approach is applied to the trilinear couplings between Higgs bosons and 3rd generation squarks, given by

$$\mathcal{L}_{\text{tri}} = -\lambda_b A_b H_d \tilde{Q}_L \tilde{b}_R^\dagger - \lambda_t A_t H_u \tilde{Q}_L \tilde{t}_R^\dagger + \text{h.c.}$$

which is part of the MSSM soft SUSY breaking potential. Here $\tilde{Q}_L$ is the $SU(2)_L$-doublet of 3rd generation squarks, $\tilde{b}_R^\dagger$ and $\tilde{t}_R^\dagger$ denote the right-handed sbottom and stop singlet fields, and H_d and H_u are the Higgs boson doublet fields. We want to limit the dimensionful parameters which multiply the Yukawa couplings $\lambda_b = m_b/v\cos\beta$ and $\lambda_t = m_t/v\sin\beta$, and μ, the Higgs mixing parameter.

2 Perturbative unitarity for scalars

The starting point is the unitarity of the S matrix,

$$S^\dagger S = 1\,, \quad \text{or equivalently} \quad -\mathrm{i}\left(T - T^\dagger\right) = T^\dagger T\,,$$

for the transition operator T with $S = 1 + \mathrm{i}\,T$. To evaluate this equation one usually restricts to $2 \to 2$ scattering (which is a good approximation in perturbation theory) and then uses angular-momentum conservation and symmetries of the model to partially diagonalize T. In our analysis we additionally restrict ourselves to scalar fields. Let $\langle f|T|i\rangle$ denote the transition matrix elements with initial state $|i\rangle$ and final state $|f\rangle$, and let $\hat{\mathcal{T}}_{fi}$ be the matrix element obtained from Feynman diagrams in momentum representation,

$$(2\pi)^4\,\delta^{(4)}(P_i - P_f)\,\hat{\mathcal{T}}_{fi}(\sqrt{s},\cos\theta) = \langle f|T|i\rangle\,,$$

evaluated in the center-of-mass system, where $\sqrt{s}$ is the total energy and θ is the scattering angle. This transition is defined by the (ordered) particle content of the two states, as well as $\sqrt{s}$ and θ, and depends on the masses m_{lk} of the particles in state $l = i, f$ via the functions $\lambda_l = \lambda(s, m_{l1}^2, m_{l2}^2)$, $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. The dependence on the scattering angle θ is eliminated by projection onto partial waves of total angular momentum $J = 0, 1, 2, \ldots$

$$\mathcal{T}_{fi}^J = \frac{1}{2}\,\frac{\lambda_f^{1/4}\lambda_i^{1/4}}{16\pi\, s}\int_{-1}^{1} \mathrm{d}\cos\theta\,\hat{\mathcal{T}}_{fi}(\sqrt{s},\cos\theta)\,P_J(\cos\theta)\,.$$

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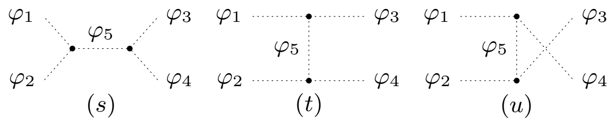

Fig. 1. Scalar $2\to 2$ tree level scattering diagrams.

The P_J are the Legendre polynomials. The factor $1/2$ is a standard convention and leads to the factor $1/2$ in Eq. (1). In this normalization, extra factors of $1/\sqrt{2}$ have to be included for each state with two identical particles. Higher partial waves usually give smaller amplitudes, so only $J=0,1$ amplitudes have to be considered in practice. The unitarity condition now reads

$$\frac{1}{2\mathrm{i}}\left(\mathcal{T}^J_{fi}-\mathcal{T}^{J*}_{if}\right)\cong\sum_h \mathcal{T}^{J*}_{hf}\,\mathcal{T}^J_{hi}\,. \tag{1}$$

The sum is taken over intermediate states. Restriction to only relevant and two-particle scalar states in the sum slightly underestimates the right-hand side and leads to conservative bounds.

The 'true', physical matrix $\mathcal{T}^J_{fi}$ is normal and can therefore be diagonalized. The same holds for the Born amplitude, which we use instead. The diagonalized matrix $\tilde{\mathcal{T}}^J_{fi}$ and thus the eigenvalues satisfy

$$\mathrm{Im}\,\tilde{\mathcal{T}}^J_{ii}\cong|\tilde{\mathcal{T}}^J_{ii}|^2\,. \tag{2}$$

The 'true' eigenvalues (for any given energy $\sqrt{s}$) must lie on the circle (called Argand diagram) given by (2): $y=x^2+y^2$ for $x=\mathrm{Re}\,\tilde{\mathcal{T}}^J_{ii}$ and $y=\mathrm{Im}\,\tilde{\mathcal{T}}^J_{ii}$ which implies $|x|\le 1/2$. For the Born approximation, the phases of the fields can be chosen such that all $2\to 2$ amplitudes are (nearly) real, if CP (nearly) holds. Approximating x by the corresponding Born amplitude yields the desired unitarity bound. The circle restricts $|x|\le 1/2$, which is the unitarity bound generally used in the literature. A Born value of $x=1/2, y=0$ needs at least[1] a correction of $\sqrt{2}-1\approx 41\%$ to become unitary. Such large corrections indicate a breakdown of perturbation theory. In addition to the perturbative unitarity bound of $|x|\le 1/2$ we therefore also consider $|x|\le 1/6$ as a condition for which the Born amplitude remains sufficiently small to trust perturbation theory.

2.1 A toy model

Figure 1 shows generic tree level Feynman graphs for $\varphi_1\varphi_2\to\varphi_3\varphi_4$ scattering with trilinear couplings of (possibly different) scalar fields φ_l $(l=1,..,5)$. The corresponding amplitudes are $A^2/(q^2-m_5^2)$, where $q^2=s,t,u$ is one of the Mandelstam variables, and A stands for the trilinear couplings. t and u depend on $\sqrt{s}$, $\cos\theta$, and the masses m_l of the exterior particles. Projecting onto partial waves, the amplitude for $J=0$ has a structure roughly like:

$$\mathcal{T}^{J=0}_{fi}\sim\frac{1}{16\pi}\,\frac{\lambda_f^{1/4}\lambda_i^{1/4}}{s}\,\frac{A^2}{\max\{s,m_5^2\}}\,, \tag{3}$$

[1] 'At least' means that this would be the minimal case where the correction directly hits the nearest circle point.

where a factor of 2 has been assumed to account for the partial wave projection. The second factor is smaller than 1 and the third factor becomes large if both the scattering energy $\sqrt{s}$ and the mass of the internal particle are small compared to the couplings A. Highest values are usually found for energies near the kinematic threshold, i.e. at energies where the model should work properly, in contrast to weak boson scattering in the SM [2,3].

2.2 Handling poles

Clearly the Born amplitudes are not sufficient to describe scattering processes where intermediate particles become on-shell. In the s channel in Figure 1 this happens when $\sqrt{s}=m_5$. Born amplitudes only will be used for unitarity considerations and s-channel poles are cut out by the condition

$$|\sqrt{s}-m|^2>a\,m\,\Gamma(Q=b\,m)\,. \tag{4}$$

Here $a,b\gtrsim 1$ are (suitably chosen) constants and the 'running width' $\Gamma(Q)$ of the internal particle φ at energy Q is approximated by the decay width via replacing its mass m with the energy Q in the phase space factor. This condition (4) has to be fulfilled for all internal particles appearing in the s channel. If this is not the case, the amplitude is set to zero, as well as the irreducible part of $\mathcal{T}^{J=0}_{fi}$ this process is in, because of possible destructive interference of matrix elements.

A width cannot be included in the Born propagator for two reasons: First, $\mathcal{T}_{fi}$ is no longer diagonalizable (at this level of approximation). Second, our $\Gamma(\sqrt{s})$ grows (linearly for large $\sqrt{s}$) with $\sqrt{s}$, which is not a good approximation to the propagator as $\sqrt{s}\gg m$.

The internal particle in the u channel of Figure 1 can also become on-shell for certain combinations of masses. This occurs e.g. if φ_1 can decay into φ_4,φ_5 and φ_2,φ_5 can fuse to φ_3 (MSSM example: $\tilde{t}_2\tilde{t}_1\to\tilde{t}_2\tilde{t}_1$ with u channel h^0 exchange when $m_{\tilde{t}_2}>m_{\tilde{t}_1}+m_h$). One obtains another possibility by switching labels $1\leftrightarrow 2$ and $3\leftrightarrow 4$ or two similar conditions for the t channel by exchange of $3\leftrightarrow 4$. The first case has the condition:

$$c\,m_1\ge m_4+m_5\ \wedge\ m_2+m_5\le c\,m_3\,. \tag{5}$$

with some suitably chosen constant $c\gtrsim 1$. Amplitudes where a condition like in (5) is fulfilled cannot be computed because the internal particle becomes on-shell for some value of the scattering angle. The constants a,b,c are chosen larger than one because in proximity of a pole one encounters unphysical enhancements of the amplitude in the pure Born approximation. Still, some enhancement can appear in special cases.

If some Born matrix elements cannot be calculated because of a t or u channel pole, the tree level matrix $\mathcal{T}^J_{fi}$ cannot be diagonalized. The solution is a partial diagonalization. Assuming time reflection invariance, we write the left-hand side of (1) as $\mathrm{Im}\,\mathcal{T}^J_{fi}$. Define the set B of all kinematically accessible states at given $\sqrt{s}$ from an irreducible part of $\mathcal{T}^J_{fi}$ and the set $C\subset B$ such that $h, l \in C$ satisfy a t or u channel pole condition for $h \to l$. Then, for states $f, i \in B \backslash C$ equation (1) becomes

$$\mathrm{Im}\, \mathcal{T}_{fi}^J \cong \sum_{h \in B\backslash C} \mathcal{T}_{hf}^{J*}\, \mathcal{T}_{hi}^J + \sum_{h \in C} \mathcal{T}_{hf}^{J*}\, \mathcal{T}_{hi}^J\,.$$

We diagonalize the sub-matrix $(\mathcal{T}_{fi}^J)|_{f,i \in B\backslash C}$ of all states accessible at a given energy $\sqrt{s}$ with a unitary matrix U and write the diagonalized part as $\tilde{\mathcal{T}}$ to obtain:

$$\mathrm{Im}\, \tilde{\mathcal{T}}_{ii}^J \cong |\tilde{\mathcal{T}}_{ii}^J|^2 + \sum_{h \in C} \left| (\mathcal{T}^J\big|_{C \times B\backslash C}\, U^{-1})_{ih} \right|^2 .$$

The second term on the right-hand side (denoted by R^2) is positive. At low energies where only a few states are kinematically allowed, R^2 typically vanishes. The circle equation $y = x^2 + y^2$ now holds for $y = \mathrm{Im}\, \tilde{\mathcal{T}}_{ii}^J$ and

$$|x| = (\,(\mathrm{Re}\, \tilde{\mathcal{T}}_{ii}^J)^2 + R^2)^{1/2}\,. \tag{6}$$

3 MSSM constraints

The MSSM has a lot of (>100) parameters, many of which are constrained by measurement of the properties of SM particles and (nearly) conserved symmetries. Here we focus on some of the SUSY breaking parameters, namely the interplay of mass terms for the scalar fields and the dimensionful A parameters of the trilinear couplings, and also μ. These parameters are constrained by lower mass bounds for the sparticles and the Higgs bosons [5]. Their phases are bounded by the absence of large CP violation. In addition, the fields must be in the observed minima of the potential and have positive masses after electroweak symmetry breaking. Complex, strong constraints from not being in charge and color breaking minima were derived in [1]. The new constraints from perturbative unitarity should be considered in addition.

3.1 Particles, parameters and calculation

In the MSSM large trilinear couplings may appear between 3rd generation squarks and the Higgs bosons and the longitudinal degrees of freedom of $W^\pm$ and Z^0. To work in the scalar sector only, the Goldstone equivalence theorem and the Feynman $R_{\xi=1}$-gauge are used, such that the Goldstone bosons $G^\pm$, G^0 represent the longitudinal polarizations of $W^\pm$, Z^0. The trilinear coupling strengths are given by vertex factors like

$$V(G^0, \tilde{t}_L, \tilde{t}_R) = g'\,(A_t - \mu^* \cot\beta)\, m_t/2m_W\,.$$

Other stop vertices with Higgs bosons are similar and contain combinations of A_t and μ (in decoupling scenarios mainly $X_t = A_t - \mu^* \cot\beta$). Likewise two sbottom Higgs boson couplings grow with A_b and μ, but are suppressed compared to the stop case, because they contain the factor $m_b/2m_W$. Some mixed stop-sbottom couplings also behave like the pure stop case, e.g. the vertex $V(G^+, \tilde{b}_L, \tilde{t}_R) = \mathrm{i}\, g'\, X_t\, m_t/\sqrt{2} m_W$.

Relevant for our analysis are the scalar Higgs bosons h^0, H^0, A^0, $H^\pm$, G^0, and $G^\pm$ and the heavy squark mass eigenstates $\tilde{b}_1$, $\tilde{b}_2$, $\tilde{t}_1$, and $\tilde{t}_2$. In addition to SM parameters, results will depend on the squark mass parameters $M_{\tilde{Q}}^2$, $M_{\tilde{b}}^2$, and $M_{\tilde{t}}^2$, the mass of the pseudoscalar Higgs boson m_A, the Higgs mixing parameter μ, the ratio of Higgs bosons VEVs $\tan\beta$ and the trilinear coupling parameters A_b and A_t. Neglecting any CP-violating effects, these parameters are chosen real in the following.

With the scalar fields listed above, two-particle states are formed. One uses charge, color, and the non-existence of three-squark-vertices to form 15 independent blocks in the scattering matrix. The biggest block (charge 0, color singlet) contains 21 states, at high enough energy, and usually supplies the largest eigenvalue. A Monte Carlo search over the scattering energy $\sqrt{s}$ is used to find the largest eigenvalue of the scattering matrix, which is obtained by numerically diagonalization. We use, by default, $\overline{\mathrm{MS}}$ quark masses (at 120 GeV) to calculate the squark vertices and masses.

Large trilinear couplings can lead to strong loop suppressions of SUSY Higgs masses [6] in regions where our analysis indicates a breakdown of perturbation theory. In such cases we ignore clashes with LEP bounds [5] and conservatively use $m_{h^0} = 120$ GeV instead.

3.2 Relevance of the method for the MSSM

Because only heavy squarks and Higgs bosons were used, no parameters for charginos and other squarks or sleptons were needed for the calculation. The results are also practically independent of these other parameters. The maximum mixing case, $X_t \gtrsim 2$ times the sfermion mass scale, is often used as a benchmark scenario. An important bound for these cases often used in literature derives from [4] (see e.g. Eq.(5) in [1]):

$$A_t^2 \le 3\, \left(M_{\tilde{Q}}^2 + M_{\tilde{t}}^2 + m_{H_u}^2 + |\mu|^2 \right), \tag{7a}$$

$$A_b^2 \le 3\, \left(M_{\tilde{Q}}^2 + M_{\tilde{b}}^2 + m_{H_d}^2 + |\mu|^2 \right), \tag{7b}$$

with $m_{H_{u(d)}}^2 + |\mu|^2 = m_A^2 \cos^2\beta\, (\sin^2\beta) \pm 1/2\, m_Z^2 \cos 2\beta$. For large $\tan\beta$ this means $|A_t|^2 \lesssim 3\, (M_{\tilde{Q}}^2 + M_{\tilde{t}}^2 - m_Z^2/2)$ and $|A_b|^2 \lesssim 3\, (M_{\tilde{Q}}^2 + M_{\tilde{b}}^2 + m_A^2 + m_Z^2/2)$. The A_t bound in (7a) is mostly determined by $M_{\tilde{Q}}^2 + M_{\tilde{t}}^2$. The unitarity bounds instead depend on the masses of the lightest particles with large trilinear couplings, which therefore can be stronger in the case of exceptionally light scalars while retaining a large value for $M_{\tilde{Q}}^2 + M_{\tilde{t}}^2$.

3.3 Examples for unitarity bounds

The approach of the unitarity bound for large trilinear couplings is illustrated in the figures. MSSM parameters for these two examples are given in the captions. Figure 2 strongly constraints μ, because A_b and A_t have been set only slightly below the bound from

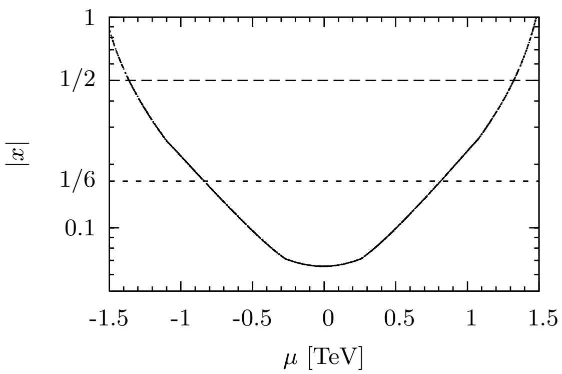

Fig. 2. μ bound example with $\tan\beta = 50$, $m_A = 100$ GeV, $M_{\tilde{Q}} = 400$ GeV, $M_{\tilde{t}} = 600$ GeV, $M_{\tilde{b}} = 630$ GeV, $A_t = 1$ TeV, and $A_b = 1.3$ TeV. 135 GeV $< m_{\tilde{b}_1} <$ 404 GeV varies with μ. $|x|$ is the maximal eigenvalue of $\mathcal{T}^0_{fi}$ as modified in Eq. (6).

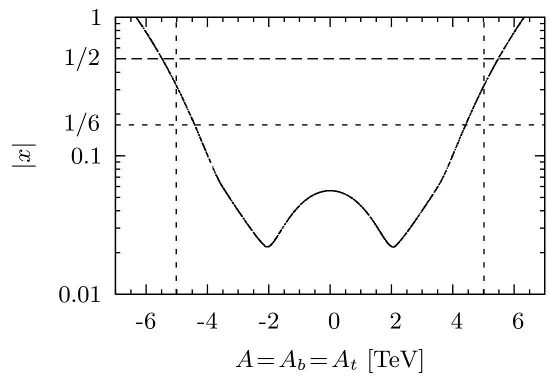

Fig. 3. A bound example, together with the bound from Eq. (7a) (vertical lines) using $\tan\beta = 30$, $m_A = 1$ TeV, $\mu = -200$ GeV, $M_{\tilde{Q}} = M_{\tilde{b}} = 2.9$ TeV with fixed $m_{\tilde{t}_1} = 100$ GeV and $m_{h^0} = 120$ GeV.

Eq. (7) and a high $\tan\beta$ has been chosen. Alternatively, one could consider A_b bounds for large, fixed μ in this scenario. In Figure 2 the diagonalized amplitude mainly results from the interfering low energy processes $\tilde{b}_1\tilde{b}_1^* \leftrightarrow h^0h^0$, $\tilde{b}_1\tilde{b}_1^* \leftrightarrow h^0H^0$, $\tilde{b}_1\tilde{b}_1^* \leftrightarrow \tilde{b}_1\tilde{b}_1^*$, and $\tilde{b}_1\tilde{b}_1^* \leftrightarrow H^0H^0$.

Figure 3 shows an example for the case of a light $\tilde{t}_1$ held at a fixed mass of 100 GeV by adjusting $M^2_{\tilde{t}}$ for each value of μ. To evade the bounds (7) for $A = A_b = A_t$, $M_{\tilde{Q}}$ is chosen high, which leads to the maximum mixing case, where the $\tilde{t}_1, \tilde{t}_1^*$ couplings to the Higgs bosons are strongly reduced. The largest amplitude is almost completely due to the $\tilde{t}_1\tilde{t}_1^* \leftrightarrow h^0h^0$ t-channel $\tilde{t}_1$ exchange diagram and is mostly independent of $\tan\beta \gg 1$, $m_A \gg m_Z$, $|\mu| \ll M_{\tilde{Q}}$ and $M_{\tilde{b}}$. In this scenario Eq. (7a) gives $|A| \lesssim 5$ TeV, while the perturbativity condition $|x| \leq 1/6$ yields $|A| \lesssim 4.4$ TeV.

4 Conclusion

A general method has been developed for constraining trilinear couplings of scalars by using perturbative unitarity. It has been implemented for the MSSM, more specifically for third generation squarks and Higgs bosons, but it can also be used for other models. Since trilinear couplings of scalars correspond to superrenormalizable dimension three operators, they produce the largest contributions to scattering amplitudes at modest center-of-mass energy. Thus, perturbative unitarity bounds are derived from scattering amplitudes at energies not very far above production threshold and they are therefore independent of the ultraviolet structure of the theory. One finds that trilinear couplings cannot be much larger than the masses of the scalar particles involved. Quantitatively, ratios of more than about a factor 5 are forbidden (see Eq. (3)).

We have analyzed two MSSM scenarios with small sbottom or stop masses, trying to constrain the soft SUSY breaking parameters A_b and A_t and the Higgs mixing parameter μ at large $\tan\beta$. Mixing effects between left- and right-handed squarks lead to a complicated picture, since large A parameters may lead to only modest trilinear couplings of the lightest scalars when the relevant mixing angle factors are small. We have built a numerical program to study these effects for two Higgs doublets and third generation squarks within the MSSM for arbitrary input parameters $\tan\beta$, μ, m_A, A_b, A_t, $M^2_{\tilde{Q}}$, $M^2_{\tilde{b}}$, and $M^2_{\tilde{t}}$. For small μ, resulting constraints are similar in strength to bounds derived from the exclusion of false vacua e.g. Eq. (7), however, the underlying arguments are quite different. Our approach therefore provides a complementary method for constraining soft SUSY breaking parameters. Given a specific MSSM scenario, specified by the Lagrangian parameters, our program gives a test whether these parameters clash with unitarity.

Since supersymmetric theories are at heart perturbative, even stronger constraints are obtained by the requirement that scattering amplitudes sufficiently far from resonances do not leave the perturbative domain. Such a test is also possible with our program, e.g. by requiring that the largest eigenvalue of the scattering matrix stays well below the unitarity bound of 1/2. Such an analysis becomes instructive when analyzing higher loop effects involving SUSY particles which potentially lead to large corrections of parameters for SM particles, like Higgs-fermion Yukawa couplings or the mass of the lightest CP-even Higgs boson. We have not yet performed such a systematic analysis.

References

1. J. A. Casas, A. Lleyda, and C. Muñoz, Nucl. Phys. B **471** (1996) 3-58.
2. B. W. Lee, C. Quigg, and G. B. Thacker, Phys. Rev. Lett. **38** (1977) 883 and Phys. Rev. D **16** (1977) 1519.
3. J. Hořejší, M. Kladiva, Eur. Phys. J. C **46** (2006) 81.
4. J. M. Frere, D. R. T. Jones, and S. Raby, Nucl. Phys. B **222** (1983) 11-19
5. ALEPH, DELPHI, L3, and OPAL Collaborations, Eur. Phys. J. C **47** (2006) 547-587.
6. S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Lett. B **455** (1999) 179.