Exploring the Landscape – Statistics of String Theory Vacua

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Introduction and Motivation

Intersecting brane models

Computational methods

Results

Conclusions

Outline

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   Statistics
   What to look for?

2 Intersecting brane models

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4 Results
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   Chiral matter
   Standard models
   Correlations

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String phenomenology

Problems

- There are zillions of possible low energy solutions. (The Landscape)
- BUT – No explicit construction that resembles the standard model is known.

Questions

- Will it be possible to predict all low energy observables from string theory?
- Might we have to invoke antropic reasoning? (cf. planetary orbits)
- Is there a fundamental principle for vacuum selection? (i.e. an "entropic principle")
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Statistics

- Explore as much of the landscape as possible, in regions as different as possible.
- Analyse results (low energy observables) statistically.
- Two possible methods:
  - Use statistics directly, calculate distributions of properties using a simplified measure on the space of solutions. [Denef, Douglas]
  - Construct solutions explicitly, analyse ensemble using statistical methods (counting).

  **IBMs:** [Blumenhagen, Douglas, FG, Honecker, Lüst, Stein, Taylor, Weigand]
  **Gepner:** [Anastasopoulos, Disjkstra, Kiritsis, Huiszoon, Schellekens]
  **Heterotic:** [Dienes, Lebedev, Lennek, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter]
What to look for?

**Common patterns**

- Look for similarities in frequency distributions of low energy observables.
- Exclude uninteresting regions of the landscape.
- Where are the huge numbers coming from – what kind of scenarios are common, which are rare?

**Correlations**

- Finding the same correlations in different regions of the landscape might lead to predictions.
- Could give hints to fundamental principles in string theory, yet to be discovered.
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Type IIA orientifolds with D6–branes at angles. [Berkooz, Douglas, Leigh]

Dual to type IIB with magnetised D9–branes.

Compactifications on $\mathbb{R}^{3,1} \times M$ to $\mathcal{N} = 1$ supersymmetric solutions in four dimensions.

$M$ compact, toroidal orbifold $T^6/G$ with $G = \mathbb{Z}_2 \times \mathbb{Z}_2$, $G = \mathbb{Z}_6$, $G = \mathbb{Z}_6'$ [Work in progress with G. Honecker].

Branes and orientifold planes wrap three–cycles $\Pi_a$ in $M$.

Possible cycles $\Pi$ given by $H_3(M, \mathbb{Z})$, which splits into parts even/odd under orientifold projection.

Chiral matter arises at intersections of brane stacks and it’s amount is computed by intersection numbers $I_{ab} = \Pi_a \circ \Pi_b$. 
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IBM's

Gauge group

- Total gauge group will be a semi–simple Lie group.
- Rank: $\sum_{a=1}^{k} N_a$ for $k$ stacks of branes with $N_a$ branes per stack.
- Factors: In general $U(N_a)$. $SO(2N)$ or $Sp(2N)$ if stack wraps the same cycle as the orientifold plane.

Chiral matter

<table>
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<tr>
<th>representations</th>
<th>multiplicity</th>
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Constraints

Supersymmetry

Calibration condition on three–cycles: sLags.

Tadpole cancellation

Cancellation of RR charge:

$$\sum_a N_a (\Pi_a + \Pi_{a'}) = R_{O6} \Pi_{O6}.$$ 

K-theory

$$\sum_a N_a \Pi_a \circ \Pi_p \equiv 0 \mod 2,$$

for any probe brane $p$ with $\Pi_p \circ \Pi_{O6} = 0$. [Uranga]
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Standard model embedding

Gauge group

\[ U(3)_a \times U(2)_b / Sp(2)_b \times U(1)_c \times U(1)_d \]

\[ U(3)_a = SU(3)_{QCD} \times U(1)_a \]

\[ U(2)_b = SU(2)_w \times U(1)_b \]

\[ U(1)_Y: \text{appropriate (massless) combination}\]

\[ Q_Y = \sum x_i Q_i \]
Computational methods

- Express the constraints in terms of algebraic equations.
- Formulate in algorithmic form.
- The full problem is NP–complete, but specific questions can be answered in polynomial time. [Douglas, Denef, Taylor]
- Large subsets of solutions however can be analysed. In the \( \mathbb{Z}_6' \) case all solutions can be computed.
- One has to be careful with the choice of subsets (unwanted bias).
Number of solutions

- In all cases it is possible to proof that the number of solutions is finite.

- Total number differs by 18 orders of magnitude: 
  \[ O(10^{10}) \text{ for } T^6/\mathbb{Z}_2 \times \mathbb{Z}_2, \]
  \[ O(10^{23}) \text{ for } T^6/\mathbb{Z}_6', \]
  \[ O(10^{28}) \text{ for } T^6/\mathbb{Z}_6. \]

- Differences can be understood from first principles (fractional cycles, constraints).
$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of the total rank $r$ of all models.
**Rank of the gauge group**

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$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of the rank of gauge group factors.
Gauge group factors

$T^6/\mathbb{Z}_6$: Frequency distribution of the rank of gauge group factors.
As a measure for the amount of chirality define the “mean chirality” as the amount of chiral matter per brane intersection:

\[ \chi := \frac{2}{k(k+1)} \sum_{a,b>a} (I_{ab} - I_{ab}') \]

(Normalisation such that a pure standard model would have \( \chi = 3. \))
$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of mean chirality.
Chiral matter

\( T^6/\mathbb{Z}_6 \): Frequency distribution of mean chirality.
$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of standard models with $g$ generations. blue: massive $U(1)$ allowed.
Number of generations

$T^6/\mathbb{Z}_6$: Frequency distribution of standard models with $g$ generations. red: non–standard Higgs allowed, blue: no exotic matter at all.
Correlation between number of bifundamental matter in $(N, N)$ and $(N, \bar{N})$ representations of the gauge groups. Left: IBMs ($\mathbb{Z}_6$), Right: Gepner models.
Conclusions

Summary

- Systematic studies of the landscape might be interesting.
- General features of specific constructions can be analysed.
- There exist non–trivial correlations.

Outlook

- Compare results from different corners of the landscape.
- Systematic search for correlating observables.
- Include more properties: Gauge– and Yukawa–couplings, etc.
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