Exploring the Landscape – Statistics of String Theory Vacua

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Outline

String phenomenology

Problems

- There are zillions of possible low energy solutions. (**The Landscape**)
- BUT – No explicit construction that resembles the standard model is known.

Questions

- Will it be possible to predict all low energy observables from string theory?
- Might we have to envoke antropic reasoning? (cf. planetary orbits)
- Is there a fundamental principle for vacuum selection? (i.e. an "entropic principle")

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Statistics

- Explore as much of the landscape as possible, in regions as different as possible.
- Analyse results (low energy observables) **statistically**.
- Two possible methods:
  - Use statistics directly, calculate distributions of properties using a simplified measure on the space of solutions. [Denef, Douglas]
  - Construct solutions explicitly, analyse ensemble using statistical methods (counting).
    IBMs: [Blumenhagen, Douglas, FG, Honecker, Lüst, Stein, Taylor, Weigand]
    Gepner: [Anastasopoulos, Disjkstra, Kiritsis, Huiszoon, Schellekens]
    Heterotic: [Dienes, Lebedev, Lennek, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter]

What to look for?

Common patterns

- Look for similarities in frequency distributions of low energy observables.
- Exclude uninteresting regions of the landscape.
- Where are the huge numbers coming from – what kind of scenarios are common, which are rare?

Correlations

- Finding the same correlations in different regions of the landscape might lead to predictions.
- Could give hints to fundamental principles in string theory, yet to be discovered.

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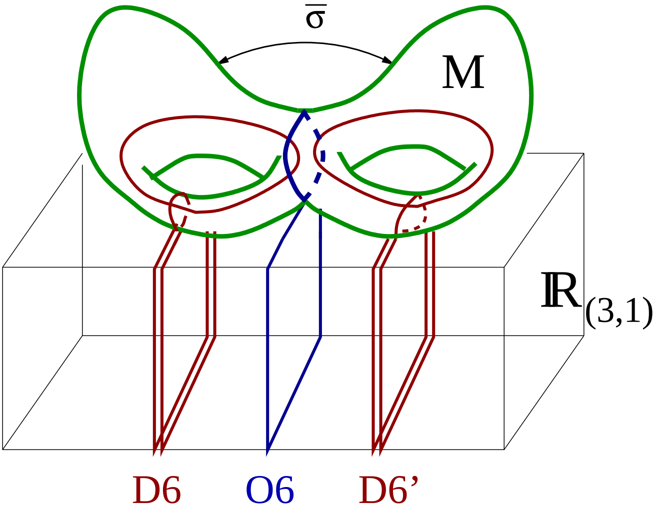
IBMs

- Type IIA orientifolds with D6–branes at angles.
  [Berkooz, Douglas, Leigh]
- Dual to type IIB with magnetised D9–branes.
- Compactifications on $\mathbb{R}^{3,1} \times M$ to $\mathcal{N} = 1$ supersymmetric solutions in four dimensions.
- M compact, toroidal orbifold T^6/G with $G = \mathbb{Z}_2 \times \mathbb{Z}_2$, $G = \mathbb{Z}_6$, $G = \mathbb{Z}_6'$ [Work in progress with G. Honecker].
- Branes and orientifold planes wrap three–cycles Π_a in M.
- Possible cycles Π given by $H_3(M, \mathbb{Z})$, which splits into parts even/odd under orientifold projection.
- Chiral matter arises at intersections of brane stacks and it's amount is computed by intersection numbers $I_{ab} = \Pi_a \circ \Pi_b$.

IBMs



IBMs

Gauge group

- Total gauge group will be a semi–simple Lie group.
- Rank: $\sum_{a=1}^{k} N_a$ for k stacks of branes with N_a branes per stack.
- Factors: In general $U(N_a)$. $SO(2N)$ or $Sp(2N)$ if stack wraps the same cycle as the orientifold plane.

Chiral matter

representations	multiplicity
$(\mathbf{N}_a, \overline{\mathbf{N}}_b)$	$\Pi_a \circ \Pi_b$
$(\mathbf{N}_a, \mathbf{N}_b)$	$\Pi_a \circ \Pi_b'$
$\mathbf{Sym}_a$	$\frac{1}{2}\left(\Pi_a \circ \Pi_{a'} - \Pi_a \circ \Pi_{O6}\right)$
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Constraints

Supersymmetry

Calibration condition on three–cycles: sLags.

Tadpole cancellation

Cancellation of RR charge:

$$\sum_a N_a \left(\Pi_a + \Pi_{a'}\right) = R_{O6} \Pi_{O6}.$$

K-theory

$$\sum_a N_a \Pi_a \circ \Pi_p \equiv 0 \mod 2,$$

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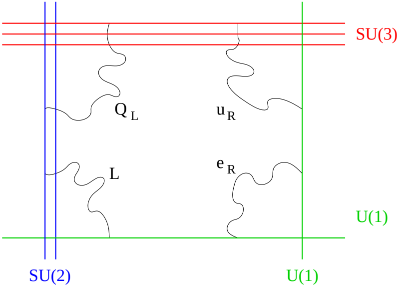
Standard model embedding

Gauge group

$$U(3)_a \times U(2)_b/Sp(2)_b \times U(1)_c \times U(1)_d$$

$U(3)_a = SU(3)_{QCD} \times U(1)_a$
$U(2)_b = SU(2)_w \times U(1)_b$
$U(1)_Y$: appropriate (massless) combination $Q_Y = \sum x_i Q_i$



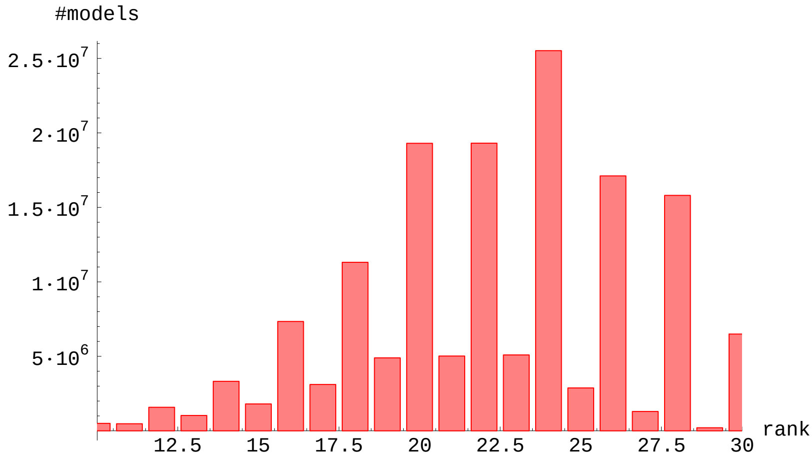
Computational methods

- Express the constraints in terms of algebraic equations.
- Formulate in algorithmic form.
- The full problem is NP–complete, but specific questions can be answered in polynomial time. [Douglas, Denef, Taylor]
- Large subsets of solutions however can be analysed. In the $\mathbb{Z}_6'$ case all solutions can be computed.
- One has to be careful with the choice of subsets (unwanted bias).

Number of solutions

- In all cases it is possible to proof that the number of solutions is finite.
- Total number differs by 18 orders of magnitude:
  $O(10^{10})$ for $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$,
  $O(10^23)$ for $T^6/\mathbb{Z}_6'$
  $O(10^{28})$ for $T^6/\mathbb{Z}_6$.
- Differences can be understood from first principles (fractional cycles, constraints).

Rank of the gauge group



$T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$: Frequency distribution of the total rank r of all models.

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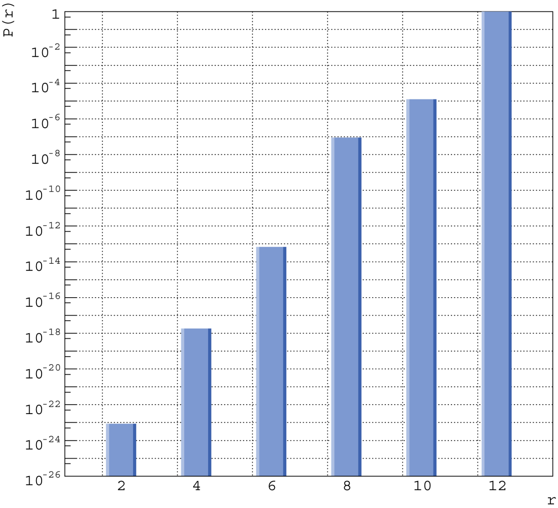
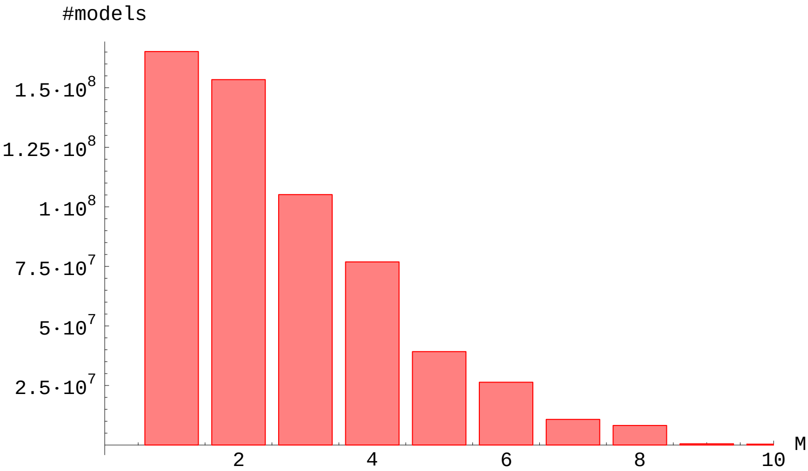


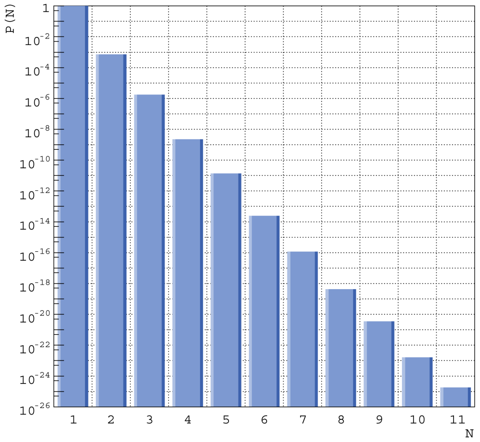
$T^6/\mathbb{Z}_6$: Frequency distribution of the total rank r of all models.

Gauge group factors



$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of the rank of gauge group factors.

Gauge group factors



$T^6/\mathbb{Z}_6$: Frequency distribution of the rank of gauge group factors.

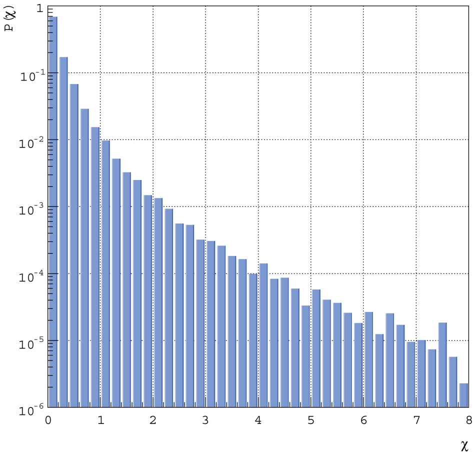
Mean chirality

As a measure for the amount of chirality define the "mean chirality" as the amount of chiral matter per brane intersection:

$$\chi := \frac{2}{k(k+1)} \sum_{a,b>a} (I_{ab} - I_{ab'})$$

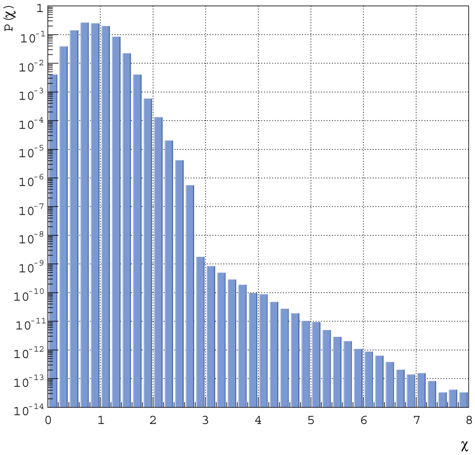
(Normalisation such that a pure standard model would have $\chi = 3$.)

Chiral matter



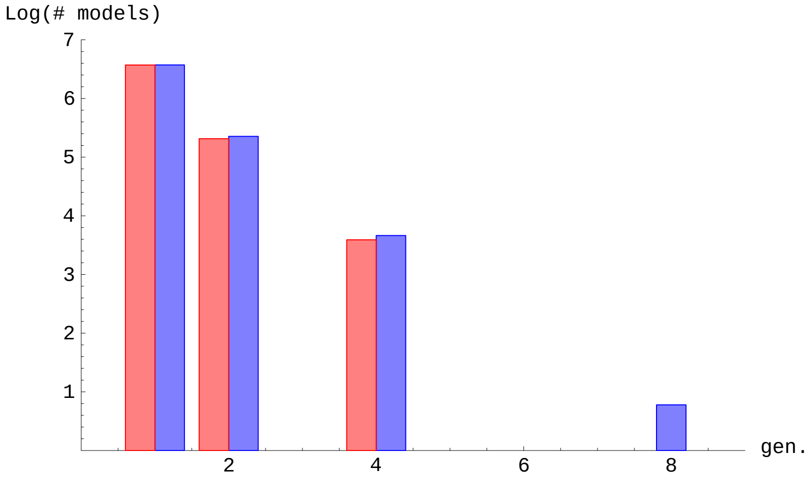
$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of mean chirality.

Chiral matter



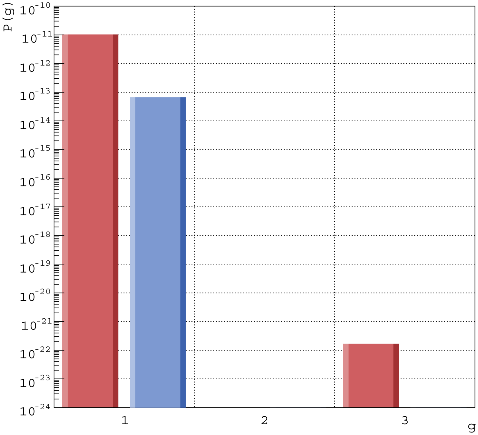
$T^6/\mathbb{Z}_6$: Frequency distribution of mean chirality.

Number of generations



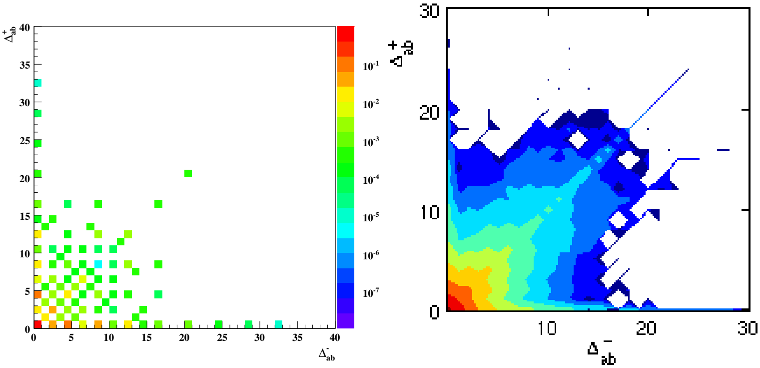
$T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$: Frequency distribution of standard models with g generations. blue: massive $U(1)$ allowed.

Number of generations



$T^6/\mathbb{Z}_6$: Frequency distribution of standard models with g generations. red: non–standard Higgs allowed, blue: no exotic matter at all.

Correlations



Correlation between number of bifundamental matter in (N, N) and $(N, \bar{N})$ representations of the gauge groups. Left: IBMs ($\mathbb{Z}_6$), Right: Gepner models.

Conclusions

Summary

- Systematic studies of the landscape might be interesting.
- General features of specific constructions can be analysed.
- There exist non–trivial correlations.

Outlook

- Compare results from different corners of the landscape.
- Systematic search for correlating observables.
- Include more properties: Gauge– and Yukawa–couplings, etc.

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