The S-parameter in (holographic) technicolor

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and additional work in progress

What is the S-parameter?

The S-parameter is the Peskin-Takeuchi electroweak parameter that can be nonzero even in fully custodially symmetric models.

$$\mathrm{Tr}\ W_{\mu\nu} H^\dagger B^{\mu\nu} H \tag{1}$$

In models with SU(2)$_L\times$ SU(2)$_R$, with hypercharge embedded in SU(2)$_R$, and H a bifundamental, the operator is

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S and technicolor

It's well-known that technicolor models tend to produce $S > 0$. There are ways of getting negative contributions, but (as far as I'm aware) every such method in the literature relies on an explicit breaking of custodial symmetry (e.g. Gates & Terning).

Our focus: neglect the custodial symmetry violation from the Standard Model gauging, and assume the new physics is fully left-right symmetric:

$$SU(2)_L \times SU(2)_R \to SU(2)_D \tag{3}$$

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Computing S

Define correlators Π_V and Π_A:

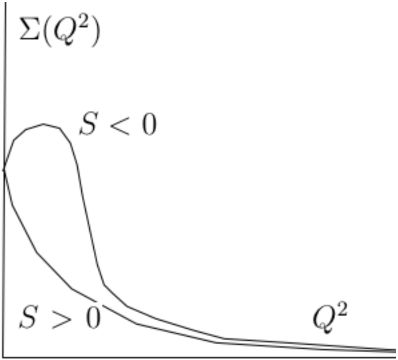
$$\delta^{ab}(q_\mu q_\nu - g_{\mu\nu}q^2)\Pi_J(q^2) = i\int d^4x\ e^{-iq\cdot x}\left\langle J^a_\mu(x)J^b_\nu(0)\right\rangle, \quad (4)$$

and consider $\Sigma(Q^2) = -Q^2(\Pi_V(Q^2) - \Pi_A(Q^2)) > 0$ (Witten).
We have $\Sigma(0) = f_\pi^2$,

$$S = -4\pi\Sigma'(0), \quad (5)$$

and $\Sigma(Q^2) \to \mathcal{O}\left(\frac{1}{Q^{2k}}\right)$ at large Q^2.

Plausibility: Why $S > 0$?



S in Randall-Sundrum

In RS2 with fermions on the IR brane, S is *negative*! With fermions on the UV brane, it is positive. This led to **ideal delocalization** (Cacciapaglia et al, Foadi et al): with approximately flat fermions, S is zero (orthogonality – fermions don't couple to KK modes).

On the other hand we really want to be free to have whatever fermion profile we like, for flexibility in studying flavor. Can we do it? Is there another way to get negative S?

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The Hirn & Sanz suggestion

Can absorb χSB effects in a different effective metric felt by vector and axial-vector fields. (hep-ph/0612239)
Convert an equation

$$-\frac{1}{w}\partial(w\partial\phi) + 2w^2v^2\phi = m^2\phi \tag{6}$$

to Schrödinger form with $\psi = \sqrt{w}\phi$ and

$$V = \frac{\partial^2\sqrt{w}}{\sqrt{w}} + w^2v^2, \tag{7}$$

so a new effective warp factor w_X is determined by

$$\frac{\partial^2\sqrt{w}}{\sqrt{w}} + w^2v^2 = \frac{\partial^2\sqrt{w_X}}{\sqrt{w_X}} \tag{8}$$

The $S < 0$ example

They were able to find effective metrics giving $S < 0$. For example, take the axial effective metric to be just Randall-Sundrum, and the vector to have warp factor

$$w(z) = \frac{1}{z} \exp\left(o_V z^4\right), \tag{9}$$

with o_V sufficiently negative.

This approach amounts to constructing two-point functions with desired properties, but tells us nothing about the details of the interacting theory. Can we get $S < 0$ in a controlled 5D effective field theory?

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$S < 0$ in a nice 5D effective theory?

Suppose we have a bulk Higgs that develops a VEV and breaks chiral symmetry. To begin with, we will only couple it with the lowest-dimension operator, so that $H^\dagger H$ becomes a mass^2 for the axial modes but not the vectors.

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The proof...

Go to the convenient coordinates (Barbieri et al) $ds^2 = e^{2\sigma}dx^2 + e^{4\sigma}dy^2$. Absorb the axial mass term into a kinetic term to show

$$S = \frac{8\pi}{g_5^2}\int_0^1 dy\ e^{2\sigma}(1-\rho^2), \tag{10}$$

where $\rho(y)$ solves

$$\partial_y^2\rho = m^2 g_5^2 e^{4\sigma}\rho, \tag{11}$$

$\rho'|_{y=1} = 0,\ \rho|_{y=0} = 1.$

It is clear that this $\rho(y)$ is strictly decreasing, so the integrand defining S is always positive.

What about higher-dimension operators?

Of course, we could also get contributions to S from higher-dimension operators in this effective theory. We can simply *add* the term $\mathrm{Tr}\ F_{L\mu\nu} H^\dagger F_R^{\mu\nu} H$ to the Lagrangian, for instance.

This term is a **kinetic** term with opposite signs for vector and axial. The trick is to make it large enough to make S negative, without creating any ghosts in the spectrum!

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Scanning parameter space

We don't have a general argument, but we can numerically scan over various choices of action in RS2 geometry and see whether making S negative always produces a ghost in the spectrum.

In particular, we can scan over power-law VEVs, $v(z) \sim z^{2+\beta}$, adding to the action a term $\frac{\alpha}{\Lambda^2} \mathrm{Tr}\ F_{L\mu\nu} H^\dagger F_R^{\mu\nu} H$ with α as large as possible without producing a ghost.

Results of scan

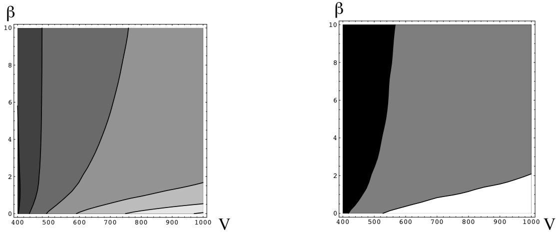

Figure 1: The contours of models with fixed values of the S-parameter due to the electroweak breaking sector. In the left panel we fix $1/R = 10^8$ GeV, while in the right $1/R = 10^{18}$ GeV. The gauge kinetic mixing parameter α is fixed to be the maximal value corresponding to the given V, β (and R' chosen such that the W mass is approximately reproduced). In the left panel the contours are $S = 1, 2, 3, 4, 5, 6$, while in the right $S = 1, 1.5, 2$.

Revisiting the plausibility argument

This is work in progress: our plausibility argument suggested that $\Sigma(Q^2)$ has a maximum as a function of real positive Q^2 if $S < 0$. Can we develop that into a general proof for $S > 0$?

Re $\Sigma(Q^2)$ is a harmonic function in the cut Q^2-plane. So it can't really have a maximum at positive Q^2: it must be increasing as we go away from the real axis. Let's follow the gradient flow out of that point.

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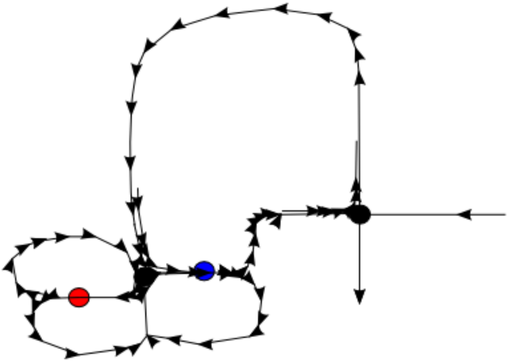
Gradient flow for negative S



Light Axial Resonance

Looking at the gradient flow strongly suggests that $S > 0$ is related to the lightest resonance being axial, not vector. (This is true in the example constructed by Hirn & Sanz, for instance.)

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Conclusions

- S is positive in all cases we analyze: a bulk Higgs gives a positive contribution
- Getting a negative S will require large effects from higher-dimension operators; loss of calculability
- Possibility that S is positive in *all* custodially symmetric models (work in progress)
- Weaker possibility: S is positive whenever ρ resonance is lighter than a_1 resonance