ffmssmsc – a C++ library for spectrum calculation and renormalization group analysis of the MSSM

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General information

Science is what we understand well enough to explain to a computer

- Main objectives
  - Given the set of low-energy (SM and QCD) "observables", and model of SUSY breaking, calculate the MSSM Lagrangian parameters.
  - Given the MSSM Lagrangian parameters, calculate physical (pole) masses of superpartners.
- Can be used from C++ and Scheme (a dialect of LISP language).
- Licence – GNU General Public Licence (GPL).
- Source code available from the public git repository at `http://theor.jinr.ru/~varg/git/hep/ffmssmsc.git`

Calculation of physical masses

- calculate running masses using well-known formulae
- add 1-loop radiative corrections (D. Pierce, J. Bagger, K. Matchev, R. Zhang, arXiv:hep-ph/9606211)
- In order to calculate mass spectrum one need to know the values of MSSM Lagrangian parameters.

Problem: RGEs with implicit boundary conditions

- In order to calculate mass spectrum one need to know the values of MSSM Lagrangian parameters.
- Since the nature of SUSY breaking is unknown, there are a lot (~ 100) arbitrary dimensionful couplings.
- In the context of certain models (e.g. minimal supergravity) there are relations between these "soft" couplings at the GUT scale.
- On the other hand, all experimental data are at the electroweak (or even lower) scale.

Implicit boundary conditions, example

In order to evaluate Yukawa coupling of the t quark from the observables, one need to calculate relation between the pole and running masses:

$$\frac{\Delta m_t}{m_t} \equiv \frac{M_t^{pole} - m_t^{\overline{\rm DR}}(\bar{\mu})}{m_t^{\overline{\rm DR}}(\bar{\mu})}$$

- The running masses of superpartners (unknown at this stage of calculation) enter this relation.
- Masses of superpartners depend on "soft" couplings, their values at the EW scale depends on gauge and Yukawa couplings (since β-functions of the "soft" couplings contain gauge and Yukawa ones). Gauge and Yukawa coupling are unknown at this stage of the calculation.

Inputs: SM and QCD "observables"

- $m_b^{5fl}(M_Z)$, $\alpha_s^{5fl}(M_Z)$, $\alpha_{em}^{5fl}(M_Z)$ $\Rightarrow m_b(M_Z), \alpha_s(M_Z), \alpha_{em}(M_Z)$
  1-loop MSSM decoupling (D. Pierce, J. Bagger, K. Matchev, R. Zhang, arXiv:hep-ph/9606211)
- M_W, M_Z, G_F $\Rightarrow g_1(M_Z)$, $g_2(M_Z)$, $v(M_Z)$
  1-loop MSSM corrections (ρ parameter: also 2-loop SM ones)
- M_t $\Rightarrow m_t(M_Z)$
  leading 1-loop MSSM corrections, 2-loop SQCD corrections (A. Bednyakov, D.I. Kazakov, AS, arXiv:hep-ph/0507139)
- M_τ $\Rightarrow m_\tau(M_Z)$
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Radiative EW symmetry breaking

- The V.E.Vs of the neutral CP-even Higgs fields satisfy the condition of minimum of the effective potential.
- This condition can be rewritten as a system of nonlinear equations on Higgs mixing parameters μ^2 and m_3^2.
- 1-loop (D. Pierce, J. Bagger, K. Matchev, R. Zhang, arXiv:hep-ph/9606211) + leading 2-loop corrections $\mathcal{O}(\alpha_s\alpha_{t,b} + \alpha_t\alpha_b + \alpha_{b,t}^2)$ (A. Dedes, G. Degrassi and P. Slavich, arXiv:hep-ph/0305127) to the MSSM effective potential are used.
- The running masses of the superpartners (unknown on this stage of the calculation) enter these equations.

SUSY breaking model and grand unification

- Gauge couplings are required to unify at the scale $\sim 10^{16}\,\mathrm{GeV}$
- *mSUGRA* conditions on soft SUSY breaking terms (other models can be easily implemented)

Sources of the errors

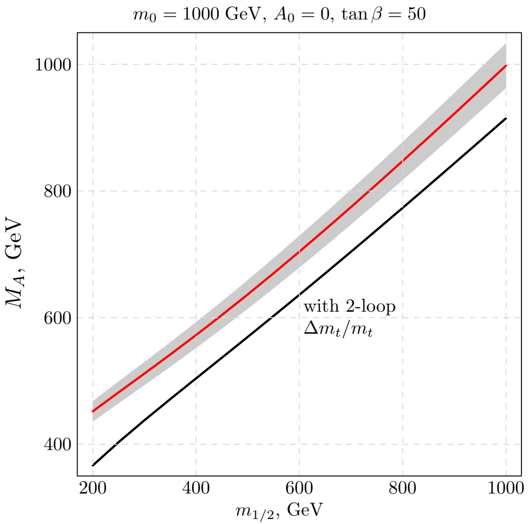
- Evaluation of the MSSM running couplings from the SM and QCD "observables": 1-loop radiative corrections (except the t quark mass).
- Radiative corrections to the masses are 1-loop (except some 2-loop contributions to the Higgs bosons masses).
- RG running is 2-loop (will not be covered here).
- Errors of numerical evaluation (mostly negligible compared to previously mentioned ones).

Uncertainties: SM & QCD matching

Two ways to estimate the uncertainties in the determination of the MSSM Lagrangian parameters:

- Make small variations of the low-energy input parameters.
- Calculate (and code) more radiative corrections to
  - the decoupling coefficient of the b quark mass m_b (A. Bednyakov, arXiv:0707.0650, required optimizations are almost done)
  - the decoupling coefficient of the strong coupling constant α_s (work in progress)
  - the t quark mass (done, A. Bednyakov, D.I. Kazakov, AS, arXiv:hep-ph/0507139)
  - the τ lepton mass
  - the effective EW mixing angle
  - the minimum condition of the MSSM effective potential

sparticles masses vs 2-loop corrections to the t quark mass



- Taking into account 2-loop (actually, α_s^2) corrections to the t quark mass changes the estimate of the value of the top Yukawa at the EW scale.
- Due to RG running *all* MSSM Lagrangian parameters get shifted.
- Thus predicted mass spectrum also changes.

Grey regions – discrepancy between different MSSM mass spectrum calculations programs (`http://cern.ch/kraml/comparison/`)

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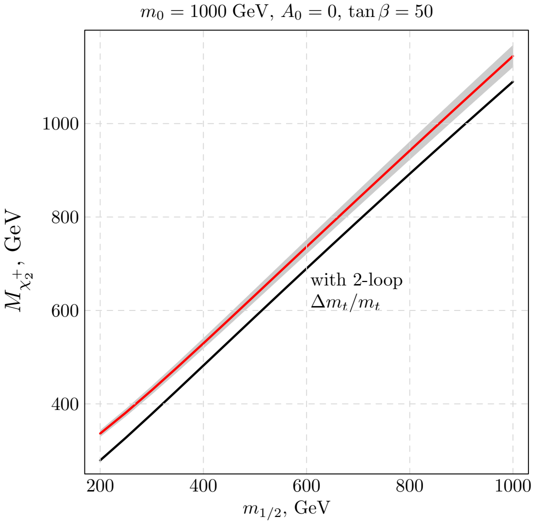

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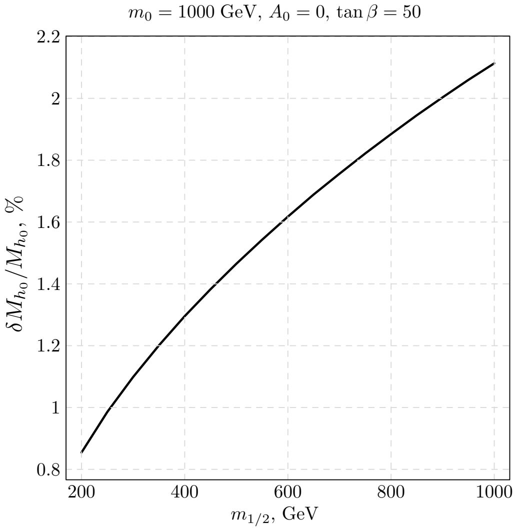

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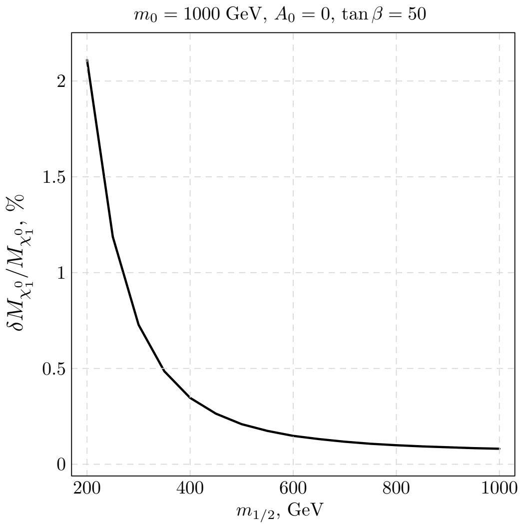

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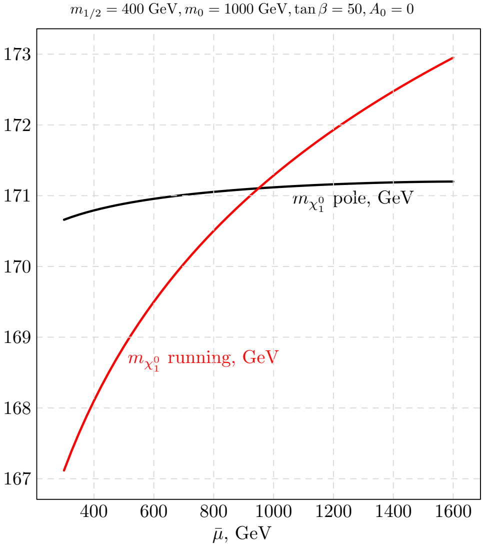
Uncertainties: rad. corrections to the sparticles masses



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- vary the scale where 1-loop ones are evaluated.
- The scale dependence of the pole mass gives an estimate of the higher-order corrections.

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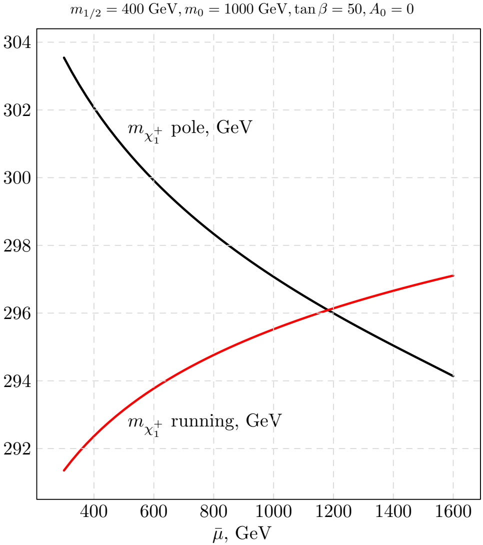

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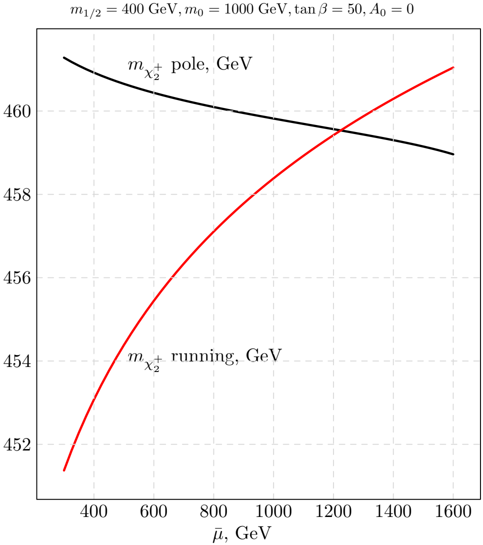

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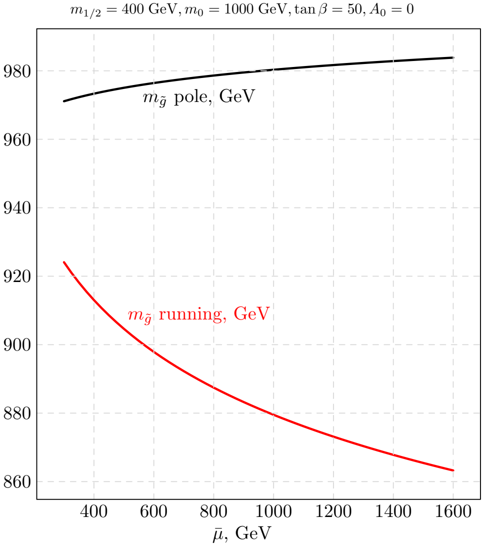

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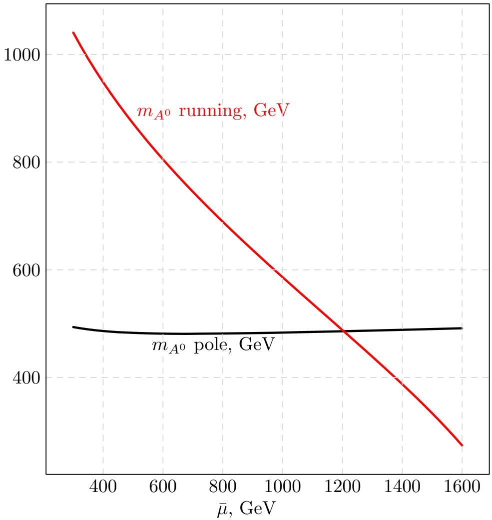
Uncertainties: rad. corrections to the sparticles masses

$m_{1/2} = 400$ GeV, $m_0 = 1000$ GeV, $\tan\beta = 50$, $A_0 = 0$



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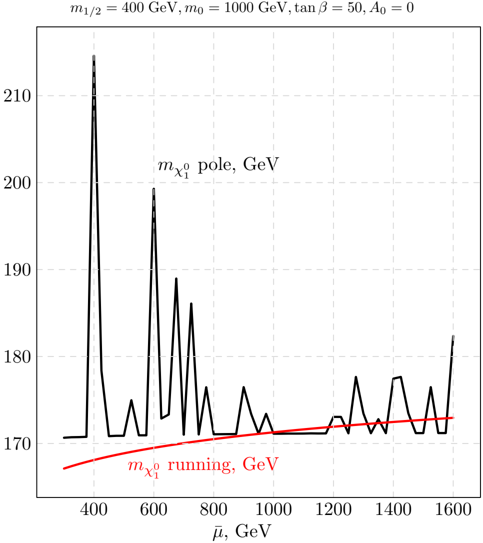
Numerical calculation errors

Method: increase the size of the mantissa of the FP (floating point) numbers (s/double/long double/g or use some arbitrary precision arithmetic library) and see how result changes.

- Numerical RGE integration: nothing to bother with. Beta-functions are very nice (polynomials in the couplings).
- Calculation of Feynman integrals. 1-loop ones are weird, but still manageable. Arbitrary precision FP arithmetics is used for 2-loop functions.

Accuracy estimates: role of FP rounding errors

Exact formula might be worse than approximate.



- If one just types in well-known expression for Passarino-Veltman B_0 function, the result is awful.
- Appropriate asymptotic expansion (A. N. Kuznetsov, F. V. Tkachov and V. V. Vlasov, arXiv:hep-th/9612037; V. A. Smirnov, Springer Tracts Mod. Phys. **177**, 1 (2002)), such as “large mass”, “large momentum”, “threshold”, improves the result.

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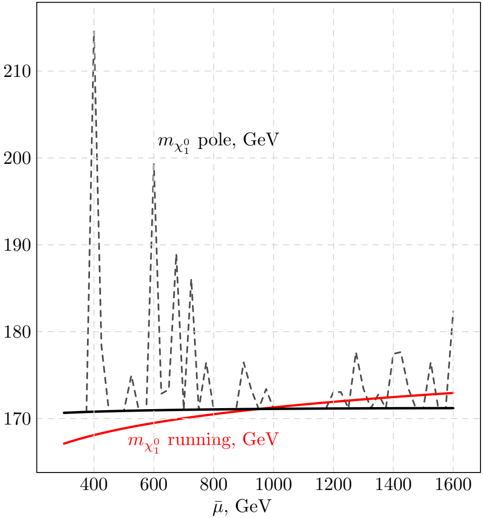

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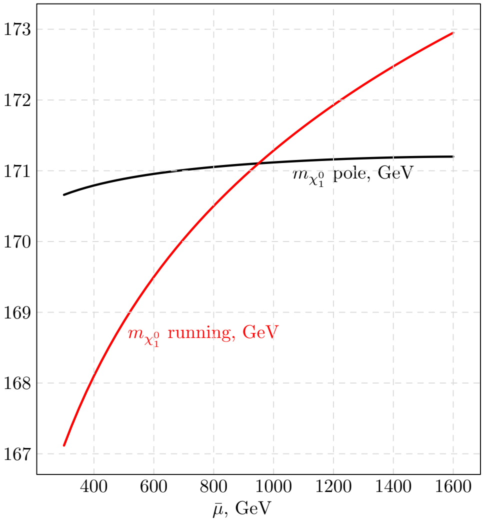

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Future work

- Analyse the effect of the 2-loop corrections to the MSSM $\longrightarrow$ QCD decoupling coefficient of the b quark mass.
- Analyse the effect of the 2-loop corrections to the MSSM $\longrightarrow$ QCD decoupling coefficient of the strong coupling α_s.
- 3-loop MSSM RGEs are known, may be run at 3 loops?
- Include more EW ($B \longrightarrow s\gamma$, a_μ, etc.) and cosmological inputs.
- Write the algorithm for finding the optimal scale for the pole mass calculation.
- Improve the documentation.
- Implement more SUSY breaking models.
- ffmssmsc is already fast, but can be (at least) 3 – 5 times faster.

Conclusion

- First free (as in "free speech") code for sparticles masses calculation.
- Given the set of SM and QCD "observables", and model of SUSY breaking, calculate the MSSM Lagrangian parameters (*fast*, errors: $\sim$ several %).
- Given the MSSM Lagrangian parameters, calculate physical (pole) masses of superpartners. (uncertainties: $\sim 10\%$ for heavy Higges and charginos, $\lesssim 5\%$ for the rest of the superpartners).

Thank you for your attention!
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