The MSSM Higgs sector & $\Delta M_{Bq}$ for large tan$\beta$

In collaboration with M. Gorbahn, S. Jäger and U. Nierste
- Soft SUSY-breaking $\rightarrow$ 2HDM-III structure at loop level:

$$\mathcal{L}_{\text{eff}} \supset \overline{d}_R^I [Y_d H_d + \epsilon_Y Y_d Y_u^\dagger Y_u H_u^c]^{IJ} \cdot Q_L$$

$\Rightarrow$ Higgs-mediated FCNC for large $\tan \beta \equiv t_\beta = v_u / v_d$:

$$\kappa_{IJ}^J \overline{d}_R^I d_L^J \left[ c_{\beta} h_u^{0*} - s_{\beta} h_d^{0*} \right] + \kappa_{JI}^J \overline{d}_L^I d_R^J \left[ c_{\beta} h_u^0 - s_{\beta} h_d^0 \right]$$

$$\kappa_{IJ} \sim \epsilon_Y t_\beta^2 m_1 / v$$

- Rich phenomenology!
Motivation

- Soft SUSY-breaking → 2HDM-III structure at loop level:

\[
\mathcal{L}_{\text{eff}} \supset \overline{d}_R^I \left[ Y_d H_d + \varepsilon_Y Y_d Y_u Y_u^+ H_u^c \right]^{IJ} \cdot Q_L^J
\]

(MFV)

\Rightarrow \text{Higgs-mediated FCNC for large } \tan\beta \equiv t_\beta = \nu_u / \nu_d:

\[
\kappa^{IJ} \overline{d}_R^I d_L^J \left[ c_\beta h_u^{0*} - s_\beta h_d^{0*} \right] + \kappa^{IJ*} \overline{d}_L^L d_R^I \left[ c_\beta h_u^{0} - s_\beta h_d^{0} \right]
\]

\[
\kappa^{IJ} \sim \varepsilon_Y t_\beta^2 m_1 / \nu
\]

- Rich phenomenology! Interesting signature within MFV: \( q = d, s \)

- Look at all (sub-)leading contributions before concluding!
Outline

I. $\Delta M_{B_q}$ anatomy

II. Matching MSSM $\rightarrow$ 2HDM

III. Higgs effects in $\Delta M_{B_q}$ versus $B_q \rightarrow \mu^+ \mu^-$
I. $\Delta M_{Bq}$ anatomy
Why the cancellation?

$$\Delta M_{q_b}^{(m_b^2)} \propto \frac{b_R - q_L}{m_b = 0}$$

[Sparrow, Kallo '00]

Sparicle masses $\gg$ Higgs masses $\Rightarrow$ effective 2HDM:

- $V = m_1^2 H^*_d H_d + m_2^2 H^*_u H_u + \{m_1 H_u \cdot H_d + h.c.\}$
- $\frac{g^2}{8} \left[ (H^*_d H_d) - (H^*_u H_u) \right]^2 + \frac{g^2}{8} (H^*_u H_d) (H^*_d H_u)$
- $\mathcal{L}_{\text{FCNC}}^{\text{Higgs}} = \kappa^{IJ} \bar{d}_R^I d_L^J \left[ c_\beta h_u^0 - s_\beta h_d^0 \right] + \kappa^{IJ^*} \bar{d}_L^J d_R^I \left[ c_\beta h_u^0 - s_\beta h_d^0 \right]$

- Dim-4 operators, $V$ at tree-level
- After SSB, for $\tan \beta \to \infty$, the theory is inv. under the PQ-type symmetry:

$$U(1)_{PQ} : Q(H_d) = Q(d_R^I) = 1, \quad Q(\text{other}) = 0$$
Why the cancellation?

\[
\Delta M_{q_b}^{(m_b^2)} \propto \begin{align*}
\bar{b}_R & \rightarrow h_d^0 \quad h_d^{0*} \rightarrow b_R \\
q_L & \rightarrow m_b \quad m_b \quad \bar{q}_L
\end{align*}
\]

\[\Delta Q = 2 \Rightarrow = 0 \text{ (LO in } 1/\tan\beta)\]

Sparticle masses \(\gg\) Higgs masses \(\Rightarrow\) effective 2HDM:

- \(V = m_{11}^2 H_d^\dagger H_d + m_{22}^2 H_u^\dagger H_u + \left\{ m_{12}^2 H_u H_d + h.c. \right\} + \frac{\tilde{g}^2}{8} \left[ (H_d^\dagger H_d) - (H_u^\dagger H_u) \right]^2 + \frac{g^2}{8} (H_u^\dagger H_d) (H_d^\dagger H_u) \)

- \(\mathcal{L}_{\text{FCNC}}^{\text{Higgs}} = \kappa^{IJ} \bar{d}_R^J d_L^I \left[ c_\beta h_u^{0*} - s_\beta h_d^{0*} \right] + \kappa^{IJ*} \bar{d}_L^J d_R^I \left[ c_\beta h_u^0 - s_\beta h_d^0 \right] \)

- Dim-4 operators, \(V\) at tree-level

- After SSB, for \(\tan\beta \rightarrow \infty\), the theory is inv. under the PQ-type symmetry:

\[U(1)_{\text{PQ}} : \quad Q(H_d) = Q(d_R^I) = 1, \quad Q(\text{other}) = 0\]
What are the leading contributions?

Look at all contributions with 1 suppression factor
What are the leading contributions?

A/ Chirality flipped contribution ("LR")

\[ \Delta Q = 0 \quad \Rightarrow \quad \Delta M_q^{LR} \propto \frac{m_b m_q}{v^2} \quad \text{decreases } \Delta M_q \]

[Buras, Chankowski, Rosiek, Sławianowska ’02]
What are the leading contributions?

A/ Chirality flipped contribution ("LR")

\[
\Delta Q = 0 \quad \Rightarrow \quad \Delta M^L_R \propto \frac{m_b m_q}{v^2} \quad \text{decreases } \Delta M_q
\]

[Buras, Chankowski, Rosiek, Sławianowska '02]

B/ Corrections to Higgs masses/mixings ("RR")

\[
\Delta Q = 2 \quad \Rightarrow \quad \Delta M^R_R \propto \frac{m_b^2}{v^2} \times \text{SUSY loop in Higgs potential}
\]

Corrections to the Higgs sector have already been extensively studied. However, contradictory statements about their effects on $B - \bar{B}$ mixing are found in the literature. We thus go through them again in part II.
Higher dimension operator contribution

\[ \Delta Q = 2 \Rightarrow \Delta M_{q}^{d6} \propto \frac{m_{b}^{2}}{v^{2}} \times \frac{v^{2}}{M_{SUSY}^{2}} \]

Higgs-FCNC are always of the type \( \overline{d}_{R}^{I}d_{L}^{J} h_{d}^{0*} / \overline{d}_{L}^{I}d_{R}^{J} h_{d}^{0} \) for large \( \tan \beta \), no matter the number of Higgs field insertions

\[ \rightarrow \text{The required breaking of the PQ symmetry cannot be produced and } \Delta M_{q}^{d6} = 0 \text{ at LO in } \tan \beta \]
C/ Higher dimension operator contribution

\[
\Delta Q = 2 \implies \Delta M_{q}^{d6} \propto \frac{m_{b}^{2}}{v^{2}} \times \frac{v^{2}}{M_{SUSY}^{2}}
\]

Higgs-FCNC are always of the type \( \bar{d}_{R}^{I}d_{L}^{J}h_{d}^{0*}/\bar{d}_{L}^{I}d_{R}^{J}h_{d}^{0} \) for large \( \tan \beta \), no matter the number of Higgs field insertions

\[\rightarrow \text{The required breaking of the PQ symmetry cannot be produced and } \Delta M_{q}^{d6} = 0 \text{ at LO in } \tan \beta\]

D/ Weak scale loop contribution

\[
\Delta Q = 0 \implies \Delta M_{q}^{WS} \propto \frac{m_{b}^{2}}{v^{2}} \times \frac{y_{b}^{2}}{16 \pi^{2}}
\]

increases \( \Delta M_{q} \), but numerically small
II. Matching
MSSM → 2HDM

(→ Corrections to Higgs masses/mixings )
Higgs potential at 1-loop level

V has the most general structure compatible with gauge symmetry:

\[ V = m_{11}^2 H_d^+ H_d + m_{22}^2 H_u^+ H_u + \left\{ m_{12}^2 H_u \cdot H_d + h.c. \right\} \]
\[ + \frac{\lambda_1}{2} (H_d^+ H_d)^2 + \frac{\lambda_2}{2} (H_u^+ H_u)^2 + \lambda_3 (H_u^+ H_u)(H_d^+ H_d) + \lambda_4 (H_u^+ H_d)(H_d^+ H_u) \]
\[ + \left\{ \frac{\lambda_5}{2} (H_u \cdot H_d)^2 - \lambda_6 (H_d^+ H_d)(H_u \cdot H_d) - \lambda_7 (H_u^+ H_u)(H_u \cdot H_d) + h.c. \right\} \]

Ex: \[ \lambda_5 = -\frac{3y_t^4 a_t^2 \mu^2}{8\pi^2 M_{\tilde{t}_R}^4} L_1 \left( \frac{M_{\tilde{Q}_L}^2}{M_{\tilde{t}_R}^2} \right) + \ldots \]
\[ L_1(x) = \frac{-1}{(1-x)^2} - \frac{(1+x)\ln x}{2(1-x)^3} \]

Note: many refs! However, due to the large cancellations at play in \( \Delta M_q \), a fully analytical treatment is desirable, and the explicit expressions for the \( \lambda \)'s we found in the literature are given for \( M_{\tilde{Q}_L}^2 = M_{\tilde{t}_R}^2 = M_{b_R}^2 \).

[Haber, Hempfling '93][Carena, Espinosa, Quirós, Wagner '95][Higgs physics at LEP-2 WG '96]
WF renormalization and definition of $\tan\beta$

\[ \mathcal{L}_{\text{Kin}} = Z_{uu}^r \partial_\mu H_u^\dagger \partial^\mu H_u + Z_{dd}^r \partial_\mu H_d^\dagger \partial^\mu H_d + \left\{ Z_{ud} \partial_\mu H_u \cdot \partial^\mu H_d + \text{h.c.} \right\} \]

\[
\begin{pmatrix}
H_u' \\
-H_d^c'
\end{pmatrix} = 
\begin{pmatrix}
1 + (\delta Z_{uu}^r + i\delta H_{uu}^r)/2 & (\delta Z_{ud}^* + i\delta H_{ud}^*)/2 \\
(\delta Z_{ud} + i\delta H_{ud})/2 & 1 + (\delta Z_{dd}^r + i\delta H_{dd}^r)/2
\end{pmatrix}
\begin{pmatrix}
H_u \\
-H_d^c
\end{pmatrix} \text{ arbitrary}
\]

$m_{11}^2, m_{22}^2, \text{Im} m_{12}^2$ are renormalized such that the bare fields in the eff. 2HDM stay at the minimum of the potential $\Rightarrow v_{i,\text{eff}} = Z_{WF,ij} v_{j,\text{tree}}$

We exploit the freedom to change the Higgs basis to
- keep the vevs real and positive
- prevent $\tan\beta$ from getting $\tan\beta$-enhanced corrections!

\[
\begin{pmatrix}
v_{u,\text{eff}} \\
v_{d,\text{eff}}
\end{pmatrix} = 
\begin{pmatrix}
1 + \delta Z_{uu}^r /2 + i t_\beta^{-1} \delta Z_{ud}^i & \delta Z_{ud}^* \\
0 & 1 + \delta Z_{dd}^r /2
\end{pmatrix}
\begin{pmatrix}
v_{u,\text{tree}} \\
v_{d,\text{tree}}
\end{pmatrix}
\]
Corrections to Higgs masses and mixings

\[ \mathcal{F} = \sin^2(\alpha - \beta) \frac{M_H^2}{M_H^2} + \cos^2(\alpha - \beta) \frac{M_h^2}{M_h^2} - \frac{1}{M_A^2} \neq 0 \]

Corrected masses and mixing angles obtained from the diagonalization of \( M_{ij} = \partial^2 V / \partial h_i^0 \partial h_j^0 \)

+ Higgs WF renormalization in the effective FCNC vertex

**Earlier approaches**

[Parry '06]: Corrections to \( \alpha, \beta, M_{h,H,A} \) using the FeynHiggs package

[Freitas, Gasser, Haisch '07]: \( \delta \mathcal{F} \propto \frac{M_h^2}{M_H^2 - M_h^2} \varepsilon_{GP} \)

This pole singularity is not present in our result

⚠️ There are many cancellations at play. These are built in in the effective Lagrangian approach.
III. Higgs effects in
\( \Delta M_{B_q} \) vs \( B_q \rightarrow \mu^+ \mu^- \)
**Final formulae**

\[
\mathcal{B}(B_{s,d} \to \mu^+ \mu^-) = \left\{ \begin{array}{c}
3.9 \cdot 10^{-5} \\
1.2 \cdot 10^{-6}
\end{array} \right\} X \frac{M_W^2}{M_A^2} \left[ \frac{\tan \beta}{50} \right]^2
\]

\[
(\Delta M - \Delta M^{SM})_{s,d} = \left\{ \begin{array}{c}
-14 \, \text{ps}^{-1} \\
\sim 0 \, \text{ps}^{-1}
\end{array} \right\} X \left[ \begin{array}{c}
m_s \\
0.06 \text{GeV}
\end{array} \right] \left[ \begin{array}{c}
m_b \\
3 \text{GeV}
\end{array} \right] \left[ \begin{array}{c}
P_{LR}^2 \\
2.56
\end{array} \right]
\]

\[
+ \left\{ \begin{array}{c}
+ 4.4 \, \text{ps}^{-1} \\
+ 0.13 \, \text{ps}^{-1}
\end{array} \right\} X \frac{M_W^2}{M_A^2} \left( -\lambda_5 + \frac{\lambda_7^2}{\lambda_2} \right) (16\pi^2) \left[ \begin{array}{c}
m_b \\
3 \text{GeV}
\end{array} \right]^2 \left[ \begin{array}{c}
P_{SLL}^{LR} \\
-1.06
\end{array} \right]
\]

\[
+ \left\{ \begin{array}{c}
0.16 \, \text{ps}^{-1} \\
0.005 \, \text{ps}^{-1}
\end{array} \right\} X y_b^2 \left[ \begin{array}{c}
m_b \\
3 \text{GeV}
\end{array} \right]^2 \left[ \begin{array}{c}
\eta_B \hat{B} \\
0.715
\end{array} \right] \quad \leftarrow \text{small!}
\]

\[
X = \frac{(\varepsilon_Y 16\pi^2)^2}{(1 + \varepsilon_3 \tan \beta)^2 (1 + \varepsilon_0 \tan \beta)^2} \frac{m_t^4}{M_W^2 M_A^2} \left[ \frac{\tan \beta}{50} \right]^4 \left\{ \begin{array}{c}
|V_{ts}| = 0.041; \quad F_{B_s} = 0.24 \text{GeV} \\
|V_{td}| = 0.0086; \quad F_{B_d} = 0.20 \text{GeV}
\end{array} \right\
\]

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[Babu, Kolda '00] [Chankowski, Sławianowska '01] [Bobeth et al '01] [Huang et al '01] [Buras et al '02] [Isidori, Retico '01]…
Sensitivity to Higgs self-couplings

\[
(\Delta M - \Delta M^{SM})_{s,d} = \begin{cases} 
-14 \, \text{ps}^{-1} \\
0 \, \text{ps}^{-1} 
\end{cases} \times \left[ \frac{m_s}{0.06 \, \text{GeV}} \right] \left[ \frac{m_b}{3 \, \text{GeV}} \right] \left[ \frac{P_2^{LR}}{2.56} \right] 
+ \begin{cases} 
+4.4 \, \text{ps}^{-1} \\
+0.13 \, \text{ps}^{-1} 
\end{cases} \times \frac{M_W^2 (-\lambda_5 + \lambda_7^2 / \lambda_2) (16\pi^2)}{M_{\lambda}^2} \times \left[ \frac{m_b}{3 \, \text{GeV}} \right]^2 \left[ \frac{P_1^{SLL}}{-1.06} \right]
\]

Typically: \( M_{\tilde{q}} = \mu = a_{t,b} \) \( \Rightarrow \frac{(y_t^2 + y_b^2)}{2} \frac{M_W^2}{M_{\lambda}^2} \)

- Sensitivity to PQ-violating Higgs self-couplings rather limited… (⊗)
- In a large part of the parameter space, the correlation to \( B(B_s \rightarrow \mu^+ \mu^-) \) remains function of \( (M_{\lambda}, \tan \beta) \) only (😊)
New effect: for small $M_A$ only

$\Delta M_s/\Delta M_s^{\text{SM (central val.)}}$

$\Delta M_d/\Delta M_d^{\text{SM (central val.)}}$

Plain: $\Delta M_q^{\text{SM+LR+RR}}$
Dashed: $\Delta M_q^{\text{SM+LR}}$
Shaded: size of th. error

$\tan \beta = 50$; $M_{\tilde{q}} = M_{\tilde{g}} = 0.8 \text{TeV}$

$a_{t,b} = 1 \text{TeV}; \mu = 1.2 \text{TeV}$

$M_{t_Lt_R}^2 = -m_t(a_t + \mu^* \cot \beta)$
$\Delta M_{B_q}$ versus $B_q \to \mu^+ \mu^-$

$M_A = 0.14 TeV; \ tan \beta = 50$
$M_{\tilde{q}} = M_{\tilde{g}} = 0.8 TeV$
$a_t = a_b \in [0, 1.5 TeV]; \ \mu = 1.2 TeV$

The exp. bound on $B(B_s \to \mu^+ \mu^-)$ excludes a decrease of $\Delta M_s$ if the only relevant NP effect is $\Delta M_s^{LR}$

[Altmannshofer, Buras, Guadagnoli '07]

This is even more so in the presence of $\Delta M_s^{RR}$. Besides, the expected increase in the $B_d$ system is correlated, and thus constrained to be quite small.

Large effects could survive in some corners of parameter space? (under investigation)
Conclusion
Conclusion

- Systematic investigation of all leading contributions to $\Delta M_q$ in the MFV-MSSM with large $\tan \beta$ and heavy sparticles.

- No new large effects are found. Still, corrections to Higgs masses/mixings can be relevant for small $M_A (< 200 \text{ GeV})$. They add to the SM contribution.

- Correlation to $\mathcal{B}(B_q \to \mu^+ \mu^-)$:

  \[
  \begin{align*}
  \Delta M_s & \text{ decreased (but less),} \\
  \Delta M_d & \text{ increased}
  \end{align*}
  \]

  \[
  \begin{align*}
  \mathcal{B}(B_q \to \mu^+ \mu^-) & \text{ increased}
  \end{align*}
  \]

- With all contributions under control: the present experimental bounds on $\mathcal{B}(B_q \to \mu^+ \mu^-)$ impede a significant decrease (increase) of $\Delta M_s (\Delta M_d)$. 