

One-loop corrections to three-body leptonic chargino decays

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Abstract. We calculate full one-loop corrections to the genuine three-body decays of the light chargino $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm \nu_\ell$ in the Minimal Supersymmetric Standard Model. We find that the corrections to the decay width can be of the order of a few percent. We show also how radiative corrections affect energy and angular distributions of the final lepton.

PACS. 14.80.Ly Supersymmetric partners of known particles – 12.15.Lk Electroweak radiative corrections

1 Introduction

Supersymmetry (SUSY) [1] is one of the most promising and best motivated extensions of the Standard Model (SM) of particle physics. The search for SUSY is one of the main goals at the present and future colliders. All SUSY theories contain charginos, the spin-1/2 superpartners of charged gauge bosons and Higgs bosons. In many scenarios charginos are expected to be light enough to be copiously produced at future high energy colliders — the Large Hadron Collider (LHC) [2] and the International Linear Collider (ILC) [3].

Once the supersymmetric particles will have been discovered it will be crucial to measure their masses, mixing, couplings and CP violating phases to reconstruct the fundamental SUSY parameters and get insight of physics at very high energy scales. To meet the requirements of very high experimental precision at the ILC it is very important to include in theoretical calculations higher-order loop corrections to physical processes and observables.

Chargino production has been thoroughly analyzed at one-loop level in the literature [4,5,6]. Recently full one-loop analysis of chargino decays was published [7], however showing only corrections to the decay widths and branching fractions of charginos.

In this note we report on the calculation of the full one-loop corrections to the genuine three-body chargino decays to the lightest neutralino, lepton and neutrino

$$\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm \nu_\ell \quad (1)$$

The calculation was performed for the complex Minimal Supersymmetric Standard Model (MSSM) and it allows inclusion of CP-violating effects in future. In the presented results we include corrections to the decay widths and to the energy and angular distributions of

the final lepton. We also show the impact of QED corrections. We compare differences between the decay to electron and to τ .

The paper is organized as follows. In Sec. 2 we recapitulate chargino and neutralino sectors of MSSM at the tree-level. In Sec. 3 we introduce renormalization scheme and analyze the structure of one-loop corrections to the decay (1). In Sec. 4 we present our numerical results and finally in Sec. 5 we summarize our findings and give outlook for future developments.

2 Gaugino/higgsino sector of the MSSM

2.1 Chargino mixing

In the MSSM, the tree-level mass matrix of the spin-1/2 partners of the charged gauge and Higgs bosons, \tilde{W}^+ and \tilde{H}^+ , takes the form

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2}m_W \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix}, \quad (2)$$

where M_2 is the SU(2) gaugino mass, μ is the higgsino mass parameter, and $\tan\beta$ is the ratio v_2/v_1 of the vacuum expectation values of the two neutral Higgs fields. By reparametrization of the fields, M_2 can be taken real and positive, while μ can be complex $\mu = |\mu| e^{i\Phi}$. Since the chargino mass matrix \mathcal{M}_C is not symmetric, two different unitary matrices are needed to diagonalize it

$$U^* \mathcal{M}_C V^\dagger = \begin{pmatrix} m_{\tilde{\chi}_1^\pm} & 0 \\ 0 & m_{\tilde{\chi}_2^\pm} \end{pmatrix}. \quad (3)$$

U and V matrices act on the left- and right-chiral $\psi_{L,R} = (\tilde{W}, \tilde{H})_{L,R}$ two-component states

$$\tilde{\chi}_j^R = U_{jk} \psi_k^R, \quad \tilde{\chi}_j^L = V_{jk} \psi_k^L, \quad (4)$$

giving two mass eigenstates $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$.

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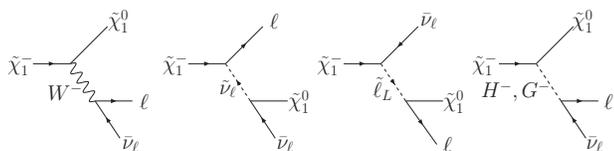


Fig. 1. Diagrams contributing to the leptonic three-body chargino decay $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm \tilde{\nu}_\ell$.

2.2 Neutralino mixing

In the MSSM, four neutralinos $\tilde{\chi}_i^0$ ($i = 1, 2, 3, 4$) are mixtures of the neutral U(1) and SU(2) gauginos, \tilde{B} and \tilde{W}^3 , and the SU(2) higgsinos, \tilde{H}_1^0 and \tilde{H}_2^0 . The neutralino mass matrix in the $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} \quad (5)$$

is built up by the fundamental SUSY parameters: the U(1) and SU(2) gaugino masses M_1 and M_2 , the higgsino mass parameter μ , and $\tan\beta$ ($c_\beta = \cos\beta$, $s_W = \sin\theta_W$ etc.). In addition to the μ parameter a non-trivial CP phase can also be attributed to the M_1 parameter. Since the matrix \mathcal{M}_N is symmetric, one unitary matrix N is sufficient to rotate the gauge eigenstate basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ to the mass eigenstate basis of the Majorana fields $\tilde{\chi}_i^0$

$$\mathcal{M}_{\text{diag}} = N^* \mathcal{M}_N N^\dagger. \quad (6)$$

The mass eigenvalues m_i ($i = 1, 2, 3, 4$) in $\mathcal{M}_{\text{diag}}$ can be chosen real and positive by a suitable definition of the unitary matrix N .

2.3 Chargino decays at tree level

At the tree level several channels contribute to the chargino decays to leptons (1). This can be W^\pm , $\tilde{\ell}_L$, $\tilde{\nu}_\ell$, H^\pm and G^\pm exchanges, see Fig. 1. For the decays to light fermions the Higgs and Goldstone boson exchange channels can be neglected because the contribution is strongly suppressed by the tiny Yukawa couplings.

3 One-loop corrections

3.1 Renormalization scheme

Since one-loop corrections introduce ultraviolet divergences we need to apply a proper renormalization scheme to obtain physically meaningful results. In this analysis we choose to work in the on-shell scheme. This means that our renormalization conditions are defined requiring the pole of the propagator and residue equal 1 at the physical masses of particles. To regularize one-loop integrals we use dimensional reduction, which preserves supersymmetry [8]. For renormalization of SM

parameters and fields we follow closely procedure given in [9].

Renormalization of the chargino and neutralino sectors is performed in the mass eigenstate basis [5]. We introduce in the Lagrangian the wave function and mass counterterms with the following substitution

$$\begin{aligned} \tilde{\chi}_i &\rightarrow (\delta_{ij} + \frac{1}{2} \delta \tilde{Z}_{ij}^L P_L + \frac{1}{2} \delta \tilde{Z}_{ij}^R P_R) \tilde{\chi}_j, \\ m_{\tilde{\chi}_i} &\rightarrow m_{\tilde{\chi}_i} + \delta m_{\tilde{\chi}_i}, \end{aligned} \quad (7)$$

where $\delta \tilde{Z}$ stands for either the chargino or the neutralino field renormalization constants, $\delta \tilde{Z}^\pm$ or $\delta \tilde{Z}^0$, respectively. Similar substitution has to be done for sleptons

$$\begin{aligned} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} &\rightarrow \begin{pmatrix} 1 + \frac{1}{2} \delta Z_{11}^{\tilde{f}} & \frac{1}{2} \delta Z_{12}^{\tilde{f}} \\ \frac{1}{2} \delta Z_{21}^{\tilde{f}} & 1 + \frac{1}{2} \delta Z_{22}^{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}, \\ m_{\tilde{f}_i}^2 &\rightarrow m_{\tilde{f}_i}^2 + \delta m_{\tilde{f}_i}^2. \end{aligned} \quad (8)$$

For the renormalization of $\tan\beta$ we take the condition that the CP-odd Higgs boson A^0 does not mix with Z boson on-shell

$$\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{m_Z \sin 2\beta} \text{Im} \left[\text{Re} \Sigma_{A^0 Z}(m_{A^0}^2) \right], \quad (9)$$

where $\Sigma_{A^0 Z}(m_{A^0}^2)$ is the self-energy for $A^0 - Z$ mixing [10].

We also have to define the chargino and neutralino rotation matrices at one-loop level. We define them in such a way that they remain unitary. Thus for charginos we have

$$\begin{aligned} \delta U_{ij} &= \frac{1}{4} \sum_{k=1}^2 \left(\delta \tilde{Z}_{ik}^{\pm, R} - (\delta \tilde{Z}_{ki}^{\pm, R})^* \right) U_{kj}, \\ \delta V_{ij} &= \frac{1}{4} \sum_{k=1}^2 \left(\delta \tilde{Z}_{ik}^{\pm, L} - (\delta \tilde{Z}_{ki}^{\pm, L})^* \right) V_{kj}, \end{aligned} \quad (10)$$

and for neutralinos

$$\delta N_{ij} = \frac{1}{4} \sum_{k=1}^4 \left(\delta \tilde{Z}_{ik}^{0, L} - \delta \tilde{Z}_{ki}^{0, R} \right) N_{kj}. \quad (11)$$

Our procedure is kept general to accommodate complex phases that may appear in the MSSM Lagrangian and to calculate CP-odd effects.

3.2 Calculation at one loop

Radiative corrections to the chargino decay include the following generic one-loop Feynman diagrams: the box diagram contributions, the virtual vertex corrections, the self-energy corrections. They are displayed in Fig. 2. To obtain finite results we also have to include proper counterterms at vertices and propagators.

Since the number of diagrams exceeds 200 an automated computation package has to be used. Generation and calculation of one-loop graphs was performed using **FeynArts 3.2** and **FormCalc 5.2** packages [11]. For numerical evaluation of loop integrals

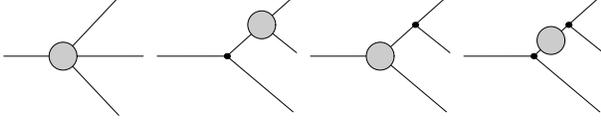


Fig. 2. Generic one-loop diagrams for chargino decays: box, vertex and self-energy corrections.

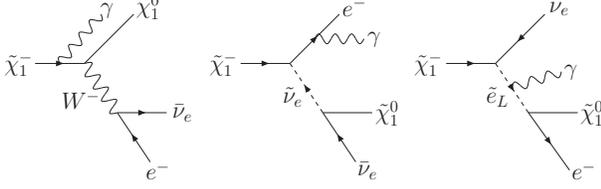


Fig. 3. Sample diagrams for photonic corrections to chargino decays.

we have used `LoopTools 2.2` [12]. For our purpose we have included in `FeynArts` model file for MSSM the necessary counterterms derived as it was described in Sec. 3.1. All results have been checked against UV-finiteness.

3.3 QED corrections

Some of the one-loop diagrams contain virtual photon exchange. Since photon is a massless particle this leads to infrared divergences in one-loop integrals. They have to be regularized using unphysical, finite photon mass. These contributions cannot be separated from the weak corrections in a gauge invariant and UV finite way. To obtain physically meaningful results one has to include photon emission from charged particles appearing at the tree-level. Sample diagrams have been depicted in Fig. 3.

To cancel IR divergences it is enough to include soft photon emission, i.e. emission of photons with energy $E_\gamma \leq \Delta E$, where ΔE is small compared to the energy scale of the process. However, this procedure gives us the result which depends on an unphysical cut-off parameter ΔE . This can be overcome by including emission of hard photons, with energy $E_\gamma > \Delta E$

$$\Gamma_{\text{brems}} = \Gamma_{\text{soft}}(\Delta E) + \Gamma_{\text{hard}}(\Delta E), \quad (12)$$

where Γ_{brems} is the total contribution to the decay width due to photon emission.

To separate QED and SUSY corrections in the decay width we follow the conventions of the Supersymmetry Parameter Analysis (SPA) [13,14]. From the sum of virtual and soft photon terms

$$\Gamma_{\text{virt}} + \Gamma_{\text{soft}} = \Gamma_{\text{logs}} + \Gamma_{\text{SUSY}} \quad (13)$$

we take Γ_{logs} which contains potentially large logarithms depending on small lepton mass m_ℓ and cut-off energy ΔE . The remaining part Γ_{SUSY} is IR and UV finite, and free from ΔE . We can now define the QED correction as

$$\Gamma_{\text{QED}} = \Gamma_{\text{logs}} + \Gamma_{\text{hard}}. \quad (14)$$

Table 1. Masses of particles in the chosen scenario.

particle	$\tilde{\chi}_1^\pm$	$\tilde{\chi}_1^0$	\tilde{e}_L	\tilde{e}_R	$\tilde{\nu}_e$
mass [GeV]	165.3	97.9	287.9	221.9	276.6
particle	$\tilde{\tau}_1$	$\tilde{\tau}_2$	\tilde{q}_L	\tilde{q}_R	H^\pm
mass [GeV]	211.9	289.0	561.3	544.3	436.4

Table 2. One-loop corrected decay widths of $\tilde{\chi}_1^-$ in keV.

decay mode	tree-level width	one-loop width
$e^- \tilde{\nu}_e \tilde{\chi}_1^0$	4.18	4.38
$\mu^- \tilde{\nu}_\mu \tilde{\chi}_1^0$	4.18	4.38
$\tau^- \tilde{\nu}_\tau \tilde{\chi}_1^0$	4.38	4.61

Using above definitions we can rewrite the complete one-loop decay width as

$$\begin{aligned} \Gamma_{\text{loop}} &= \Gamma_{\text{tree}} + \Gamma_{\text{virt}} + \Gamma_{\text{brems}} \\ &= \Gamma_{\text{tree}} + \Gamma_{\text{SUSY}} + \Gamma_{\text{QED}}. \end{aligned} \quad (15)$$

4 Numerical analysis

For the numerical analysis we take a modified SPS1a' point [13]. The modification is needed since in the original parameter set there are two-body decay channels open for the light chargino. To close these decay modes we decreased M_2 parameter and increased slepton masses. The resulting spectrum is given in Tab. 1.

Results for one-loop corrections to the decay width in the three-body leptonic chargino decays have been shown in Tab. 2. In our case corrections are of the order $\sim 5\%$, however in other scenarios they can reach $\sim 10\%$ [7]. We also checked the results for the scenario introduced in [7] and found good agreement¹.

In Fig. 4 we show the energy distributions of electron and τ in the decay (1) at the one-loop level. We also show separately the impact of QED and genuine SUSY corrections, Eq. (15), for these distributions. As can be seen, the shape of QED corrections is different for electron and τ due to the difference in their masses. This difference results in a shift of one-loop corrected energy distribution for electrons towards lower energies. We also note that corrections for electrons are in principle larger. Generally both QED and SUSY corrections can be of the order $\sim 10\%$ but due to the opposite relative sign we observe a partial cancelation of these contributions. In Fig. 5 we show the electron angular distribution with respect to the chargino polarization vector. For some values of $\cos\theta_e$ the correction can reach 10%.

Figure 6 shows the dependence of the one-loop corrected decay width $\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 e^- \tilde{\nu}$ on the CP phase of the bino mass parameter M_1 , which enters in the neutralino couplings. This parameter has the strong influence on Γ and can change it by an order of magnitude.

¹ We note that there is an evident misprint in the third column of Tab. VII of Ref. [7].

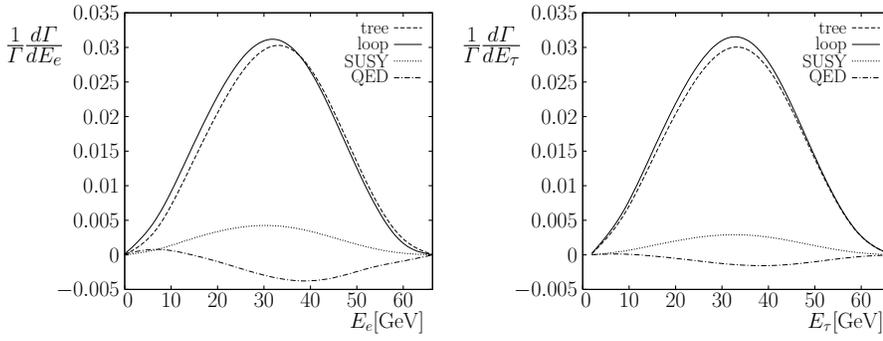


Fig. 4. One-loop corrected energy distributions for electron (left) and τ (right) in the decay (1) together with various contributions as in Eq. (15).

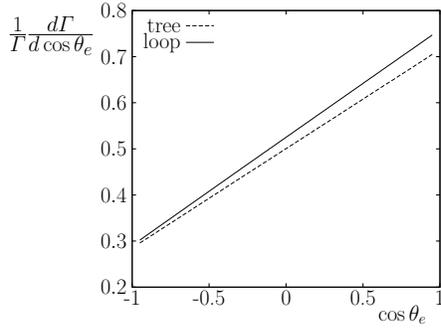


Fig. 5. One-loop corrected electron angular distribution with respect to the chargino polarization vector in the decay (1).

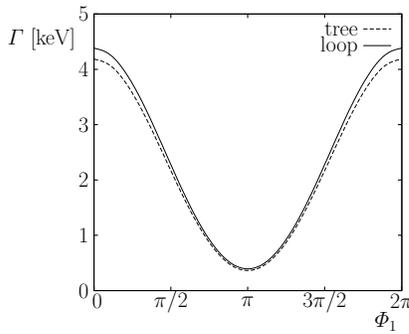


Fig. 6. Tree-level and one-loop corrected decay width for $\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^+ e^- \bar{\nu}$ as a function of the phase of M_1 parameter.

Although the decay width is not a CP-odd observable this feature can provide some information on the CP phase in the neutralino sector due to its influence on the branching fractions of light chargino decay modes.

5 Summary and outlook

We have calculated the one-loop corrections to the three-body leptonic chargino decays in the complex MSSM. This corrections may turn out to be important for precision physics at the future linear collider. The next step will be the inclusion of the hadronic decay modes and incorporation of one-loop corrections to the full production-decay process.

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