The Higgs sector in the MSSM with CP-phases at higher orders

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Outline

- Higgs sector in the MSSM with CP-phases
- Mass of the lightest Higgs boson
- Effective couplings

Higgs bosons

At Born level: no CP-violation:

- one phase in the Higgs potential: $V_{\text{Higgs}} = \cdots + \epsilon_{ij} |m_3^2| e^{i\varphi_{m_3^2}} H_1^i H_2^j + \ldots$
  elimination via Peccei-Quinn transformation
- phase difference of Higgs doublets:
  vanishes because of minimum condition

Physical mass eigenstates (at Born level):

- 5 Higgs bosons: 3 neutral H^0, h^0, A^0; 2 charged $H^\pm$

Masses of the Higgs bosons:

- not all independent: here: $H^\pm$-mass $M_{H^\pm}$ (and $\tan\beta$) as free parameter
  $\tan\beta = \frac{v_2}{v_1}$: ratio of the Higgs vac. expect. values
- lightest Higgs boson: h^0

Mass of the lightest Higgs boson

Upper theoretical Born mass bound: $M_{h^0} \leq M_Z = 91$ GeV

with quantum corrections of higher orders: $M_{h^0} \lesssim 135$ GeV

dependent on the MSSM parameters:
particularly on parameter phases

- Discovery of the Higgs boson:

accurate measurement → mass of the Higgs boson
precise prediction → mass of the Higgs boson

$\Rightarrow$ strong bounds on the MSSM parameter, e.g. on $\tan\beta = \frac{v_2}{v_1}$

(precise prediction:) i.e. inclusion of higher order corrections with full phase dependence

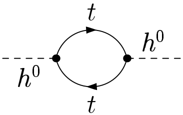
- Now, before the discovery: Exclusion of parts of the parameter space

Determination of the Higgs masses

Two-point-function: $-i\hat{\Gamma}(p^2) = p^2 - \mathbf{M}(\mathbf{p^2})$

with the matrix:

$$\mathbf{M}(\mathbf{p^2}) = \begin{pmatrix} M^2_{H^0_{\rm Born}} - \hat{\Sigma}_{H^0H^0}(p^2) & -\hat{\Sigma}_{H^0h^0}(p^2) & -\hat{\Sigma}_{H^0A^0}(p^2) \\ -\hat{\Sigma}_{H^0h^0}(p^2) & M^2_{h^0_{\rm Born}} - \hat{\Sigma}_{h^0h^0}(p^2) & -\hat{\Sigma}_{h^0A^0}(p^2) \\ -\hat{\Sigma}_{H^0A^0}(p^2) & -\hat{\Sigma}_{h^0A^0}(p^2) & M^2_{A^0_{\rm Born}} - \hat{\Sigma}_{A^0A^0}(p^2) \end{pmatrix}$$



Real parameters:

$$\hat{\Sigma}_{H^0A^0}(p^2) = \hat{\Sigma}_{h^0A^0}(p^2) = 0$$

no mixing between CP-even and CP-odd states

Calculate the zeros of the determinant of $\hat{\Gamma}$: $\det[p^2 - \mathbf{M}(\mathbf{p^2})] = 0$

$\Rightarrow$ Higgs masses M_{h_1}, M_{h_2}, M_{h_3}

Higgs masses at higher orders (incl. CP-phases)

Status:

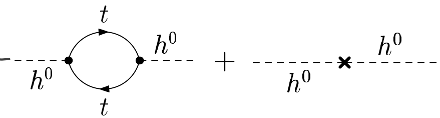
- Higgs masses at higher order without CP-phases ⇒ good shape (up to leading 3-loop [S. Martin 07])

Including CP-phases:

- Eff. potential approach, up to two-loop leading-log contributions (sfermionic/fermionic contributions) [Pilaftsis, Wagner], [Demir], [Choi, Drees, Lee], [Carena, Ellis, Pilaftsis, Wagner]
- Gaugino contributions [Ibrahim, Nath]
- Effects of imaginary parts at one-loop [Ellis, Lee, Pilaftsis], [Choi, Kalinowski, Liao, Zerwas], [Bernabeu, Binosi, Papavassiliou]

Here: full one-loop [M. Frank, et al.] + dominant two-loop (Feynman diagrammatic)

Renormalized Higgs self energies at one-loop



Parameters of the Higgs sector need to be defined at one-loop:

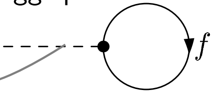
- define the $H^{\pm}$-, W- as well as the Z-mass as **pole mass**

  $\Rightarrow$ directly related to a **physical observable**

  $$\delta M_X^{(1)} = \mathrm{Re}\Sigma_{XX}(M_X^2), \qquad X = \{H^{\pm}, W, Z\}$$

- **no** shift of the minimum of the Higgs potential

  $$\delta t_\phi^{(1)} = -T_\phi$$



- $\overline{\textbf{DR}}$-scheme for field and $\tan\beta$ renormalization

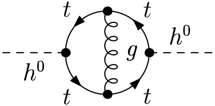
  $$\delta \tan^{(1)}\beta = \delta\tan\beta^{\overline{\mathrm{DR}}}, \qquad \delta Z_{H_i}^{(1)} = \delta Z_{H_i}^{\overline{\mathrm{DR}}}$$

Renormalized Higgs self energies at two-loop

Calculation of the dominant two-loop contributions $\mathcal{O}(\alpha_t \alpha_s)$ ($\alpha_t = \lambda_t^2/(4\pi)$):

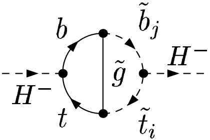
Yukawa coupling

- Extraction of the relevant terms (equiv. to eff. potential approach):
  - use vanishing external momenta $\hat{\Sigma}^{(2)}(0)$
  - use vanishing electroweak gauge couplings g, g'



- Parameters of the Higgs sector defined at two-loop:
  - no shift of the minimum of the Higgs potential
  - define the $H^{\pm}$-mass $M_{H^{\pm}}$ as the **pole mass**

    $\Rightarrow$ directly related to a **physical observable**



- Parameters of the top (bottom) sector defined at one-loop:
  - top quark mass and top squark masses on-shell
  - generalization of the mixing angle condition: $\widetilde{\text{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\text{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_2}) = 0$

Phases in other sectors

- Sfermion sector:
  - phase φ_{A_f} of the trilinear coupling A_f
- Higgsino sector:
  - phase of μ (small), μ: Higgsino mass parameter

    constraints from measurements of electr. dipole moments

soft breaking parameter

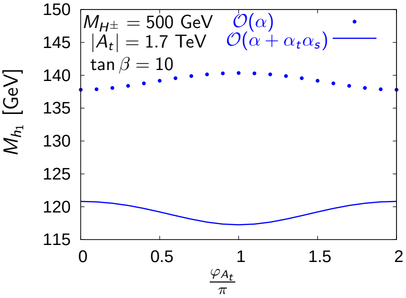
- Gaugino sector:
  - phases of the gaugino mass parameters M_1, M_2, M_3
    - — one phase can be eliminated (R-Transformation), often φ_{M_2}
    - — phase φ_{M_3} is the phase of the gluino mass parameter

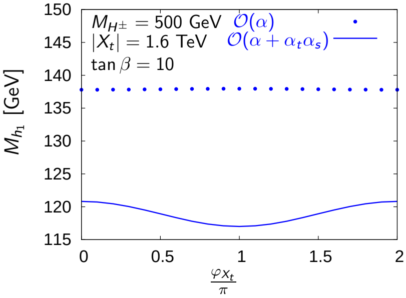
      $\Rightarrow$ enters into the Higgs sector at two-loop level

Results: φ_{A_t}- versus φ_{X_t}-dependence (large $M_{H^\pm}$)

size of the squark mixing:

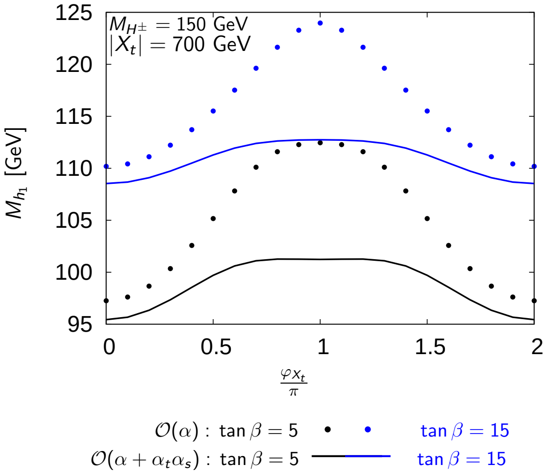
$X_t := A_t - \mu^* \cot\beta$





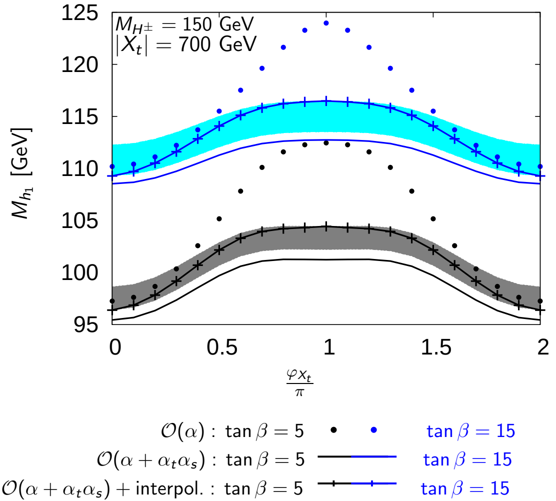
- The qualitative behaviour of M_{h_1} can change with the inclusion of quantum corrections of $\mathcal{O}(\alpha_t\alpha_s)$.
- Quantum corrections tend to be smaller for constant absolute value of of the squark mixing, $|X_t| = \text{const}$.

Results: φ_{X_t}-dependence (small $M_{H^\pm}$)



- Higgs mass M_{h_1} does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- One-loop corrections are more sensitive to φ_{X_t} for small $M_{H^\pm}$.

Results: φ_{X_t}-dependence (small $M_{H^\pm}$)



Bands: Estimate of the size of the corrections of $\mathcal{O}(\alpha_b\alpha_s + \alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ [Slavich et al.], FeynHiggs

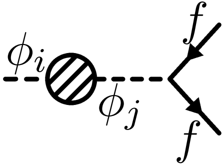
Interpolation: Size of above corrections known for the MSSM with **real** parameters: Evaluate for $\varphi_{X_t} = 0$ and $\varphi_{X_t} = \pi$ and interpolate

- Higgs mass M_{h_1} does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- One-loop corrections are more sensitive to φ_{X_t} for small $M_{H^\pm}$.

Amplitudes with external Higgs bosons

Mixing between the Higgs bosons:

($\overline{\text{DR}}$/on-shell scheme) $\phi_{\{i,j\}} = H^0,\ h^0,\ A^0$



Finite wave function normalization factors needed:

$$\sqrt{\hat{Z}_i}(\Gamma_i + \hat{Z}_{ij}\Gamma_j + \hat{Z}_{ik}\Gamma_k)$$

- $\hat{Z}_i$ ensures that residuum is set to 1
- $\hat{Z}_{ij}$ describes transition $i \to j$

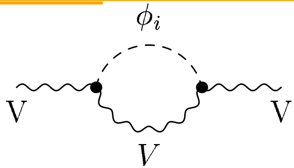
Definition of mixing matrix ($\hat{Z}_{ii} = 1$): $\tilde{\mathbf{Z}}_{ij} = \sqrt{\hat{Z}_i}\hat{Z}_{ij}$

Vertex with external Higgs boson:

$$\tilde{\mathbf{Z}}_{ii}\Gamma_i + \tilde{\mathbf{Z}}_{ij}\Gamma_j + \tilde{\mathbf{Z}}_{ik}\Gamma_k$$

Amplitudes with internal Higgs bosons

Diagrams with internal Higgs bosons enter precision observables (W-mass, ...):



- Calculation with Born states $\phi_i = H^0,\, h^0,\, A^0$: no problem
- Calculation with $\phi_i = h_1,\, h_2,\, h_3 \Rightarrow$ Inclusion of higher order effects:

One possibility: Use of effective couplings:

Consider $\tilde{\mathbf{Z}}_{ij}$ as mixing matrix:

Problem: $\tilde{\mathbf{Z}}_{ij}$ is a non-unitary matrix (no rotation matrix)

Further approximations:

effective potential approach: $\tilde{\mathbf{Z}}(\hat{\Sigma}(p^2)) \to \tilde{\mathbf{Z}}(\hat{\Sigma}(0)) = \mathcal{R}$

on-shell approximation: $\tilde{\mathbf{Z}}(\hat{\Sigma}(p^2)) \to \tilde{\mathbf{Z}}(\mathrm{Re}\hat{\Sigma}(p^2_{\mathrm{OS}})) = \mathcal{U}$ $\hat{\Sigma}_{ii}(p^2_{\mathrm{OS}} = M^2_{i_{\mathbf{Born}}})$ $\hat{\Sigma}_{ij}(p^2_{\mathrm{OS}} = (M^2_{i_{\mathbf{Born}}} + M^2_{j_{\mathbf{Born}}})/2)$

Couplings

One example:

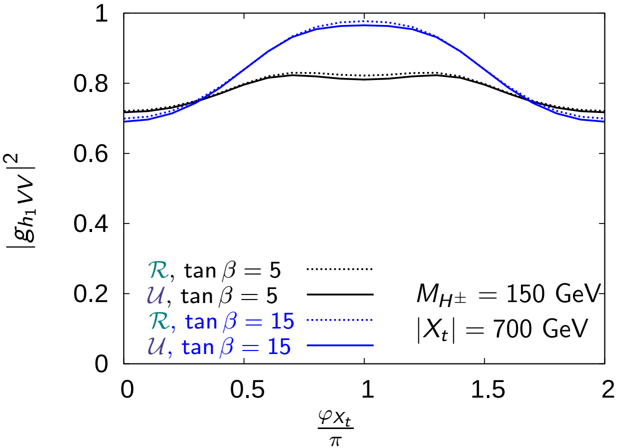
Coupling of two gauge bosons ($V = W,\ Z$) and one Higgs boson:

$$g_{h_i VV} = [U_{i1} \cos(\beta - \alpha) + U_{i2} \sin(\beta - \alpha)]\, g_{H_{\rm SM} VV}$$

standard model coupling

- only CP-even components of the Higgs bosons couple to V
- all three Higgs bosons can have a CP-even component

Results: φ_{X_t}-dependence of couplings



- Here: $g_{h_1 VV}$ is normalized to the standard model coupling.
- $|g_{h_1 VV}|^2$ does depend on the phase φ_{X_t}, $|X_t| = 700$ GeV.
- $\mathcal{R}_{p^2=0}$ and $\mathcal{U}_{p_{\rm OS}}$ give similar results with only tiny differences.

Summary

- At **Born** level: **no** CP-violation in the Higgs sector
- **Quantum corrections** can **induce** CP-violation.
- **Quantum corrections** have to be taken into account:
  - prediction of Higgs boson masses
  - amplitudes with external Higgs bosons $\Rightarrow$ $\tilde{\mathbf{Z}}$
  - amplitudes with internal Higgs bosons $\Rightarrow$ $\mathcal{R}$, $\mathcal{U}$
- The dominant two-loop contributions $\mathcal{O}(\alpha_t \alpha_s)$ with complete phase dependence are **included** into FeynHiggs (talk by T. Hahn).