# Enhanced symmetry points and metastable supersymmetry breaking along pseudo-runaway directions

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**Abstract.** We construct a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. The metastable vacua appear along a pseudo-runaway direction near a point of enhanced symmetry as a result of a balance between non-perturbative and perturbative quantum effects. We show that metastable supersymmetry breaking is a rather generic feature near certain enhanced symmetry points of gauge theory moduli spaces.

**PACS.** 11.30.Pb Supersymmetry – 11.30.Qc Spontaneous and radiative symmetry breaking

## 1 Introduction

Intriligator, Seiberg and Shih (ISS) [1] showed that nonsupersymmetric vacua are rather generic if one requires them to be only local, rather than global, minima of the potential. One lesson from ISS is that certain properties of moduli spaces can hint at the existence of metastable vacua. In their case, it was the existence of supersymmetric vacua coming in from infinity that signaled an approximate R-symmetry. Here we will point out that one should also look for another feature, namely, enhanced symmetry points, which are defined by the appearance of massless particles. We claim that if the moduli space has certain coincident enhanced symmetry points, metastable vacua with all the relevant couplings arising by dimensional transmutation may be obtained [2].

The model considered here consists of two SQCD sectors, each with independent rank and number of flavors, coupled by a singlet. It involves only marginal operators with all scales generated dynamically. At the origin of moduli space, the singlet vanishes and the quarks of both sectors become massless simultaneously. There are thus two coincident enhanced symmetry points at the origin. While one of the SQCD sectors is in the electric range and has a runaway, the other has a magnetic dual description as an O'Raifeartaigh-like model. Near the enhanced symmetry point, the Coleman-Weinberg corrections stabilize the nonperturbative instability producing a long-lived metastable vacuum. We refer to a runaway direction stabilized by perturbative quantum corrections as a "pseudo-runaway". A feature of our model is that it may be possible to gauge parts of its large global symmetry to obtain renormalizable, natural models of direct gauge mediated supersymmetry breaking with a singlet. R-symmetry is broken both spontaneously and explicitly in our model; the spontaneous breaking allows for non-zero gaugino masses, while the explicit breaking allows for a non-zero $R$-axion mass.

## 2 Models with Moduli Dependent Masses

The matter content of the models considered here consists of two copies of supersymmetric QCD, each with independent rank and number of flavors, and a single gauge singlet chiral superfield:

$$\begin{array}{ccccc} & SU(N_c) & SU(N_c') & & \\ Q_i & \Box & 1 & i=1,\ldots,N_f & \\ \overline{Q}_i & \overline{\Box} & 1 & & \\ P_{i'} & 1 & \Box & i'=1,\ldots,N_f' & \\ \overline{P}_{i'} & 1 & \overline{\Box} & & \\ \Phi & 1 & 1 & & \end{array} \qquad (1)$$

The most general tree-level superpotential with only relevant or marginal terms in four dimensions for the matter content (1) with $N_c$, $N_c' \geq 4$ is

$$W = (\lambda_{ij}\Phi + \xi_{ij})Q_i\overline{Q}_j + (\lambda'_{i'j'}\Phi + \xi'_{i'j'})P_{i'}\overline{P}_{j'} + w(\Phi)\,, \qquad (2)$$

where $w(\Phi)$ is a cubic polynomial in $\Phi$. We shall find metastable vacua even in the simplest case of $w(\Phi) = 0$, which we assume from now on. The general situation is discussed in Sec. 4 (in [3], the case $w(\Phi) = \kappa\Phi^3$ was used to stabilize $\Phi$ supersymmetrically).

At the classical level this superpotential has an accidental $U(1)_R \times U(1)_V \times U(1)_V'$ global symmetry. In the quantum theory the $U(1)_R$ symmetry is anomalous with respect to the $SU(N_c)$ and $SU(N_c')$ gauge dynamics. In the $SU(N_f)_V \times SU(N_f')_V$ global symmetry limit the superpotential (2) (with $w(\Phi) = 0$)

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reduces to

$$W = (\lambda\Phi + \xi)\mathrm{tr}(Q\overline{Q}) + (\lambda'\Phi + \xi')\mathrm{tr}(P\overline{P})\,. \quad (3)$$

For $\xi = \xi'$ both masses may simultaneously be absorbed into a shift of $\Phi$, and the tree level superpotential in this case reduces to

$$W = \lambda\Phi\ \mathrm{tr}(Q\overline{Q}) + \lambda'\Phi\ \mathrm{tr}(P\overline{P})\,. \quad (4)$$

The classical moduli space of vacua is as usual parameterized by baryon and meson invariants, see [2]. It is lifted by nonperturbative effects in the quantum theory. Since the metastable supersymmetry breaking vacua discussed below arise for $\Phi \neq 0$, only this branch of the moduli space will be considered in detail. On this branch, holomorphy, symmetries, and limits fix the exact superpotential written in terms of invariants, to be

$$\begin{aligned} W =\ & \lambda\Phi\ \mathrm{Tr}M + (N_c - N_f)\left[\frac{\Lambda^{3N_c-N_f}}{\det\ M}\right]^{1/(N_c-N_f)} \quad (5)\\ & + \lambda'\Phi\ \mathrm{Tr}M' + (N_c' - N_f')\left[\frac{\Lambda'^{3N_c'-N_f'}}{\det\ M'}\right]^{1/(N_c'-N_f')}\,. \end{aligned}$$

For gauge sectors in the free magnetic range, the nonperturbative contribution refers to the Seiberg dual. Since the meson invariants are lifted on this branch, they may be eliminated by equations of motion to give the exact superpotential in terms of the classical modulus $\Phi$

$$\begin{aligned} W = & N_c\left[(\lambda\Phi)^{N_f}\Lambda^{3N_c-N_f}\right]^{1/N_c}\\ & + N_c'\left[(\lambda'\Phi)^{N_f'}\Lambda'^{3N_c'-N_f'}\right]^{1/N_c'}\,. \end{aligned} \quad (6)$$

The supersymmetric minima are given by stationary points of the superpotential, $\partial W/\partial\Phi = 0$.

The metastable vacua found below occur when one of the SQCD sectors (the unprimed sector) is in the free magnetic range, i.e. $N_c + 1 \le N_f < \frac{3}{2}N_c$, and the other (primed) sector exhibits an ADS runaway behavior, i.e. $N_f' < N_c'$ [4]. Moreover, we work in the range $\Lambda' \ll E \ll \Lambda$, where the primed sector is weakly interacting while the unprimed sector has a dual weakly coupled description in terms of the magnetic gauge group $SU(\tilde{N}_c)$ with $\tilde{N}_c{=}N_f - N_c$, $N_f^2$ singlets $M_{ij}$, and $N_f$ magnetic quarks $(q_i,\, \tilde{q}_j)$. In terms of this description, the full nonperturbative superpotential reads

$$\begin{aligned} W =\ & m\Phi\ \mathrm{tr}\ M + h\ \mathrm{tr}\ qM\tilde{q} + \lambda'\Phi\ \mathrm{tr}\ P\bar{P}\\ & + (N_c' - N_f')\left(\frac{\Lambda'^{3N_c'-N_f'}}{\det P\bar{P}}\right)^{1/(N_c'-N_f')}\\ & + (N_f - N_c)\left(\frac{\det M}{\tilde{\Lambda}^{3N_c-2N_f}}\right)^{1/(N_f-N_c)}\,. \end{aligned} \quad (7)$$

Hereafter, $M_{ij} = Q_i\bar{Q}_j/\Lambda$, and $m := \lambda\Lambda$. The magnetic sector has a Landau pole at $\tilde{\Lambda} = \Lambda$.

In this description, the meson $M$ and the primed quarks $(P,\,\bar{P})$ become massless at $\Phi = 0$. $M = 0$ is also an enhanced symmetry point since here the magnetic quarks $(q,\,\tilde{q})$ become massless.

## 3 Metastability near enhanced symmetry points

Starting from the superpotential (7), the discussion is simplified by taking the limit $\tilde{\Lambda} \to \infty$, while keeping $m$ fixed. The nonperturbative $\det M$ term is only relevant for generating supersymmetric vacua, and not important for the details of the metastable vacua that will arise near $M = 0$. Thus, for $M/\tilde{\Lambda} \to 0$ and $\Phi/\tilde{\Lambda} \to 0$, it is enough to consider the superpotential

$$\begin{aligned} W = & m\Phi\ \mathrm{tr}\ M + h\ \mathrm{tr}\ qM\tilde{q} + \lambda'\Phi\,\mathrm{tr}\,P\bar{P}\\ & + (N_c' - N_f')\left(\frac{\Lambda'^{3N_c'-N_f'}}{\det P\bar{P}}\right)^{1/(N_c'-N_f')}\,. \end{aligned} \quad (8)$$

As discussed in [2], metastable vacua can only appear in the region near the enhanced symmetry point $M = 0$. This still has a runaway, but it turns out that one-loop corrections stop it; this novel effect is characterized as a "pseudo-runaway". The reason for this is that the Coleman-Weinberg formula

$$V_{CW} = \frac{1}{64\pi^2}\,\mathrm{Str}\,M^4\ \ln\ M^2 \quad (9)$$

will have polynomial (instead of logarithmic) dependence. This will be explained next.

In the region $\Phi \neq 0$, $(P, \bar{P})$ may be integrated out by equations of motion provided that $\Lambda' \ll \lambda'\Phi$. This is a good description if we are not exactly at the origin but near it, as given by $\Phi/\tilde{\Lambda} \ll 1$. Taking, as before, $\tilde{\Lambda} \to \infty$ and $m$ fixed, the superpotential reads

$$\begin{aligned} W = & m\Phi\,\mathrm{tr}\,M + h\,\mathrm{tr}\,qM\tilde{q}\\ & + N_c'\left[\lambda'^{N_f'}\,\Lambda'^{3N_c'-N_f'}\,\Phi^{N_f'}\right]^{1/N_c'}\,. \end{aligned} \quad (10)$$

This description corresponds to an O'Raifeartaigh - type model in terms of magnetic variables but with no flat directions.

Given that $\phi = \langle\Phi\rangle \neq 0$, we will expand around the point of maximal symmetry

$$\begin{aligned} q = \begin{pmatrix} q_0 & 0 \end{pmatrix},\ \tilde{q} = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix},\\ M = \begin{pmatrix} 0 & 0 \\ 0 & 0 + X\cdot I_{N_c\times N_c} \end{pmatrix}. \end{aligned} \quad (11)$$

Here $q_0$ and $\tilde{q}_0$ are $\tilde{N}_c \times \tilde{N}_c$ matrices satisfying

$$hq_{0i}\tilde{q}_{0j} = -m\phi\,\delta_{ij}\ ,\ i,j = \tilde{N}_c + 1, \ldots N_f\,, \quad (12)$$

and the nonzero block matrix in $M$ has been taken to be proportional to the identity; indeed, only $\mathrm{tr}\,M$ appears in the potential. This minimizes $W_M$ and sets $W_q = W_{\tilde{q}} = 0$. The spectrum of fluctuations around (11) was studied in detail in [2], where it was shown that the lightest degrees of freedom correspond to $(\phi, X)$ with mass given by $m$. The effective potential derived from (10) is

$$\begin{aligned} V(\phi, X) = & \left| mN_cX + N_f'\lambda'^{N_f'/N_c'}\left(\frac{\Lambda'^{3N_c'-N_f'}}{\phi^{N_c'-N_f'}}\right)^{1/N_c'}\right|^2\\ & + N_cm^2|\phi|^2 + V_{CW}(\phi, X)\,, \end{aligned} \quad (13)$$

where the first term comes from $W_\phi$. The Coleman-Weinberg contribution will be discussed shortly.

As a starting point, set $X = 0$ and $V_{CW} \to 0$. Minimizing $V(\phi, X = 0)$ gives

$$|\phi_0|^{(2N_c'-N_f')/N_c'} = \sqrt{\frac{N_c'-N_f'}{N_c N_c'}}\; N_f' \frac{\lambda'^{N_f'/N_c'}}{m} \Lambda'^{\frac{3N_c'-N_f'}{N_c'}} \tag{14}$$

and since $W_{\phi\phi} \sim m$, $V(\phi_0+\delta\phi, X = 0)$ corresponds to a parabola of curvature $m$. The nonperturbative term only affects $\phi_0$ but not the curvature $m$.

Next, allowing $X$ to fluctuate (but still keeping $V_{CW} \to 0$), $V(\phi_0, X)$ gives a parabola centered at

$$X_{W_\phi=0} = -\sqrt{\frac{N_c'}{N_c(N_c'-N_f')}}|\phi_0| \tag{15}$$

and curvature $m$. In other words, $X = 0$ is on the side of a hill of curvature $m$ and height $V(\phi_0, 0) \sim m^2|\phi_0|^2$.

To create a minimum near $X = 0$, $V_{CW}$ should contain a term $m_{CW}^2|X|^2$, with $m_{CW} \gg m$; this would overwhelm the classical curvature. As explained in [2], the massive degrees of freedom giving the dominant contribution to $V_{CW}$ come from integrating out the massive fluctuations along $q_0$ and $\tilde{q}_0$. The result is

$$V_{CW} = N_c b h^3 m|\phi||X|^2 + \ldots \tag{16}$$

with $b$=$(\log 4 - 1)/8\pi^2\tilde{N}_c$ [1], and '...' represent contributions that are unimportant for the present discussion. In this computation, $X$ and $\phi$ are taken as background fields. The quadratic $X$ dependence appears because $X = 0$ is an enhanced symmetry point.

In order to be able to produce a local minimum, the marginal parameters $(\lambda, \lambda')$ will have to be tuned to satisfy

$$\epsilon \equiv \frac{m^2}{m_{CW}^2} = \frac{m}{bh^3|\phi|} \ll 1\,. \tag{17}$$

In this approximation, the value of $\phi$ at the minimum is still given by (14); also, $X$ is stabilized at the nonzero value

$$|X_0| = \sqrt{\frac{N_c N_c'}{N_c'-N_f'}}\,\frac{m}{bh^3}\,, \tag{18}$$

with $|X_0| \ll X_{W_\phi=0}$. At the minimum, (17) gives

$$(m/\Lambda')^{3N_c'-N_f'} \ll (bh^3)^{(2N_c'-N_f')/N_c'}\lambda'^{N_f'} \tag{19}$$

so the Yukawa coupling $\lambda$ in $m = \lambda\Lambda$ must be taken small for the analysis to be self-consistent. The calculability condition $\Lambda' \ll \lambda'\Phi$ follows as a consequence of this. At the minimum, $X_0 \ll \phi_0$. The F-terms are given by $W_\phi \sim m\phi_0 \sim W_X$, and from (14) the scale of supersymmetry breaking is thus controlled by the dynamical scales of both gauge sectors.

Thus the model has a metastable vacuum near the origin, created by a combination of quantum corrections and nonperturbative gauge effects. The pseudo-runaway towards $X = X_{W_\phi=0}$ has been lifted by the Coleman-Weinberg contribution, as anticipated. This

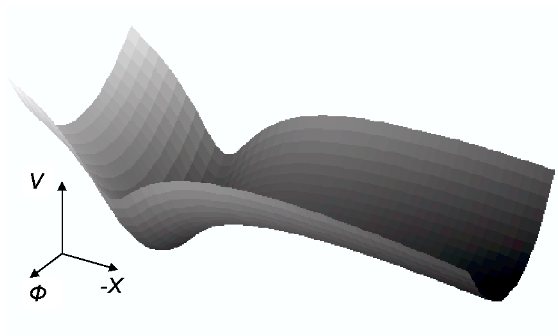


**Fig. 1.** A plot showing the shape of the potential, including the one-loop Coleman-Weinberg corrections, near the metastable minimum. In the $\phi$-direction the potential is a parabola, whereas in the $X$-direction it is a side of a hill with a minimum created due to quantum corrections. This plot was generated with the help of [5].

is the origin of the $1/b$ dependence in (18). The local minimum is depicted in Fig. 1.

The metastable vacuum appears from a competing effect between a runaway behavior in the primed sector and one loop corrections for the meson field $X$. One is naturally led to ask if, under these circumstances, other quantum effects are under control. These include higher loop terms from the massive particles producing $V_{CW}$ as well as perturbative $g'$ corrections. In [2] it was shown that such corrections are under control and do not destabilize the metastable vacuum.

The metastable non-supersymmetric vacuum can be made parametrically long-lived by taking the parameter $\epsilon \equiv \frac{m}{bh^3|\phi_0|}$ sufficiently small. The direction in field space to tunnel out of the false vacuum is towards negative $X$ with fixed $|\phi| = |\phi_0|$ [2]; the one-dimensional potential $V(X) \equiv V(|\phi|{=}|\phi_0|,X)$ is shown in Fig. 2. The bounce action scales as

$$B \sim \frac{\tilde{X}^4}{V(X_{peak}) - V(X_0)} \sim b\,h^3\,\frac{1}{\epsilon^2}, \tag{20}$$

where the field tunnels to $\tilde{X}$, and $V(X)$ reaches a local maximum at $X_{peak}$. Thus $B \to \infty$ as $\epsilon \to 0$.

To have gaugino masses, any R-symmetry must be broken, explicitly and/or spontaneously. The low energy superpotential (10) has the $U(1)_R$ symmetry

$$R_\phi = 2\frac{N_c'}{N_f'}\,,\; R_X = 2\frac{N_f'-N_c'}{N_f'}\,,\; R_q = R_{\tilde{q}} = \frac{N_c'}{N_f'}\,. \tag{21}$$

Since the VEV's of these fields are nonzero in the metastable vacuum, the R-symmetry is spontaneously broken, and there is an R-axion $a$. For finite $\tilde{\Lambda}$, the det $X$ contributions need to be taken into account, and the $U(1)_R$ symmetry becomes anomalous. This explicit breaking gives a mass to the R-axion,

$$m_a^2 \sim m^2\left(\left[\frac{\lambda}{bh^3}\right]^{2\,\frac{3N_c-2N_f}{N_f-N_c}}\frac{\epsilon}{bh^3}\right) \ll m^2\,, \tag{22}$$

where $\lambda$ is the Yukawa coupling appearing in $m = \lambda\Lambda$.

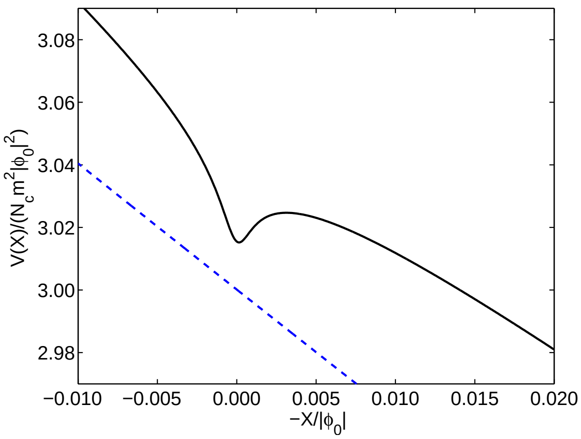


**Fig. 2.** The classical potential (dashed line) and the total potential including one-loop corrections (solid line) for fixed $|\phi| = |\phi_0|$, where $|\phi_0|$ is the value of $\phi$ at the metastable minimum (14). In the figure, $N_f = 3$, $N_c = 2$, $N'_f = 1$ and $N'_c = 2$, and $W_\phi = 0$ has been scaled to equal 1 on both axes; the metastable minimum lies at $\sim 10^{-4}$. This plot was generated with the help of [5].

## 4 Meta-Stability Near Generic Points of Enhanced Symmetry

A generic structure in the landscape of effective field theories corresponds to a gauge theory with vector-like matter and mass given by a singlet, whose dynamics is related to another sector. The superpotential may be written as

$$W = f(\Phi) + \lambda\,\Phi \mathrm{tr}(Q\bar{Q})\,. \tag{23}$$

Here, $(Q, \bar{Q})$ are $N_f$ quarks in $SU(N_c)$ SQCD; $f(\Phi)$ may be generated, for instance, from a flux superpotential, by nonrenormalizable interactions, or by another gauge sector. Next, it is required that the SQCD sector be in the free magnetic range, which is still a generic situation. The dual magnetic description is weakly coupled near the enhanced symmetry point $\Phi = 0$, where the superpotential reads

$$W = f(\Phi) + m\Phi\,\mathrm{tr}\;M + h\,\mathrm{tr}\;qM\tilde{q}\,. \tag{24}$$

The question that will be addressed here is: what restrictions need to be imposed on $f(\Phi)$, so that the one loop potential $V_{CW}$ can create a metastable vacuum near $M = 0$? Since we are interested in the novel effect of pseudo-runaway directions we will demand $f'(\Phi) \neq 0$. The model in [3] would correspond to $f'(\Phi) = 0$.

As discussed in Sec. 3, this is possible only if

$$m^2_{CW} := N_c b h^3 m|\phi| \gg m^2 \tag{25}$$

where $\phi$ denotes the expectation value of $\Phi$ at the metastable vacuum. Further, one needs to impose that

$$h^2|X|^2 \ll m|\phi| \tag{26}$$

in order for the Taylor expansion of $V_{CW}$ around $X = 0$ to converge. Evaluating the potential as in (13),

$$V = N_c m^2|\phi|^2 + \left|f'(\phi) + mN_c\,X\right|^2 + m^2_{CW}|X|^2\,. \tag{27}$$

The rank condition, an essential ingredient in the discussion, just follows from having SQCD in the free magnetic range. This fixes the first term, which comes from $W_M$, and the block structure of the matrix $M$; $X$ was defined in (11).

The minima of $V(\phi, X)$ are

$$N_c m^2\phi = -f'(\phi)\,f''(\phi)^*,\; m^2_{CW}\,X = -N_c m f'(\phi)\,. \tag{28}$$

It is possible to combine the conditions (25) and (26) with the values at the metastable vacuum (28) to derive constraints on $f(\phi)$: (25) now reads

$$\frac{|f'(\phi)f''(\phi)|}{m^3} \gg \frac{1}{bh^3}\,, \tag{29}$$

while (26) gives

$$h^2|f'(\phi)|^2 \ll m(bh^3)^2|\phi|^3\,. \tag{30}$$

The necessary conditions for metastable vacua near $X = 0$ to exist are (29) and (30). These conditions may require fine-tuning the coefficients of $f(\phi)$, for example if one has two gauge sectors as in (3) with non-coincident enhanced symmetry points [2]. In the case of coincident enhanced symmetry points, where there are no relevant scales, no fine-tuning is required.

In summary, we constructed a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. The model has the desirable features of an explicitly and spontaneously broken $R$-symmetry, a singlet, a large global symmetry, naturalness and renormalizability. The metastable vacua are produced near a point of enhanced symmetry by a combination of nonperturbative gauge effects and, crucially, perturbative effects coming from the one-loop Coleman-Weinberg potential. A salient feature is the existence of pseudo-runaway directions, which correspond to a runaway behavior that is lifted by one loop quantum corrections. It would be interesting to study the implications of this for the landscape of supersymmetric gauge theories.

*Acknowledgments:*

We thank S. Thomas for suggesting this problem. RE would like to thank the organizers of SUSY07 for the opportunity to present this work. This research is supported by the Department of Physics and Astronomy at Rutgers University.

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