Inhomogeneous preheating in multi-field models of cosmological perturbation

Tomohiro Matsuda[1] [a][b]

Saitama Institute of Technology

Abstract. We consider inhomogeneous preheating in multi-field models of cosmological perturbation. After preheating, two fields are trapped at an enhanced symmetric point. One field is an oscillating field and the other is a light field that plays an important role in generating perturbation. In this presentation, we consider two types of potential for the light field. Unlike the usual modulated (p)reheating scenario, there is no moduli problem because moduli-dependent couplings are not needed. Since there is no moduli problem the inflation scale can be lowered.

PACS. 98.80.Cq Particle-theory models of the early Universe – 11.25.-w Strings and branes

1 Introduction

Motivation for the "Alternatives" to the traditional inflation
We consider the following:

- Traditional Inflaton
  According to the traditional inflationary scenario, the spectrum of the curvature perturbation is generated by the inflaton field. The spectrum is determined by the inflation model alone.
- Alternatives to the traditional inflation
  The primordial density perturbation may instead originate from the vacuum fluctuation of a "non-inflaton" field.

Considering alternatives to the traditional scenario, the inflation model can be eliminated as the source of the generation of the curvature perturbation. Hence, we can expect that inflation may be separated from the problems related to the generation of the cosmological perturbation. For the treatment of nontraditional cosmological perturbation, we introduce "ϕ_2" as the non-inflaton field.

Problem in low-scale inflation (typical)
For traditional inflation, the spectrum of the curvature perturbation is given by

$$\mathcal{P}_\mathcal{R}(k) = \frac{1}{24\pi^2 M_p^4}\frac{V_I}{\epsilon_I} \quad \text{(Traditional Inflation)} \quad (1)$$

Assuming that the scale of the inflation is much smaller than the Planck scale, serious fine-tuning is required. For example:

$$V_I^{1/4} < 10^7 GeV \quad \rightarrow \epsilon_I < 10^{-13}. \quad (2)$$

[a] *Email:* matsuda@sit.ac.jp
[b] *Present address:* Fusaiji, Okabe-machi, Saitama, Japan

Since a low-scale gravity model is still an important possibility that could be tested in future experiments, it is important to consider how low-scale inflation takes place. Future experiments (like BH production in LHC) may or may not put an indirect bound on $H_I^{1/4}$, such as $H_I \leq \mathcal{O}(\text{TeV})$.

Examples of the "Alternatives"

- Curvatons[1,2,3]
- Modulated (p)reheating[4,5,6]
- Inhomogeneous preheating[7,8]
- Generating δN_e at the end of inflation[9,10]
- Others(equally important)

In this talk, we mainly consider inhomogeneous preheating combined with curvatons or δN_e generation at the end of inflation. We do not consider either moduli-dependent couplings nor fluctuation of moduli that may lead to serious moduli problem.

What is inhomogeneous preheating?
To explain inhomogeneous preheating, we consider first simple preheating and then introduce ϕ_2 field to develop "multi-field preheating". We then explain how multi-field preheating induces inhomogeneous preheating. Finally, we discuss generation of the cosmological perturbation from inhomogeneous preheating combined with curvatons or δN_e generation at the end of inflation. Preheating[11] is induced by an oscillating field ϕ_1 (usually an inflaton).(See Fig.1) We assume an interaction given by

$$\mathcal{L} = -\frac{1}{2}g^2|\phi_1|^2\chi^2, \quad (3)$$

which induces a mass term for the preheat field χ. At the enhanced symmetric point (ESP), the effective mass of the preheat field vanishes and non-adiabatic excitation of χ occurs, which induces efficient generation of the preheat field χ. The number density of the
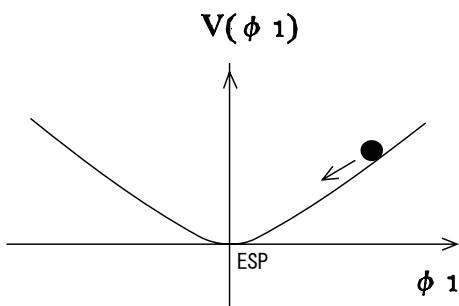
Fig. 1. Oscillation after inflation that induces preheating at the ESP.

preheat field (n_χ) that is generated at the first scattering is given by

$$n_\chi = \frac{(g|\dot\phi_1(t_*)|)^{3/2}}{8\pi^3} \exp\left[-\frac{\pi m_\chi^2}{g|\dot\phi_1(t_*)|}\right]. \qquad (4)$$

Multi-field extension
In addition to the oscillating field ϕ_1, we may add ϕ_2 that has the same coupling as ϕ_1. This leads to multi-field preheating. If the potential $V(\phi_1, \phi_2)$ is symmetric for rotation in (ϕ_1, ϕ_2) space, the potential looks like that shown in Fig.2. On the other hand, if the
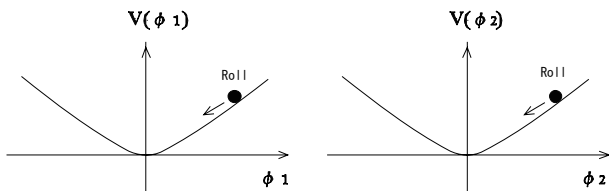

Fig. 2. Symmetric potential.

global symmetry is badly broken (i.e., there is a hierarchical mass difference $m_1 \gg m_2$), the potential looks like that in Fig.3. Depending on the situation,
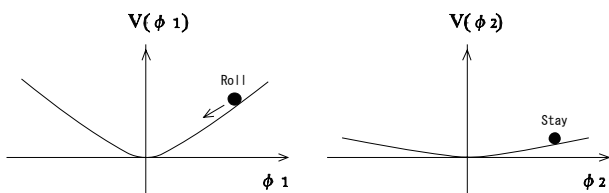

Fig. 3. Potential with a hierarchical mass difference.

the trajectory of the multi-field oscillation becomes

– "A straight line that precisely hits the ESP" for a symmetric potential. (first picture in Fig.4)
– "A curved line that does not hit the ESP" for slightly broken symmetry. (lower left in Fig.4)
– "An almost straight line that does not hit the ESP" for a hierarchical mass difference. (right-hand side in Fig.4)
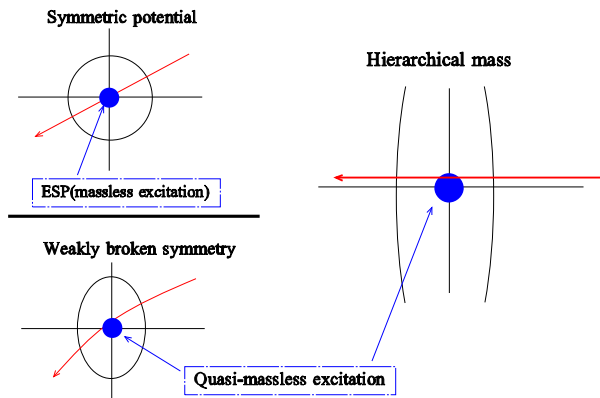

Fig. 4. Trajectories of inflaton motion.

2 Inhomogeneous preheating; origin of the fluctuation in multi-field preheating

Except for the symmetric potential, for which the trajectory precisely hits the origin, the effective mass (m_χ) does not vanish during preheating. For a potential with a slightly broken symmetry, m_χ depends on the initial angle and is given by $m_\chi(\theta)$. For a potential with hierarchical masses, m_χ depends on the initial value of the light field ϕ_2 and is given by $m_\chi(\phi_2)$. Therefore,

– For the "slightly broken symmetry" the trajectory is determined by the initial angular parameter θ. The origin of the fluctuation is denoted by $\delta\theta$, where θ is the $U(1)$ angle of the symmetry.
– For "hierarchical mass" the distance from the ESP is determined by the initial value of ϕ_2. The origin of the fluctuation is denoted by $\delta\phi_2$, where ϕ_2 is the additional light field of multi-field preheating.

In the hierarchical mass model, the light field ϕ_2 gives mass to the preheat field at the ESP:

$$m_\chi|_{ESP} \simeq g\phi_2. \qquad (5)$$

Therefore, the primordial fluctuation $\delta\phi_2$ leads to fluctuation of the mass δm_χ, which finally induces inhomogeneous preheating and $\delta n_\chi \neq 0$. It is important to note that both the magnitude and typical length scale of the fluctuation $\delta\phi_2$ is determined by the primordial inflation.

Previous approaches
In the following earlier previous approaches, "instant decay" has been assumed for the preheat field:

– Slightly broken symmetry[7]
  E. W. Kolb, A. Riotto, A. Vallinotto.
  The origin of the fluctuation is $\delta\theta$.
– Hierarchical mass difference(badly broken symmetry)[8]
  T. Matsuda.
  The origin of the fluctuation is $\delta\phi_2$.

For instant decay, generation of the cosmological perturbation and reheating occurs just after the inhomogeneous preheating. This requires

$$\left.\frac{\rho_\chi}{\rho_{total}}\right|_{ini} \sim 1, \qquad (6)$$

which puts a lower bound on the coupling constant.

Now we consider what happens if χ does not decay instantaneously and whether it is possible to remove the condition $\left.\frac{\rho_\chi}{\rho_{total}}\right|_{ini} \sim 1$ that leads to $g \sim 1$ in the previous approaches.

Back reaction from the preheat field

The effective potential induced by a stable χ-field induces an attractive confining force to both ϕ_1 and ϕ_2. For single-field preheating, a similar situation has been discussed for moduli trapping in string theory[12]. According to this treatment, the preheat field induces effective potential

$$V_c(\phi_i) \sim gn_\chi|\phi_i|, \qquad (7)$$

which leads to an attractive confining force proportional to the distance from the ESP. This trapping can generate the cosmological perturbation from inhomogeneous preheating. The obvious differences from the previous approaches are (1) No instant decay is assumed (2) The preheat field does not dominate the energy density just after preheating.

<u>Model 1</u> : **Generating δN_e at the end of trapping inflation**

First, we discuss how to generate δN_e at the end of trapping inflation[12] with the potential

$$V(\phi_2) = -\frac{1}{2}m^2\phi_2^2 + \lambda\frac{\phi_2^n}{M^{n-4}}. \qquad (8)$$

Note that the ϕ_2-potential is inverted.(See Fig.5)
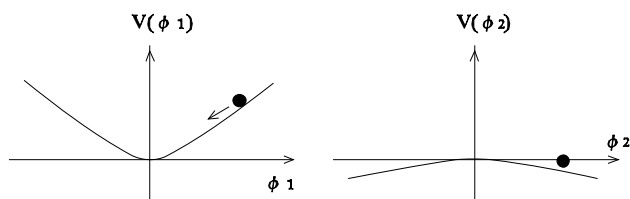

Fig. 5. Potential for trapping inflation after inhomogeneous preheating

This potential may remind the reader of thermal inflation induced by thermal trapping. Note that in our model, trapping inflation is induced by trapping after preheating but before reheating. Because of the back reaction from the preheat field, the effective potential near the origin is significantly altered.(See Fig.6.)

Let us summarize the process for generating the curvature perturbation in this scenario.

1. Preheating occurs due to ϕ_1-oscillation while trapping occurs for both fields.
2. ϕ_2 is trapped at the local minimum at the origin.
3. The potential barrier ΔV decreases as $\Delta V \propto n_\chi^2$.
4. Trapping inflation ends with the ϕ_2-tunneling.

Generating δN_e from δn_χ.

Fig.7 shows that the start-line of trapping inflation is independent of the fluctuation δn_χ and is given by a flat surface (the straight line at $N_e = 0$). On the other
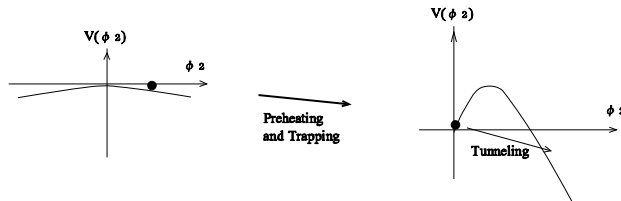

Fig. 6. Back reaction from the preheat field.

hand, the end-line is determined by the number density of the preheat field χ, which has the fluctuation δn_χ. Note that trapping inflation is not the primary inflation but an additional inflationary stage that starts after preheating.

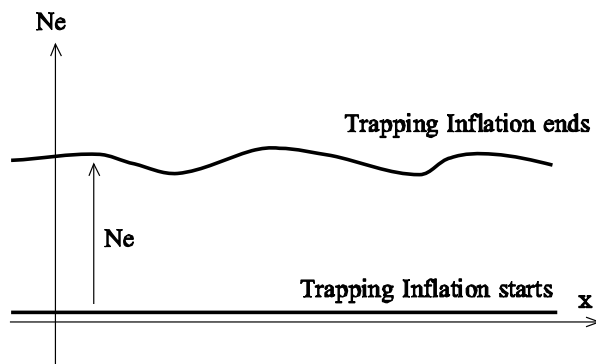**Fig. 7.** Generating δN_e from δn_χ

Calculation

During trapping inflation, $V^{eff}(\phi_2)$ is given by

$$V_2^{eff}(\phi_2) = V_0 - \frac{1}{2}m^2\phi_2^2 + \frac{\lambda|\phi_2|^{n_2}}{M_2^{n_2-4}} + gn_\chi|\phi_2|. \quad (9)$$

For the effective potential near the origin, the effective potential for $\phi_2 > 0$ is written as

$$V_2^{eff}(\phi_2) \simeq V_0 - \frac{1}{2}m^2\left(\phi_2 - \frac{gn_\chi}{m^2}\right)^2 + \frac{g^2n_\chi^2}{2m^2}. \quad (10)$$

The number of e-foldings that elapse during the trapping inflation is given by

$$N_e \sim \frac{1}{3}\ln\left(\frac{n_\chi(t_i)}{n_\chi(t_e)}\right). \qquad (11)$$

δN_e generated at the end of inflation is

$$\delta N_e \sim \frac{g\phi_2\delta\phi_2}{v}, \qquad (12)$$

where v is the velocity of the oscillating field at the ESP. Our result shows that: (1) Low-scale inflation ($H_I \sim GeV$) is successful. (2) The non-Gaussian parameter is always large, $|f_{NL}| > 1$. Unfortunately, these results depend crucially on the initial condition.

<u>Model 2</u> : **Weak trapping and non-oscillating**

(NO) curvatons

We consider the quintessential potential for ϕ_2:

$$V(\phi_2) = \frac{\Lambda}{(\phi_2)^n}. \tag{13}$$

Note that the attractive force from the preheat field acts on ϕ_2 to prevent ϕ_2 from rolling down the potential.
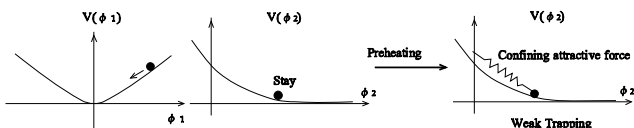

Fig. 8. Attractive force acting on ϕ_2

Here we consider the situation

– The preheat Field (χ) is identified with the curvaton.
– There is the back reaction from the preheat field (i.e., from the curvaton).

The late-time evolution is found from the force-balance equation

$$gn_\chi(t) - \frac{nM^{n+4}}{\phi_2^{n+1}} = 0, \tag{14}$$

which leads to the evolution of the expectation value $\phi_2(t)$,

$$\phi_2(t) = M\left(\frac{nM^3}{gn_\chi(t)}\right)^{1/(n+1)}. \tag{15}$$

From these equations we can calculate the ratio of ρ_χ to $V(\phi_2)$,

$$\frac{\rho_\chi}{V(\phi_2)} = n. \tag{16}$$

Since the number density n_χ evolves as $n_\chi \propto a^{-3}$, the energy density of the preheat field ρ_χ evolves as

$$\rho_\chi \propto a^{-3(1-\frac{1}{n+1})}. \tag{17}$$

Note that the mass of the curvaton m_χ grows as

$$m_\chi \simeq g\phi_2(t) \propto a^{\frac{3}{n+1}}. \tag{18}$$

As a consequence, we find clear differences from the normal curvaton;

1. The time when the curvaton starts to oscillate is determined by the mass of the oscillating field ϕ_1, which is independent of the curvaton mass (m_χ). Note that ϕ_1 may or may not be the inflaton.
2. The time when the curvaton decays is determined by $m_\chi(t)$, which increases with time.
3. The density of the NO curvaton decreases slower than the matter density.

As a result, the cosmological bound for the NO curvaton is very different from the usual curvatons.

Let us consider an example. For the Quintessential potential

$$V(\phi_2) = \frac{M^8}{(\phi_2)^4}, \qquad M = 10^2 GeV \tag{19}$$

we obtain $T_R \simeq 1\text{MeV}$. There is no obvious bound for the Hubble parameter above $H_I \sim \mathcal{O}(\text{GeV})$, but again, the results depend crucially on the initial condition.

Conclusions and summary

We have presented two multi-field models of cosmological perturbation. The first model deals with a typical double-well potential, which has the same form as the one that has been used for thermal inflation. We showed that a combination of inhomogeneous preheating and trapping inflation leads to the generation of the curvaton perturbation. We conclude that inhomogeneous preheating is an interesting possibility and that the traditional scenario for generating cosmological perturbations can be replaced by our proposed alternatives. Future cosmological observations should help to determine which of these alternatives is the most suitable; non-Gaussianity in particular may be a key observation.[1] However, a more efficient method is required to properly identify which model is most appropriate, and we are currently conducting research in this direction.

References

1. A. D. Linde and V. F. Mukhanov, Phys.Rev. **D56**, (1997) 535; D. H. Lyth, D. Wands, Phys.Lett. **B524**, (2002) 5; T. Moroi, T. Takahashi, Phys.Lett. **B522**, (2001) 215.
2. T. Matsuda, Class.Quant.Grav. **21**, (2004) L11; K. Dimopoulos, D. H. Lyth and Y. Rodriguez, JHEP **0502**, (2005) 055.
3. T. Matsuda, Phys.Rev. **D72**, (2005) 123508.
4. L. Kofman, [astro-ph/0303614]
5. G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys.Rev. **D69**, (2004) 023505.
6. F. Vernizzi, Phys.Rev.**D69**, (2004) 083526.
7. E. W. Kolb, A. Riotto and A. Vallinotto, Phys.Rev. **D71**, (2005) 043513.
8. T. Matsuda, JCAP **0703**, (2007) 003; T. Matsuda, JCAP **0706**, (2007) 029. T. Matsuda, JHEP **0707**, (2007) 035.
9. D. H. Lyth, JCAP **0511**, (2005) 006; T. Matsuda, JHEP **0703**, (2007) 096; T. Matsuda, Phys.Rev. **D67**, (2003) 083519.
10. T. Matsuda, JCAP **0609**, (2006) 003; D. H. Lyth and Antonio Riotto, Phys.Rev.Lett. **97**, (2006) 121301.
11. L. Kofman, A. D. Linde and A. A. Starobinsky, Phys.Rev. **D56**, (1997) 3258.
12. L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein JHEP **030**, (2004) 0405; S. Watson, Phys.Rev. **D70**, (2004) 066005.
13. T. Matsuda, JHEP **09**, (2007) 027; T. Matsuda, Phys.Rev. **D67**, (2003) 083519.

---

[1] See also Ref.[13].