R-parity and see-saw neutrinos from the heterotic string

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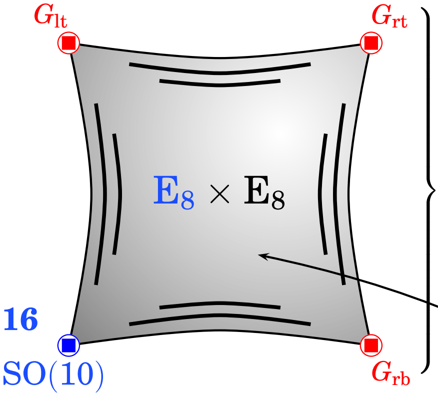
CERN

SUSY '07
Karlsruhe, July 29

Based on:

- O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter, Phys. Lett. B 645, 88-94 (2006)
- W. Buchmüller, K. Hamaguchi, O.L., S. Ramos-Sánchez, M. Ratz, hep-ph/0703078 (to appear in PRL)
- O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter, in preparation

Local grand unification



W. Buchmüller, K. Hamaguchi, O.L., M. Ratz (2004-2006)

'low-energy' → effective theory

standard model as an intersection of $G_{\mathrm{rb}}, G_{\mathrm{rt}}, G_{\mathrm{lt}}$ & $\mathrm{SO}(10)$ in G

SM generation(s):
localized in region with $\mathrm{SO}(10)$ symmetry

Higgs doublets:
live in the bulk

Exact MSSM spectra from strings on orbifolds

Guided by the idea of local grand unification we have obtained $\mathcal{O}(100)$ models with the following features:

cf. Peter Nilles' talk

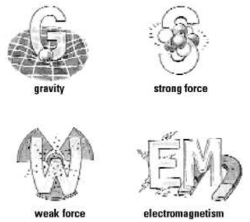
1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$

No exotics

Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ **models with:** cf. Peter Nilles' talk

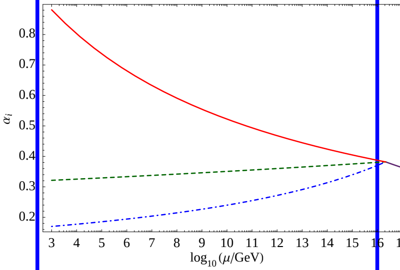
1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$



Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ models with: cf. Peter Nilles' talk

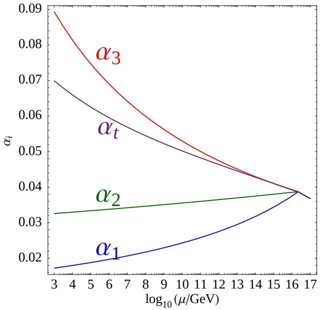
1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. unification



Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ models with: cf. Peter Nilles' talk

1. $3 \times \mathbf{16}$ + Higgs + nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. unification
4. $y_t \simeq g$ @ M_{GUT} & qualitatively realistic flavor structures



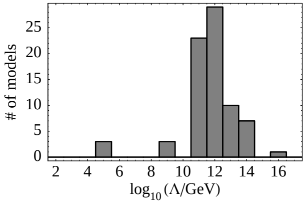
Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ models with: cf. Peter Nilles' talk

1. $3 \times \mathbf{16}$ + Higgs + nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. unification
4. $y_t \simeq g$ @ M_{GUT} & qualitatively realistic flavor structures
5. hidden sector gaugino condensation

➥ Spontaneously broken SUSY with TeV scale soft masses

☹ no time to discuss



$$m_{3/2} \sim \frac{\Lambda^3}{M_{\mathrm{P}}^2}$$

Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ models with: cf. Peter Nilles' talk

1. $3 \times \mathbf{16} +$ Higgs $+$ nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
3. unification
4. $y_t \simeq g$ @ M_{GUT} & qualitatively realistic flavor structures
5. TeV scale soft masses

Topics of this talk:

6. R-parity

~~$\bar{u}\bar{u}\bar{d}$~~ ~~$q\bar{d}\ell$~~

~~$\ell\ell\bar{e}$~~ ~~$\ell\bar{\phi}$~~

Exact MSSM spectra from strings on orbifolds

$\mathcal{O}(100)$ models with: cf. Peter Nilles' talk

1. $3 \times \mathbf{16}$ + Higgs + nothing
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$
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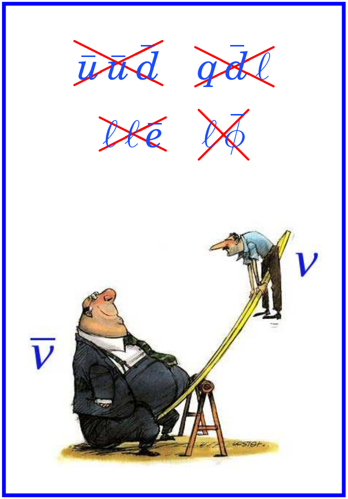
Topics of this talk:

6. R-parity
7. See-saw



Matter parity or effective R-parity from $\mathrm{U}(1)_{B-L}$

- $\mathrm{U}(1)_{B-L} \subset \mathrm{SO}(10)$ yields standard charges for matter

$$\begin{aligned}
\mathrm{SO}(10) &\rightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-L} \\
\mathbf{16} &\rightarrow (\mathbf{3},\mathbf{2})_{1/6,1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3,-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3,-1/3} \\
&\quad \oplus (\mathbf{1},\mathbf{1})_{1,1} \oplus (\mathbf{1},\mathbf{2})_{-1/2,-1} \oplus (\mathbf{1},\mathbf{1})_{0,1}
\end{aligned}$$

Matter parity or effective R-parity from $\mathrm{U}(1)_{B-L}$

- $\mathrm{U}(1)_{B-L} \subset \mathrm{SO}(10)$ yields standard charges for matter

$$\begin{aligned} \mathrm{SO}(10) \quad &\rightarrow \quad \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-L} \\ \mathbf{16} \quad &\rightarrow \quad (\mathbf{3},\mathbf{2})_{1/6,1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3,-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3,-1/3} \\ &\quad\quad \oplus (\mathbf{1},\mathbf{1})_{1,1} \oplus (\mathbf{1},\mathbf{2})_{-1/2,-1} \oplus (\mathbf{1},\mathbf{1})_{0,1} \end{aligned}$$

- How to define $B-L \subset \mathrm{E}_8 \times \mathrm{E}_8$?

W. Buchmüller, K. Hamaguchi, O.L. & M. Ratz (2006)

O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter (2006)

1. q_{B-L}(members of **16**-plet) $\stackrel{!}{=}$ standard
2. spectrum $\stackrel{!}{=}$ 3 generations + vector-like w.r.t. $G_{\mathrm{SM}} \times \mathrm{U}(1)_{B-L}$

Matter parity or effective R-parity from $\mathrm{U}(1)_{B-L}$

- $\mathrm{U}(1)_{B-L} \subset \mathrm{SO}(10)$ yields standard charges for matter

$$\begin{array}{rcl} \mathrm{SO}(10) & \to & \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-L} \\ \mathbf{16} & \to & (\mathbf{3},\mathbf{2})_{1/6,1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3,-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3,-1/3} \\ & & \oplus (\mathbf{1},\mathbf{1})_{1,1} \oplus (\mathbf{1},\mathbf{2})_{-1/2,-1} \oplus (\mathbf{1},\mathbf{1})_{0,1} \end{array}$$

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1. q_{B-L}(members of $\mathbf{16}$-plet) $\overset{!}{=}$ standard
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in many models of the Mini-Landscape:
existence of SM singlets with $q_{B-L} = \pm 2$!

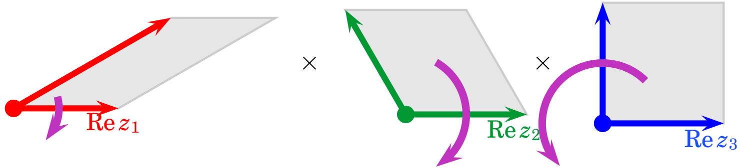
➤

Matter parity:
$\mathrm{U}(1)_{B-L} \to \mathbb{Z}_2^R$

An explicit example

O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter (2006)

☞ Input = geometry, shift & Wilson lines



$$V = \left(\tfrac{1}{3}, -\tfrac{1}{2}, -\tfrac{1}{2}, 0, 0, 0, 0, 0\right) \left(\tfrac{1}{2}, -\tfrac{1}{6}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$W_2 = \left(\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, -\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}\right) \left(1, -1, -\tfrac{5}{2}, -\tfrac{3}{2}, -\tfrac{1}{2}, -\tfrac{5}{2}, -\tfrac{3}{2}, \tfrac{3}{2}\right)$$

$$W_3 = \left(-\tfrac{1}{2}, -\tfrac{1}{2}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}, \tfrac{1}{6}\right) \left(\tfrac{10}{3}, 0, -6, -\tfrac{7}{3}, -\tfrac{4}{3}, -5, -3, 3\right)$$

An explicit example

O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter (2006)

- Input = geometry, shift & Wilson lines

- ➥ Gauge group

$$G = [\overbrace{\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y}^{\subset\, \mathrm{SU}(5)\, \subset\, \mathrm{SO}(10)} \times \mathrm{U}(1)_{B-L}] \times [\mathrm{SO}(8) \times \mathrm{SU}(2)] \times \mathrm{U}(1)^6$$

GUT normalization ➡ gauge coupling unification

$$\mathsf{t}_Y = \left(0,0,0,\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3}\right)\ (0,0,0,0,0,0,0,0)$$

$$\mathsf{t}_{B-L} = \left(1,1,0,0,0,-\tfrac{2}{3},-\tfrac{2}{3},-\tfrac{2}{3}\right)\ \left(\tfrac{1}{2},\tfrac{1}{2},0,\tfrac{1}{2},\tfrac{1}{2},0,0,0\right)$$

normalization not as in $\mathrm{SO}(10)$

An explicit example

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- Input = geometry, shift & Wilson lines
- Gauge group

$$G = [\overbrace{\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y}^{\subset\,\mathrm{SU}(5)\,\subset\,\mathrm{SO}(10)}\times\mathrm{U}(1)_{B-L}]\times[\mathrm{SO}(8)\times\mathrm{SU}(2)]\times\mathrm{U}(1)^6$$

- Spectrum

$$\text{spectrum} = 3\times\text{generation}+\text{vector-like w.r.t. } G_{\mathrm{SM}}\times\mathrm{U}(1)_{B-L}$$

Spectrum @ orbifold point

#	irrep	label	#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$			
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	ϕ_i	$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
$3+12$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$	12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$	3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+	20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,0)}$	h_i	2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{(0,0)}$	y_i
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(1/2,1)}$	x_i^+	2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(-1/2,-1)}$	x_i^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i	18	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,-1/2)}$	f_i	2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i	4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

Spectrum @ orbifold point

#	irrep	label	#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$			
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	ϕ_i	$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
$3+12$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$	12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$	3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+	20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
20	(1				
2	(1				
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i	18	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,-1/2)}$	f_i	2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i	4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

spectrum = 3 generations + vector-like

Spectrum @ orbifold point

#	irrep	label		#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i		3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$				
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$		1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	ϕ_i		$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
$3+12$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$		12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$		3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+		20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$		3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
20	([illegible]	[illegible]		[illegible]	[illegible]	[illegible]
2	([illegible]	[illegible]		[illegible]	[illegible]	[illegible]
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i		18	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$		4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,-1/2)}$	f_i		2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i		4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

spectrum = 3 generations + vector-like

Spectrum @ orbifold point

#	irrep	label	#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i	3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$			
3 + 1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$	1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
3 + 1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i	1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
1 + 3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	ϕ_i	1 + 3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
3 + 12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$	12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$	3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+	20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$	3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
20	([illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	([illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i	18	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
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5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i	4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

spectrum = 3 generations + vector-like

Spectrum @ orbifold point

#	irrep	label		#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i		3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$				
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$		1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$	ϕ_i		$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,0)}$	$\bar{\phi}_i$
$3+12$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$	$\bar{n}_i$		12	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$	$\bar{\delta}_i$		3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i^+		20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$					η_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$					y_i
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})$					x_i^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})$					s_i^0
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$		4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,-1/2)}$	f_i		2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i		4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

$B-L$ allows to discriminate

- between lepton and Higgs fields
- between neutrinos and other singlets

Spectrum @ orbifold point

#	irrep	label		#	irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/6,1/3)}$	q_i		3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-2/3,-1/3)}$	$\bar{u}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}_i$				
$3+1$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,-1/3)}$	$\bar{d}_i$		1	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/3,1/3)}$	d_i
$3+1$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ_i		1	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(1/2,1)}$	$\bar{\ell}_i$
$1+3$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(-1/2,0)}$				$)_{(1/2,0)}$	$\bar{\phi}_i$
$3+12$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,1)}$				$)_{(0,-1)}$	n_i
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/3,2/3)}$				$)_{(-1/3,-2/3)}$	δ_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/2,*)}$	s_i		20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/2,*)}$	s_i^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,1)}$	$\bar{\eta}_i$		3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,-1)}$	η_i
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(0,0)}$	h_i		2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{(0,0)}$	y_i
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(1/2,1)}$	x_i^+		2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{(-1/2,-1)}$	x_i^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,\pm 2)}$	χ_i		18	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{(0,0)}$	s_i^0
4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{(-1/6,*)}$	$\bar{v}_i$		4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{(1/6,*)}$	v_i
2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,-1/2)}$	f_i		2	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,1/2)}$	$\bar{f}_i$
5	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{(0,0)}$	w_i		4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{(0,*)}$	m_i

crucial:

existence of SM singlets with $q_{B-L}=\pm 2$

Spectrum in MSSM vacua

☞ Decoupling of exotics

$$X_i \overline{X}_j \quad \underbrace{s_{i_1} \dots s_{i_n}}_{\text{vev}\to\text{mass term}}$$

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1. exotics' mass matrices have **full rank** with

$$s_i \;=\; G_\mathrm{SM} \times \mathrm{SO}(8) \text{ singlets with } q_{B\text{–}L} = 0 \text{ or } \pm 2$$

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remainder of this talk : neutrino masses

cf. Giedt, Kane, Langacker, Nelson (2005)

What is a ('right-handed') neutrino?

☞ 4D GUTs: $\bar{\nu}$ member of $\mathbf{16}$-plet

$$\begin{aligned}\mathrm{SO}(10) \quad &\longrightarrow \quad \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_\mathrm{Y}\times\mathrm{U}(1)_{B-L}\\ \mathbf{16} \quad &\longrightarrow \quad (\mathbf{3},\mathbf{2})_{1/6,1/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{-2/3,-1/3}\oplus(\overline{\mathbf{3}},\mathbf{1})_{1/3,-1/3}\\ &\qquad \oplus(\mathbf{1},\mathbf{1})_{1,1}\oplus(\mathbf{1},\mathbf{2})_{-1/2,-1}\oplus(\mathbf{1},\mathbf{1})_{0,1}\end{aligned}$$

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\mathbf{16} \quad &\rightarrow \quad (\mathbf{3},\mathbf{2})_{1/6,1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3,-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3,-1/3} \\
&\qquad \oplus (\mathbf{1},\mathbf{1})_{1,1} \oplus (\mathbf{1},\mathbf{2})_{-1/2,-1} \oplus (\mathbf{1},\mathbf{1})_{0,1}
\end{aligned}
$$

Higher-dimensional GUTs/Strings:

$\bar{\nu} = G_\mathrm{SM}$ singlet which is odd under matter parity

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Higher-dimensional GUTs/Strings:

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- remark: we get 39 neutrinos in the example

$$n_i \;\&\; \bar{n}_i = (\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0,\mp1}$$

$$\bar{\eta}_1 = \begin{pmatrix}\bar{n}_{16}\\ \bar{n}_{17}\end{pmatrix},\dots\bar{\eta}_3 = \begin{pmatrix}\bar{n}_{20}\\ \bar{n}_{21}\end{pmatrix};\ \eta_1 = \begin{pmatrix}n_{13}\\ n_{14}\end{pmatrix},\dots\eta_3 = \begin{pmatrix}n_{17}\\ n_{18}\end{pmatrix}$$

$$\{\nu_i\}_{i=1}^{39} = \{n_i\}_{i=1}^{21}\cup\{\bar{n}_i\}_{i=1}^{18}$$

See-saw couplings

☞ see-saw couplings: $W_{\text{see-saw}} = Y_\nu^{ij}\,\bar{\phi}\,\ell_i\,\bar{\nu}_j + M_{ij}\,\bar{\nu}_i\,\bar{\nu}_j$

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- ➥ see-saw mass matrix

$$W_{\text{see-saw}} \xrightarrow{\phi_u \to v} (\nu, \bar{\nu}) \begin{pmatrix} 0 & y_\nu\, v \\ y_\nu\, v & M \end{pmatrix} \begin{pmatrix} \nu \\ \bar{\nu} \end{pmatrix} \simeq \frac{y_\nu^2\, v^2}{M}\,\nu\,\nu + M\,\bar{\nu}\,\bar{\nu}$$

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- naive GUT expectation:
  $m_\nu \sim (100\,\text{GeV})^2/10^{16}\,\text{GeV} \sim 10^{-3}\,\text{eV}$

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... suspiciously close to observed values

$$\sqrt{\Delta m^2_{\text{atm}}} \simeq 0.04\,\text{eV} \quad \& \quad \sqrt{\Delta m^2_{\text{sol}}} \simeq 0.008\,\text{eV}$$

See-saw neutrinos from the heterotic string

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See-saw is a **generic feature** in heterotic MSSM vacua:

Y_ν and M exist with M & $m_\nu = v^2\, Y_\nu^T\, M^{-1}\, Y_\nu$ having full rank

$$\mathcal{M}_{nn} = \left(\begin{array}{cccccccccccccccccc}
0 & 0 & 0 & \widetilde{s}^2 & 0 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \widetilde{s}^6 & 0 & \widetilde{s}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\widetilde{s}^2 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\widetilde{s}^6 & \widetilde{s}^2 & 0 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
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0 & \widetilde{s}^3 & 0 & 0 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^2 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 \\
0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^3 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 \\
\widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\widetilde{s} & \widetilde{s}^2 & \widetilde{s}^3 & \widetilde{s}^5 & \widetilde{s}^4 & \widetilde{s}^4 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 \\
\widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\widetilde{s}^5 & \widetilde{s}^4 & \widetilde{s}^4 & \widetilde{s} & \widetilde{s}^2 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 \\
0 & 0 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 \\
0 & 0 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 \\
\widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & \widetilde{s}^5 & \widetilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & \widetilde{s}^5 & \widetilde{s}^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \widetilde{s}^3 & 0 & 0 & \widetilde{s}^3 & 0 & \widetilde{s}^2 & \widetilde{s}^6 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & \widetilde{s}^2 & \widetilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \widetilde{s}^3 & 0 & 0 & \widetilde{s}^3 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 \\
0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 & 0 & 0 & 0 & 0
\end{array}\right)
$$

See-saw neutrinos from the heterotic string

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0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\widetilde{s}^4&0&0&0&0&0&0&0&0&0&0&0&0&\widetilde{s}^3&\widetilde{s}^3&0&0&0&0\\
0&\widetilde{s}^4&0&0&0&0&0&0&0&0&0&0&0&0&\widetilde{s}^3&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\widetilde{s}^4&0&0&0&0&0&0&0&0&0&\widetilde{s}^3&\widetilde{s}^3&0&0&0&0\\
0&0&0&0&\widetilde{s}^4&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\widetilde{s}^3&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\widetilde{s}^3&0&0&\widetilde{s}^3&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\widetilde{s}^3&0&0&\widetilde{s}^3&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}
$$

See-saw neutrinos from the heterotic string

W. Buchmüller, K. Hamaguchi, O.L., M. Ratz (2006)

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O.L., H.P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. Vaudrevange, A. Wingerter (in preparation)

See-saw is a **generic feature** in heterotic MSSM vacua:

Y_ν and M exist with M & $m_\nu = v^2\, Y_\nu^T\, M^{-1}\, Y_\nu$ having full rank

$$\mathcal{M}_{\bar{\nu}\bar{\nu}} = \begin{pmatrix} \mathcal{M}_{\bar{n}\bar{n}} & \mathcal{M}_{n\bar{n}}^T \\ \mathcal{M}_{n\bar{n}} & \mathcal{M}_{nn} \end{pmatrix}$$

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$$Y_{\bar{n}} = \begin{pmatrix} 0&\widetilde{s}^6&0&0&\widetilde{s}^6&0&0&0&0&0&\widetilde{s}&\widetilde{s}&0&0&0&\widetilde{s}^4&\widetilde{s}^4&0&0&0&0\\ 0&\widetilde{s}^6&0&0&\widetilde{s}^6&0&0&0&0&0&\widetilde{s}&\widetilde{s}&0&0&0&\widetilde{s}^5&\widetilde{s}^5&0&0&0&0\\ 0&\widetilde{s}^2&\widetilde{s}^6&0&\widetilde{s}^2&\widetilde{s}^6&0&0&0&0&\widetilde{s}^6&\widetilde{s}^6&0&0&0&\widetilde{s}^5&\widetilde{s}^5&0&0&0&0\\ 0&\widetilde{s}^2&\widetilde{s}^6&0&\widetilde{s}^2&\widetilde{s}^6&0&0&0&0&\widetilde{s}^6&\widetilde{s}^6&0&0&0&\widetilde{s}^5&\widetilde{s}^5&0&0&0&0 \end{pmatrix}$$

$$Y_\nu = (Y_{\bar{n}}, Y_\nu)$$

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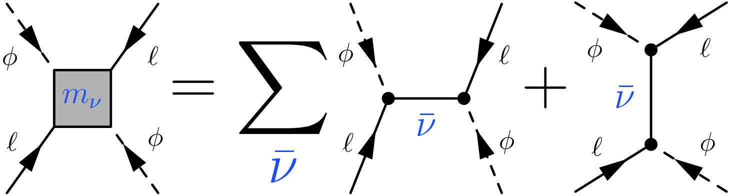

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- ☞ there are $\mathcal{O}(100)$ neutrinos ($=$ R-parity odd SM singlets)
- ➥ $\mathcal{O}(100)$ contributions to the (effective) neutrino mass operator
- ➥ effective suppression of the see-saw scale

$$m_\nu \sim \frac{v^2}{M_*} \qquad M_* \sim \frac{M_\mathrm{GUT}}{10 \dots 100}$$

... seems consistent with observation

$$\left(\sqrt{\Delta m^2_\mathrm{atm}} \simeq 0.04\,\mathrm{eV} \;\&\; \sqrt{\Delta m^2_\mathrm{sol}} \simeq 0.008\,\mathrm{eV}\right)$$

Some features of Mini-Landscape vacua

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1. ***R*-parity** as $\mathbb{Z}_2$ subgroup of $\mathrm{U}(1)_{B\text{–}L}$

it is quite likely that there are other possibilities to get R-parity

~~$\bar{u}\,\bar{u}\,\bar{d}$~~ ~~$q\,\bar{d}\,\ell$~~

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$\mathcal{O}(100)$ neutrinos effectively lower the see-saw scale

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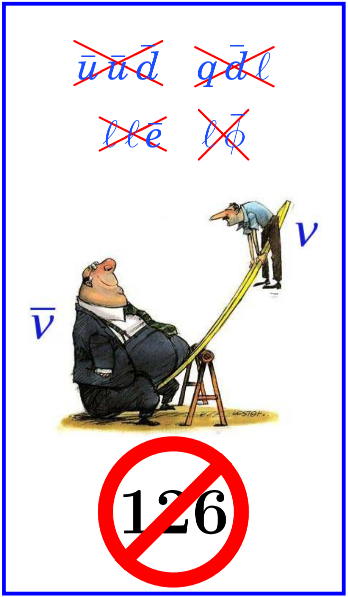
- Remark: we get all these features without an $\mathbf{126}$-plet of $\mathrm{SO}(10)$



‘Appendix’

F-flat directions vs. F-flat points

- One possible approach: search for F- and D-flat **directions**

cf. Giedt, Kane, Langacker, Nelson (2005)

F-flat directions vs. F-flat points

- One possible approach: search for F- and D-flat **directions**

  cf. Giedt, Kane, Langacker, Nelson (2005)

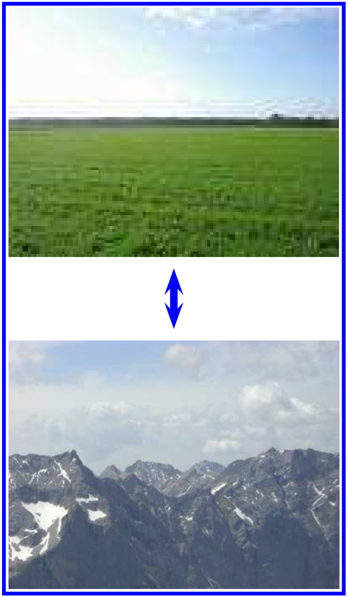
- **However**: given a set of fields entering holomorphic gauge invariant monomials It is possible to 'rescale' solutions of $\partial W/\partial\phi_i = 0$ to $V_D = 0$ by 'complexified gauge transformations'

  Ovrut, Wess (1982)

➥ leads to local solutions to $V_F = V_D = 0$

- note: the superpotential also depends on the geometric moduli, receives non-perturbative corrections etc.

  cf. e.g. W. Buchmüller, K. Hamaguchi, O.L., M. Ratz (2006)



Yukawa couplings

$$Y_u \sim \begin{pmatrix} \widetilde{s}^2 & \widetilde{s}^5 & \widetilde{s}^5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad Y_d = \begin{pmatrix} \widetilde{s} & \widetilde{s}^4 & 0 \\ 1 & \widetilde{s}^3 & 0 \\ 1 & \widetilde{s}^3 & 0 \end{pmatrix} , \quad Y_e = \begin{pmatrix} \widetilde{s} & 1 & 1 \\ \widetilde{s}^2 & \widetilde{s} & \widetilde{s} \\ \widetilde{s}^6 & 0 & 0 \end{pmatrix}$$

Higgs mass matrices

$$\mathcal{M}_{\bar{\phi}\phi} = \begin{pmatrix} \widetilde{s}^4 & 0 & 0 & \widetilde{s} \\ \widetilde{s} & \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^6 \\ \widetilde{s}^5 & 0 & 0 & \widetilde{s}^3 \\ \widetilde{s} & 0 & 0 & \widetilde{s}^3 \end{pmatrix}$$

back

Mass matrices (cont'd)

$$\mathcal{M}_{\bar{\ell}\ell} = \begin{pmatrix} \widetilde{s}^2 & \widetilde{s}^2 & \widetilde{s}^3 & \widetilde{s}^3 \end{pmatrix}$$

$$\mathcal{M}_{d\bar{d}} = \begin{pmatrix} \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 \end{pmatrix}$$

$$\mathcal{M}_{mm} = \begin{pmatrix} 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 \\ 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 \\ \widetilde{s}^6 & \widetilde{s}^6 & 0 & \widetilde{s}^6 \\ \widetilde{s}^6 & \widetilde{s}^6 & \widetilde{s}^6 & 0 \end{pmatrix}$$

$$\mathcal{M}_{\delta\bar{\delta}} = \begin{pmatrix} \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 \\ \widetilde{s}^3 & \widetilde{s}^3 & \widetilde{s}^3 \\ 0 & \widetilde{s}^3 & \widetilde{s}^3 \end{pmatrix}$$

$$\mathcal{M}_{yy} = \begin{pmatrix} \widetilde{s}^1 & \widetilde{s}^5 \\ \widetilde{s}^5 & \widetilde{s}^1 \end{pmatrix}$$

$$\mathcal{M}_{v\bar{v}} = \begin{pmatrix} \widetilde{s} & \widetilde{s}^5 & 0 & 0 \\ \widetilde{s}^5 & \widetilde{s} & 0 & 0 \\ 0 & 0 & \widetilde{s}^5 & \widetilde{s}^5 \\ 0 & 0 & \widetilde{s}^5 & \widetilde{s}^5 \end{pmatrix}$$

Mass matrices (cont'd)

$$
\begin{aligned}
\mathcal{M}_{x^+x^-} &= \begin{pmatrix} \widetilde{s}^5 & \widetilde{s}^5 \\ \widetilde{s}^5 & \widetilde{s}^5 \end{pmatrix} \\
\mathcal{M}_{f\bar{f}} &= \begin{pmatrix} 0 & \widetilde{s}^3 \\ 0 & \widetilde{s}^3 \end{pmatrix} \\
\mathcal{M}_{ww} &= \begin{pmatrix} \widetilde{s} & \widetilde{s}^5 & 0 & \widetilde{s}^5 & \widetilde{s}^5 \\ \widetilde{s}^5 & \widetilde{s} & 0 & \widetilde{s}^5 & \widetilde{s}^5 \\ 0 & 0 & 0 & \widetilde{s}^3 & \widetilde{s}^3 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^3 & \widetilde{s}^6 & \widetilde{s}^6 \end{pmatrix}
\end{aligned}
$$

back

Mass matrices (cont'd)

$$
\mathcal{M}_{s^+s^-} \quad = \quad \left(\begin{array}{cccccccccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^6 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & \tilde{s} & 0 & 0 & \tilde{s}^5 & 0 & \tilde{s} & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 \\
\tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & \tilde{s}^5 & 0 & 0 & \tilde{s} & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s} & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^6 & \tilde{s}^5 \\
\tilde{s}^5 & 0 & \tilde{s}^6 & \tilde{s}^5 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tilde{s}^6 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^5 & 0 & 0 & \tilde{s}^5 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\
0 & 0 & 0 & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & 0 & 0 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 & 0 & \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5
\end{array}\right)
$$

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